Notes on the Application of Algorithmic Principles to Organizational Analysis

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Abstract

Professional managers have observed that organizational performance under certain circumstances has exhibited non-linear characteristics as a function of increasing task complexities. Certain organizational activities are characterized by a search for a solution to a problem. If these search activities could be analyzed using algorithmic analysis, in much the same way that algorithmic performance of a computer program is analyzed, then some qualitative, and possibly quantitative, insight could be derived into the way that organizations solve problems. This in turn could lead to the optimization of the parametric issues confronting working organizations. A heuristic proof will be offered that indicates that these non-linearities as a function of problem complexity do indeed exist, are unavoidable, and can severely impact the organization's efficiency and performance, if not appropriately optimized. A general model, using algorithmic principles, is derived for organizational problem solving, and several optimizations presented. It is the general conclusion of this presentation that problem solving organizations do indeed exhibit non-linear time/resource requirements in response to complexity issues, and that time and/or resources, may be optimized to provide the best organizational design. Further, it is shown that ignoring these non-linear issues, and operating non-optimally, can lead to organizational dysfunction. Finally, it is shown that the interface coordination between organizational members is, singularly, the limiting issue in organizational performance and as such is the most important issue in organizational optimization. Throughout the text, metric methods are suggested, for measurement of parameters, that effect organizational performance and optimization. It is the basic premiss that if it is assumed that problem solving is algorithmical, then problem solving must obey the rules of algorithmical dynamics, and if problem solving obeys the rules of algorithmical dynamics, then problem solving can be analyzed and is therefore predictable. Any other premiss essentially states that problem solving is not a systematic procedure. This presentation explores the nature of organizational problem solving with algorithmic analysis.

Preface

The concepts presented herein were developed over a period of 25 years and are not innovational. That organizations doing problem solving must have disproportionately more resource allocation to handle more complex problems is not a new concept-military theorist have known that for millennia. That interface coordination in the group is important is a concept as old as humanity itself. What is new here is that these issues can be quantified and subjected to numerical methods, or metrics, to arrive at an optimized operating situation for the organization, and that qualitative issues of organizational design can be addressed analytically using only remedial engineering optimization techniques, in a deterministic fashion. Further, in this age of computer automation of the organization, it can, within reasonable accuracies, be done automatically.

It is not a difficult task, particularly with today's system accounting techniques that are available in contemporary operating systems, to write a program that can tell how many, for how long, and for what, commands were issued in a given interval of a project task. The analysis of these activities determine the parametric values, which can be optimized, using the techniques presented herein. Indeed, the issue of what complexity really means is probably better addressed by the amount of computer resources dispensed to a project, than the subjective evaluation of those responsible for the project.

Is such collection of data an ethical issue? I always took the courtesy of informing people that they were being monitored, and most agreed, particularly if they could participate in the analysis of the data. But the ethical issue probably is still on the table. Most were intrigued when told that the interface coordination issue, which as it will be shown, is centrally the most important issue confronting the organization-and not only that, the issue is endemic to the problem solving process of the organization itself.

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Chapter 1

General Analysis of Organizations

1.1 Introduction

When confronted with an organization that takes disproportionately more time and resources to solve problems of increasing complexity, (ie., the organization's time and resource commitment to solve a problem is a non-linear function of the problem's complexity,) managers often, erroneously, jump to the conclusion that the organization is non-productive, inefficient, or mismanaged. It is the premiss of this presentation that these organizational non-linearities not only happen, but are the very essence of the way organizations operate. On the one hand, the assumption that these non-linearities exist complicates the understanding of how to operate an organization, however on the other hand, since these complexities are inherently non-linear in nature, they lend well to optimization techniques that can be used to increase organizational efficiency, productivity, and performance. It will be shown in this chapter that, in at least complex organizations, it is not the completion of tasks that is an issue, but the process by which the tasks are completed that determines how effective an organization is at solving problems. Throughout this presentation, the emphasis will be on analyzing operational organizations that must perform services on a continuing basis, as opposed to project management.

An algorithm is a method for solving a problem. The formal foundations for algorithmic analysis were established during the axiomatization of the theory of mathematical operators and set theory developed during the first half of this century as mathematicians established the theoretical foundations of mathematics itself. In lay terms, an algorithm is a procedure that searches for the solution to a problem. Algorithmic analysis is the study of the performance and methodologies of these procedures, and is a well established science, dating back to the early numerical analysis techniques used by Newton. Although the concept of the algorithm is generally associated with computer technology, the application of the concept to computer science is relatively recent. This discussion is about the application of algorithmic analysis to the problem of organizational performance.

1.2 The Organizational Performance Problem

Consider an organization, composed of many people, with many tasks to do. If the organization's objective is to search for a solution to some problem, then algorithmic analysis may be used to analyze the organization's operations[1]. A good example of this type of organization is a marketing organization that is searching for a solution on how to introduce a new product into the marketplace, or searching for a solution on how to increase sales. Another example is an engineering development organization that is searching for a technological solution in new product development. Since these organizations are searching for a solution to a problem, algorithmic analysis dictates that as a function of the problem's complexity, the problem's solution will probably require an exponential solution time. In technical parlance, the "search for solution" problem is NP in nature, (and probably NP-hard in the examples given above.) The organization's attempt at solving the problem is therefore constrained by the characteristic nature of how NP problems lend there self to solution.

Software organizations are familiar with the non-linear characteristics of development time as a function of increasing task complexities, and the phenomena is well documented. More importantly, these non-linearities create software cost and time overruns which are well known in the industry. The complexity of software is an essential property, not an accidental one. Software systems have orders-of-magnitude more states than computers do. It is not surprising that much effort in the industry is focused on automating the complexity, ie., the development process, with such things as CASE and concurrent engineering. As mentioned above, complexity issues must be addressed by organizations other than software developers. Software organizations, however, have been the subject of many published empirical studies on these complexity issues, which will be referenced throughout this analysis, and are therefore of interest when studying how organizations, in general, handle complexity issues.

1.3 Derivation of the Organizational Model

For tutorial purposes, the organization will be modeled, starting with a single person organization, then one person will be added to the organization, with the tasks divided equally between the two persons. Then compensation for coordination issues between the two persons will be added. Finally, the general case will be analyzed.

[1]Note that not all organizations fall into this category. Organizations that process things in an administrative or mechanically predetermined fashion may, but not necessarily. The key issue is the words "searching for a solution".

