Notes on the Fractal Analysis of Various Market Segments in the North American Electronics Industry

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July 12, 2006

Abstract

This manuscript presents some personal notes on a fractal analysis of various market segments in the North American electronics industry. Although a very simple model is presented to analyze the dynamics of the industry's markets, there is, probably, reasonable evidence presented that the market segments do, indeed, have fractal characteristics. Although the model presented does not offer significant advantages over other quantitative methodologies, the qualitative analysis, without having access to any other data other than the time series of the market's rate of revenue returns, would seem to predict that:

- Research, development and infrastructural investments seem reasonable at about 12 to 20 percent of the rate of revenue returns for the market segments analyzed. This seems consistent with the industry.
- Venture success rates at 60 months seems reasonable at about 1 in 12, which is commensurate with the industry.
- Project success rates, of 8 month duration, are about 1 in 3, which is consistent with numbers from the Application Specific Integrated Circuit business, which could be considered as "representative."
- The "80/20 rule" that 80% of an organization's revenue comes from only a few, 3 was shown to be typical, products is really, probably, 84.13%, or one standard deviation—which is consistent through the industry.
- The "80/20 rule" that 80% of an organization's products should be "industry standard," and the remainder "proprietary" is probably, one standard deviation, or 84.13%.
- Although the prediction of product life cycle will be shown to be "pessimistic," it is none, the less, depending on the reader's point of view, reasonable, and was fairly consistent with industry averages.
- The inventory control dynamics presented seem to be consistent with the markets analyzed.
- The failure rate of Fortune 500 Companies seems consistent with predicted failure rate of organizations in the markets analyzed, although the rate of failure will be shown to be "optimistic," when related to re-investment strategy.
- The calculated number of companies participating in the markets analyzed is reasonably close to the industry numbers, and there is inferential evidence that they are operating optimally—at least in the entropic sense as defined in Chapter 2—which seems consistent with the economic theory that the companies that operate the most optimally or efficiently will, eventually, dominate the market. (The calculated number of companies participating in the various markets varied between 6 and 28, with an average of 10, and with Shannon probabilities for the individual company's market time series varying between 0.54 and 0.6, with an average of 0.57, which, interestingly, is close, within approximately 5%, to the Shannon probability for the various company's stock price time series.)
- The variance in the aggregate market time series is smaller than the the variance of the time series for any company participating in the market, which is consistent with the industries analyzed.
- It would seem that there is some supporting evidence that optimizing a company's fiscal strategy to achieve maximum market growth and optimizing a company's fiscal strategy to optimize capital growth may be mutually exclusive, which has, traditionally, been the case in the industries analyzed. Additionally, it would seem that, at least in the markets analyzed, the fiscal strategies deployed would tend to be optimizing market growth, which seems consistent with author's experience in these industries.

Additionally, it would seem to be shown that visibility into the future, regarding rate of revenue returns, was only a few months, at best. This would seem to be in disagreement with the prevailing concept that “strategic planning” should be “long term.” An interesting interpretation of this may be that these industries require a more dynamic management methodology, perhaps using “rolling” budgets, etc. to approximate an immediate feedback mechanism. But this would seem to be inconsistent with methodologies where objectives are monitored on an annual basis—it would seem that profit and loss issues are very dynamic, and, probably, require detailed attention at no more than a monthly rate, including inventory and project management issues.

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Preface

This manuscript is a compilation of my un-edited personal notes, written over two decades in which I was involved in operational management in the electronics industry. One of the issues addressed in industrial operations is the dynamics of the market place. This manuscript describes an attempt at "modeling," at least in a qualitative sense, the dynamics of the North American Electronics Industry. Being an engineer by academic tenure, I chose fractal methodologies, since a large formal infrastructure exists, and it is a "method of choice," where applicable. The latter part of my career has been spent in the financial community, where such entropic methods have been used, controversially, in such areas as "programmed trading" in the capital markets. The reader should be advised that such methods remain controversial, and applications to industrial markets, will no doubt, add to the controversy. How applicable the methodology is found to be will be left to the reader's discretion.

Most of the presentation is in chronological order. The derivations in chapters 2 and 3 were done over the span of the last decade and a half, and the sections are, roughly, in temporal order. The reader should be advised that there were paradigm shifts and symbol ambiguities as time progressed, and that chapters 2 and 3 are in need of a total rework—they are offered in that context, and the reader should be advised that there are, almost inevitably, errors in the presentation. (Specifically, Section 2.4 in Chapter 2 regarding time sampled time series should be regarded with scepticism and should be used with caution—it is unfortunate that a clear derivation of the effect of sampling on time series has not been addressed adequately, since there are many issues left open at this time.)

The programs used in the analysis were written by the author, and should be considered as "fragile," since there has never been any intention to turn them into "commercial" products. They are offered in that context, and the reader should be advised that there is little or no provision for handling numerical exceptions, and no attention given to quality assurance. The "C" language sources to the programs are available, as is, by sending an electronic mail to john-archive-request@johncon.johncon.com with a subject of "archive get fractal". The source distribution also contains the LaTeX sources to this manuscript.

The other programs used in the composition of this manuscript are all freely available by anonymous ftp. For the list of programs, and availability on the Internet, see the colophon.

—J. C. C.

July, 2006

Chapter 1

A Simple Industrial Market Model

This chapter presents a simple industrial market model, and assumptions, that should, in principle, be applicable to analysis by fractal methodologies. It should be advised that the model and assumptions are very simple, and probably not adequately sophisticated for accurate or precision analysis of industrial markets. The attempt is to present a methodology that is applicable to analyzing market dynamics—which makes fractal methodologies the method of choice. Because of the simplistic models, it would be inappropriate to appropriate to consider this presentation financial advice.

1.1 Simplified Assumptions

The simple model is outlined as follows:

- The paradigm is that unless an organization updates goods and services rendered to an industrial market place, to contemporary standards, the organization's rate of revenue returns will decrease.
- However, deciding what must be done to update the goods and services rendered is a speculative process, requiring speculation on what the market place will favor in the future. It is assumed that the desires of the market place in the future is not predictable, and has, at least to some extent, a degree of "randomness."
- It is assumed that the updating of the goods and services rendered is a continuing, iterated process that will constitute a significant portion of the organization's resources.
- The objective is presumed to be to maximize the rate of revenue returns.

Note that this "model," albeit simple, is a "prescription" for a fractal process. For a brief tutorial on fractal processes and analysis, see appendix A.

If it can be shown, and that remains to be seen, that industrial markets exhibit fractal characteristics, then there is a large existing mathematical and economic infrastructure that can be exploited to, perhaps, optimize industrial operations using entropic methodologies. The purpose of this manuscript is to offer a suggestion that this may, indeed, be the case—although with the un-sophisticated models and methodologies that are presented, there may be no intrinsic qualitative value.

1.2 Comparison of the Simplified Model with Previous Works

[Rez94, pp. 450] citing J. L. Kelly, Jr[1] suggests an interesting model which presents the problem of the rate of transmission of information in a different way:

> Consider the case of a gambler with a private wire who places bets on the outcomes of a game of chance. We assume that the side information which he receives has a probability p of being true and $1 - p$ of being false. Let the capital of the gambler be V_0 and V_k his capital after the Kth betting. Since the gambler is not certain that the side information is entirely reliable, he places only a fraction e of his capital on each bet. Thus, subsequent to n bettings, assuming the independence of the successive tips . . . The problem with which the gambler is faced is the determination of e leading to the maximum of the average exponential rate of growth of his capital . . . Thus, under these rather natural hypotheses, the maximum possible average exponential gain of the gambler coincides with the numerical value of the channel capacity. If the channel were noiseless, the gambler would obviously risk all his capital at each betting . . . Also, if he knew the value of p beforehand, he would be able to use this knowledge to his advantage and bet all his capital (or none). But the reliability of the tip is not known to him.
>
> According to Kelly, here we have an example of a real-life situation where considerations similar to the concept of source, channel, rate of transinformation, and channel capacity are valid. In the above reference, Kelly extends these results to more general cases of a gambler placing bets on outcomes of several games of chance. The gambler receives independent tips on each game conditional on the result of another game. The situation is analogous to a discrete independent source driving a discrete memoryless noisy channel.
>
> In conclusion, our acquired knowledge of information theory, which was based primarily on Shannon's communication model, can well be applied to other mathematical models arising from real-life problems.

By similar reasoning, the simple model outlined in Section 1.1 has the same mechanism suggested by Kelly, however, the gambler's capital is market size, measured in revenue returns, and the "tips" come from the market itself with a probability, p which can be computed from the fluctuations in revenue returns over time.

1.2.1 Compatability with more Sophisticated Economic Models

The proposed model is shown to be consistent as a first order approximation to the logistic function used in modeling industrial markets in the literature [Mod92] in Chapter 2, Section 2.8.

More recently, [Art, pp. 8] argues that these types of scenarios are created by multiple agents acting on inadequate information in the marketplace. These agents form an inductive reasoning system that consists of a multitude of "elements" in the form of belief-modes or hypotheses that adapt to the aggregate economic ecological system. This concept is further developed in Chapter 2, Section 2.9. Thus, it qualifies as an *adaptive complex* system. These systems can be characterized by economic increasing-returns [Art89] of a dynamic process with random events—an economy based on positive feedback [Art89]. For general implications to the business environment, see [Art96]. A system that consists of dynamic processes, with random events can be characterized as a random walk [Art95, pp. 6], [Art88, pp. 6, 10, 16], [BdL95, pp. 29, 42] fractal—identically as above. See Chapter 2, Section 2.8 for additional comments.

1.3 Examples of Simplified Assumptions

As a simplified example of the assumptions presented in Section 1.1, consider a shoe manufacturer. The shoe company manufactures blue shoes, and red shoes. The operational agenda would require speculation on how many pairs of red shoes, and how many pairs of blue shoes should be manufactured each month, for sales, marketing and distribution

[1] J. L. Kelly, Jr., "*A New Interpretation of Information Rate,*" Bell System Tech. J., vol. 35, pp. 917-926, 1956.

next month. If the “forecast” for blue shoes is incorrect, say, perhaps, too large, then not only does the company loose the money invested in the manufacture of the blue shoes, but also looses market share to the companies that forecasted the demand for blue shoes correctly—and the company’s rate of revenue returns would be expected to decrease next month. It is important to note that, at least as far as this simplified model is concerned, that the rate of revenue returns is actually a cumulative sum of all past decisions and investments in infrastructure, made by the company, in catering to the market place[2]

As a related example, a company attempts a development project for a new product, and if the development is successful and well received in the market place, the company’s rate of revenue returns increase. If not, the company looses the money on the development investment, and in addition, the company’s rate of revenue returns decrease.

It should be pointed out that these examples illustrate the usage of a very simple “model,” to describe the operations of very complex industries, which is probably not be adequate for commercial purposes. Additionally, the model may have the disadvantage of abstracting the operational environment in such a manner that causality may be difficult to establish, and relating the model’s parameters to accounting metrics may be difficult. However, there is some possibility that the methodology outlined in this manuscript can be used with Operations Research, as a forecasting methodology.

1.4 General Concepts in the Industrial Market Modeling Methodology

As a simple conceptual model, the industrial markets will be modeled as a simple tossed unfair coin game, which has characteristics of classical Brownian motion, as presented in appendix A. The analytical derivation is presented in Chapter 2, and the analytical fractal methodology used will is presented in Chapter 3. A brief description of the software programs used in the implementation of the analytical fractal methodology is presented in appendix B. The analysis of the markets is presented in appendix C, and the concept of modeling the industrial market pro forma with classical Brownian motion expanded to models using fractional Brownian motion.

The concept of using classical Brownian motion to model markets was chosen because it is the simplest of all fractal systems that can be used to analyze speculative markets. Naturally, the quality of such an analysis must be subject to appropriate scrutiny, and there are issues which can not be handled with such simple models. However, possibly, the simple models can be used to approximate industrial market process with accuracies that are adequate, hopefully, for months, and, possibly, but not likely, perhaps years.

The classical Brownian motion model assumes that fixed increments will approximate the, probably Gaussian distributed increments, and simulations will be presented to offer a qualitative view of the accuracies, and inaccuracies, of such a simple models. For more in depth methods of addressing optimizations of markets that exhibit Gaussian distributed increments, the reader is referred to the bibliography.

Chapter 4 will offer several conclusions concerning the analysis presented in appendix markets, and is offered in the context that there may be, perhaps, some value in using fractal analysis as a methodology, probably, at least, in a qualitative sense, in analyzing industrial markets. There is no insinuation that the methodology could be used for analyzing industrial markets in a qualitative sense at this time.

[2]The concept is subtile—that the derivative of the cumulative revenue returns, the rate of revenue returns, is actually a cumulative sum, or, in some sense, an integral of past market pro forma of the company. Note that this is reasonable, in some abstract sense, because if many bad forecasts were made in the past, we would expect the company’s rate of revenue returns to be much smaller than a company that made more fortunate decisions. The attempt there is to establish an isomorphism between a company’s rate of revenue returns and a cumulative sum process of a random variable—ie., a fractal. At least in principle, it is possible, using fractal analysis, to have some degree of confidence that a market’s characteristics are fractal. See [Cro95, pp. 244].

Chapter 2

General Derivation for Fractal Time Series

This chapter presents a general derivation of the optimization of betting strategies in speculative markets. It is offered in academic perspective, and under no circumstances would it be appropriate to consider it financial advice. It can serve, however, as an introduction to the contemporary economic theory of speculative markets. Rigorous and sophisticated approaches that address the issues of investing in speculative markets are contained in the bibliography.

2.1 General Derivation

Consider that investing in a speculative market is an iterated process[1], with the objective of maximizing the value of the investment's cumulative returns, R, and that the process operates according to the following principles for each and every iteration:

1. For any iteration, "wagers" are made that are a fraction, f, of the investment's cumulative returns, R. Of course, $0 \leq f \leq 1$, and the amount wagered is $f \cdot R$.

2. The investment's returns, for the current iteration, will occur in the next iteration, and will be determined by a random process, F, which will determine whether the investment's return is a loss or a gain[2], and the amount wagered, $f \cdot R$, which will determine the amount of the loss or gain.

3. The investment's returns, losses and gains, for each iteration are summed to the cumulative returns, R.

then, for the n^{th} iteration:

$$R_{n+1} = R_n + (R_n \cdot f_n \cdot F_n) \tag{2.1}$$

$$= R_n \left((f_n \cdot F_n) + 1\right) \tag{2.2}$$

[1]It is an important concept that in many cases, the iterated speculative investment process is implicit. For example, in the stock market, an investment can obviously have losses or gains, over time, without one actually making any additional physical investments, ie., "letting the investment ride" constitutes a speculative investment in itself.

[2]If the iteration's random process, F, is either $+1$ or -1 for gains and losses, respectively, then the random process is termed "Brownian." However, if the random process, F, can assume other values, besides $+1$ and -1, and, furthermore, these values have a Gaussian distribution, then the random process is termed "fractional Brownian," [Fed88, pp. 164, 172], [Cro95, pp. 232]. In either case, the random process may be unfairly biased. For example, there can be more iterations that have gains than have losses, on average, in the iterative process. Such processes could, potentially, offer a knowledgeable investor an exploitable advantage—that is why they are termed "unfairly biased." Apparently, many speculative markets exhibit such phenomena. For example, many capital markets are alleged to have unfair bias, [Pet91, pp. 81], [Sch91, pp. 127], [Cas94, pp. 255], [Rez94, pp. 450], [Pie80, pp. 270].

and rearranging Equation 2.1:

$$\frac{R_{n+1} - R_n}{R_n} = F_n \cdot f_n \tag{2.3}$$

therefore, the rules for deriving the values of $F_n \cdot f_n$, for an iteration are:

1. Subtract the value of the last iteration's cumulative returns from the value of the current iteration's cumulative returns to calculate the current iteration's incremental difference in cumulative returns.
2. Divide the current iteration's incremental difference in cumulative returns by the value of the last iteration's cumulative returns.
3. The result is the random process, F_n, multiplied by the wager fraction, f_n, for the last iteration.

Separating an iteration's wager fraction, f_n, from the iteration's random process, F_n, is difficult for incremental differences that are characterized by Gaussian distributions, ie., fractional Brownian random processes. However, for Brownian random processes, the sign of the iteration's incremental difference in cumulative returns, ie., whether the iteration's incremental difference was a loss or a gain, can be used to derive the value of the random process, F_n. Taking the absolute value[3] of the iteration's incremental difference, in the specific case of Brownian random processes, is the amount that was wagered, f_n, (provided, of course, that any unfair bias in the random process was sufficiently small, which is usually the case.)

2.1.1 Fibonacci Sequence of a Time Series

Interestingly, the normalized increments, when constructed in this manner, is simply the Fibonacci sequence of V_t, minus unity. The recursive representation of constructing the normalized increments is:

$$V_t = V_{t-1}(1 + f_t F_t) \tag{2.4}$$

and subtracting V_{t-1} from both sides:

$$V_t - V_{t-1} = V_{t-1}(1 + f_t F_t) - V_{t-1} \tag{2.5}$$

and dividing both sides by V_{t-1}:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_{t-1}(1 + f_t F_t) - V_{t-1}}{V_{t-1}} \tag{2.6}$$

and combining:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = (1 + f_t F_t) - 1 = f_t F_t \tag{2.7}$$

but the left side of Equation 2.7 is:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_t}{V_{t-1}} - 1 \tag{2.8}$$

which is the Fibonacci sequence of V_t, minus unity. The *tsmath* program can be used to add unity to the time series of the normalized increments of a time series to construct the Fibonacci sequence of the equity's value. Additionally, the time series could be time sampled with the *tssample* program prior to constructing the normalized increments to possibly investigate for any cyclic phenomena [Sch91, pp. 49].

[3] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

2.2 Construction of Random Process

The strict definition of Brownian motion is the cumulative sum of a random variable with a Gaussian distribution, [Fed88, pp. 164], [PJS92, pp. 481], [Cro95, pp. 232]. However, [Sch91, pp. 145] mentions a Brownian process with fixed increments, also called a Markov—Wiener process, and, further [Sch91, pp. 125], mentions that Brown noise is generated by summing independent random numbers, with no mention of the distribution, other than on [Sch91, pp. 128] where Brown noise is generated by independent increments, and states that the fluctuating capital of the gambler is is also, Brown noise, deducing that the probability of winning a coin is p, and a probability of loosing the coin is $p - 1$, which at first consideration does not seem to present the process of a cumulative sum of a random variable with a *Gaussian* distribution.

To reconcile the issue, consider the pseudo code for a simple Brownian noise generator time series, of n many samples, as proposed by [Fed88, pp. 164], [PJS92, pp. 481], and [Cro95, pp. 232]. From [PJS92, pp. 484], using 6, 6 sided dice, for n many samples in the time series:

```
cumulative sum = 0
for i = 1 to n
    throw 6 die
    cumulative sum = cumulative sum + total spots on all six die
    print cumulative sum
```

where, citing the content of the Central Limit Theorem, a Gaussian distribution is approximated since the process was a cumulative sum of independent and similar random events, ie., summing many independent and similar random events—the spots on 6 dice in this case—will produce a list of numbers with a Gaussian distribution. Summing these numbers will produce Brownian noise.