1.3.1 Variables used in Derivations

c is the linear factor of complexity. If c is divided by two, the task is exactly half as complex. This variable should also reflect the ability of the organization to handle complex tasks. An organization that is twice as efficient as another organization would have half the c. (It may be appropriate to add a factor of $\frac{1}{s}$ to c, where s represents an organization's skill level which may change do to learning.)

e is 2.718...

k is a constant of proportionality with the same dimension as t below.

n is the number of persons in the organization.

p is the probability, multiplied by a constant of proportionality, that a coordination issue will increase the time required to solve the problem. In some sense, p reflects the quality of the coordination process. A p of zero means that there are no coordination issues. The constant of proportionality reflects how sensitive the system is to problems created by coordination issues.

t is the execution time required for an organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time.

r is the total time allocation for an organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time, and is more pertinent as an indicator of the total resources allocated to solve the problem, ie., man hours.

1.3.2 Derivation of the One Person Organization Model

Assuming that the time, t_1, required for completion of the search for solution to a problem is exponential on complexity

$$t(1) = ke^c \tag{1.1}$$

where k is a constant of proportionality, and c is proportional to the complexity of the problem. Likewise, in this simple case

$$r(1) = ke^c \tag{1.2}$$

1.3.3 Derivation of the Two Person Organization Model

Assuming that the task can be divided equally between the two persons, and each has the same technology and skills in searching for a solution to the problem

$$t(2) = ke^{\frac{c}{2}} \tag{1.3}$$

and likewise for the resources required

$$r(2) = ke^{\frac{c}{2}} + ke^{\frac{c}{2}} = 2ke^{\frac{c}{2}} \tag{1.4}$$

Note that the task was divided equally, with each person addressing half of the complexities. However, a new situation exists in that the work done by one person may not coordinate with the work done by the other, and visa versa, perhaps requiring a rework of some tasks, etc. If we assume that searching for, and solving, the problem created by the coordination issue is itself NP, then the previous equation must be multiplied by a factor of e^p where p is the probability that a coordination issue will occur multiplied by a constant that represents the affect that the issue has on the organization's efficiency[2]. Therefore,

$$t(2) = ke^{\frac{c}{2}}e^p = ke^{\frac{c}{2}+p} \tag{1.5}$$

and likewise for the resources required

$$r(2) = 2ke^{\frac{c}{2}}e^p = 2ke^{\frac{c}{2}+p} \tag{1.6}$$

1.3.4 Derivation of the n Person Organizational Model

The general solution for the problem's solution time can be implied

$$t(n) = ke^{\frac{c}{n}+(n-1)p} \tag{1.7}$$

and likewise for the resources required

$$r(n) = nke^{\frac{c}{n}+(n-1)p} \tag{1.8}$$

The reason for the $(n - 1)$ multiplier in the exponent is because the probability that a coordination issue will happen is proportional to the number of interfaces that can occur. It is assumed that each person interfaces with all others in the organization, and each interface has an equal probability of creating a coordination issue.

1.4 Summary

Equation 1.7 and 1.8 express the time and committed resources, respectively, for an organization to solve a problem, where n is the number of persons in the organization, c represents the complexity of the problem, p represents the quality of the organization's interface coordination, and k is a constant of proportionality. The metrics on p can be obtained, for comparative studies, by solving equations 1.7 and 1.8 with $p = 0$, which

[2]Note that the objective here is consistent with other research which presents the following equation

$$\text{Actual Productivity} = \text{Potential Productivity} - \text{Losses Due to Faulty Process}$$

The "Actual Productivity" in this case would be $\frac{c}{2} - p$.

would be an idealized organization with no interface issues-naturally, a real organization must operate less efficiently, but how close the organization is to an idealized organization can be expressed numerically. The metrics on problem complexity, c is a more difficult proposition because complexity is somewhat subjective. However, it can be measured, post facto, by knowing p, n, and the time required to solve the problem. As it turns out, if the organization is large and it is desired to operate the organization optimally, these parameters can be ascertained indirectly.

Chapter 2

The Flat Organization

2.1 Introduction

The flat organization is an organization without division of task responsibility, and is therefore the simplest organization structure to analyze. Although performance comparisons will be made with structured organizations, the comparisons are presented in the more general sense that the predictions, derived in this section, are a pessimistic theoretical limit. An optimistic theoretical limit will be derived in Chapter 3 which analyzes the performance of structured organizations. The performance of real organizations lay between these two theoretical limits since no organization is completely flat, or completely structured. The optimizations derived in this section are useful when analyzing the performance of the various flat organizations that are the constituent parts of a structured organization.

2.1.1 Variables used in Derivations

c as before, is the linear factor of complexity. If c is divided by two, the task is exactly half as complex. This variable should also reflect the ability of the organization to handle complex tasks. An organization that is twice as efficient as another organization would have half the c. (It may be appropriate to add a factor of $\frac{1}{s}$ to c, where s represents an organization's skill level which may change do to learning.)

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k as before, is a constant of proportionality with the same dimension as t below.

n as before, is the number of persons in the organization.

$n_f|_{opt}$ is the number of persons in the flat organization that will be required to achieve the economic optimum operating situation for the organization.

$n_f|_{t=\min}$ is the number of persons in the flat organization that will be required to achieve the minimum solution time for a problem.

$n_f|_{r=\min}$ is the number of persons in the flat organization that will be required to achieve the minimum resource allocation to solve a problem.

p as before, is the probability, multiplied by a constant of proportionality, that a coordination issue will increase the time required to solve the problem. In some sense, p reflects the quality of the coordination process. A p of zero means that there are no coordination issues. The constant of proportionality reflects how sensitive the system is to problems created by coordination issues.

t_f is the execution time required for a flat organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time.

r_f is the total time allocation for an flat organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time, and is more pertinent as an indicator of the total resources allocated to solve the problem, ie., man hours.

2.2 The Case to Increase Organizational Size

The question can be posed, when can a person, or persons, be added to an organization, presumably to decrease the time required to find a solution to a problem. By analyzing the ratio of the time required to solve a problem with an additional person to the time required to solve the same problem without the additional person, some insight can be obtained on whether it is an advisable proposition, or from equation 1.7

$$\frac{t_f(n+1)}{t_f(n)} = \frac{ke^{\frac{c}{n+1}+np}}{ke^{\frac{c}{n}+(n-1)p}} \tag{2.1}$$

and reducing

$$\frac{t_f(n+1)}{t_f(n)} = e^{\frac{c}{n+1}-\frac{c}{n}+p} \tag{2.2}$$

Note that for a decrease in the time to solve the problem to occur

$$p + c\left(\frac{1}{n+1} - \frac{1}{n}\right) \tag{2.3}$$

must be less than zero. If it is equal to zero, ie.,

$$p = c\left(\frac{1}{n} - \frac{1}{n+1}\right) \tag{2.4}$$

then adding a person will have no effect on the time to solve the problem. Likewise, if equation 2.3 is greater than zero, then the situation exists where adding a person can actually increase the time required to solve a problem. (Perhaps this is the mathematical definition of bureaucracy.) Combining terms in equation 2.4

$$p = c\left(\frac{n+1-n}{n\,(n+1)}\right) = c\left(\frac{1}{n\,(n+1)}\right) = \frac{c}{n^2+n} \tag{2.5}$$

then if $n >> 1$, ie., the organization is large, and solving for n

$$n_f\,|_{opt} \approx \sqrt{\frac{c}{p}} \tag{2.6}$$

2.2.1 A Possible Economic Optimum

In some sense, it is apparent that adding persons to an organization without altering p will result in diminishing returns after a certain point, and to maintain p small will probably require automating the coordination interface. It is interesting to note that this is one of the basic premises of concurrent engineering. If it is determined that equation 2.4 really is the onset of diminishing returns, then this may possibly constitute the optimum economic operating point for the organization.

2.3 Possible Optimum Staffing Levels

The possible economic optimum operating point for the organization suggests that an optimum staffing level exists that will minimize the time to solve a problem, or alternately, minimize the resources that must be committed to solve the problem.

2.3.1 Optimum Staffing Level for Minimum Time

Taking the derivative of equation 1.7 with respect to the number of persons working on solving a problem will possibly yield the minimum solution time to solve the problem

$$\frac{dt_f\,(n)}{dn} = ke^{\frac{c}{n}+(n-1)p}\left(p - \frac{c}{n^2}\right) \tag{2.7}$$

and by equating equation 2.7 to zero, the minima occurs when

$$\frac{c}{n^2} = p \tag{2.8}$$

or solving for n

$$n_f\,|_{t=\min} = \sqrt{\frac{c}{p}} \tag{2.9}$$

It is not clear whether this minima is of any practical interest, but it agrees nicely with equation 2.6, at least for large organizations.

2.3.2 Optimum Staffing Level for Minimum Resources

Taking the derivative of equation 1.8 with respect to the number of persons working on solving a problem will possibly yield the minimum resource solution to solve the problem

$$\frac{dr_f\,(n)}{dn} = nke^{\frac{c}{n}+(n-1)p}\left(p - \frac{c}{n^2}\right) + ke^{\frac{c}{n}+(n-1)p} \tag{2.10}$$

and by equating equation 2.10 to zero, the minima occurs when

$$n\left(p - \frac{c}{n^2}\right) + 1 = 0 \tag{2.11}$$

or

$$np - \frac{c}{n} + 1 = 0 \tag{2.12}$$

and multiplying by n

$$pn^2 + n - c = 0 \tag{2.13}$$

then by the quadratic equation

$$n = \frac{\sqrt{1+4pc} - 1}{2p} \tag{2.14}$$

Again, it is not clear whether this minima is of any practical interest. If $n >> 1$, ie., the organization is large ($\sqrt{4pc} >> 2p$), then

$$n_f\,|_{r=\min} \approx \frac{\sqrt{4pc}}{2p} \tag{2.15}$$

or

$$n_f\,|_{r=\min} \approx \sqrt{\frac{c}{p}} \tag{2.16}$$

This agrees nicely with equation 2.9, at least for large organizations, which was the optimization by which equations 2.19 and 2.23 were derived.

2.4 Discussion

Severable multiple optimization alternatives exist. The first is the minimum time optimum. The next is the minimum time with optimal resources, and finally, the minimum time and optimal resources.

2.4.1 Minimum Time Optimum

Substituting p from equation 2.8 into equation 1.7 yields

$$t_f(n)|_{\min} = ke^{\frac{c}{n}+(n-1)\frac{c}{n^2}} \tag{2.17}$$

and combining terms

$$t_f(n)|_{\min} = ke^{\frac{c}{n}+\frac{c}{n}-\frac{c}{n^2}} = ke^{\frac{c}{n}(2-\frac{1}{n})} \tag{2.18}$$

then if $n >> 1$, ie., the organization is large,

$$t_f(n)|_{\min} \approx ke^{2\frac{c}{n}} \tag{2.19}$$

or by substituting equation 2.9

$$t_f(n)|_{\min} \approx ke^{2\sqrt{pc}} \tag{2.20}$$

Interestingly, comparing equation 2.19 to equation 1.7 with $p = 0$, (ie., a perfect organization that has no interface coordination issues,) states that if the organization is operating optimally, that only 50 percent of the organization's efforts are directed at problem solving, and the remaining 50 percent is directed at interface coordination issues. Possibly, this could account for the time slippages of 50 percent cited in the literature. Although the references assumed a constant linear time slippage of 100 percent in the PERT elements, this analysis would indicate a 100 percent increase in the amount of work that had to be accomplished, which would not necessarily be reflected, unless extrapolated, in a linear time slippage, (assuming that the organizations were operating optimally.) This may well be a question of semantics.

2.4.2 Minimum Time Optimum Resources

Likewise, substituting p from equation 2.8 into equation 1.8 yields

$$r_f(n)|_{t=\min} = nke^{\frac{c}{n}+(n-1)\frac{c}{n^2}} \tag{2.21}$$

and again combining terms

$$r_f(n)|_{t=\min} = nke^{\frac{c}{n}+\frac{c}{n}-\frac{c}{n^2}} = nke^{\frac{c}{n}(2-\frac{1}{n})} \tag{2.22}$$

then if $n >> 1$, ie., the organization is large,

$$r_f(n)|_{t=\min} \approx nke^{2\frac{c}{n}} \tag{2.23}$$

or by substituting equation 2.9

$$r_f(n)|_{t=\min} \approx nke^{2\sqrt{pc}} \tag{2.24}$$

If it is assumed that the organizations studied in the literature are operating optimally and that the project complexity is proportional to the number of lines of instructions coded.