Now consider the following pseudo code, which is much the same, but instead of using 6 dice, only one die is used, and the cumulative sum sampled every 6th roll of the die:

```
cumulative sum = 0
for i = 1 to 6n
    throw the die
    cumulative sum = cumulative sum + total spots on the die
    if modulus i, 6 is zero
        print cumulative sum
```

Note that the two algorithms give, approximately, identical results. Apparently, sampling a sum of independent and similar random numbers, ie., white noise, is identical to a sum of random numbers with a Gaussian distribution.

Now, consider the original works of Hurst cited in [Fed88, pp. 154], using a "Monte Carlo" simulation for a random process of independent random variables obtained by tossing m many coins n many times and taking the random variable to be the number of heads minus the number of tails for each toss of the m many coins. The probability of obtaining k heads by throwing m coins is $(\frac{1}{2})^m(\frac{m!}{k!(m-k)!})$. If the coin set is tossed n times, then k, and the the incremental differences in the cumulative sum, are given by the binomial distribution, which approaches a Gaussian distribution for large n and m. The pseudo code for such a process, again using a single 6 sided die instead of a coin for compatability with the previous pseudo code, is:

```
cumulative sum = 0
for i = 1 to n
    die sum = 0
    for j = 1 to m
        throw the die
        if the number of spots on the die > 3
            increment the die sum by one
        else
            decrement the die sum by one
    cumulative sum = cumulative sum + die sum
    print cumulative sum
```

which can further be simplified by sampling the cumulative sum every mth roll of the die:

```
cumulative sum = 0
for i = 1 to nm
    throw the die
    if the number of spots on the die > 3
        increment the the cumulative sum by one
    else
        decrement the cumulative sum by one
    if modulus i, m is zero
```

print cumulative sum

As mentioned in [Fed88, pp. 156], this process is asymptotic with a Brownian motion of random variables with a Gaussian distribution, provided m and n are sufficiently large.

2.2.1 Conclusion

The three methods of generating a time series for Brownian motion with an incremental difference distribution that is Gaussian are approximately identical, provided:

- The sampling rate is sufficiently low to allow many iterations of the uniform distributed random process to be added to the cumulative sum between samples.
- The fractional Brownian time series is sufficiently close to classical, (ie., a random walk with a Gaussian distribution step length,) Brownian motion, ie., the time series' Hurst coefficient is sufficiently near 0.5.

2.3 A Simple Analysis

Consider the following fragment of a time series:

0.2
-0.2
0.2
-0.2
0.2

and if the fragment is replicated many times to produce a time series data file containing many records that "oscillates," on a period of 5, with a Shannon probability of $3/5 = 0.6$, since $f = 2P - 1$, where $P = 0.6$, and $f = 0.2$.

The rationale is as follows:

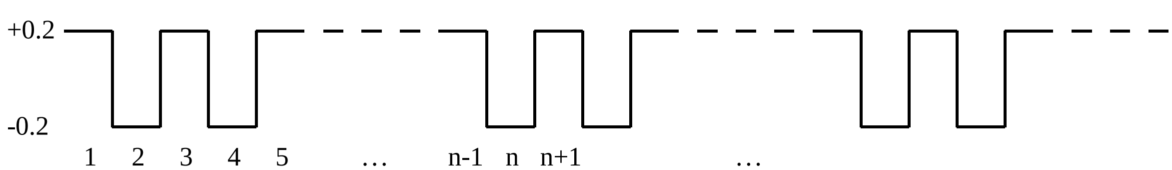

Figure 2.1: Win or Loss Fraction of Cumulative Sum.

2.3.1 "Average" Exponential Returns

In Figure 2.1, there are 3 $+0.2$'s for every 2 $-.2$'s, for an average of $+0.2$ per 5 time units, for an average of $+0.2/5 = +0.04$. The reason for the numbers, $+0.2$ and -0.2, is that it is the optimum for a Shannon probability of

0.6, since $0.2 = 2P - 1$, (which also equals $P - (1 - P)$,) where $2 \cdot 0.6 - 1 = 0.2$, which is the optimal amount of the cumulative returns to wager with an unfair coin that has a probability of 0.6 of a win, ie., 3 out of 5. If the $n - 1$'th value in the time series is subtracted from the n'th value, and the value of this subtraction divided by the $n - 1$'th value, then the quotient should be $+0.2$ or -0.2 depending on the whether the wager was won or lost. It is an important insight that the "average" returns is not useful, and creates substantial errors. See Section 2.3.2, below.

Under this scenario, $P = 0.6$, and the returns are:

$$2^{0.029049406} = e^{0.020135514} \tag{2.9}$$

which can be verified with the program *tsshannon*, which is briefly described in appendix B, and is consistent with [Sch91, pp. 128].

But, using the "average" value, of 0.04:

$$2^{0.056583528} = e^{0.039220713} \tag{2.10}$$

which is in substantial error.

2.3.2 Exponential Returns

Using *tsunfairbrownian*, which is also described in appendix B, with arguments of -f 0.2 will construct an exponential data time series that is known to be optimum, ie., a Shannon probability of 0.6, with an optimal wager fraction of 0.2, with an "approximate" Brownian motion noise content-albeit not random. It is useful for evaluating analytical methodologies. The derivation of the exponential is as follows:
assume an exponential function:

$$f(t) = e^{kt} \tag{2.11}$$

then:

$$\frac{f(t) - f(t-1)}{f(t)} = A = \frac{e^{kt} - e^{k(t-1)}}{e^{k(t-1)}} \tag{2.12}$$

or:

$$A = \frac{e^{kt} - e^{kt-k}}{e^{kt-k}} = \frac{e^{kt} - e^{kt}e{-k}}{e^{kt}e^{-k}} = \frac{1 - e^{-k}}{e^{-k}} = e^{k} - 1 \tag{2.13}$$

where:

$$k = \ln(A + 1) \tag{2.14}$$

As a useful equation in data reduction involving Shannon probability, which requires an argument of logarithmic returns in bits:

$$2^{bits \cdot t} = (A + 1)^{t} \tag{2.15}$$

or:

$$(bits \cdot t)\ln(2) = t \cdot \ln(A + 1) \tag{2.16}$$

and dividing both sides of the equation by t and solving for the number of bits:

$$bits = \frac{\ln(A + 1)}{\ln(2)} \tag{2.17}$$

Now, consider the iterated cycles in Figure 2.1, beginning with cumulative returns of 1:

- after the first, the cumulative returns are 1.2, since it it was a "win."
- after the second, the cumulative returns are $1.2 \cdot 0.8 = 0.96$, since it was a "loss."
- after the third, the cumulative returns are $1.2 \cdot 0.8 \cdot 1.2 = 1.152$, since it was a "win."
- after the fourth, the cumulative returns are $1.2 \cdot 0.8 \cdot 1.2 \cdot 0.8 = 0.9216$, since it was a "loss."
- after the fifth, the cumulative returns are $1.2 \cdot 0.8 \cdot 1.2 \cdot 0.8 \cdot 1.2 = 1.10592$, since it was a "win."

Therefore:

$$R_{n+5} = R_n \cdot (1.2 \cdot 0.8 \cdot 1.2 \cdot 0.8 \cdot 1.2) = 1.10592 R_n \tag{2.18}$$

and finding the average gains per one iteration period of the "game:"

$$R_{n+1} = Rn \cdot 1.10592^{\frac{1}{5}} = R_n \cdot 1.020339601 \tag{2.19}$$

or the incremental revenue gain, from one time period to the next is:

$$\frac{R_{n+1}}{R_n} = 1.020339601 \tag{2.20}$$

or:

$$\frac{R_{n+1}}{R_n} - 1 = 1.020339601 - 10.020339601 = A \tag{2.21}$$

where A is from Equation 2.13. From Equation 2.14:

$$k = \ln(A+1) = \ln(1.020339601) = 0.020135514 \tag{2.22}$$

Therefore, the formula for the exponential cumulative returns function is, from Equation 2.11:

$$f(t) = e^{kt} = e^{0.020135514t} \tag{2.23}$$

which is Equation 2.9. The time series analyzed in this section is simulated in appendix C, Section C.20. It is an important insight that the "average" returns derived in Section 2.3.1 is not useful, and creates substantial errors.

2.3.3 General Analysis of Exponential Returns

Consider a time series, similar to Figure 2.1, but with a true random distribution. As in Section 2.3.1, the Shannon probability, P, is $P \leq 0 \leq 1$. The fraction of the cumulative returns, R, that is wagered with each iteration of the game is f, where $0 \leq f \leq 1$, and F is an independent random process:

$$F = \begin{cases} +1, & \text{with a probability of P} \\ -1, & \text{with a probability of 1 - P} \end{cases} \tag{2.24}$$

Then in general, the returns after N many iterations would be:

$$\begin{aligned} R_N = \; & R_0 \cdot ((1 + F_1 \cdot f_1) \cdot (1 + F_2 \cdot f_2) \cdot (1 + F_3 \cdot f_3) \cdot \quad \cdots \\ & \cdot \left(1 + F_{n-1} \cdot f_{n-1}\right) \cdot (1 + F_n \cdot f_n) \cdot \left(1 + F_{n+1} \cdot f_{n+1}\right) \cdot \quad \cdots \\ & \cdot \left(1 + F_{N-1} \cdot f_{N-1}\right) \cdot (1 + F_N \cdot f_N)) \end{aligned} \tag{2.25}$$

where R_0 is the cumulative returns at time 0, f_n is the fraction of the cumulative returns wagered in the n'th iteration of the game, and F_n determines whether the n'th wager was one or lost, ie., added or subtracted from the cumulative returns.

Out of N many iterations, there will be $P \cdot N$ many wins, and $(1 - P) \cdot N$ many losses. Therefore, if f is constant, [SW49, pp. 38], [Rez94, pp. 114, pp. 450], [Pie80, pp. 270], [KF88, pp. 155], [Ash65, pp. 9], Equation 2.25 reduces to:

$$R_N = R_0 \left(1 + f\right)^{PN} \left(1 - f\right)^{(1-P)N} \tag{2.26}$$

or:

$$\frac{R_N}{R_0} = \left(1 + f\right)^{PN} \left(1 - f\right)^{(1-P)N} \tag{2.27}$$

if N is sufficiently large, then the average increase in cumulative returns for one iteration of the game, $\frac{R_{n+1}}{R_n}$, is:

$$\begin{aligned}
\left(\frac{R_{n+1}}{R_n}\right)^N &= \frac{R_N}{R_0} && (2.28)\\
&= \left(1 + f\right)^{PN} \left(1 - f\right)^{(1-P)N} && (2.29)\\
&= \left[\left(1 + f\right)^{P} \left(1 - f\right)^{(1-P)}\right]^N && (2.30)
\end{aligned}$$

and:

$$\frac{R_{n+1}}{R_n} = \left(1 + f\right)^{P} \left(1 - f\right)^{(1-P)} \tag{2.31}$$

and from Equation 2.14:

$$k = \ln\left(\left(1 + f\right)^{P} \left(1 - f\right)^{(1-P)} - 1\right) \tag{2.32}$$

where the equation for the cumulative returns, as a function of time, $R(t)$, from Equation 2.11, is:

$$\begin{aligned}
R\left(t\right) &= e^{\left(\ln\left(\left(1+f\right)^{P}\left(1-f\right)^{(1-P)} - 1\right)\right)t} && (2.33)\\
&= \left[\left(1 + f\right)^{P} \left(1 - f\right)^{1-P}\right]^t && (2.34)
\end{aligned}$$

Using, for example, $P = 0.6$ and $f = 0.2$, the cumulative returns, as a function of time, would be:

$$\begin{aligned}
R\left(t\right) &= \left[\left(1 + 0.2\right)^{0.6} \left(1 - 0.2\right)^{(1-0.6)}\right]^t && (2.35)\\
&= \left[1.2^{0.6} \cdot 0.8^{0.4}\right]^t && (2.36)\\
&= \left[1.115600622 \cdot 0.914610104\right]^t && (2.37)\\
&= \left[1.020339601\right]^t && (2.38)
\end{aligned}$$

which is in agreement with Equation 2.19

Optimization of Returns

From Equation 2.27, the average increase in the cumulative returns for one iteration of the game would be:

$$R_N = R_0 (1+f)^{PN} (1-f)^{(1-P)N} \tag{2.39}$$

$$\frac{R_N}{R_0} = (1+f)^{PN} (1-f)^{(1-P)N} \tag{2.40}$$

$$\frac{R_1}{R_0} = (1+f)^{P} (1-f)^{(1-P)} \tag{2.41}$$

which can be maximized to maximize the growth in the cumulative returns, G, for each iteration of the game. Considering G as a function of f, the fraction of the the cumulative returns wagered on each iteration of the game:

$$G(f) = \frac{R_1}{R_0} = (1+f)^{P} (1-f)^{(1-P)} \tag{2.42}$$

and taking the derivative, and equating to 0 to find the maxima:

$$\frac{dG(f)}{df} = P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{1-P-1}(1+f)^{P} = 0 \tag{2.43}$$

and combining terms:

$$P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{-P}(1+f)^{P} = 0 \tag{2.44}$$

and splitting:

$$P(1+f)^{P-1}(1-f)^{1-P} = (1-P)(1-f)^{-P}(1+f)^{P} \tag{2.45}$$

and taking the natural logarithm of both sides of the equation:

$$\ln(P) + (P-1)\ln(1+f) + (1-P)\ln(1-f) = \ln(1-P) - P\ln(1-f) + P\ln(1+f) \tag{2.46}$$

and combining terms:

$$((P-1)\ln(1+f) - P\ln(1+f) + (1-P)\ln(1-f) + P\ln(1-f) = \ln(1-P) - \ln(P) \tag{2.47}$$

$$\ln(1-f) - ln(1+f) = \ln(1-P) - \ln(P) \tag{2.48}$$

$$\tag{2.49}$$

and performing the logarithmic operations:

$$\ln\left(\frac{1-f}{1+f}\right) = \ln\left(\frac{1-P}{P}\right) \tag{2.50}$$

and exponentiating:

$$\frac{1-f}{1+f} = \frac{1-P}{P} \tag{2.51}$$

$$P(1-f) = (1-P)(1+f) \tag{2.52}$$

$$P - Pf = 1 - Pf - P + f \tag{2.53}$$

$$2P = 1 + f \tag{2.54}$$

and finally:

$$f = 2P - 1 \tag{2.55}$$

which is identical to Equations A.1 and A.11, and agrees with [Sch91, pp. 128, 151]. A more elegant approach to maximization of the cumulative returns, using information—theoretic techniques, is presented in [SW49, pp. 38], [Rez94, pp. 114, pp. 450], [Pie80, pp. 270], [KF88, pp. 155], [Ash65, pp. 9].

Note that, referring to Figure 2.1 and Equations 2.24 and 2.25, that the root mean square, rms, of Equation 2.24 is simply f—assuming f is constant and N is sufficiently large, or:

$$rms = f \tag{2.56}$$

and the average avg will be $P - (1 - P)$ time this value, since there will be P many f's and $P - 1$ many $-f$'s, or:

$$avg = rms\,[P - (1 - P)] = rms\,(2P - 1) \tag{2.57}$$

Note that if the game is not being run at the optimum, where $2P - 1 = f$, then f must be increased by a factor which can be calculated from measuring avg and rms. Letting K be the amount that f must be multiplied by so that the f will be optimum, f_{opt}, from Equation 2.57:

$$\frac{avg}{rms} = (2P - 1) \tag{2.58}$$

which will be true whether f is optimum or not. But $2P - 1 = f_{opt}$ when optimum, therefore:

$$\frac{avg}{rms} = f_{opt} \tag{2.59}$$

but from Equation 2.56, $rms = f$, and after dividing both sides by $rms = f$:

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{2.60}$$

2.3.4 Ancillary derivation of the Shannon Probability, P

As an interesting manipulation to Equation 2.31,

$$\frac{R_{n+1}}{R_n} = (1 + f)^P\,(1 - f)^{(1-P)} \tag{2.61}$$

or:

$$\frac{R_{n+1}}{R_n} = \frac{(1 + f)^P\,(1 - f)}{(1 - f)^P} \tag{2.62}$$

and:

$$\frac{\frac{R_{n+1}}{R_n}}{(1 - f)} = \frac{(1 + f)^P}{(1 - f)^P} \tag{2.63}$$

$$= \left(\frac{(1 + f)}{(1 - f)}\right)^P \tag{2.64}$$

and taking the natural logarithm of both sides:

$$\ln\left(\frac{\frac{R_{n+1}}{R_n}}{(1-f)}\right) = P\ln\left(\frac{(1+f)}{(1-f)}\right) \tag{2.65}$$

and solving for P:

$$P = \frac{\ln\left(\frac{\frac{R_{n+1}}{R_n}}{(1-f)}\right)}{\ln\left(\frac{(1+f)}{(1-f)}\right)} \tag{2.66}$$

Equation 2.66 presents an important relationship, since metric methodologies can be used to quantize P, $\frac{R_{n+1}}{R_n}$, and f.

2.4 The Case of the Time Sampled Time Series

In many cases, the time series data under consideration is time sampled. For example, the data may be by month, which is the aggregate of many time series, in a fashion similar to the situation outlined in this chapter, Section 2.2.

Referring to Figure 2.1 and Equations 2.24 and 2.25, the root mean square, rms, of Equation 2.24 is simply f—assuming f is constant and N is sufficiently large. From Equation 2.57:

$$avg = rms\,[P - (1-P)] = rms\,(2P-1) \tag{2.67}$$

and letting rms = f:

$$avg = f\,(2P-1) \tag{2.68}$$

But from Equation A.11:

$$F = 2P - 1 \tag{2.69}$$

where f is F with the signs removed, following reasoning similar to those in Equations 2.24 and 2.25, or:

$$f = 2P - 1 \tag{2.70}$$

Inserting into Equation 2.68:

$$avg = f^2 = rms^2 \tag{2.71}$$

It would be desirable to consider avg, f, and rms as functions of the number of iterations in a time sample, N:

$$avg\,(N) = f\,(N)^2 = rms\,(N)^2 \tag{2.72}$$

Since the root mean square value, rms will combine the iterations in a sample time interval in a root mean square fashion, it would be expected that rms would be proportional to the square root of the number of iterations in a time sample[4], N, or rms as a function of N:

$$rms\,(N) = rms_0\sqrt{N} \tag{2.73}$$

[4]Technically, the results of the iterations are multiplied together, as described in Equation 2.25. However, it can be reasoned that these multiplications can be broken down into additions in a fashion similar to multiplying 3 by 2. The operation can be performed by adding 2, three times, $2+2+2=6$. The reasoning is that root mean square operations can be used for multiplying variables with a normal distribution together, in a manner of implementing Equation 2.25 with the methodologies outlined in this chapter, Section 2.2. The reasoning is that the random process, summed over N many intervals will have the characteristics of a Central Limit process.

where rms_0 is a constant of proportionality and is the value of rms when the number of intervals in a time sample is one. From this, it can be concluded that the value of $(2P - 1)$ in Equation 2.68 must be proportional to the square root of N, also. Solving for the function $P(N)$:

$$2P\left(N\right) - 1 = rms_0 \sqrt{N} \tag{2.74}$$

and solving for $P(N)$:

$$P\left(N\right) = \frac{rms_0 \sqrt{N} + 1}{2} \tag{2.75}$$

From Equation 2.72:

$$avg\left(N\right) = \left(rms_0 \sqrt{N}\right)^2 = rms_0^2 N \tag{2.76}$$

or avg will vary linearly with N.

As a simulated example, the variables rms, $mean$, which is avg above, and P can be plotted. In the directory graphics/probability is a collection of files, generated by the script file "probability", which uses P with an initial value of 0.51 and constructs files that are sampled on intervals N = 1 through 100. The graphs of the functions are superimposed with the computed theoretical values, in Figure 2.2.

As another simulated example, the output of the programs *tsrms*, *tslsq*, *tsshannon*, *tslogreturns*, and *tsnormal* are plotted as a function of the sample interval in the directory /graphics/brownian as generated by the script file "brownian," which uses P with an initial value of 0.501 and constructs files that are sampled on intervals N = 1 through 100. The graphs of the functions are superimposed with the computed theoretical values, in Figure 2.3.

As interesting aside, the file "mean," produced by the "probability.vs.N" script, should be linear as a function of N, at least in principle. The general formula for these files, as a function of the sampling period, N, is $mean = rms(2P-1)$. If the input file is time sampled, then the formula is $meanN = rms\sqrt{(N)}(2P-1)\sqrt{(N)}$, and the "mean" file should be linear, with a slope of the mean as a function of N. To exercise this, see the file "probability.vs.N" in the directory ../markets/tscoin.tssample.

2.4.1 Simulation of Simultaneous Games

Consider playing N many coin tossing games, simultaneously. This can be simulated as a binomial distribution. Let f = fraction of capital wagered in each unit of time (this is the normalized increment of the total capital, where the total capital is the instantaneous value of the sum of the capital for all games.) Then, letting avg = the average of f, rms = the root mean square of f, and P_f = the fundamental Shannon probability of each coin, in each game, (assumed to be identical,) then:

$$avg = \left(2P_f + 1\right) Nf \tag{2.77}$$

and:

$$rms = f\sqrt{N} \tag{2.78}$$

where f, rms, and avg are measured, and represent the fraction of capital wagered, the root mean square, and the average of the normalized increments of the time series of the total capital, ie., playing N many games, simultaneously:

$$rms = \frac{f}{\sqrt{N}} \tag{2.79}$$

and:

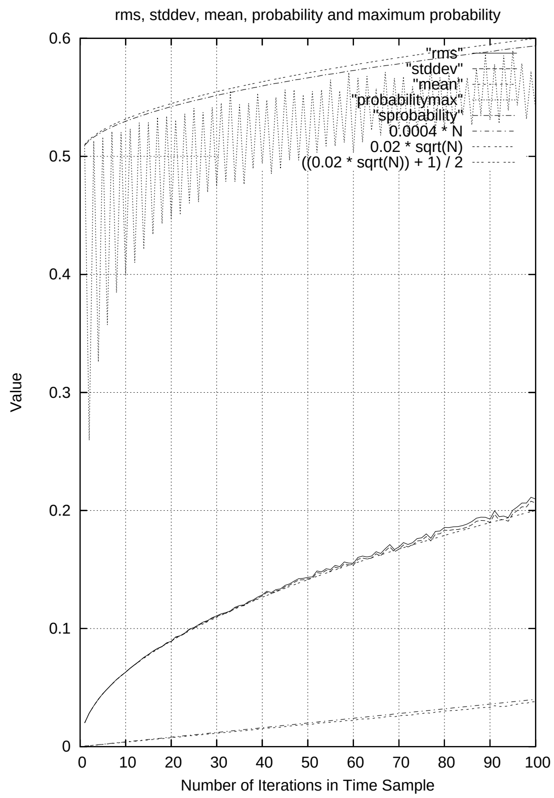

Figure 2.2: Example probability, as a function of sample intervals, root mean square, mean, and standard deviation, of Equation 2.25 with $P = 0.51$.

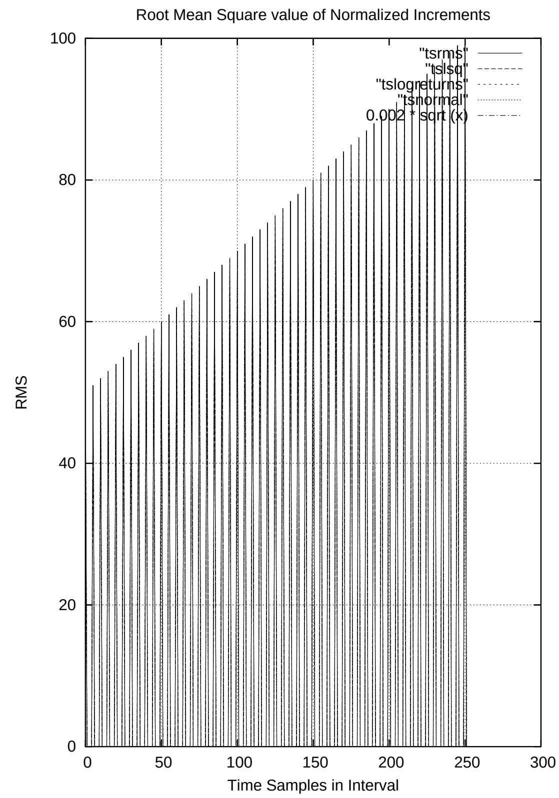

Figure 2.3: Example program outputs as a function of sample intervals.

$$avg = \left(2P_f - 1\right) f \tag{2.80}$$

What this means is that the average, avg, adds linearly, and the rms root mean square, where the wager on each of the simultaneous games is f/N.

It makes no difference whether many coins are thrown at once, or one coin thrown many times.

In other words, playing many games simultaneously, each with coins of equal Shannon probability, will not effect the growth of the total capital. The volatility of the total capital will go down by the square root of the number of games played.

It is rather computationally inefficient, but this can be verified with the *tsbinomial* program, using the -r option.

2.5 Analysis of Brownian Motion with Fixed Increments

Consider a time series, similar to Figure 2.1, but with a true random distribution. As in Section 2.3.1, the Shannon probability, P, is $P \leq 0 \leq 1$. The fraction of the cumulative returns, R, that is wagered with each iteration of the game is f, where $0 \leq f \leq 1$, and F is an independent random process, from Equation 2.24:

$$F = \begin{cases} +1, & \text{with a probability of P} \\ -1, & \text{with a probability of 1 - P} \end{cases} \tag{2.81}$$

Then in general, the returns after N many iterations would be, from Equation 2.25:

$$\begin{aligned} R_N &= R_0 \cdot ((1 + F_1 \cdot f_1) \cdot (1 + F_2 \cdot f_2) \cdot (1 + F_3 \cdot f_3) \cdot \quad \cdots \\ &\cdot \left(1 + F_{n-1} \cdot f_{n-1}\right) \cdot (1 + F_n \cdot f_n) \cdot \left(1 + F_{n+1} \cdot f_{n+1}\right) \cdot \quad \cdots \\ &\cdot \left(1 + F_{N-1} \cdot f_{N-1}\right) \cdot (1 + F_N \cdot f_N)\big) \end{aligned} \tag{2.82}$$

which, after performing a process similar to the operations described in Chapter 3, Section 3.1, would result in Equation 2.3:

$$\frac{R_{n+1} - R_n}{R_n} = F_n \cdot f_n \tag{2.83}$$

which is the increments of the cumulative returns.

Standard Deviation of Increments of the Cumulative Returns

The standard deviation, σ, of the increments of the cumulative returns is:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (F_n f_n - avg)^2 \tag{2.84}$$

where avg is the average of the increments of the cumulative returns, from Equation 2.57:

$$avg = rms\,[P - (1 - P)] = rms\,(2P - 1) \tag{2.85}$$

and from Equation 2.56:

$$rms = f \tag{2.86}$$

or:

$$avg = rms\,(2P - 1) = f\,(2P - 1) \tag{2.87}$$

and Equation 2.84 becomes, if f_n is constant:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (F_n f - f\,(2P - 1))^2 \tag{2.88}$$

and factoring f:

$$\sigma^2 = \frac{f^2}{N} \sum_{i=1}^{N} (F_n - (2P - 1))^2 \tag{2.89}$$

and additionally, the summation series will contain PN many cases where $F_n = 1$, and $(1 - P)N$ many cases where $F_n = -1$, or:

$$\sigma^2 = \frac{f^2}{N} \left(PN \left(1 - (2P - 1)\right)^2 + (1 - P) N \left(-1 - (2P - 1)\right)^2 \right) \tag{2.90}$$

and factoring N:

$$\begin{aligned}
\sigma^2 &= \frac{Nf^2}{N} \left(P \left(1 - (2P - 1)\right)^2 + (1 - P) \left(-1 - (2P - 1)\right)^2 \right) && (2.91)\\
&= f^2 \left(P \left(1 - (2P - 1)\right)^2 + (1 - P) \left(-1 - (2P - 1)\right)^2 \right) && (2.92)\\
&= f^2 \left(P \left(1 - 2P + 1\right)^2 + (1 - P) \left(-1 - 2P + 1\right)^2 \right) && (2.93)\\
&= f^2 \left(P \left(2 - 2P\right)^2 + (1 - P) \left(2P\right)^2 \right) && (2.94)\\
&= f^2 \left(P \left(2 - 2P\right)^2 + (1 - P) \left(2P\right)^2 \right) && (2.95)\\
&= 4f^2 \left(P \left(1 - P\right)^2 + (1 - P) \left(P\right)^2 \right) && (2.96)\\
&= 4Pf^2 \left(\left(1 - P\right)^2 + (1 - P) \left(P\right) \right) && (2.97)\\
&= 4Pf^2 \left(\left(1 - P\right)^2 + P \left(1 - P\right) \right) && (2.98)\\
&= 4P(1 - P)f^2 \left((1 - P) + P \right) && (2.99)\\
&= 4P \left(1 - P\right) f^2 && (2.100)
\end{aligned}$$

or:

$$\sigma^2 = 4f^2 P \left(1 - P\right) \tag{2.101}$$

and solving Equation 2.101 for f:

$$\frac{\sigma^2}{4P \left(1 - P\right)} = f^2 \tag{2.102}$$

or:

$$f = \frac{\sigma}{\sqrt{4P \left(1 - P\right)}} = \frac{\sigma}{2\sqrt{P \left(1 - P\right)}} \tag{2.103}$$

and substituting into Equation 2.57

$$avg = rms \left(2P - 1\right) = \frac{\sigma \left(2P - 1\right)}{2\sqrt{P \left(1 - P\right)}} \tag{2.104}$$

which states the relationship between the average, or mean, the standard deviation, and the root mean square of the increments of the returns for a time series representing Brownian motion with fixed increments. Furthermore, metrics for these values can be determined using the methodology outlined in Chapter 3, Section 3.1.

2.6 Analysis of a Fractal Time Series with Many Contributing Agents

In many market and financial instrument historical time series, there are many contributing agents. For example, an industrial market time series may be the aggregate of the production of many participating companies. From Equation 2.60:

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{2.105}$$

where avg is the average of the normalized increments of the time series and is related to the long term exponential growth of the market, $(1 + avg)^n$, where n is the number of time units, and rms is the root mean square of the normalized increments of the time series, and is related to the “volatility,” or variance of the market. If the following “model” of a company participating in the market is assumed:

1. Each company acts independently, and will receive cash flow from the market.
2. Some of this cash flow will be diverted into new product manufacturing, development, etc., which in turn will go back into the market, which in turn will create cash flow, and so on—but there is a random element in this process.
3. Analysis of various markets, (see Appendix C,) yields that they are probably a fractal, (fractional Brownian variety,) with a fairly accurate distribution of the normalized increments that appears to be Gaussian in nature, a range that appears to increase with the square root of time, and an exponential curvature. These are indicative of system that can be modeled by as a gambler’s capital in an unfair coin toss game, or Brownian fractal.

Under these assumptions, it would seem reasonable that:

1. If a market that is supplied by a single company. The time series for the market could be represented, at least statistically, as an unfair coin tossing game, (see the *tscoins* program,) with each time unit of manufacturing going into the marketplace, the marketplace returning cash to the company’s P & L, which is distributed to the company’s operations to manufacture more product, and so on. But there is an element of randomness in this process that represents the aggregate of customer desires and market forces—this is assumed be a central limit phenomena, ie., it can be represented as a random variable with a normal, (Gaussian,) distribution. Note, that like the gambler, the company’s operations managers are continually wagering on the future—and each wager may, or may not prove to be a successful. It is further assumed that the company will commit capital to enhancing its market position, (ie., increase manufacturing capacity, develop new products, etc.,) and, as above, the decision to do so will contain an element of risk, and will sometimes work out, and sometimes not.
2. Now consider that another company decides to participate in the marketplace—under the same scenario of as above. If everything else is equal, we would expect the market, eventually, to be divided equally between the two companies, or each company would have half the market. When the second company was added to the market, the first company’s contribution to the marketplace was cut in half—and its root mean square value of its normalized increments contribution to the marketplace was also cut in half. The second company’s contribution to the marketplace is the remaining one half, and its contribution to the root mean square value of its normalized increments is the same as the first company’s. (The point is that the contributions to the marketplace add linearly, but the contribution of to the normalized increments of the marketplace add root mean square—so we would expect the root mean square value of the normalized increments to decrease when the number of participants in the marketplace changes from one to two—since the value of the normalized increments for each company is proportional to the contribution to its the market.) Conceptually, think of it as a Gaussian noise generator. If we cut the root mean square value (amplitude,) of the noise generator in one half, and add an identical noise generator, the resulting noise output of both generators will be the square root of two, divided by two.
3. Or in general, the root mean square value of the normalized increments of a marketplace time series will be proportional to one over the square root of the number of companies in the market.

Figure 2.4 is a schematic representation of a company’s relation to the market of the above scenario. $G(t)$ represents the company’s random element in the system, and is a function of a random variable with a Gaussian distribution. $V(n)$ is the market, and $v(n)$ is the companies contribution to the market in the n’th time interval. Note that the various companies in the system have P&L’s that are dependent on their own random element, and the market as a whole, which, in addition, has randomness that is dependent on the random element and fiscal strategy of all of the companies.

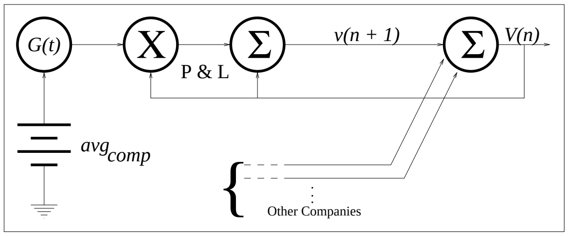

Figure 2.4: Schematic representation of a company in a market.

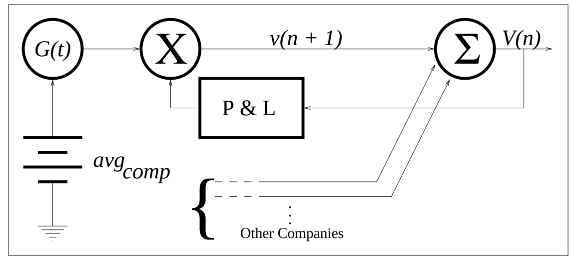

Figure 2.5: Alternative schematic representation of a company in a market.

It should be mentioned that "models" proposed in Figures 2.4 and 2.5 are not the only possible implementations. For example, Appendix C, Section C.26, suggests another of many alternatives.

Or, generalizing, letting rms_{ind} and avg_{ind} be the root mean square and average value of the normalized increments of the market time series for the aggregate industry, and rms_{comp} be the root mean square of the normalized increments for each individual company, (which in this simple analysis are assumed to be equal for all companies participating in the marketplace,) and if it is further assumed that all of the companies are operating optimally, then, from Equation 2.71:

$$avg_{comp} = rms_{comp}^2 \tag{2.106}$$

and summing the variances for N many companies to obtain the average and variance of the aggregate:

$$rms_{ind}^2 = \overbrace{\frac{rms_{comp}{}^2}{N} + \cdots + \frac{rms_{comp}{}^2}{N}}^{N many} \tag{2.107}$$

or:

$$rms_{ind}^2 = \frac{n}{N^2} rms_{comp} = \frac{1}{N} rms_{comp} \tag{2.108}$$

or the number of companies participating in the market place, N, is:

$$N = \frac{avg_{ind}}{rms_{ind}^2} \tag{2.109}$$

under the above approximations and assumptions, and where $avg_{ind} = avg_{comp} + \cdots + avg_{comp}$. It should be noted that this derivation assumes a characteristic Brownian motion time series, which is only an approximation, and assumes a Hurst exponent of 0.5. As pointed out in [Sch91, pp. 157], this assumption must be used with care, and can lead to incorrect conclusions—but with the simplified assumptions used, it will probably suffice for "general" analysis. A simulation program, *tsmarket*, which is briefly described in appendix B can be used to investigate the validity of the assumptions.

Note that it would be anticipated, under the above assumptions, that an individual company's variance would be larger than the variance of the industry as a whole by an amount $\sqrt{N}$. This means that the Shannon probability, P for an "average" company in the industry, from Equation 2.57, would be:

$$P = \frac{\frac{avg_{ind}}{\sqrt{N} rms_{ind}} + 1}{2} \tag{2.110}$$

which would be smaller than the Shannon probability for the aggregate industry.