This presentation yields a pessimistic prediction of these organization's performance. The significance is that the performance of structured organizations are being compared to the predicted performance of a flat organization. (Chapter 3 will analyze the structured organization, which will predict a performance that is too optimistic-the performance of real organizations will be between these two theoretical limits.)

Taking the derivative of equation 2.22 with respect to the number of persons working on solving a problem will possibly yield the minimum solution resources to solve the problem

$$\frac{dr_f\left(n\right)|_{t=\min}}{dn} = ke^{2\frac{c}{n}-\frac{c}{n^2}} + nke^{2\frac{c}{n}-2\frac{c}{n^2}}\left(-2\frac{c}{n^2}+2\frac{c}{n^3}\right) \tag{2.25}$$

and by equating equation 2.25 to zero, the minima occurs when

$$0 = 1 + n\left(2\frac{c}{n^3} - 2\frac{c}{n^2}\right) = 1 + \left(\frac{2c}{n^2} - \frac{2c}{n}\right) \tag{2.26}$$

and multiplying by n^2

$$0 = n^2 - 2cn + 2c \tag{2.27}$$

then by the quadratic equation

$$n = \frac{2c + \sqrt{4c^2 - 8c}}{2} = c + \sqrt{c^2 - 2c} = c + \sqrt{c}\sqrt{c-1} \tag{2.28}$$

then if $n \;>>\; 1$, ie., the organization is large, and solving for n

$$n_f\,|_{t=\min, r=\min} \approx 2c \tag{2.29}$$

2.4.3 Minimum Time and Optimum Resources

Substituting equation 2.29 into equation 2.19 yields

$$t_f\left(n\right)|_{t=\min, r=\min} \approx ke^{\frac{2c}{2c}} \approx ke \tag{2.30}$$

and into equation 2.23 yields

$$r_f\left(n\right)|_{t=\min, r=\min} \approx nke^{\frac{2c}{2c}} \approx nke \approx 2cke \tag{2.31}$$

These optimal solutions exist when, by substituting equation 2.29 into equation 2.8

$$p = \frac{c}{(2c)^2} = \frac{c}{4c^2} = \frac{1}{4c} = \frac{1}{2n} \tag{2.32}$$

To reiterate, with the variables p, c, and n

$$n_f\,|_{t=\min,r=\min} = \frac{1}{2p} \tag{2.33}$$

$$c = \frac{1}{4p} \tag{2.34}$$

$$t_f\,(n)\,|_{t=\min,r=\min} = ke \tag{2.35}$$

$$r_f\,(n)\,|_{t=\min,r=\min} = \frac{1}{2p}ke \tag{2.36}$$

2.4.4 Importance of Interface Coordination

Of particular interest is the fact that as the organization grows, the coordination interface quality must improve, ie., p must get smaller, if adding additional persons is to impact the time to solve a problem. Consider equation 2.4 for a given size of organization, then calculate what p has to be for an organization that is 10 times as large, ie., p must decrease, approximately, with $\frac{1}{n^2}$ for large organizations. In point of fact, it may be wise to do metrics on the value of p by some management scheme as a qualitative indicator of how well the organization's coordination interface is implemented.

2.5 Summary

It was suggested in this chapter that equation 2.4 is a mathematical definition of the onset of organizational bureaucracy and this is a possible economic optimum operating point for the organization. Equation 2.9 suggests that the quality of the organization's interface coordination process must improve (ie., p decreases) with the square of the number of persons in the organization as the organization expands. Equation 2.9 further suggests that an organization can operate optimally, as far as time to problem solution is concerned. Additionally, 2.14 further suggests that an organization can operate optimally, as far as minimization of resources for a problem solution is concerned.

If the organization is large, then from equation 2.6, 2.9, 2.16, and 2.29, the organization's optimum operating situation is when

$$n_f\,|_{t=\min,r=\min} \approx \sqrt{\frac{c}{p}} \approx 2c$$

Under these circumstances, from equations 2.19, 2.20, 2.23, 2.24, and 2.31, the time to complete a project, and the resources required will be, respectively,

$$t_f\,(n)\,|_{t=\min,r=\min} \approx ke^{2\frac{c}{n}} \approx ke^{2\sqrt{pc}} \approx ke$$

and

$$r_f(n)\,|_{t=\min, r=\min} \approx nke^{2\frac{c}{n}} \approx nke^{2\sqrt{pc}} \approx nke$$

The organization will then be operating at the economic optimum, minimal time to solution optimum, and minimal resources for solution optimum, ie., at operational optimum[1]. This agrees favorably, but pessimistically, with historical presentations in the literature.

The interpretation of the operational optimum is that the organization must operate at the point of diminishing returns, which allows the use of conventional economic/operational metrics to evaluate organizational dynamics. The point of diminishing returns can be measured dynamically by increasing the number of persons, or complexity issues in the organization, and observing the organizational output. If the organizational output is inversely effected, then the point of diminishing returns has been attained. Otherwise, it hasn't. If the point of diminishing returns has been attained, the only alternative is to affect the interface coordination process of the organization. The interface coordination process, or quality thereof, can be optimized, by instituting changes and then observing if the point of diminishing returns has been altered, for better or worse.

[1]Perhaps this is an organizational minimax optimum. If the organization's product environment is stochastic in nature, the project would have required the minimum commitment of resources to complete in the minimum amount of time, allowing quick market introduction to maximize revenues with the least amount of risk of resource commitment.

Chapter 3

The Structured Organization

3.1 Introduction

This analysis will consider the hierarchical organization. The hierarchical organization will be considered to be made up, at the lowest level, of n many persons, each reporting into a team leader. The team leader is a member of another organization which, for simplicity, consists of n many persons. This organization reports into a team leader that is a member of yet another higher level organization that also consists of n many persons, and so on. It is assumed that organizational interface is vertically, through the team leaders to the individual teams, and not laterally between members of different organizations.

3.1.1 Variables used in Derivations

c as before, is the linear factor of complexity that each organization in the hierarchy must address. If c is divided by two, the task is exactly half as complex. This variable should also reflect the ability of the organization to handle complex tasks. An organization that is twice as efficient as another organization would have half the c. (As before, it may be appropriate to add a factor of $\frac{1}{s}$ to c, where s represents an organization's skill level which may change do to learning.

C is the linear factor of complexity that the entire hierarchical organization must address. If C is divided by two, the task is exactly half as complex. This variable should also reflect the ability of the organization to handle complex tasks. An organization that is twice as efficient as another organization would have half the C.

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k as before, is a constant of proportionality with the same dimension as t below.

n as before, is the number of persons in each organization of the hierarchy.

$n_h|_{opt}$ is the number of persons in the hierarchical organization that will be required to achieve the economic optimum operating situation for the organization.

$n_h|_{t=\min}$ is the number of persons in the hierarchical organization that will be required to achieve the minimum solution time for a problem.

$n_h|_{r=\min}$ is the number of persons in the hierarchical organization that will be required to achieve the minimum resource allocation to solve a problem.

N is the number of persons in the entire hierarchical organization.

O is the number of organizations in the hierarchy.

p as before, is the probability, multiplied by a constant of proportionality, that a coordination issue will increase the time required to solve the problem. In some sense, p reflects the quality of the coordination process. A p of zero means that there are no coordination issues. The constant of proportionality reflects how sensitive the system is to problems created by coordination issues.

t_h is the execution time required for an organization in the hierarchy, of n many persons to solve a problem of complexity, c. It would generally have the dimension of time. If all organizations in the hierarchy solve problems equally fast (ie., there is no wasted time) then t is also the execution time required for the entire hierarchical organization consisting of N many persons, T, to solve a problem of complexity, C.

T see t above.

r_h is the total time allocation for an organization in the hierarchy, of n many persons to solve a problem of complexity, c. It would generally have the dimension of time, and is more pertinent as an indicator of the total resources allocated to solve the problem, ie., man hours.

R is the total time allocation for the entire hierarchical organization, of N many persons to solve a problem of complexity, C. It would generally have the dimension of time, and is more pertinent as an indicator of the total resources allocated to solve the problem, ie., man hours.

3.1.2 Derivation of the Hierarchical Organizational Model

For an organization in the middle of the hierarchy, each team member has to interface with the other $n - 1$ team members in the organization, and n persons in the reporting organization, plus one person in the organization that this organization reports to, for a total of $2n$ interfaces. It is assumed that inter-organizational interface issues, between organizations in the hierarchy, are handled by the by the next higher level organization in the hierarchy. Then with reasoning similar to that used in the derivation of equation 1.7, a problem's solution time can be implied

$$t_h\left(n\right) = T = ke^{\frac{c}{n}+2np} \tag{3.1}$$

and with likewise similar reasoning to that used in the derivation of equation 1.8, the resources required

$$r_h\left(n\right) = nke^{\frac{c}{n}+2np} \tag{3.2}$$

Additionally, the number of organizations in the hierarchy is the total number of persons in the entire hierarchical organization divided by the number of persons in each organization in the hierarchy

$$O = \frac{N}{n} \tag{3.3}$$

and the complexity that each organization in the hierarchy must address is the total complexity divided by the number of organizations in the hierarchy

$$c = \frac{C}{O} \tag{3.4}$$

Solving equation 3.6 for O and equating to equation 3.3

$$\frac{C}{c} = \frac{N}{n} \tag{3.5}$$

and solving for c

$$c = \frac{Cn}{N} \tag{3.6}$$

which is the complexity that each organization in the hierarchy must address. Substituting equation 3.6 into equation 3.1

$$T = ke^{\frac{C}{N}+2np} \tag{3.7}$$

and likewise, substituting into equation 3.2

$$r_h\left(n\right) = nke^{\frac{C}{N}+2np} \tag{3.8}$$

Since all organizations are assumed to operate in parallel to solve a problem, the total resources for the entire hierarchical organization would be the resources expended by each organization in the hierarchy multiplied by the number of organizations in the hierarchy

$$R = Onke^{\frac{C}{N}+2np} \tag{3.9}$$

and substituting equation 3.3

$$R = Nke^{\frac{C}{N}+2np} \tag{3.10}$$

3.2 The Case to Increase Organizational Size

By analyzing the ratio of the time required to solve a problem with an additional person to the time required to solve the same problem without the additional person, some insight can be obtained on whether it is an advisable proposition to increase the size of an organization in the hierarchy, or from equation 3.1

$$\frac{t_h\,(n+1)}{t_h\,(n)} = \frac{k e^{\frac{c}{n+1}+2(n+1)p}}{k e^{\frac{c}{n}+2np}} \tag{3.11}$$

and reducing

$$\frac{t_h\,(n+1)}{t_h\,(n)} = e^{\frac{c}{n+1}-\frac{c}{n}+2p} \tag{3.12}$$

Note that for a decrease in the time to solve the problem to occur

$$2p + c\left(\frac{1}{n+1} - \frac{1}{n}\right) \tag{3.13}$$

must be less than zero. If it is equal to zero, ie.,

$$2p = c\left(\frac{1}{n} - \frac{1}{n+1}\right) \tag{3.14}$$

then adding a person will have no effect on the time to solve the problem. Likewise, if equation 3.13 is greater than zero, then the situation exists where adding a person can actually increase the time required to solve a problem. (Perhaps this is the mathematical definition of bureaucracy.) Combining terms in equation 3.14

$$p = \frac{c}{2}\left(\frac{n+1-n}{n\,(n+1)}\right) = \frac{c}{2}\left(\frac{1}{n\,(n+1)}\right) = \frac{\frac{c}{2}}{n^2+n} \tag{3.15}$$

then if $n \;>>\; 1$, ie., the organization is large, and solving for n

$$n_h|_{opt} \approx \sqrt{\frac{c}{2p}} \tag{3.16}$$

3.2.1 A Possible Economic Optimum

If it is determined that equation 3.14 really is the onset of diminishing returns (similar to equation 2.4), then this may possibly constitute the optimum economic operating point for the organization. It is probably important to note that in adding one person to an organization in the middle of the hierarchy is equivalent to adding an organization to the next level down in the hierarchy, and such organizational designs are subject to diminishing returns.

3.3 Possible Optimum Staffing Levels

Using reasoning similar to that used in chapter 2 to optimize the number of persons in the hierarchical organization for minimum time and resources will provide the optimum operating situation for the hierarchical organization.