2.6.1 Optimization to maximize the P&L

Figure 2.4 presents another optimization alternative. It is generally assumed that optimizing cash flow in a corporation's P&L to maximize market growth is the objective—and that this in turn will maximize the P&L. However, that may not necessarily be the case. Consider the following, where, in a manner similar to that described schematically in Figure 2.4, the P&L is the capital on hand at time t:

- Let $I(t)$ be the amount of capital at time t, ie., the value of the P&L.
- Let $W(t)$ be the amount of the capital wagered at time t, ie., the amount of the the capital distributed through the company for manufacturing operations, new product development, etc., or in other words, what is "wagered" on the future.
- Let $V(t)$ be the value of the industrial market at time t.

then, letting f be the fraction of the capital, or P&L wagered in a fashion similar to Section 2.5:

$$W(t) = fI(t-1) \tag{2.111}$$

where f is presumed not to be a function of time. Then:

$$I(t) = I(t-1) + W(t)\frac{V(t) - V(t-1)}{V(t-1)} \tag{2.112}$$

and substituting:

$$I(t) = I(t-1) + fI(t-1)\frac{V(t) - V(t-1)}{V(t-1)} \tag{2.113}$$

or:

$$\frac{I(t)}{I(t-1)} = 1 + f\frac{V(t) - V(t-1)}{V(t-1)} \tag{2.114}$$

If it is assumed that the stock's price time series can be represented as a Brownian noise fractal, then the optimum value of f would be, from Equation 2.55:

$$f = 2P - 1 \tag{2.115}$$

where P is the Shannon probability of the market time series, found by:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{2.116}$$

where, as in Equation 2.58, avg is the average, and rms is the root mean square, of the normalized increments of the market's time series, which can be calculated by:

$$\frac{V(t) - V(t-1)}{V(t-1)} \tag{2.117}$$

for each data point in the market's time series.

Since the market's time series already has a value rms as the root mean square of the normalized increments, for the optimal wagering strategy, the fraction should be divided by rms to provide a multiplier:

$$multiplier = fraction/rms \tag{2.118}$$

so that:

$$\frac{I(t)}{I(t-1)} = 1 + multiplier \frac{V(t) - V(t-1)}{V(t-1)} \tag{2.119}$$

This is similar to the "wagering" strategies used in entropic financial instrument trading. What this means is that if you have capital, (ie, a portfolio,) $I(t)$, the fraction of $I(t)$ that should be wagered with each iteration of the "game", (ie., time unit,) would be twice the Shannon probability minus unity, where the capital, (or portfolio,) is the sum total of cash on hand, $C(t)$, and the current value investment in the industrial market, ie., the inventory. Or, to maximize the P&L growth:

$$f = 2P - 1 \tag{2.120}$$

where the Shannon probability, P is calculated from Equation 2.116.

This is interesting, at least according to the simplified model used in this section, because maximizing market segment growth and maximizing the P&L will be mutually exclusive except when the market consists of exactly one company.

Figure 2.5 is a schematic representation of a company's relation to the market in this scenario.

2.7 Statistical Estimation of Required Data Set Size

Consider the following formula for determination of the Shannon Probability, P, of an equity market time series, using the average and root mean square of the normalized increments, avg, and, rms, respectively, by rearranging Equation 2.58:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{2.121}$$

which is useful in the determination of the optimal fraction of capital, f, to invest in a stock, from Equation 2.55:

$$f = 2P - 1 \tag{2.122}$$

The objective is to estimate how large the data set has to be for determining P to a given accuracy, possibly using statistical estimates of how many data points are required for a given confidence level that the error is less than a specific value.

Suppose we have a confidence level, $0 < c < 1$, that a value is within, plus or minus, an error level, e. What this means, for example if $c = 0.9$, and $e = 0.1$, is that for 90% of the cases, the value will be within the limits of $\pm e$, or, 5% of the time, on the average, it will be less than $-e$, and 5% of the time more than $+e$.

The error level for avg, e_{avg},for a given confidence level, will be:

$$e_{avg} = k \frac{rms}{\sqrt{n}} \tag{2.123}$$

where n is the number of records in the data set, and k is a function involving a normal distribution. The error level for rms, for the same given confidence level, will be:

$$e_{rms} = k \frac{rms}{\sqrt{2n}} \tag{2.124}$$

where k is identical in both cases. Also, the number of records required for a given error level would be:

$$n_{avg} = \left(\frac{(rms \cdot k)}{e_{rms}} \right)^2 \tag{2.125}$$

and

$$n_{rms} = \frac{1}{2}\left(\frac{(rms \cdot k)}{e_{rms}}\right)^2 \tag{2.126}$$

where k is the same as above.

For equity market indices, a typical value for rms would be 0.01, and 0.0003 for avg. This is probably typical for many stocks, however, high gain stocks, in a "bull" market can have an rms of 0.04, and an avg of 0.005.

The value of k can be determined from standard statistical tables, as shown in table 2.1, where $k =$ sigma level, for a confidence level, c.

Table 2.1: Confidence Level vs. σ Level.

Confidence Level, c	σ level
(%)	
50	0.67
68.27	1.00
80	1.28
90	1.64
95	1.96
95.45	2.00
99	2.58
99.73	3.00

Note that for a given confidence level:

$$\frac{avg}{rms} = \frac{avg \pm k\frac{rms}{\sqrt{n}}}{rms \pm k\frac{rms}{\sqrt{2n}}} \tag{2.127}$$

$$= \frac{\frac{avg}{rms} \pm k\frac{1}{\sqrt{n}}}{1 \pm k\frac{1}{4\sqrt{n}}} \tag{2.128}$$

Now, consider the specific example of avg and rms for an exponential function. In this specific case, $avg = rms$, and $\frac{avg}{rms} = 1$. Since k is assumed to be a function of a normally distributed random variable, the error in the ratio $\frac{avg}{rms}$ as a function of the data set size, n, can be found by superposition, and adding the contributing error values as a function of n for both rms and avg root mean square, or:

$$\sqrt{1^2 + \left(\frac{1}{4}\right)^2} = 1.030776406 \tag{2.129}$$

or:

$$\frac{avg}{rms} \sim \frac{avg}{rms} \pm 1.03\frac{1}{\sqrt{n}}k \sim \frac{avg}{rms} \pm \frac{1}{\sqrt{n}}k \tag{2.130}$$

where k is determined from the table, above. In this specific case, where $avg = rms$:

$$\frac{avg}{rms} \sim \frac{avg}{rms}\left(1 \pm \frac{1}{\sqrt{n}}k\right) \tag{2.131}$$

An interpretation of what this means is that, given a data set size, n, and a confidence level of, say 90%, then 90% of the time, our measurements of $\frac{avg}{rms}$, would fall within an error level of $\pm 1.64\frac{1}{\sqrt{n}}$, ie., 5% of the time it would be greater than the error value, and 5% of the time, it would be lower than the error value. In general, the concern is with the lower error value since from the equation:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{2.132}$$

(at least in this specific case where $avg = rms$,) that a 90% confidence level would imply that there is a 5% chance of the real value $\frac{avg}{rms}$ being zero is where:

$$\frac{k}{\sqrt{n}} = 1 \tag{2.133}$$

or:

$$\frac{1.64}{\sqrt{n}} = 1 \tag{2.134}$$

or $n = 2.6896 \sim 3$.

What this means is that, if we repeat the experiment of finding 3 records in a row that have $rms = avg$, with neither equal to zero, many times, that we would loose money in 5% of the cases, making the measured Shannon probability, P, unity, and the estimated Shannon probability, 0.95, eg., we should consider the Shannon probability as 0.95 in this specific case—ie., it would be ill advised to invest all of the capital in such a scenario, since, sooner or later, all of the capital would be lost, (on average, by the 20'th game.)

This implies a simple methodology. Measure avg and rms, and compute the Shannon probability. Decease that probability by a factor—ie., one minus the confidence level, divided by two—that the wager could be a loosing proposition, based on the estimates that avg could be zero, (which is a function of the confidence level, and the number of records in the data set.) This, conceivably, could provide a quantitative estimate on the number of records required in a data set.

Note that if $\frac{avg}{rms}$ is measured at 0.9, then:

$$\frac{1.64}{\sqrt{n}} = 0.9 \tag{2.135}$$

for the same confidence level of 0.9, or

$$n = 3.32 \tag{2.136}$$

and:

for the same confidence level 0.9. What the table means is that if you have a stock price time series of 67 records, then the minimum measured Shannon probability must be at least 0.6—and the wagering strategy should use the Shannon probability of 0.57—and the minimum number of records used to measure avg and rms is 67. Additionally, a stock time series with a Shannon probability of 0.53 should be measured using not less than 1076 records, and no wager should be made, unless the measurements involve substantially more than 1076 records. In general, the Shannon probability of almost all stock time series fall, inclusively, in this range. 67 business days is, approximately, 13.4 weeks, or little more than a calendar quarter. 1076 business days is slightly longer than four calendar years.

Note that [Pet91, pp. 83] referencing [Fed88, pp. 179], the claim is made that 2500 records is the minimum size of the data set for using fractal analytical methodologies. Note that a data set of this size would have, with an $\frac{avg}{rms}$ of 0.5—which is "typical" for a stock time series, a Shannon probability error level that is approximately 1%, since it lies between 2 and 3 sigma, and c would be approximately 0.99. This would seem to be consistent with the empirical arguments of both Peters and Feder, although Peters implies that less could be used if the system being analyzed is "chaotic" in nature, and one "cycle" of the system's, apparently, "strange attractor" is less than 2500 time units. This analysis would seem to be consistent with the observations of these authors, provided that it is a requirement that the measured Shannon probability be used to calculate the optimum wager fraction.

Table 2.2: Shannon Probability vs. Data Set Size.

$\frac{avg}{rms}$	n	$P_{measured}$	P
1.0	2.7	1.00	0.95
0.9	3.3	0.95	0.90
0.8	4.2	0.90	0.86
0.7	5.5	0.85	0.81
0.6	7.5	0.80	0.76
0.5	10.8	0.75	0.71
0.4	16.8	0.70	0.67
0.3	29.9	0.65	0.62
0.2	67.2	0.60	0.57
0.1	268.9	0.55	0.52
0.05	1075.8	0.53	0.50

What this analysis would tend to suggest is that, although Feder's and Peter's arguments seem to be confirmed, that there may, also, be other viable solutions for data sets, (or fragments thereof,) that are very much smaller, provided that the measured Shannon probability of the data set, or segment, is sufficiently large—for example, a stock that has a time series fragment that has 5 out of 6 upward movements may prove to be a viable investment opportunity at a measured Shannon probability that is greater than 0.85, ($\frac{5}{6}$ = a Shannon probability of 0.833 $\sim$ 0.85,) if played at a Shannon probability as high as 0.8, but no higher.

For example, using a Shannon probability, P, of 0.51 for the *tscoins* and *tsfraction* programs, to provide an input fractal time series for the *tsstatest* program, and iterating, indicates that for a standard deviation of 0.020000, with a confidence level of 0.960784 that the error did not exceed 0.020000, 3 samples would be required.

Since the Shannon probability is calculated directly from the standard deviation, (ie., rms = root mean square of the normalized increments,) the maximum error can be calculated:

$$\frac{0.5}{0.51} = 0.980392157 \tag{2.137}$$

which means that a confidence level of 0.960784314 that the error level in the standard deviation is less than 0.02 because standard deviation $= rms = 0.02 - 0.02 = 0$, which would correspond to a Shannon probability, P, of 0.5, and since half the errors outside the range of 0.02 would be negative, (and the other half positive,) the confidence level required would be $1 - ((1 - 0.980392157) \cdot 2)$.

What this means is that $((1 - 0.960784314)/2) \cdot 100$ percent of the time, the actual rms value will be sufficiently small to make P equal to, or less than 0.5. This means that P must be decreased by 1.960784300 percent. The reasoning is that after many iterations, the measured P would be too small by 1.90784300% of the time, on average, making the measured P, over all of the iterations, 0.5.

This suggests a dynamic rule: do not wager unless the Shannon probability, P, is strictly greater than 0.51, as measured on strictly more than 3 time units. Interestingly, the Hurst Coefficient, as measured by the *tshurst* program, graph of a random walk, Brownian motion, or fractional Brownian motion fractals indicates that there is significant near term correlations for 4 or less time units. This suggests a dynamic trading methodology for equities.

Similar reasoning would indicate that using a value of $P = 0.6$ for the *tscoins* and *tsfraction* programs to provide input to the *tsstatest* program with a confidence level of 0.8, and an error of 0.12, (ie., 10% of the time the value of P would be less than $0.9 \cdot 0.6 = 0.54$, where $0.2 - 0.12 = 0.08$, and $0.54 = \frac{0.08+1}{2}$,) would require a minimum of 3 records. The fraction of capital wagered should be $2 \cdot 0.54 - 1 = 0.08$.

2.8 Non-linear extensions

As mentioned in Section 2.5, consider a time series, similar to Figure 2.1, but with a true random distribution. As in Section 2.3.1, the Shannon probability, P, is $P \leq 0 \leq 1$. The fraction of the cumulative returns, R, that is wagered with each iteration of the game is f, where $0 \leq f \leq 1$, and F is an independent random process, from Equation 2.24:

$$F = \begin{cases} +1, & \text{with a probability of P} \\ -1, & \text{with a probability of 1 - P} \end{cases} \tag{2.138}$$

Then in general, the returns after N many iterations would be, from Equation 2.25:

$$\begin{aligned} R_N \quad = \quad & R_0 \cdot ((1 + F_1 \cdot f_1) \cdot (1 + F_2 \cdot f_2) \cdot (1 + F_3 \cdot f_3) \cdot \quad \cdots \\ & \cdot \left(1 + F_{n-1} \cdot f_{n-1}\right) \cdot (1 + F_n \cdot f_n) \cdot \left(1 + F_{n+1} \cdot f_{n+1}\right) \cdot \quad \cdots \\ & \cdot \left(1 + F_{N-1} \cdot f_{N-1}\right) \cdot (1 + F_N \cdot f_N)) \end{aligned} \tag{2.139}$$

which, after performing a process similar to the operations described in Chapter 3, Section 3.1, would result in Equation 2.3:

$$\frac{R_{n+1} - R_n}{R_n} = F_n \cdot f_n \tag{2.140}$$

which is the general formula for deriving the cumulative returns of the next iteration from the current iteration. Rearranging:

$$R_{n+1} = R_n + R_n \cdot F_n \cdot f_n \tag{2.141}$$

and adding a constant, n, which represents the magnitude of the second order part of the time series:

$$R_{n+1} = R_n + R_n \cdot F_n \cdot f_n + n \cdot R_n^2 \tag{2.142}$$

or:

$$R_{n+1} = R_n \left(1 + F_n \cdot f_n + n \cdot R_n\right) \tag{2.143}$$

which is the logistic equation, with Brownian noise, ie., Equations 2.83 and 2.141 are first order approximations to logistic equation.

Letting $1 + \overline{F_n} \cdot \overline{f_n}$ be a where $\overline{F_n}$ and $\overline{f_n}$ are the average of the independent random process and wager fraction, respectively:

$$R_{n+1} = R_n \left(a + b \cdot R_n\right) \tag{2.144}$$

where $n = b$ for compatability with the *tsdlogistic* program. Equation 2.144 is identical to the algorithm used in the program *tsdlogistic* which implements the discreet time logistic function. The programs *tscoin* and *tscoins* also have options to implement the logistic function. A brief description of the programs appears in Appendix B.

Note that the logistic function has been used to model market dynamics in the literature, [Mod92, pp. 55, 124, 188, 239-274]. Interestingly, the sign of the non-linear term is insignificant, provided the Shannon probability, P is sufficiently close to $1/2$, and $n = b$ is sufficiently small. The reason for this is that the independent random process, F_n, has values of ± 1, and dominates the equation.

Continuing from Equation 2.144, by subtracting R_n from both sides:

$$R_{n+1} - R_n = R_n \left(a + b \cdot R_n\right) - R_n \tag{2.145}$$

and dividing both sides by R_n:

$$\frac{R_{n+1} - R_n}{R_n} = a + b \cdot R_n - 1 \tag{2.146}$$

which is the formula for the normalized increments of the discreet time logistic function. Note that the right side of the formula is linear, ie., it is possible to derive the constants of the discreet time logistic function by using the program *tsfraction* to make a time series of the normalized increments of a time series, and, possibly, using the program *tslsq* with the -p option to provide the formula for the least squares linear fit. The least squares fit is of the form:

$$f(t) = P - Qt \tag{2.147}$$

where:

$$a = P + 1 \tag{2.148}$$

and:

$$b = Q \tag{2.149}$$

could be used as the arguments to the *tsdlogistic* program.

The presumption used in Equations 2.83 and 2.141 is that the non-linear term of the logistic equation is insignificant when studying new growth industry markets—ie., it can be ignored when concern is the left side of the logistic equation. This is inappropriate in the study of long term market phenomena, such as saturation, or limitation of resources.

Equity/asset market pricing may exhibit logistical non-linearities [Mod92, pp. 156].

More recently, [Art88] has proposed that industrial markets exhibit economic non-linear increasing returns phenomena. Although specifically addressing market share of competing technologies, [Art88, pp. 6] offers the argument that such scenarios will have a time series that is a random walk, ie., fixed increment Brownian motion fractal, in the case of diminishing and constant returns. Further, it is shown, [Art88, pp. 16] that the case of increasing returns also exhibits random walk characteristics—at least where the agent's expectations are that that will be the case. See [Art] for arguments concerning the nature of expectations in a multi-agent economic environment. These environments are not ergodic, (ie., not mean reverting,) and exhibit unpredictability, and are not necessarily path efficient [Art88, pp. 7]. However, if it is assumed that an industrial market is a sum of the many random walk time series of the many agent's market share, and it is assumed that the sum of many random walk time series is a random walk, then the market time series could, possibly, be analyzed by the methodology proposed in this chapter. However, in the case of increasing returns, it is doubtful that the constants in the time series could be derived with adequate precision for analysis of market share, but, perhaps, the market itself may be analyzed. As a case in point, in [Art95, pp. 6] argues that individual agents operating in the equity markets exhibit behavior that is derived from *inductive* reasoning, but the market itself exhibits random walk characteristics. Quoting from [Art95, Abstract]:

> Actions taken by economic decision makers are typically predicated upon hypotheses or predictions about future states of a world that is itself in part the consequence of these hypotheses or predictions. When we attempt to model how such predictions might be generated we become stymied: the predictions some economic agents might form depend on the predictions they believe others might form; and the predictions these might form depend upon the predictions *they* believe the original group might form. Predictions or expectations can then become self-referential and deductively indeterminate. This indeterminacy pervades economics and game theory.

The three papers, [Art95], [Art], and [Art88] present compelling arguments for a new economic paradigm. In practical multi-agent economic scenarios, some models of the economy will seem to be correct, for a while, only to be discarded later. In [Art95] the argument is presented that some models will be stable, ie., if it is believed that a model works, the economy will operate according to that model. In [Art88] it is shown that the random walk hypothesis is one such model—at least in the case of competing technologies. It is not clear whether all multi-agent systems exhibit random walk characteristics, but it would seem reasonable to assume so, and [Art95, [pp. 9] presents simulation results that tend to confirm the assumption. If that is the case, then the random walk analytical methodology presented in this chapter could be applicable.

2.9 Generalization

To reiterate the general concepts presented so far, a fractal is a cumulative sum of a random process. In the literature, it is sometimes called a Brownian motion, or "random walk," process since, at any time, the next element in the process time series is a random increment added to the current element in the time series. From [Fed88, pp. 164]:

> We emphasize that in Brownian motion it is not the position of the particle at one time that is independent of the position of the particle at another; it is the displacement of that particle in one time interval that is independent of the displacement of the particle during another time interval.