3.3.1 Optimum Staffing Level for Minimum Time

Taking the derivative of equation 3.7 with respect to the number of persons in each hierarchical organization will possibly yield the minimum solution time to solve the problem

$$\frac{dT(n)}{dn} = ke^{\frac{C}{N}+2np}\left(2p - \frac{c}{n^2}\right) \tag{3.17}$$

and by equating equation 3.17 to zero, the minima occurs when

$$\frac{c}{n^2} = 2p \tag{3.18}$$

or solving for n

$$n_h|_{t=\min} = \sqrt{\frac{c}{2p}} \tag{3.19}$$

3.3.2 Optimum Staffing Level for Minimum Resources

Taking the derivative of equation 3.10 with respect to the number of persons in each hierarchical organization will possibly yield the minimum resource solution to solve a problem

$$\frac{dR(n)}{dn} = nke^{\frac{c}{n}+2p}\left(2p - \frac{c}{n^2}\right) + ke^{\frac{c}{n}+(n-1)p} \tag{3.20}$$

and by equating equation 3.20 to zero, the minima occurs when

$$n\left(2p - \frac{c}{n^2}\right) + 1 = 0 \tag{3.21}$$

or

$$2np - \frac{c}{n} + 1 = 0 \tag{3.22}$$

and multiplying by n

$$2pn^2 + n - c = 0 \tag{3.23}$$

then by the quadratic equation

$$n = \frac{\sqrt{1+8pc} - 1}{4p} \tag{3.24}$$

Again, it is not clear whether this minima is of any practical interest. If $n >> 1$, ie., the organization is large ($\sqrt{4pc} >> 2p$), then

$$n_h|_{r=\min} \approx \sqrt{\frac{c}{2p}} \tag{3.25}$$

which agrees nicely with equation 3.19.

3.4 Discussion

The total resources for the entire organization can be derived by solving equation 3.18 for $2np$ and substituting in equation 3.10

$$R = Nke^{\frac{C}{N}+\frac{c}{n}} \tag{3.26}$$

then solving 3.5 for $\frac{c}{n}$ and substituting

$$R(N) = Nke^{2\frac{C}{N}} \tag{3.27}$$

Taking the derivative of equation 3.27 with respect to the total number of persons in the organization will possibly yield the minimum resource solution to solve a problem

$$\frac{dR(N)}{dN} = ke^{2\frac{C}{N}} + kNe^{2\frac{C}{N}}\left(-2\frac{C}{N^2}\right) \tag{3.28}$$

and by equating equation 3.28 to zero, the minima occurs when

$$\frac{C}{N} = \frac{1}{2} \tag{3.29}$$

Substituting equation 3.29 into equation 3.27 yields

$$R|_{\min} = Nke = 2Cke \tag{3.30}$$

Solving equation 3.18 for $2np$ and substituting into equation 3.7 yields

$$T = ke^{\frac{C}{N}+\frac{c}{n}} \tag{3.31}$$

and solving equation 3.6 for $\frac{c}{n}$ and substituting

$$T = ke^{2\frac{C}{N}} \tag{3.32}$$

and substituting equation 3.29

$$T|_{R=\min} = ke \tag{3.33}$$

Solving equation 3.18 for $2np$ and substituting into equation 3.8

$$r_h = nke^{\frac{C}{N}+\frac{c}{n}} \tag{3.34}$$

and solving equation 3.6 for $\frac{c}{n}$ and substituting

$$r_h = kne^{2\frac{C}{N}} \tag{3.35}$$

and substituting equation 3.29

$$r_h|_{R=\min} = kne \tag{3.36}$$

3.4.1 Minimum Time Optimum

Substituting $2np$ from equation 3.18 into equation 3.7 yields

$$T = ke^{\frac{C}{N}+2n\frac{c}{2n^2}} = ke^{\frac{C}{N}+\frac{c}{n}} \tag{3.37}$$

and solving 3.5 for $\frac{c}{n}$ and substituting

$$T|_{\min} = ke^{2\frac{C}{N}} \tag{3.38}$$

3.4.2 Minimum Time Optimum Resources

Likewise, substituting p from equation 3.18 into equation 3.10 yields

$$R|_{t=\min} = nke^{\frac{C}{N}+2n\frac{c}{2n^2}} = nke^{\frac{C}{N}+\frac{c}{n}} \tag{3.39}$$

and solving 3.5 for $\frac{c}{n}$ and substituting

$$R|_{t=\min} = nke^{2\frac{C}{N}} \tag{3.40}$$

Substituting n from equation 3.19 and N from equation 3.29 into equation 3.3 yields

$$O = \frac{\frac{2}{C}}{\sqrt{\frac{c}{2p}}} \tag{3.41}$$

and substituting c from equation 3.4

$$O = \frac{2C}{\sqrt{\frac{C}{2Op}}} = 2\sqrt{2COp} \tag{3.42}$$

and squaring both sides

$$O^2 = 8COp \tag{3.43}$$

or

$$O = 8Cp \tag{3.44}$$

which is the optimum number of organizations to solve a problem of complexity C. Interestingly, it is a linear relationship on complexity. Substituting c from equation 3.4 into equation 3.19 yields

$$n_h|_{R=\min,T=\min} = \sqrt{\frac{C}{2Op}} \tag{3.45}$$

and substituting O from equation 3.44

$$n_h|_{R=\min,T=\min} = \sqrt{\frac{C}{16Cp^2}} = \sqrt{\frac{1}{16p^2}} = \frac{1}{4p} \tag{3.46}$$

which is the optimum number of persons in each organization of the hierarchy.

Equating 3.19 and 3.46

$$\frac{1}{4p} = \sqrt{\frac{c}{2p}} \tag{3.47}$$

and squaring

$$\frac{1}{16p^2} = \frac{c}{2p} \tag{3.48}$$

or

$$p = \frac{1}{8c} \tag{3.49}$$

and substituting into equation 3.46

$$n = \frac{1}{4\frac{1}{8c}} = 2c \tag{3.50}$$

and substituting into equation 3.36

$$r = 2cke \tag{3.51}$$

and solving equation 3.49 for c and substituting

$$r = 2\frac{1}{8p}ke = \frac{1}{4p}ke \tag{3.52}$$

Solving equation 3.49 for c and substituting into equation 3.50 yields

$$n = 2\frac{1}{8p} = \frac{1}{4p} \tag{3.53}$$

and solving 3.49 for c

$$c = \frac{1}{8p} \tag{3.54}$$

The last three equations represent the optimum organization size in the hierarchical structure, with only p as a parameter.