This is a subtile concept. Note that the term "cumulative sum" really means that in any time interval, the position of the particle is dependent only on the position of the particle in the previous time interval, and a random displacement. But the position in the previous time interval was dependent only on the position in the time interval prior to that, and another displacement, and so on, ie., to make a fractal process, we need only know where the particle is at the current time, and add a displacement to it, for each interval in time. The subtilty is that we need only know where the particle is, and not where it has been to calculate where it will be.

This section will use this concept, and expand the concept of the random process to include game-theoretic issues by introducing iterated two player mixed strategy games, then a simple self-referencing game where no formal strategy can exist, and finally multi-player games, where the random process is generated by the inconsistency of the self-referential, inductive reasoning among the players. In all cases, the iterated time series of such games will be argued to be fractal, in nature.

2.9.1 The Game of Mora

A simple coin tossing game was analyzed in Section 2.2. In this section, those concepts will be expanded to include games of strategy. The game of Mora, following [Bro73, pp. 434], is very old, (being mentioned in Sanskrit,) and is played between two players and, in its simplest version, goes as follows. The two players move simultaneously. Each shows either one or two fingers, and at the same time guesses whether the other player is showing one or two fingers. If both players guess right, or both guess wrong, no money changes hands. However, if only one player guesses right, the player wins from the other as many coins as the two players together showed fingers. The possible outcomes of any game are as follows if your call is right, and your opponent's wrong:

1. Guessing your opponent will show 1 finger and showing 1 finger you will win 2 coins.
2. Guessing your opponent will show 2 fingers and showing 1 finger you will win 3 coins.
3. Guessing your opponent will show 1 finger and showing 2 fingers you will win 3 coins.
4. Guessing your opponent will show 2 fingers and showing 2 fingers you will win 4 coins.

The game is fair, but a player who knows the right strategy will, with average luck, win against one who does not. The right strategy is to ignore courses 1) and 4), and to play courses 2) and 3) in the ratio of 7 to 5, ie., the right strategy is, in any 12 iterations of the game, to play course 2) on the average 7 times, and course 3) on the average 5 times. Obviously, your opponent must not know which course you are going to play, so the two courses must be intermixed randomly.

The game is zero-sum, meaning that what one player wins, the other looses. The mathematical method by which the best strategy was found is called game theory. However, it is not hard to verify that the strategy is effective by calculating what happens when your opponent counters by using course 1), 2), 3), or 4), above. Namely, if your opponent chooses course:

1. Course 1), will, on the average, win 7 times out of 12, and will win only 2 coins for each win; whereas losses will occur 5 times out of 12, and those losses will be 3 coins for each loss—making an average loss of 1 coin in 12 iterations of the game.

2. Course 2), will have no coins change hands, since either both players are right, or both are wrong.

3. Course 3), will have no coins change hands, since either both players are right, or both are wrong.

4. Course 4), will, on the average, win 5 times out of 12, and will win 4 coins for each win; whereas losses will be occur 7 times out of 12, and those losses will be 3 coins for each loss—making an average loss of 1 coin in 12 iterations of the game.

As in Section 2.2, the objective of each player is to maximize the number of coins won over many iterations of the game, ie., to maximize the cumulative returns of the game. Note that each player's capital, will fluctuate, depending on the outcome of a particular iteration-and that fluctuation will be random, and either 0, 2, 3, or 4 coins. We would expect that the time series representing the fluctuations in a player's capital to be a random walk, which could be represented by a formula similar to Equation 2.1.

It is often convenient to represent the game as a table, which lists all the possibilities of the courses for both players, and how much the each player would win or loose for each course, ie., a *payoff matrix*, where one player's alternatives are represented by the columns in Table refMORA:POM, and the other player's alternatives are represented by the rows. The payoff to a particular game solution is the intersection of the row and column of the course played by the two players.

Table 2.3: The Game of Mora, Payoff Matrix.

Finger, Guess	1,1	1,2	2,1	2,2
1,1	0	2	-3	0
1,2	-2	0	0	3
2,1	3	0	0	-4
2,2	0	-3	4	0

The optimal strategy for a game as simple as Mora can be derived by game-theoretic methodology[5] [LR57, pp. 56], [Hil90, pp. 441], [DSS58, pp. 419], [Saa59, pp. 209], [Sin68, pp. 127], [Str88, pp. 435], [NT93, pp. 258], [Kar91, pp. 67], [Kap82, pp. 105], but in many games of interest, the rules are too complicated, and may even change over time[6]. In these scenarios, the strategy can be derived empirically, over time, using *adaptive control* computational methodologies. For example, if the strategy of Mora was not known, the optimal ratio of courses could be determined by varying the ratio, and observing the effect on the cumulative reserves over many iterations of the game. Note that such a methodology can be problematical since your opponent may be doing the same thing. An example of such a scenario is presented in the next section.

[5] These methodologies are often called *operations research*. The algorithm of choice used to derive the optimal game play seems to be the *simplex algorithm*—at least for games with a small payoff matrix. The simplex algorithm is one of a class of algorithms that are implemented using *linear algebra*.

[6] In the game of Mora, the optimal strategy does not depend on the strategy of the opposing player. In more sophisticated games, this is not true.

2.9.2 Prisoner's Dilemma

A simple mixed strategy zero-sum game was analyzed in the previous section. In the game of Mora, the optimal strategy does not depend on how your opponent plays the game over time. The prisoner's dilemma game is qualitatively different. It is also one of the most commonly studied scenarios in game theory[7] [LR57, pp. 94], [Pou92], [Wal92, pp. 262], [Cas94, pp. 262], [Cas89a, pp. 295], [Cas89b, pp. 199] [Cas90, pp. 297], [Str88, pp. 439], and [Kap82, pp. 155] [Dav91, pp. 170].

The rules of the game are simple. There are two players, and each player has only two choices for each iteration of the "game," and those choices are to chose either "A" or "B." If both players pick "A," then each wins 3 coins. If one picks "A," and the other "B," then the player picking "B" wins 6 coins, and the other player gets nothing. However, if both players pick "B," then both win 1 coin.

The payoff matrix for the prisoner's dilemma game is shown in Table 2.4, where, as before, one player's alternatives are represented by the columns, the other player's alternatives are represented by the rows. The payoff to a particular game solution is the intersection of the row and column of the course played by the two players.

Table 2.4: The Prisoner's Dilemma Game, Payoff Matrix.

Choice	A	B
A	3,3	6,0
B	6,0	1,1

The prisoner's dilemma is not a zero-sum game—neither player can ever loose any money. So there is an incentive to always play. The choice "A" is known as a "cooperation strategy," and the choice "B" is known as the "defection strategy" for each player. It is a very subtile and devious game. Here is why, and the logic you would go through. Just before you played an iteration of the game, you would think:

1. If you choose "A," there are two possible scenarios:
   (a) If your opponent chooses "A," you would get 3 coins, and your opponent would get 3 coins.
   (b) If your opponent chooses "B," you would get 1 coin, and your opponent would get 6 coins.
2. If you choose "B," there are also two possible scenarios:
   (a) If your opponent chooses "A," you would get 6 coins, and your opponent would get nothing.
   (b) If your opponent chooses "B," you would get one coin, and your opponent would get one coin.

Note that by choosing "A," the best you could do is to win 3 coins, and the worst is to win nothing. But, by choosing "B," the best you could make is 6 coins, and the worst is one coin. It would appear, at least initially, that "B," is the best choice, irregardless of what you opponent does.

But now the logic of the game gets subtile. Your opponent will determine the same strategy, and will never play "A." So you both make one coin with every iteration of the the game. But you could make 3 coins—if you cooperated, by both playing "A." But if you do that, there is an incentive for either player to play "B," if he knows the other player is going to play "A," and thus make 6 coins. And we are right back where we started. Indeed, a very diabolical game.

It is an important concept that you will be basing your decision whether to cooperate, ie., choose "A," or defect, ie., choose "B," based on how you think your opponent is going to play. But your opponent's decision will be based on

[7] The prisoner's dilemma has generated much interest since it is a game that is simple to understand, and has all of the intrigue and strategy of many human social dilemmas—for example, John Von Neumann, the inventor of game theory, once said that the reason we do not find intelligent beings in the universe is that they probably existed, but did not solve the prisoner's dilemma problem and destroyed their self. The prisoner's dilemma has been used to model such scenarios as the nuclear arms race, battle of the sexes, etc.

consideration of how you are going to play. Which, in turn, will be based on how you think your opponent will play, ad infinitum. It is circular logic, or more correctly, the game strategy is *self-referential* [Hof89, pp. 17, pp. 465] [Cas90, pp. 361, pp. 379], [Cas89b, pp. 335], [Cas89a, pp. 356], [Hod83, pp. 84, pp. 103, pp. 215], [Pen89, pp. 101][8].

This presents a problem in defining an optimal strategy for playing the game of the iterated prisoner's dilemma since no "theory of operation" of a self-referential system can ever be proposed that will be both consistent and complete, ie., whatever theory is proposed, it will not cover all circumstances, or provide inconsistent results in other circumstances [Hof89, pp. 465, pp. 471], [Art95]. The best way to play the game is deductively indeterminate. This indeterminacy pervades economics and game theory [Art95, Abstract].

However, just because such problems do not have axiomatized, provably robust solutions does not mean that good strategies do not exist. For example, the *tit-for-tat* strategy [Pou92, pp. 239] has been shown to be a very effective. The objective is to avoid letting the game degenerate into both players playing defection strategies. It is very simple, and consists of cooperating, ie., playing "A," on the first iteration of the game, and then do whatever the other player did on the previous iteration[9]. Note that it is a "nice" strategy, (in the jargon of game theory, a "nice" strategy is one that never defects first.) It is also a "provocable" strategy—it defects in response to a defection by the opponent. It is also a "forgiving" strategy—the opponent can implicitly "learn" that there is an incentive for cooperating after a defection[10]. An important concept of the tit-for-tat strategy is that, unlike the game of Mora, the strategy does not have to be kept secret. When one is faced by an opponent that is playing tit-for-tat, one can do no better than to cooperate. This makes tit-for-tat a stable strategy.

Unfortunately, tit-for-tat does not do so well when the opponent occasionally defects, and then returns to a generally cooperative strategy. Neither does it do well when the other player is playing a random strategy. As in the case of the game of Mora, the strategy can be derived empirically, over time, using adaptive control computational methodologies. The subject of *inductive reasoning* as an adaptive control methodology is considered in Section 2.9.3.

As in Section 2.2, the objective of each player is to maximize the number of coins won over many iterations of the game, ie., to maximize the cumulative returns of the game. Note that each player's capital, will fluctuate, depending on the outcome of a particular iteration-and that fluctuation will be random, and either 0, 1, 3, or 6 coins. We would expect that the time series representing the fluctuations in a player's capital to be a random walk, which could be represented by a formula similar to Equation 2.1[11]. Computer simulations of the co-evolving strategies of iterated multi-player prisoner dilemma scenarios where the individual players "learn" how to cooperate further support the hypothesis [Dav91, pp. 170].

2.9.3 Multi-Player Games

A simple coin tossing game was analyzed in Section 2.2. In Section 2.9.1, those concepts were expanded to include zero-sum games of mixed strategy, using the game of Mora as an example. It was shown in these types of games, the optimal strategy does not depend on how your opponent plays the game over time. In Section 2.9.2, a nonzero-sum game, the prisoner's dilemma, was analyzed and it was shown that the strategy for the game is deductively indeterminate since the game's logic is self-referential. The reason for this was that one player's strategy depended on how the other

[8] The Penrose citation, referencing Russell's paradox, is a very good example of logical contradiction in a self-referential system. Consider a library of books. The librarian notes that some books in the library contain their titles, and some do not, and wants to add two index books to the library, labeled "A" and "B," respectively; the "A" book will contain the list of all of the titles of books in the library that contain their titles; and the "B" book will contain the list of all of the titles of the books in the library that do not contain their titles. Now, clearly, all book titles will go into either the "A" book, or the "B" book, respectively, depending on whether it contains its title, or not. Now, consider in which book, the "A" book or the "B" book, the title of the "B" book is going to be placed—no matter in which book the title is placed, it will be contradictory with the rules. And, if you leave it out, the two books will be incomplete.)

[9] The tit-for-tat strategy sounds like a human social strategy between two people—as well it should. It is known to work well with human subjects [Pou92, pp. 239]. It is also strict military dogma, and has formed the strategy of arbitration of the complexity of power in many marriages.

[10] Tit-for-tat is kind of a "do unto others as you would have them do unto you—or else," strategy. The tit-for-tat strategy in human relationships is very old. Another ancient proverb illustrating tit-for-tat is "an eye for an eye, a tooth for a tooth."

[11] Assuming that one player, or the other, will, at least occasionally, alter strategy in an attempt to gain an advantage—in this case, for example, two players, each playing tit-for-tat will "lock" in to either a defection strategy, or cooperation strategy. This is considered a degenerate case of Equation 2.1.

player plays the game over time. In both cases, the cumulative sum of winnings of a player was shown to have characteristics of a random walk, Brownian motion fractal. In this section, these concepts will be expanded to include multi-player games, where the players use inductive reasoning to determine a set of perceptions, expectations, and beliefs concerning the best way to play the game. These types of scenarios are typical of industrial manufacturing and equity markets.

Inductive Reasoning

Paraphrasing[12] [Art95], actions taken by economic decision makers are typically a predicated on hypotheses or predictions about future states of the world that is itself, in part, the consequence of these hypotheses or predictions. Predictions or expectations can then become self-referential and deductively indeterminate. In such situations, agents predict not deductively, but inductively. They form subjective expectations or hypotheses about what determines the world they face. These expectations are formulated, used, tested, modified in a world that forms from others' subjective expectations. This results in individual expectations trying to prove themselves against others' expectations. The result is an ecology of co-evolving expectations that can often only be analyzed by computational means. This co-evolution of expectations explains phenomena seen in real equity markets that appear as anomalies to standard finance theory [Art95], [Art].

This concept views such "games" in psychological terms: as a collection of beliefs, anticipations, expectations, cognitions, and interpretations; with decision-making and strategizing and action-taking predicated upon beliefs and expectations. Of course this view and the standard economic views are related—activities follow from beliefs and expectations, which are mediated by the physical economy [Art95].

This is a very useful concept because it essentially states that economic agents make their choices based upon their current beliefs or hypothesis about future prices, interest rates, or a competitors' future move in a market. These choices, when aggregated, in turn shape the prices, interest rates, market strategies, etc., that the agents face. These beliefs or hypotheses of the agents are largely individual, subjective, and private. They are constantly tested and modified in a world that forms from their's and others' actions [Art95].

In the aggregate, the economy will consist of a vast collection of these beliefs or hypotheses, constantly being formulated, acted upon, changed and discarded; all interacting and competing and evolving and co-evolving. Beyond the simplest problems in economics, this ecological view of the economy becomes inevitable [Art95].

The "standard way" to handle predictive beliefs in economics is to assume identical agents who possess perfect rationality and arrive at shared, logical conclusions about the economic environment. When these these expectations are validated as predictions, then they are in equilibrium, and are called *rational expectations*. Rational expectations often are not robust since many agents can arrive at different conclusions from the same data, causing some to deviate in their expectations, causing others to predict something different and then deviate too [Art95].

[Art95] cites the "El Farol Bar" problem as an example. Assume one hundred people must decide independently each week whether go to the bar. The rule is that if a person predicts that more than, say, 60 will attend, it will be too crowded, and the person will stay home; if less than 60 is predicted, the person will go to the bar. As trivial as this seems, it destroys the possibility of long-run shared, rational expectations. If all believe *few* will go, then *all* will go, thus invalidating the expectations. And, if all believe *many* will go, then *none* will go, likewise invalidating those expectations. Like the iterated prisoner's dilemma, predictions of how many will attend depend on others' predictions, and others' predictions of others' predictions. Once again, there is no rational means to arrive at deduced *a-priori* predictions. The important concept is that expectation formation is a self-referential process in systems involving many agents with incomplete information about the future behavior of the other agents. The problem of logically forming expectations then becomes ill-defined, and rational deduction, can not be consistent or complete. This indeterminacy of expectation-formation is by no means an anomaly within the real economy. On the contrary, it pervades all of economics and game theory [Art95].

It is an important concept that this view of industrial and financial markets address such notions as market "psychology," "moods," and "jitters." Markets do turn out to be reasonably efficient, as predicted by standard financial

[12] Actually, plagiarize would be a more appropriate choice of wording. This entire section is a condensed version of the text from [Art95] and [Art].

theory, but the statistics show that trading volume and price volatility in real markets are a great deal higher than the standard theories predict. Statistical tests also show that technical trading can produce consistent, if modest, long-run profits. And the crash of 1987 showed dramatically that sudden price changes do not always reflect rational adjustments to news in the market [Art95].

In this market model, inductive reasoning prevails as the "engine" of the market since no deductive hypothesis is possible because of the Gödelian issues of self-referential arbitrage.

It should be pointed out that inductive reasoning in such scenarios is not an exact process, and usually relies, to some extent, on correlation between events in the economy. In self-referential processes, single simplex statistical evaluations are not possible, and this can lead to misinterpretation of the significance of the statistics of the events [Cas90, pp. 50][13].

A multi-player, self-referential model of an equities market

Suppose that throughout a trading day, agents line up to buy or sell a stock. When a particular agents' turn comes, the agent has the option to try to increase or decrease the price of the stock from the transaction price of the previous agent, (by lowering the price to sell stock the agent owns, or raising the price to buy stock from another agent.) The agent will have to make this decision based on beliefs concerning the beliefs of the agents in the rest of the market. This decision process will vary as different agents post their transaction through the day, based on their personal set of beliefs, cognitions, and hypothesis concerning the market. We would expect that the time series representing the fluctuations in a stock's price to be a random walk, which could be represented by a formula similar to Equation 2.1 [Art95, pp. 8]. Empirical analysis of many stocks tend to support the hypothesis that stock prices can be "modeled" as a random walk, or fractional Brownian motion fractal. Additionally, computer models of stock market asset pricing under inductive reasoning with many agents has been initiated and further support the hypothesis [Art95, pp. 8].

Stability Issues

In section 2.8 and 2.9.2 the issues of process stability were mentioned. Note that not all processes are stable. For example, consider a stock market scenario that historically had cyclic or periodic increases and decreases in price. The value at the bottom of the cycle would increase, (because the agents in the market could exploit a "buy low, sell high" strategy that would be predictable,) and the price advantage would be arbitrated away, and the cyclic phenomena would disappear. Cyclic phenomena would then be considered as an unstable process—similar to the El Farol Bar problem mentioned above. However, note that if the agents in the market believed that their financial position could be improved by altering their investment strategy, by buying or selling of stocks, then, as outlined in the previous section, the stock price would fluctuate similar to a random walk, and this would be stable since it is a self reinforcing situation.

Extensibility Speculations

Interestingly, the arguments presented in this section are possibly extensible into other areas. For example, the Stanford economist Kenneth Arrow has shown that the ranking of priorities in a group is intransitive[LKS91, pp. 1] [LR57, pp. 327] [Hof93, pp. 213]. What this means is that there exists no way to use deductive rationality to rank priorities in a society. If it is assumed that it is necessary to do so, then inductive reasoning would have to be used. If it is further assumed that such a situation is self-referential, which seems reasonable by arguments similar to those presented in this section, then the same issues outlined in this section could be applicable to social welfare issues, etc. This would tend to imply that political issues were fractal in nature, and the political process justified—which is contrary to the thinking of many. The arguments presented in [Art95], and [Art] may well be extensible into other fields of interest. Other speculations could involve theoretical interests in the dynamics of democratic process, legal process[14], and

[13] Additionally, there are issues concerning causality. Cause and effect may not be discernable from each other.

[14] Could the legal system be optimized? Or is that an oxymoron?

organizational process[15]. There are probably other applications[16].

As another interesting aside, the arguments presented in this section side-stepped the issue of utility theory.

Conclusion

In this section, it was shown that markets would be expected to exhibit self-referential processes, which can not be analyzed by deductive rationality. However, when players rely on inductive reasoning to formulate strategies to execute their market agenda, the result is that the market will exhibit fractal dynamics. Previously, in this chapter, it was shown that the fractal dynamics can be exploited and optimized. Interestingly, in some sense, there appears to be a convergence of game-theoretic, information-theoretic, non-linear dynamical systems theory, and fractal/chaos-theoretic concepts. Further, there also appears to be a convergence of these concepts with the cognitive sciences.

2.10 Conclusion

In this chapter, a very simple, first order, model of an industrial market has been proposed. The reader should be aware that a paradigm is involved. This chapter proposes that industrial markets can be adequately modeled as a fixed increment Brownian, perhaps time sampled, fractal. In general, markets exhibit fractional Brownian fractal characteristics. A method of optimization of market operations is proposed that uses fixed increment Brownian fractal methodologies to analyze industrial markets. The analysis of various markets in Appendix C would tend to offer supporting evidence that the paradigm is practical. However, the application of Equation 2.55, $f = 2P - 1$ to fractional Brownian fractal time series remains an open formal issue. The model is extended with non-linear terms, and shown to be a logistic function, which is compatible with other literature.

The first order model is then extended into iterated game-theoretic scenarios, as a mixed strategy zero-sum game, and then as a nonzero-sum game, that is self-referential. In both cases, the capital of a player in the game is argued to show random walk, or Brownian motion fluctuations.

Finally, the model is extended into a multi-player market scenario, where the agents use inductive reasoning to cope with the self-referential characteristics of the marketplace. An argument is presented that the time series of the market will have fluctuations that are similar to a random walk, or Brownian motion fractal.

2.11 Summary

If we consider capital, V, invested in a savings account, and calculate the growth of the capital over time:

$$V_t = V_{t-1}(1 + a_t) \tag{2.150}$$

where a_t is the interest rate at time t, (usually a constant[17].) In equities, a_t is not constant, and varies—perhaps being

[15] For example, [Sen90, pp. 81] has a diagram of the sales department process in an organization. It has the same schema as Figures 2.4 and 2.5. If it could be shown that organizational complexity is an NP problem [SvW88, pp. 313], [GJ79, pp. 13], then there there could be some reasonable formalization of the observations presented in [Bro82] and [Ula91].

[16] Others feel a bit more epistemological about the issue—see [Ruc93, pp. 178], the chapter entitled "Life is a Fractal in Hilbert Space."

[17] For example, if $a = 0.06$, or 6%, then at the end of the first time interval the capital would have increased to 1.06 times its initial value. At the end of the second time interval it would be $(1.06)^2$, and so on. What Equation 2.150 states is that the way to get the value, V in the next time interval is to multiply the current value by 1.06. Equation 2.150 is nothing more than a "prescription," or a process to make an exponential, or "compound interest" mechanism. In general, exponentials can always be constructed by multiplying the current value of the exponential by a constant, to get the next value, which in turn, would be multiplied by the same constant to get the next value, and so on. Equation 2.150 is nothing more than a construction of $V(t) = e^{kt}$ where $k = \ln(1 + a)$. The advantage of representing exponentials by the "prescription" defined in Equation 2.150 is analytical expediency. For example, if you have data that is an exponential, the parameters, or constants, in Equation 2.150 can be determined by simply reversing the "prescription," ie., subtracting the previous value, (at time $t - 1$,) from the current value, and dividing by the previous value would give the exponentiating constant, $(1 + a_t)$. This process of reversing the "prescription" is termed calculating the "normalized increments." (Increments are simply the difference between two values in the exponential, and normalized increments are this difference divided by the value of

negative at certain times, (meaning that the value of the equity decreased.) This fluctuation in an equity's value can be represented by modifying a_t in Equation 2.150:

$$a_t = f_t F_t \tag{2.151}$$

where the product $f_t \cdot F_t$ is the fluctuation in the equity's value at time t.

An equity's value, over time, is similar to a simple tossed coin game [Sch91, pp. 128], where f_t is the fraction of a gambler's capital wagered on a toss of the coin, at time t, and F_t is a random variable[18], signifying whether the game was a win, or a loss, ie., whether the gambler's capital increased or decreased, and by how much. The amount the gambler's capital increased or decreased is $f_t \cdot F_t$. In general, F_t is a function of a random variable, with an average, over time, of avg_f, and a root mean square value, rms_f, of unity. Note that for simple, time invariant, compound interest, F_t has an average and root mean square, both being unity, and f_t is simply the interest rate, which is assumed to be constant. For a simple, single coin game, F_t is a fixed increment, (ie., either $+1$ or -1,) random generator. From an analytical perspective, it would be advantageous to measure the the statistical characteristics of the generator. Substituting Equation 2.151 into Equation 2.150[19]:

$$V_t = V_{t-1}(1 + f_t F_t) \tag{2.152}$$

and subtracting V_{t-1} from both sides:

$$V_t - V_{t-1} = V_{t-1}(1 + f_t F_t) - V_{t-1} \tag{2.153}$$

and dividing both sides by V_{t-1}:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_{t-1}(1 + f_t F_t) - V_{t-1}}{V_{t-1}} \tag{2.154}$$

and combining:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = (1 + f_t F_t) - 1 = f_t F_t \tag{2.155}$$

We now have a "prescription," or process, for calculating the characteristics of the random process that determines an equity's value. That process is, for each unit of time, subtract the value of the of the equity at the previous time from the value of the equity at the current time, and divide this by the value of the equity at the previous time. The

the exponential.) Naturally, since one usually has many data points over a time interval, the values can be averaged for better precision—there is a large mathematical infrastructure dedicated to precision enhancement, for example, least squares approximation to the normalized increments, and statistical estimation.

[18]"Random variable" means that the process, F_t, is random in nature, ie., there is no possibility of determining what the next value will be. However, F_t can be analyzed using statistical methods [Fed88, pp. 163], [Sch91, pp. 128]. For example, F_t typically has a Gaussian distribution for equity values [Cro95, pp. 249], in which case the it is termed a "fractional Brownian motion," or simply a "fractal" process. In the case of a single tossed coin, it is termed "fixed increment fractal," "Brownian," or "random walk" process. In any case, determination of the statistical characteristics of F_t are the essence of analysis. Fortunately, there is a large mathematical infrastructure dedicated to the subject. For example, F_t could be verified as having a Gaussian distribution using Chi—Square techniques. Frequently, it is convenient, from an analytical standpoint, to "model" F_t using a mathematically simpler process [Sch91, pp. 128]. For example, multiple iterations of tossing a coin can be used to approximate a Gaussian distribution, since the distribution of many tosses of a coin is binomial—which if the number of tosses is sufficient will represent a Gaussian distribution to within any required precision [Sch91, pp. 144], [Fed88, pp. 154].

[19]Equation 2.152 is interesting in many other respects. For example, adding a single term, $m \cdot V_{t-1}$, to the equation results in $V_t = V_{t-1}\left(1 + f_t F_t + m \cdot V_{t-1}\right)$ which is the "logistic," or 'S' curve equation, (formally termed the "discreet time quadratic equation,") and has been used successfully in many unrelated fields such as manufacturing operations, market and economic forecasting, and analyzing disease epidemics [Mod92, pp. 131]. There is continuing research into the application of an additional "non-linear" term in Equation 2.152 to model equity value non-linearities. Although there have been modest successes, to date, the successes have not proved to be exploitable in a systematic fashion [Pet91, pp. 133]. The reason for the interest is that the logistic equation can exhibit a wide variety of behaviors, among them, "chaotic." Interestingly, chaotic behavior is mechanistic, but not "long term" predictable into the future. A good example of such a system is the weather. It is an important concept that compound interest, the logistic function, and fractals are all closely related.

root mean square[20] of these values are the root mean square of the random process. The average of these values are the average of the random process, avg_f. The root mean square of these values can be calculated by any convenient means, and will be represented by rms. The average of these values can be found by any convenient means, and will be represented by avg[21]. Therefore, if $f_t = f$, and does not vary over time:

$$rms = f \tag{2.156}$$

which, if there are sufficiently many samples, is a metric of the equity value's "volatility," and:

$$avg = f \cdot F_t \tag{2.157}$$

and if there are sufficiently many samples, the average of F_t is simply avg_f, or:

$$avg = f \cdot avg_f \tag{2.158}$$

which is a metric on the equity value's rate of "growth." Note that this is the "effective" compound interest rate from Equation 2.150.

Equations B.128 and B.130 are important equations, since they can be used in portfolio management. For example, Equation B.128 states that the volatility of the capital invested in many equities, simultaneously, is calculated as the root mean square of the individual volatility of the equities. Equation B.130 states that the growths in the same equity values add together linearly[22].

Dividing Equation 2.158 by Equation 2.156 results in the two f's canceling, or:

$$\frac{avg}{rms} = avg_f \tag{2.159}$$

[20]In this section, "root mean square" is used to mean the variance of the normalized increments. In Brownian motion fractals, this is computed by $\sigma_t otal^2 = \sigma_1^2 + \sigma_2^2 + \cdots$ However, in many fractals, the variances are not calculated by adding the squares, (ie., a power of 2,) of the values—the power may be "fractional," ie., $3/2$ instead of 2, for example [Sch91, pp. 130], [Fed88, pp. 178]. However, as a first order approximation, the variances of the normalized increments of equity values can successfully be added root mean square [Cro95, kpp. 250]. The so called "Hurst" coefficient, which can be measured, determines the process to be used. The Hurst coefficient is range of the equity values over a time interval, divided by the standard deviation of the values over the interval, and its determination is commonly called "R/S" analysis. As pointed out in [Sch91, pp. 157] the errors committed in such simplified assumptions can be significant—however, for analysis of equities, squaring the variances seems to be a reasonable simplification.

[21]For example, many calculators have averaging and root mean square functionality, as do many spreadsheet programs—additionally, there are computer source codes available for both. See the programs *tsrms* and *tsavg*. The method used is not consequential.

[22]There are significant implications do to the fact that equity volatilities are calculated root mean square. For example, if capital is invested in N many equities, concurrently, then the volatility of the capital will be $\frac{1}{\sqrt{N}} \cdot rms$ of an individual equity's volatility, rms, provided all the equites have similar statistical characteristics. But the growth in the capital will be unaffected, ie., it would be statistically similar to investing all the capital in only one equity. What this means is that capital, or portfolio, volatility can be minimized without effecting portfolio growth—ie., volatility risk can addressed. Further, it does not make any difference, as far as portfolio value growth is concerned, whether the individual equities are invested in concurrently, or serially, ie., if one invested in 10 different equities for 100 days, concurrently, or one could invest in only one equity, for 10 days, and then the next equity for the next 10 days, and so on. The capital growth would have the same characteristics for both agendas. (Note that the concurrent agenda is superior since the volatility of the capital will be the root mean square of the individual equity volatilities divided by the square root of the number of equities. In the serial agenda, the volatility of the capital will be simply the root mean square of the individual equity volatilities.) Almost all equity wagering strategies will consist of optimizing variations on combinations of serial and concurrent agendas. There are further applications. For example, Equation B.127 could be modified by dividing both the normalized increments, and the square of the normalized increments by the daily trading volume. The quotient of the normalized increments divided by the trading volume is the instantaneous growth, avg_f, of the equity, on a per-share basis. Likewise, the square root of the square of the normalized increments divided by the daily trading volume is the instantaneous root mean square, rms_f, of the equity on a per-share basis, ie., its instantaneous volatility of the equity. (Note that these instantaneous values are the statistical characteristics of the equity on a per-share bases, similar to a coin toss, and not on time.) Additionally, it can be shown that the range—the maximum minus the minimum—of an equity's value over a time interval will increase with the square root of of the size of the interval of time [Fed88, pp. 178]. Also, it can be shown that the number of expected stock value "high and low" transitions scales with the square root of time, meaning that the probability of an equity value "high or low" exceeding a given time interval is proportional to the square root of the time interval [Sch91, pp. 153].

There may be analytical advantages to "model" avg_f as a simple tossed coin game, (either played with a single coin, or multiple coins, ie., many coins played at one time, or a single coin played many times[23].) The number of wins minus the number of losses, in many iterations of a single coin tossing game would be:

$$P - (1 - P) = 2P - 1 \tag{2.160}$$

where P is the probability of a win for the tossed coin. (This probability is traditionally termed, the "Shannon probability" of a win.) Note that from the definition of F_t above, that $P = avg_f$. For a fair coin, (ie., one that comes up with a win 50% of the time,) $P = 0.5$, and there is no advantage, in the long run, to playing the game. However, if $P > 0.5$, then the optimal fraction of capital wagered on each iteration of the single coin tossing game, f, would be $2P - 1$. Note that if multiple coins were used for each iteration of the game, we would expect that the volatility of the gambler's capital to increase as the square root of the number of coins used, and the growth to increase linearly with the number of coins used, irregardless of whether many coins were tossed at once, or one coin was tossed many times, (ie., our random generator, F_t would assume a binomial distribution—and if the number of coins was very large, then F_t would assume, essentially, a Gaussian distribution.) Many equities have a Gaussian distribution for the random process, F_t. It may be advantageous to determine the Shannon probability to analyze equity investment strategies. From Equation 2.159:

$$\frac{avg}{rms} = avg_f = 2P - 1 \tag{2.161}$$

or:

$$\frac{avg}{rms} + 1 = 2P \tag{2.162}$$

and:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{2.163}$$

where only the average and root mean square of the normalized increments need to be measured, using the "prescription" or process outlined above.

Interestingly, what Equation 2.161 states is that the "best" equity investment is not, necessarily, the equity that has the largest average growth, avg_f. The best equity investment is the equity that has the largest growth, while simultaneously having the smallest volatility. In point of fact, the optimal decision criteria is to choose the equity that has the largest *ratio* of growth to volatility, where the volatility is measured by computing the root mean square of the normalized increments, and the growth is computed by averaging the normalized increments.

We now have a "first order prescription" that enables us to analyze fluctuations in equity values, although we have not explained why equity values fluctuate. For a formal presentation on the subject, see the bibliography in [Art95] which, also, offers non-mathematical insight into the explanation.

[23]Here the "model" is to consider two black boxes, one with a stock "ticker" in it, and the other with a casino game of a tossed coin in it. One could then either invest in the equity, or, alternatively, invest in the tossed coin game by buying many casino chips, which constitutes the starting capital for the tossed coin game. Later, either the equity is sold, or the chips "cashed in." If the statistics of the equity value over time is similar to the statistics of the coin game's capital, over time, then there is no way to determine which box has the equity, or the tossed coin game. The advantage of this model is that gambling games, such as the tossed coin, have a large analytical infrastructure, which, if the two black boxes are statistically the same, can be used in the analysis of equities. The concept is that if the value of the equity, over time, is statistically similar to the coin game's capital, over time, then the analysis of the coin game can be used on equity values. Note that in the case of the equity, the terms in $f_t \cdot F_t$ can not be separated. In this case, $f = rms$ is the fraction of the equity's value, at any time, that is "at risk," of being lost, ie., this is the portion of a equity's value that is to be "risk managed." This is usually addressed through probabilistic methods, as outlined below in the discussion of Shannon probabilities, where an optimal wagering strategy is determined. In the case of the tossed coin game, the optimal wagering strategy is to bet a fraction of the capital that is equal to $f = rms = 2P - 1$ [Sch91, pp. 128, 151], where P is the Shannon probability. In the case of the equity, since $f = rms$ is not subject to manipulation, the strategy is to select equities that closely approximate this optimization, and the equity's value, over time, on the average, would increase in a similar fashion to the coin game. The growth of either investment would be equal to $avg = rms^2$, on average, for each iteration of the coin game, or time unit of equity investment. This is an interesting concept from risk management since it maximizes the gain in the capital, while, simultaneously, minimizing risk exposure to the capital.

Consider a very simple equity market, with only two people holding equities. Equity value "arbitration" (ie., how equity values are determined,) is handled by one person posting (to a bulletin board,) a willingness to sell a given number of stocks at a given price, to the other person. There is no other communication between the two people. If the other person buys the stock, then that is the value of the stock at that time. Obviously, the other person will not buy the stock if the price posted is too high—even if ownership of the stock is desired. For example, the other person could simply decide to wait in hopes that a favorable price will be offered in the future. So the stock seller must not post a price that the other person would consider too high, and the other person would not buy at the price if it is reasoned that the seller's pricing strategy will be to lower the offering price in the future, which would be a reasonable deduction if the posted price is considered too high. What this means is that the seller must consider not only the behavior of the other person, but what the other person thinks the seller's behavior will be, ie., the seller must base the pricing strategy on the seller's pricing strategy. Such convoluted logical processes are termed "self referential," and the implication is that the market can never operate in a consistent fashion that can be the subject of deductive analysis [Pen89, pp. 101][24]. As pointed out by [Art95, Abstract], these types of indeterminacies pervade economics.

What the two players do, in absence of a deductively consistent and complete theory of the market, is to rely on inductive reasoning. They form subjective expectations or hypotheses about how the market operates. These expectations and hypothesis are constantly formulated and changed, in a world that forms from others' subjective expectations. What this means is that equity values will fluctuate as the expectations and hypothesis concerning the future of equity values change[25]. The fluctuations created by these indeterminacies in the equity market are represented by the term $f_t F_t$ in Equation 2.152, and since there are many such indeterminacies, we would anticipate F_t to have a Gaussian distribution.

This is a rather interesting conclusion, since analyzing the actions of aggregately many "agents," each operating on subjective hypothesis in a market that is deductively indeterminate, can result in a system that can not only be analyzed, but optimized.