Chapter 4

The Polynomial Organization

4.1 Introduction

The flat organization with exponential responses to complexity was analyzed in chapter 2, and was found to lead to pessimistic predictions of organizational performance. The flat organization with simple polynomial, ie. c^2, response to complexity will be analyzed in this section and comparisons made to the derivations in chapter 2. The outline of the derivations will follow those presented in chapters 1 and 2.

4.1.1 Variables used in Derivations

c is the linear factor of complexity. If c is divided by two, the task is exactly half as complex. This variable should also reflect the ability of the organization to handle complex tasks. An organization that is twice as efficient as another organization would have half the c. (It may be appropriate to add a factor of $\frac{1}{s}$ to c, where s represents an organization's skill level which may change do to learning.)

e is 2.718...

k is a constant of proportionality with the same dimension as t below.

n is the number of persons in the organization.

$n_p|_{opt}$ is the number of persons in the polynomial organization that will be required to achieve the economic optimum operating situation for the organization.

$n_p|_{t=\text{min}}$ is the number of persons in the polynomial organization that will be required to achieve the minimum solution time for a problem.

$n_p|_{r=\text{min}}$ is the number of persons in the polynomial organization that will be required to achieve the minimum resource allocation to solve a problem.

p is the probability, multiplied by a constant of proportionality, that a coordination issue will increase the time required to solve the problem. In some sense, p reflects the quality of the coordination process. A p of zero means that there are no coordination issues. The constant of proportionality reflects how sensitive the system is to problems created by coordination issues.

t_p is the execution time required for an organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time.

r_p is the total time allocation for an organization of n many persons to solve a problem of complexity, c. It would generally have the dimension of time, and is more pertinent as an indicator of the total resources allocated to solve the problem, ie., man hours.

4.1.2 Derivation of the One Person Organization Model

Assuming that the time, $t(1)$, required for completion of the search for solution to a problem is a simple polynomial on complexity

$$t_p(1) = kc^2 \tag{4.1}$$

where k is a constant of proportionality, and c is proportional to the complexity of the problem. Likewise, in this simple case

$$r_p(1) = kc^2 \tag{4.2}$$

4.1.3 Derivation of the Two Person Organization Model

Assuming that the task can be divided equally between the two persons, and each has the same technology and skills in searching for a solution to the problem

$$t_p(2) = k\left(\frac{c}{2}\right)^2 \tag{4.3}$$

and likewise for the resources required

$$r_p(2) = k\left(\frac{c}{2}\right)^2 + k\left(\frac{c}{2}\right)^2 = 2k\left(\frac{c}{2}\right)^2 \tag{4.4}$$

Note that the task was divided equally, with each person addressing half of the complexities. However, a new situation exists in that the work done by one person may not coordinate with the work done by the other, and visa versa, perhaps requiring a rework of some tasks, etc. If we assume that searching for, and solving, the problem created by the coordination issue is itself polynomial, then the previous equation must have an additional value of p added, where p is the probability that a coordination issue will occur multiplied by a constant that represents the affect that the issue has on the organization's efficiency. Therefore,

$$t_p(2) = k\left(\frac{c}{2} + p\right)^2 \tag{4.5}$$

and likewise for the resources required

$$r_p(2) = 2k\left(\frac{c}{2} + p\right)^2 \tag{4.6}$$

4.1.4 Derivation of the n Person Organizational Model

The general solution for the problem's solution time can be implied

$$t_p(n) = k\left(\frac{c}{n} + (n-1)\,p\right)^2 \tag{4.7}$$

and likewise for the resources required

$$r_p(n) = nk\left(\frac{c}{n} + (n-1)\,p\right)^2 \tag{4.8}$$

The reason for the $(n - 1)$ multiplier in the exponent is because the probability that a coordination issue will happen is proportional to the number of interfaces that can occur. It is assumed that each person interfaces with all others in the organization, and each interface has an equal probability of creating a coordination issue.

4.2 The Case to Increase Organizational Size

The question can be posed, when can a person, or persons, be added to an organization, presumably to decrease the time required to find a solution to a problem. By analyzing the ratio of the time required to solve a problem with an additional person to the time required to solve the same problem without the additional person, some insight can be obtained on whether it is an advisable proposition, or from equation 4.7

$$\frac{t_p(n+1)}{t_p(n)} = \frac{k\left(\frac{c}{n+1} + pn\right)^2}{k\left(\frac{c}{n} + p\,(n-1)\right)^2} \tag{4.9}$$

and for convenience taking the square root of both sides and reducing

$$\sqrt{\frac{t_p(n+1)}{t_p(n)}} = \frac{\frac{c}{n+1} + pn}{\frac{c}{n} + p\,(n-1)} \tag{4.10}$$

Note that for a decrease in the time to solve the problem to occur, the numerator of equation 4.10 must be less than the denominator. If they are equal, ie.,

$$\frac{c}{n+1} + pn = \frac{c}{n} + p\,(n-1) \tag{4.11}$$

or

$$\frac{c}{n+1} + pn = \frac{c}{n} + pn - p \tag{4.12}$$

and reducing

$$\frac{c}{n+1} = \frac{c}{n} - p \tag{4.13}$$

and combining terms of c

$$c\left(\frac{1}{n+1} - \frac{1}{n}\right) = -p \tag{4.14}$$

and finding the lowest common denominator

$$c\left(\frac{n-(n+1)}{n\,(n+1)}\right) = -p \tag{4.15}$$

and reducing

$$c\left(\frac{1}{n\,(n+1)}\right) = p \tag{4.16}$$

and expanding

$$c\left(\frac{1}{n^2+n}\right) = p \tag{4.17}$$

then if $n \;>> \; 1$, ie., the organization is large, and solving for n

$$n_p|_{opt} \approx \sqrt{\frac{c}{p}} \tag{4.18}$$

which is in agreement with equation 2.6.

4.2.1 A Possible Economic Optimum

In some sense, it is apparent that adding persons to an organization without altering p will result in diminishing returns after a certain point. If it is determined that this really is the onset of diminishing returns, then this may possibly constitute the optimum economic operating point for the organization.

4.3 Possible Optimum Staffing Levels

The possible economic optimum operating point for the organization suggests that an optimum staffing level exists that will minimize the time to solve a problem, or alternately, minimize the resources that must be committed to solve the problem.