The only remaining derivation is to show that the optimal wagering strategy is, as cited above:

$$f = rms = 2P - 1 \tag{2.164}$$

where f is the fraction of a gambler's capital wagered on each toss of a coin that has a Shannon probability, P, of winning.

Following [Rez94, pp. 450], consider that the gambler has a private wire into the future who places wagers on the outcomes of a game of chance. We assume that the side information which he receives has a probability, P, of being true, and of $1 - P$, of being false. Let the original capital of gambler be $V(0)$, and $V(n)$ his capital after the n'th wager. Since the gambler is not certain that the side information is entirely reliable, he places only a fraction, f, of his capital on each wager. Thus, subsequent to n many wagers, assuming the independence of successive tips from the future, his capital is:

$$V(n) = (1 + f)^w (1 - f)^l V(0) \tag{2.165}$$

where w is the number of times he won, and $l = n - w$, the number of times he lost. These numbers are, in general, values taken by two random variables, denoted by W and L. According to the law of large numbers:

[24]Penrose, referencing Russell's paradox, presents a very good example of logical contradiction in a self-referential system. Consider a library of books. The librarian notes that some books in the library contain their titles, and some do not, and wants to add two index books to the library, labeled "A" and "B," respectively; the "A" book will contain the list of all of the titles of books in the library that contain their titles; and the "B" book will contain the list of all of the titles of the books in the library that do not contain their titles. Now, clearly, all book titles will go into either the "A" book, or the "B" book, respectively, depending on whether it contains its title, or not. Now, consider in which book, the "A" book or the "B" book, the title of the "B" book is going to be placed—no matter which book the title is placed, it will be contradictory with the rules. And, if you leave it out, the two books will be incomplete.)

[25]Interestingly, the system described is a stable system, ie., if the players have a hypothesis that changing equity positions may be of benefit, then the equity values will fluctuate—a self fulfilling prophecy. Not all such systems are stable, however. Suppose that one or both players suddenly discover that equity values can be "timed," ie., there are certain times when equities can be purchased, and chances are that the equity values will increase in the very near future. This means that at certain times, the equites would have more value, which would soon be arbitrated away. Such a scenario would not be stable.

$$\lim_{n\to\infty} \frac{1}{n} W = P \tag{2.166}$$

and:

$$\lim_{n\to\infty} \frac{1}{n} L = q = 1 - P \tag{2.167}$$

The problem with which the gambler is faced is the determination of f leading to the maximum of the average exponential rate of growth of his capital. That is, he wishes to maximize the value of:

$$G = \lim_{n\to\infty} \frac{1}{n} \ln \frac{V(n)}{V(0)} \tag{2.168}$$

with respect to f, assuming a fixed original capital and specified P:

$$G = \lim_{n\to\infty} \frac{W}{n} \ln(1+f) + \frac{L}{n} \ln(1-f) \tag{2.169}$$

or:

$$G = P \ln(1+f) + q \ln(1-f) \tag{2.170}$$

which, by taking the derivative with respect to f, and equating to zero, can be shown to have a maxima when:

$$\frac{dG}{df} = P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{1-P-1}(1+f)^{P} = 0 \tag{2.171}$$

combining terms:

$$P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{P}(1+f)^{P} = 0 \tag{2.172}$$

and splitting:

$$P(1+f)^{P-1}(1-f)^{1-P} = (1-P)(1-f)^{P}(1+f)^{P} \tag{2.173}$$

then taking the logarithm of both sides:

$$\ln(P) + (P-1)\ln(1+f) + (1-P)\ln(1-f) = \ln(1-P) - P\ln(1-f) + P\ln(1+f) \tag{2.174}$$

and combining terms:

$$(P-1)\ln(1+f) - P\ln(1+f) + (1-P)\ln(1-f) + P\ln(1-f) = \ln(1-P) - \ln(P) \tag{2.175}$$

or:

$$\ln(1-f) - \ln(1+f) = \ln(1-P) - \ln(P) \tag{2.176}$$

and performing the logarithmic operations:

$$\ln\left(\frac{1-f}{1+f}\right) = \ln\left(\frac{1-P}{P}\right) \tag{2.177}$$

and exponentiating:

$$\frac{1-f}{1+f} = \frac{1-P}{P} \tag{2.178}$$

which reduces to:

$$P\,(1-f) = (1-P)\,(1+f) \tag{2.179}$$

and expanding:

$$P - Pf = 1 - Pf - P + f \tag{2.180}$$

or:

$$P = 1 - P + f \tag{2.181}$$

and, finally:

$$f = 2P - 1 \tag{2.182}$$

Chapter 3

A Metric Methodology

This chapter outlines the methodology used in the construction of the data presented in appendix C. The reader is assumed to have remedial knowledge of computing concepts, statistics, and manipulation of time series data sets.

3.1 General Concepts

Consider a time series, of interest because it appears to have exponentially increasing rate of revenue returns, presumably as a result of a process similar to that discussed in Section 2.3.1 in Chapter 2, and, further, appears to have characteristics of a random process as discussed in Chapter 2 in Section A.4. Then, from Equation 2.3:

$$\frac{R_{n+1} - R_n}{R_n} = F_n \cdot f_n \tag{3.1}$$

where $\frac{R_{n+1}-R_n}{R_n}$ can be calculated from the time series by the following algorithm:

sequentially, for each value in the time series:

> **subtract the last value in the time series from the this, the current, value in the time series**
>
> **divide this difference by the last value in the time series**
>
> **print the quotient**

This will make a new time series, similar to the series shown schematically in Figure 2.1, and defined by Equations 2.24 and 2.25. This new time series is the time series of the increments. It is important to note that this process "decomposes" the fractal time series into the time series of the underlying mechanisms that created the time series. The program *tsfraction*, described briefly in appendix B, can perform this function. It is important to note that the new time series contains the fraction of the rate of revenue returns, won or lost in each iteration of the game, $\frac{R_{n+1}-R_n}{R_n}$. $R_{n+1} - R_n$ is the amount won or lost, depending on whether R_{n+1} is larger, or smaller than R_n, respectively, and dividing this value by R_n calculates the fraction of the rate of revenue returns won or lost in the iteration. An initial assumption of this section is that the "wins" and "losses" are the result of a random process. Averaging all values, of $\frac{R_{n+1}-R_n}{R_n}$ will give the average fraction of the rate of revenue returns won for all iterations in the time series. This can be calculated by finding the mean, perhaps using the program *tsnormal*, or by a least squares fit of the new time series, perhaps using the program *tslsq*, both of which are described in appendix B. Additionally, since, from Equation 2.27:

$$\frac{R_N}{R_0} = (1+f)^{PN}(1-f)^{(1-P)N} \tag{3.2}$$

for N many records in the time series, and, on the average, from Equation 2.34,

$$R(t) = \left[(1+f)^P(1-f)^{1-P}\right]^t \tag{3.3}$$

where $R(t)$ can be used to find the constants in the general form of Equation 2.11:

$$f(t) = e^{kt} \tag{3.4}$$

which was an initial assumption in this section. The program *tslogreturns*, or perhaps tslsq using the exponential fit argument, -p, can be also be used to find the average exponential fit to the cumulative returns represented by the original time series.

Additionally, it is important to note that since the new time series, derived above in this section, contains the fraction of rate of revenue returns won or lost in each iteration of the game, that the absolute value of this time series is the time series of the fraction of the rate of revenue returns wagered in each iteration of the game[1], assuming the original time series has characteristics of fractional Brownian motion, or could be "modeled" by Brownian motion with fixed increments. The absolute value, for each increment could be calculated by simply removing the all negative signs, and then averaging with, perhaps, the programs *tsnormal* or *tslsq*. Alternately, the root mean square value of the time series may be calculated, perhaps using the program *tsrms*, which is described in appendix B. The average value or root mean square value is the parameter f in Equations 3.1 and 3.3, assuming that f is constant.

Note, also, that if the total number of records, N is sufficiently large, then the probability of a "win" in any iteration, P, can be determined by counting the number of positive values in the new time series of the increments, and dividing this number by the total number of records, N, in the time series.

All of these values, $\frac{R_{n+1}-R_n}{R_n}$, f, and P, are related by Equation 2.66:

$$P = \frac{\ln\left(\frac{\frac{R_{n+1}}{R_n}}{(1-f)}\right)}{\ln\left(\frac{(1+f)}{(1-f)}\right)} \tag{3.5}$$

which can be used as a subjective evaluation of how accurate the "model" is. Additionally, the program *tsnormal* can be used to plot a histogram of the increments for evaluation of the distribution of the increments, consistent with the presentation in Chapter 2, Section 2.2.

As an additional metric for the Shannon probability, P, from Equation 2.57:

$$avg = rms\,[P - (1-P)] = rms\,(2P-1) \tag{3.6}$$

where avg and rms can be measured, or:

$$\frac{avg}{rms} = 2P - 1 \tag{3.7}$$

or:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{3.8}$$

[1]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

3.2 Procedure

The following procedure is executed in the ..x/markets directory from a "Makefile" using the Unix utility make(1). The programs *tsfraction*, *tslsq*, *tsnormal*, *tsrms*, *tshurst*, *tshcalc*, *tsunfairbrownian*, *tsshannonmax*, *tsshannon*, and *tslogreturns* are described briefly in appendix B, and in addition have online manual pages which can be viewed by the Unix utility man(1). In depth descriptions of the programs is available in the program sources.

Note that many of the parametric values in the analysis of the fractal time series data set are derived by different methodologies. This is for comparative consistency verification. See Section 3.4.

1. Run the program *tsfraction* on the fractal time series data set to produce a time series of the increments.

2. Run the program *tslsq*, with the -p option, on the time series of the increments to produce the least squares fit formula for the average of the increments in the time series of the increments.

3. Run the program *tsnormal*, with the -p option, on the time series of the increments to produce the mean and standard deviation for the average of the increments in the time series of the increments.

4. Run the program *tsrms* on the time series of the increments to produce a time series of the root mean square of the time series of the increments.

5. Run the program *tsrms*, with the -p option, on the time series of the increments to produce the root mean square of the time series of the increments.

6. Using the Unix utility sed(1), remove any negative signs from the time series of the increments to produce a time series of the absolute value of the time series of the increments.

7. Run the program *tslsq*, with the -p option, on the time series of the absolute value of the increments to produce the least squares fit formula for the absolute value of the increments.

8. Run the program *tsnormal*, with the -p option, on the time series of the absolute value of the increments to produce the mean and standard deviation for the average of the time series of the absolute value of the increments.

9. Run the program *tsnormal*, with the options -t -s 30, on the time series of the increments to produce a time series graph of the bell curve of the distribution of the increments in the time series of the increments.

10. Run the program *tsnormal*, with the options -t -s 30 -f, on the time series of the increments to produce a time series graph of the distribution of the increments in the time series of the increments.

11. Run the program *tsXsquared* on the distribution of the increments to produce a χ^2 confidence level that the distribution of the increments does have a Gaussian distribution.

12. Run the program *tsstatest* on the distribution of the increments to produce an estimation of the size of the required data set for reasonable accuracy.

13. Run the program *tsderivative* on the time series of the increments to produce the first derivative of the time series of the increments. Additionally, run the program *tsnormal*, with the options -t -s 30, and -t -s 30 -f to produce a time series graph of the distribution of the first derivative of the increments.

14. Run the program *tsderivative* on the time series of the increments to produce the second derivative of the time series of the increments. Additionally, run the program *tsnormal*, with the options -t -s 30, and -t -s 30 -f to produce a time series graph of the distribution of the second derivative of the increments.

15. Run the program *tshurst* on the time series to produce a graph of the Hurst coefficient of the time series.

16. Run the program *tslsq*, with the -p option, on the graph of the Hurst coefficient of the time series to produce the least squares fit formula for the Hurst coefficient of the time series.

17. Run the program *tshcalc* on the time series to produce a graph of the H parameter of the time series of the increments.

18. Run the program *tslsq*, with the -p option, on the graph of the H parameter of the time series of the increments to produce the least squares fit formula for the H parameter of the time series of the increments.

19. Run the program *tsunfairbrownian*, with the -f option and the root mean square value of the time series of the increments, on the fractal time series data set to produce a simulation of the fractal time series data set.

20. Run the program *tsfraction* on the simulation of the fractal time series data set to produce a time series of the increments of the simulation of the fractal time series data set.

21. Run the program *tsnormal*, with the -p option, on the simulation of the time series of the increments to produce the mean and standard deviation for the average of the increments in the simulation of the time series of the increments.

22. Run the program *tsnormal*, with the options -t -s 30, on the simulation of the time series of the increments to produce a time series graph of the bell curve of the distribution of the increments in the simulation of the time series of the increments.

23. Run the program *tsnormal*, with the options -t -s 30 -f, on the simulation of the time series of the increments to produce a time series graph of the distribution of the increments in the simulation of the time series of the increments.

24. Run the program *tsshannonmax* on the fractal time series data set to produce a graph of the maximum Shannon probability for the fractal time series data set.

25. Run the program *tsshannonmax*, with the -p option, on the fractal time series data set to produce the value of the maximum Shannon probability for the fractal time series data set.

26. Run the program *tslogreturns*, with the -p option, on the fractal time series data set to produce the value of the logarithmic returns of the fractal time series data set.

27. Run the program *tsshannon* with the value of the logarithmic returns of the fractal time series data set to produce the value of the Shannon probability for the fractal time series data set.

28. Run the program *tslsq*, with the -e -p options, on the fractal time series data set to produce the value of the coefficient of the exponential returns for the fractal time series data set.

29. Run the program *tsshannon* with the value of the coefficient of the exponential returns of the fractal time series data set to produce the value of the Shannon probability for the fractal time series data set.

30. Use the Unix utility egrep(1) with the argument "-e -" on the time series of the increments to "filter" records containing a negative sign. Pipe this time series to the Unix utility wc(1) to produce a count of the records in the time series of the increments with negative signs.

31. Use the Unix utility wc(1) on the time series of the increments to produce a count of the records in the time series of the increments.

32. Use the Unix utility awk(1) divide the count of the records in the time series of the increments with negative signs, by the count of the records in the time series of the increments, and subtracting from unity, to produce the value of the maximum Shannon probability for the time series of the increments.

In addition, the Unix utility awk(1) is used to parse and reformat data from this procedure into LaTeX macros for direct import into this manuscript. Appendix C is machine generated.

3.3 Description of the Usage of the Programs in the Procedure

The time series of the increments was made using the program *tsfraction*, which essentially implements the formula of Equation 3.1, for the fractal time series data set.

The root mean square of the time series of the increments is calculated using the program *tsrms*. The program, also, is used to produce a running graph of the root mean square process.

The average, ie., mean, and standard deviation of the increments of the time series of the increments is calculated using the program *tsnormal*.

The root mean square, mean, and standard deviation of the time series of the increments are the metric values of Equation 3.6. The Shannon probability was calculated by three different methods. One, using the program *shannonmax*, which produces the maximum Shannon probability from the original fractal time series data set; another, by counting the number of records with negative signs in the time series of the increments; and last, by using the program *tsshannon*, which calculates the Shannon probability from the logarithmic returns of the fractal time series data set.

The normalized histogram and bell curve of the time series graphs, in each analysis, was made by first using the program *tsfraction* to find the increments of the time series. Then the program *tsnormal* was run on the increment time series, with 30 intervals, ie., -s 30 argument, to provide the data for the bell curve. The program *tsnormal* was executed again, with -f -s 30 arguments, to provide the histogram. See [Cro95, pp. 250] for the rationale. If the bell curve and the histogram are approximately the same, then Gaussian increment property, “property 2” as described in [Cro95, pp. 245], is validated, and the time series probably represents a fractional Brownian motion. Additionally, the program is used with the options -t -s 30, and -t -s 30 -f to produce a time series graph of the distribution of the first and second derivative of the increments—these are useful for comparison of the distributions with the standard “white noise” for a “qualitative” verification of the cumulative sum process in empirical data.

The Hurst coefficient graph, in each analysis, was made by running the program *tshurst* on the time series. See [Fed88, pp. 153], [Cas94, pp. 253], [PJS92, pp. 493], [Ç93, pp. 172], [Pet91, pp. 62], or, [Sch91, pp. 129], for the rationale. The program *tslsq* was used on this Hurst coefficient data, with the -p argument, to calculate the least squares approximation to the Hurst coefficient.

The H parameter graph, in each analysis, was made by running the program *tshcalc* on the time series. See [Cro95, pp. 249] for the rationale. The program *tslsq* was used on the H parameter data, with the -p argument, to calculate the least squares approximation to the H parameter data.

For the optimum fiscal strategy, the program *tslogreturns* was run on the fractal time series data set with the -p option to print the formulas for the logarithmic returns. As an alternative, the program *tslsq* was used with the -e and -p options to print the formulas for a least squares exponential fit to the fractal time series data set. This renders a slightly more accurate set of formulas, but was not used in the analysis to be consistent with [Pet91, pp. 81]

For the calculation of the Shannon probability, the program *tsshannon* was run with the formulas derived from the *tslogreturns* program, above. The formulas were parsed with a Bourne shell script, using the Unix stream editor, sed(1), and presented to the program *tsshannon* via the command line.

For the simulations, the program *tsunfairbrownian* was used. This program performs the inverse function of *tsfraction*. Given a Shannon probability, or alternatively, the fraction of the cumulative sum to be “wagered” on each element in the time series of the increments of the fractal time series data set, a simulated fractal time series data set can be produced that has the “wager” altered to the metric values calculated above. This simulation can be analyzed using the procedure outlined herein, and the characteristics of the simulation compared against the original.

In this way, the data analysis and reduction were largely automated for each of the individual markets studied. It should be reiterated that the data analysis methodology presented here is remedial by contemporary standards for such issues. A more formal approach has been suggested by [Cro95, pp. 259] using Fourier analysis do derive the spectral exponent of a time series. The program *tsdft*, described briefly in appendix B, can perform this function. The Hurst coefficient is related to the spectral exponent, by the relation:

$$\beta = 2H + 1 \tag{3.9}$$

where β is the Fourier spectral exponent, and H is the Hurst coefficient, [Sch91, pp. 130], [Cro95, pp. 262], [Ç93, pp. 207]. This methodology is difficult to implement without manual intervention, but produces superior accuracies.

Additionally, the Hurst coefficient is related to the Shannon probability of a time series as derived in Chapter 2. A Shannon probability of 0.5 should give a far term Hurst coefficient of 0.5. Other values of Shannon probability and Hurst coefficient are related, however, there are known accuracy issues with the methodology of deriving the Hurst coefficient. See [Fed88, pp. 156], [BdL95, pp. 27].

Additionally, there are methods of fractal analysis that address concepts of fractal dimension. The fractal dimension of a time series is related to the Hurst coefficient by the following relationship, [Fed88, pp. 196], [PJS92, pp. 495]:

$$D = 2 - H \tag{3.10}$$

where D is the fractal dimension, and H is the Hurst coefficient.