4.3.1 Optimum Staffing Level for Minimum Time

Taking the derivative of equation 4.7 with respect to the number of persons working on solving a problem will possibly yield the minimum solution time to solve the problem

$$\frac{dt_p(n)}{dn} = 2k\left(\frac{c}{n} + (n-1)p\right)\left(p - \frac{c}{n^2}\right) \tag{4.19}$$

and by equating equation 4.19 to zero, the minima occurs when

$$\frac{c}{n^2} = p \tag{4.20}$$

or solving for n

$$n_p|_{t=\text{min}} = \sqrt{\frac{c}{p}} \tag{4.21}$$

It is not clear whether this minima is of any practical interest, but it agrees nicely with equation 4.18, at least for large organizations.

4.3.2 Optimum Staffing Level for Minimum Resources

Taking the derivative of equation 4.8 with respect to the number of persons working on solving a problem will possibly yield the minimum resource solution to solve the problem

$$\frac{dr_p(n)}{dn} = k\left(\frac{c}{n} + (n-1)p\right)^2 + 2nk\left(\frac{c}{n} + (n-1)p\right)\left(p - \frac{c}{n^2}\right) \tag{4.22}$$

and by equating equation 4.22 to zero, the minima occurs when

$$\frac{c}{n} + p(n-1) + 2n\left(p - \frac{c}{n^2}\right) = 0 \tag{4.23}$$

or expanding and combining terms

$$\frac{c}{n} + pn - p - \frac{2nc}{n^2} + 2np = 0 \tag{4.24}$$

and combining terms

$$\frac{c}{n} + 3pn - p - \frac{2c}{n} = -\frac{c}{n} + 3pn - p = 0 \tag{4.25}$$

and multiplying by n

$$3pn^2 + -pn - c = 0 \tag{4.26}$$

then by the quadratic equation

$$n = \frac{p + \sqrt{p^2 + 12pc}}{6p} \tag{4.27}$$

or by multiplying the numerator and denominator by p

$$n = \frac{1 + \sqrt{1 + \frac{12p}{c}}}{6} = \frac{1}{6} + \sqrt{\frac{1}{36} + \frac{c}{3p}} \tag{4.28}$$

Again, it is not clear whether this minima is of any practical interest. If $n >> 1$, ie., the organization is large, then

$$n_p|_{r=\min} \approx \sqrt{\frac{c}{3p}} \tag{4.29}$$

And substituting n from equation 4.29 into equation 4.7 yields

$$t_p(n)|_{r=\min} = k\left(\frac{c}{\sqrt{\frac{c}{3p}}} + p\left(\sqrt{c}3p - 1\right)\right)^2 \tag{4.30}$$

and combining terms

$$t_p(n)|_{r=\min} = k\left(\sqrt{3cp} + \sqrt{\frac{cp}{3}} - p\right)^2 = k\left(2\sqrt{cp} - p\right)^2 \tag{4.31}$$

and factoring

$$t_p(n)|_{r=\min} = k\left(\left(\sqrt{3} + \sqrt{\frac{1}{3}}\right)\sqrt{pc} - p\right)^2 \tag{4.32}$$

then if $n >> 1$, ie., the organization is large, then

$$t_p(n)|_{r=\min} \approx k\left(\left(\sqrt{3} + \sqrt{\frac{1}{3}}\right)\sqrt{pc}\right)^2 \approx 5.333kpc \tag{4.33}$$

4.4 Discussion

4.4.1 Minimum Time Optimum

Substituting p from equation 4.20 into equation 4.7 yields

$$t_p(n)|_{\min} = k\left(\frac{c}{n} + (n-1)\frac{c}{n^2}\right)^2 \tag{4.34}$$

and combining terms

$$t_p(n)|_{\min} = k\left(\frac{c}{n} + \frac{c}{n} - \frac{c}{n^2}\right)^2 = k\left(\frac{c}{n}\left(2 - \frac{1}{n}\right)\right)^2 \tag{4.35}$$

then if $n >> 1$, ie., the organization is large,

$$t_p(n)|_{\min} \approx k\left(2\frac{c}{n}\right)^2 \tag{4.36}$$

Interestingly, comparing equation 4.36 to equation 1.7 with $p = 0$, (ie., a perfect organization that has no interface coordination issues,) states that if the organization is operating optimally, that only 50 percent of the organization's efforts are directed at problem solving, and the remaining 50 percent is directed at interface coordination issues.

Alternately, substituting n from equation 4.21 into equation 4.7 yields

$$t_p(n)|_{\min} = k\left(\frac{c}{\sqrt{\frac{c}{p}}} + p\left(\sqrt{c}p - 1\right)\right)^2 \tag{4.37}$$

and combining terms

$$t_p(n)|_{\min} = k\left(\sqrt{cp} + \sqrt{cp} - p\right)^2 = k\left(2\sqrt{cp} - p\right)^2 \tag{4.38}$$

then if $n >> 1$, ie., the organization is large, then

$$t_p(n)|_{\min} \approx k\left(2\sqrt{cp}\right)^2 \approx 4kcp \tag{4.39}$$

4.4.2 Minimum Time Optimum Resources

Likewise, substituting p from equation 4.20 into equation 4.8

$$r_p(n)|_{t=\min} = nk\left(\frac{c}{n} + (n-1)\frac{c}{n^2}\right)^2 \tag{4.40}$$

and again combining terms

$$r_p(n)|_{t=\min} = nk\left(\frac{c}{n} + \frac{c}{n} - \frac{c}{n^2}\right)^2 = nk\left(\frac{c}{n}\left(2 - \frac{1}{n}\right)\right)^2 \tag{4.41}$$

then if $n >> 1$, ie., the organization is large,

$$r_p(n)|_{t=\min} \approx nk\left(2\frac{c}{n}\right)^2 \tag{4.42}$$

4.5 Summary

Interestingly, in contrast with the optimizations derived in chapter 2, the optimized time to solve a problem is linear on complexity. Additionally, an option exists on whether to allocate minimum resources to solve the problem, or solve the problem in minimum time. If the problem is solved with minimum resources, it will take approximately $\frac{1}{3}$ longer, than if the problem is solved in minimum time, but require about $\frac{1}{3}$ fewer resources. Interestingly, the optimizations still exhibit diminishing returns at the economic optimum, and any other operating situation will exhibit a non-linear solution time and resource requirement.