3.4 Verification Methodology

As a cursory verification methodology:

1. Using the mean and root mean square values of the normalized increments of the time series data, and the Shannon probability as calculated by counting the total number of records that the market movement was positive, in relation to the total number of records in the data set, verify the accuracy of the equality in Equation 3.8.

2. Compare the Shannon probability, as found by the *tsshannonmax* program to the value of the Shannon probability as calculated by counting the total number of records that the market movement was positive, in relation to the total number of records in the data set

3. Compare the four methods of calculating the logarithmic returns:

   - By calculation based on the mean of the normalized increments.
   - By the calculation of the constant in the least squares approximation to the normalized increments.
   - By the calculation of the exponential least squares fit to the original time series data set, with the program *tslsq*.
   - By the calculation of the logarithmic returns, with the program *tslogreturns*.

4. Using the mean, standard deviation, and the root mean square of the normalized increments, and the Shannon probability as calculated by counting the total number of records that the market movement was positive, in relation to the total number of records in the data set, verify the accuracy of the equality of Equation 2.104.

5. Compare the accuracy of the equality of the absolute value and root mean square of the normalized increments[2].

Note that the numerical manipulations are relatively simple, and can be implemented with simple awk(1) scripts.

[2]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

Chapter 4

Conclusions and Observations

This chapter presents various qualitative conclusions, as analyzed in appendix C and appendix D, concerning the Fractal Analysis of Various Market Segments in the North American Electronics Industry. It should not be concluded that these industries are representative of industries in general, and is offered in academic perspective, and under no circumstances would it be appropriate to consider it financial advice.

4.1 Comparison of Derived Relationships with Industrial Observations

There are some interesting relationships that were presented, and, although they may be coincidental, taken in the larger context that many of the relationships are very close to what industry analysts have used as “rules of thumb,” or “bench marks” derived through years of experience, it would seem that using fractal analysis on industry or market place historical data sets may possibly provide an additional analytical “tool” for optimizing industrial pro forma issues. As a partial selection of the relationships:

- Research,development, and infrastructural investments seem reasonable at about 12 to 20 percent of the rate of revenue returns for the market segments analyzed. This seems consistent with the industry.
- Venture success rates at 60 months seems reasonable at about 1 in 11, which is commensurate with the industry.
- Project success rates, of 8 month duration, are about 1 in 3, which is consistent with numbers from the Application Specific Integrated Circuit business, which could be considered as “representative.”
- The “80/20 rule” that 80% of an organization’s revenue comes from only a few, 3 was shown to be typical, products is really, probably, 84.13%, or one standard deviation—which is consistent through the industry.
- The “80/20 rule” that 80% of an organization’s products should be “industry standard,” and the remainder “proprietary” is probably, one standard deviation, or 84.13%.
- Although prediction of product life cycle in the operations sections proved to be “pessimistic,” it was, none the less, depending on the reader’s point of view, reasonable, and was fairly consistent with industry averages.
- The inventory control dynamics presented in the operations section, seem to be consistent with the markets analyzed.
- The failure rate of Fortune 500 Companies seems consistent with predicted failure rate of organizations in the markets analyzed, although the rate of failure was shown to be “optimistic,” when related to re-investment strategy.

- The calculated number of companies participating in the markets analyzed is reasonably close to the industry numbers, and there is inferential evidence that they are operating optimally—at least in the entropic sense as defined in Chapter 2—which seems consistent with the economic theory that the companies that operate the most optimally or efficiently will, eventually, dominate the market. (The calculated number of companies participating in the various markets varied between 6 and 28, with an average of 10, and with Shannon probabilities for the individual company's market time series varying between 0.54 and 0.6, with an average of 0.57, which, interestingly, is close, within approximately 5%, to the Shannon probability for the various company's stock price time series.)

- The variance in the aggregate market time series is smaller than the the variance of the time series for any company participating in the market, which is consistent with the industries analyzed.

- It would seem that there is some supporting evidence that optimizing a company's fiscal strategy to achieve maximum market growth and optimizing a company's fiscal strategy to optimize capital growth may be mutually exclusive, which has, traditionally, been the case in the industries analyzed. Additionally, it would seem that, at least in the markets analyzed, the fiscal strategies deployed would tend to be optimizing market growth, which seems consistent with author's experience in these industries.

4.2 Asides and Speculations

Several issues that were not addressed are the relationship between a company's valuation, perhaps calculated by the value of its stock, and company's rate of revenue returns. If their is a causality, it would seem that there would be, in general, a reason offered as to why the Dow Jones Average is rising exponentially. Additionally, it would seem that there would be a correlation that could be confirmed in employment figures, economic indicators, flow of money, etc. These speculations are offered as a suggestion for further investigation into the applicability of fractal analysis to industrial markets—as a possible means of induction.

There is some possibility that fractal analysis can be used in conjuction with other contemporary methodologies of operations research. For example, possibly, failure analysis could be used to optimize the expected life of a company vs. the growth rate of a company as alluded to in the optimally maximal fiscal strategy sections of appendix C. Additionally, perhaps, fractal analysis could be used in forecasting market dynamics in conjunction with mathematical methods and linear programming optimization of corporate operations, specifically inventory control.

There are remaining issues that, although addressed, were not addressed to the author's satisfaction. Specifically, there was no reason offered as to why the companies analyzed were not operating closer to the maximum Shannon probabilities, as presented in the simulation and maximization sections of appendix C. Additionally, it would seem that visibility into the future, regarding rate of revenue returns, was only a few months, at best. This would seem to be in disagreement with the prevailing concept that "strategic planning" should be "long term." An interesting interpretation of this may be that these industries require a more dynamic management methodology, perhaps using "rolling" budgets, etc. But this would seem to be inconsistent with methodologies where objectives are monitored on an annual basis. It would seem that, looking at the graphs of the normalized increments in all sections presented in appendix C, that profit and loss issues are very dynamic, and, probably, require detailed attention at no more than a monthly rate—these graphs show that a lot of dynamic changes can occur in a year, or even a quarter.

4.3 Conclusion

Overall, taken in context, it would seem, depending on the reader's point of view, that fractal analysis could provide additional insight into market and industrial operations—perhaps offering appropriate optimizations in specific circumstances. Granted, this is a controversial usage of the methodology, and there are interpretations that were made, which may or may not be considered appropriate—the text states that, in all cases where such interpretations were made, that it was an “interesting interpretation,” in an attempt not to mislead the reader. The application of fractal analysis to the optimization of industrial operations should be considered as “novel,” and, although there may be academic value, it would be inappropriate to accept any conclusions presented in this manuscript as “factual” at this time.

Appendix A

Tutorial on Fractal Time Series

This appendix presents a remedial tutorial on the optimization of betting strategies in speculative markets. It is offered in academic perspective, and under no circumstances would it be appropriate to consider it financial advice. It can serve, however, as an introduction to the contemporary economic theory of speculative markets. Rigorous and sophisticated approaches that address the issues of investing in speculative markets are contained in the bibliography.

This section begins with the analysis of a very simple speculative game, that of tossing coins. The analysis will then be expanded by permitting the use of unfair coins in the game, roughly following [Sch91, pp. 128]. An optimal betting strategy will be developed for the game, and this strategy will then be generalized and extended to include remedial betting strategies in certain speculative markets.

A.1 The Coin Tossing Game

Consider a coin tossing game, where a player makes a wager, and then, flips a coin. If the coin comes up heads, then the player wins twice the original wager, (ie., makes back the original wager plus an amount equal to the original wager from the "bank.") But if the coin comes up tails, then the player looses the wager to the bank. The game is iterated, many times, until the player decides not to play any more, or goes "bust."

A.1.1 Strategic Considerations in the Iterated Coin Tossing Game

The player has an initial cash reserve, to which are added the cumulative returns, and from which each wager is made, and the cumulative returns will increase by the amount of the wager each time the player wins, and, likewise the cumulative returns will decrease by the amount of the wager each time the player looses. The objective of the player is, obviously, to maximize the magnitude of the cumulative returns, over time. Note that this is a speculative game, in that the player speculates on the likelihood that the coin will come up heads on next iteration of the game, and adjusts the wager accordingly, betting zero if the outcome of the next coin toss is anticipated to be tails.

Description of "Time Series" and "Fractal"

Time Series: If the player makes a list, recording the time, and magnitude of cumulative returns, for each iteration of the game, such a table is called the *time series* of the cumulative returns [Sch91, pp. 223], [Ç93, pp. 199].

Fractal: If the player plots a graph of the time series, time on the X—axis and magnitude of cumulative returns on the Y—axis, this graph will exhibit *fractal*[1] characteristics, which means that it represents a system that has the

[1] Technically, the term, as used here, should be *Random Fractal*, or *Fractional Brownian Motion*, [Fed88, pp. 170].

characteristics of a cumulative sum, (ie., integrative process,) of random events, coin tosses in this case [Cro95, pp. 229], [Fed88, pp. 163].

It is an important concept that the magnitude of the cumulative returns, at any time during the iterated game, is the cumulative sum of all of the wins and losses of wagers made in the previous iterations of the game, starting with the initial cash reserves, ie., it is an integrative process. If the player does not have "a priory" knowledge of the outcome of the future coin tosses, then optimizing this integrative process is the strategic objective of playing a rational game.

For example, it would be foolish for the player to always wager zero, since, although there would never be a loss, there would never be a win, either. Likewise, it would be foolish for the player to wager a large percentage of the cumulative returns, since a few losses in succession would deplete the cash reserves and cumulative returns to zero, and the player would "go bust," thus ending the game. Obviously, the player could wager too much, or too little of the cumulative returns on a single game iteration. The series optimum wagers, is termed the *optimal betting strategy*, [Sch91, pp. 128], [Rez94, pp. 450], [Pie80, pp. 270].

A.2 Optimal Betting Strategy in the Iterated Coin Tossing Game

If the coin is a fair coin, ie., it has a 50% chance of coming up heads and a 50% chance of coming up tails, then the player should elect not to play. The rationale for this statement is that, in the long run, some iterated games will be won, and some lost, with the amount of money won equal to the amount of money lost—so there is no financial incentive to play the game. However, suppose that there is a 60% chance for the coin to come up heads, on any single iteration of the game, and a 40% chance of coming up tails. It turns out that the fractal characteristics of the game can be exploited to determine the optimal betting strategy[2]. The optimal betting strategy, in this case, is for the player to wager 20% of the cumulative returns, every iteration of the game. As it turns out, this will maximize the growth of the player's cumulative returns [Sch91, pp. 128], [Rez94, pp. 450], [Pie80, pp. 270]. The way that this was computed was from the formula:

$$F = 2P - 1 \tag{A.1}$$

where F is the fraction of the player's cumulative returns that should be wagered on an iteration of a game with a P chance of winning the game [Sch91, pp. 151], [Rez94, pp. 450], [Pie80, pp. 270]. In the above case, with a 60%, (ie., $p = 0.6$,) chance of winning:

$$F = (2 \cdot 0.6) - 1 \tag{A.2}$$

$$F = 1.2 - 1 \tag{A.3}$$

$$F = 0.2 \tag{A.4}$$

or F is 20% of the player's cumulative returns[3]. Playing this betting strategy, the player can expect an average of 2% increase in the magnitude of cumulative returns on each toss of the coin[4].

[2] Fortunately, there is a large analytical infrastructure in mathematics and economics available that addresses these issues. The answers are provided by *Information Theory*, and the applications of these *entropic* principles are a firmly entrenched discipline in the field of economics [Sch91, pp. 127], [Rez94, pp. 450], [Pie80, pp. 270].

[3] This will maximize the logarithmic growth of the player's cumulative returns, and is the highest value that can be attained, as given by Shannon's *information capacity*, $C(P) = 1 - H(P)$ of a binary symmetric channel with an error probability of P. Here, $H(P)$ is the entropy function, $H(P) = -[P \ln P + (1 - P) \ln(1 - P)$ [Sch91, pp. 128, 151], [SW49, pp. 38], [Rez94, pp. 114, pp. 450], [Pie80, pp. 270], [KF88, pp. 155], [Ash65, pp. 9].

[4] This value is computed by taking the logarithm to the base 2, $H(P) = 0.97$ bits per game, and $2^C = 1.02$, or 2% each game [Sch91, pp. 128], [SW49, pp. 36], [Ash65, pp. 30], [Rez94, pp. 450], [Pie80, pp. 270].

For those wishing to experiment with optimal betting strategies, the unfair coin can be simulated with a six sided die. After the wager, the die is rolled, and if the die comes up 1, 2, 3, or 4 the player wins. But if it comes up 5 or 6, the player loses. The probability of winning, P, in this case is 0.66 since the player will win 4 times out of 6, on average. The optimal wager will be $F = (2 \cdot 0.66) - 1 = 0.33$, or 33% of the cumulative returns should be wagered on each iteration of the game. It is interesting to play many iterations of the game, particularly using different betting strategies—for example change the wager fraction to 20%, or 40% of the cumulative returns—and see how the long term cumulative returns change in response to the different betting strategies.

The program, *tsunfaircoin*, which is briefly described in Appendix B, uses a random number generator to simulate the unfair coin in an iterated coin tossing game. The program's command line options control the wager fraction, F, and the Shannon probability, P, and the number of iterations of the game. The cumulative returns for each iteration of the game are printed to the terminal, and may be plotted, to show that there is indeed an optimum value of wager fraction, F, for any value of Shannon probability, P, provided P is greater than $\frac{1}{2}$.

A.3 Important Intuitive Concepts of Speculative Games

The unfair coin tossing game is probably one of the simplest speculative games. It is important to develop an intuitive concept based on the fundamentals of this simple game. Speculative games have the following characteristics:

- Speculative games are iterated. A wager is made from the player's cumulative returns for the game, and depending on the outcome of the iteration of the game, the player either wins or looses the wager for that iteration. The winnings or losses, for each iteration, are summed to the player's cumulative returns.
- The outcome of a particular iteration has random characteristics, ie., the outcome of a particular iteration is not "predictable."
- The objective of the game is to maximize the value of the player's cumulative returns.

As it turns out, these simple concepts have many applications, for example, they can be used to model and analyze the capital markets [Pet91, pp. 81].

A.4 An Analytical Approach to the Iterated Unfair Coin Tossing Game

In Section A.2 it was assumed that the player had knowledge about the probability of a tossed coin coming up heads. In most speculative games, knowledge of the random mechanism is not available. For a simple game, like tossing an unfair coin, the coin could be tossed many times, and the probability of it coming up heads measured. The methodology would be to toss the coin, say, 100 times, and count how many times it came up heads. Say it comes up heads 60 times out of the 100 tosses. Then the probability that the coin will come up heads on any particular iteration of the game would be 60%, and the player could arrange a betting strategy, accordingly. It turns out that this concept is very extensible.

In many speculative games, there is no knowledge available about the characteristics of the random process of the game. As a simple example, assume that no knowledge is available about the underlying random process of the unfair coin tossing game. Like the capital markets, we have only historical data about the wagering process, ie., what was won, and what was lost during each iteration of the game. If we look at the historical time series of the game, we would observe that since the cumulative returns are increasing, that the game is unfair. It would be desirable gain some insight into the random process that controls the outcome of an iteration of the game, so a betting strategy can be formulated. Referring to the preceeding paragraph, when the coin was tossed a hundred times to count how many times it came up heads, it should be realized that this was a cumulative sum of number of times the coin came up heads over a hundred iterations.

Being formal, in n many tosses of the coin, it would be expected that the coin came up heads, $P \cdot n$ many times, and come up tails, $(1 - P) \cdot n$ many times, where P is the probability of the coin coming up heads. If a counting process is started, tallied in C, by which, if the coin is tossed, and it comes up heads, we increase the count by one, and if it comes up tails, we decrease the count by one, then it would be expected, after n many tosses[5]:

$$C = P \cdot n - (1 - P) \cdot n \tag{A.5}$$

Notice that C was derived empirically, and from C, we can compute the probability, P, of the coin coming up heads in any iteration of the game. Rearranging:

$$C = P \cdot n - n + P \cdot n \tag{A.6}$$

and dividing both sides of the equation by n:

$$\frac{C}{n} = P - 1 + P = 2 \cdot P - 1 \tag{A.7}$$

and solving for P:

$$P = \frac{\frac{C}{n} + 1}{2} \tag{A.8}$$

noting that $\frac{C}{n}$ is the "average" C.

The same methodology can be used in general. Access to the unfair coin to measure the probability of it coming up heads on any iteration is not necessary—this information can be deduced from the historical files of a game where the coin was used. For example, we can take the historical time series of a unfair coin tossing game, and for each iteration, subtract the value of the cumulative returns of the previous iteration from the value of the cumulative returns of the next iteration, dividing the result of the subtraction by the value of the cumulative returns in the previous iteration, making a new time series. This is a very powerful concept in the strategy of speculative games. The new time series contains the fraction of the cumulative returns that was won or lost on each iteration of the game.

Using our example of the unfair coin tossing game, we would observe that the new time series would be a list of numbers, containing either $+F$, if the wager was won in an iteration, or $-F$, if the wager was lost, (assuming that F was constant throughout the game.) The important concept here is that, given a specific iteration, the fraction of the cumulative returns wagered can be deduced, and whether the wager was won or lost. It is an important concept that we can reconstruct the characteristics of the random mechanism, and the fraction of the cumulative returns wagered from the historical data of a speculative game, without having knowledge of the random mechanism[6]. As before, we do a cumulative sum on the random game's process, only instead of it being a tossed coin, it is the new time series that contains the fraction of the cumulative returns that was won or lost in each iteration of the game. Formalizing, using Equation A.8, and replacing $\frac{C}{n}$, the average value of C, with the "average" value of F, found by summing all of the values in the new time series, and dividing by the number of iterations:

$$P = \frac{1}{2} + \frac{1}{2n} \sum_{i=0}^{n} F(i) \tag{A.9}$$

[5]For computational reasons, it is advantageous to implement counting of the number of heads in a series of coin tosses in this manner, which finds the "average" C by summing both heads and tails, with differing signs. In the unfair coin tossing game, the random mechanism can be analyzed by simply counting the number of times heads comes up in series of tosses. However, in speculative games, in general, the random mechanisms are much more sophisticated, requiring an "average" to be taken. This methodology provides a means of extensibility to these types of systems.

[6]It is not a complicated concept, actually, if you look at the process by which the historical time series was made. A wager is made, that is a fraction of the cumulative returns, and the wager was either added or subtracted from the cumulative returns for the game, depending on the results of a random process. When we subtract the value of the cumulative returns of a previous iteration from the value of the cumulative returns of the next iteration, and dividing by the value of the cumulative returns in the previous iteration, we are actually "undoing" the cumulative returns process of the game—kind of working backward to create the underlying random process and betting strategy.

or more generally:

$$P = \frac{1}{2} + \frac{1}{2t} \int_0^t F(n)\, dn \tag{A.10}$$

Interestingly, if we want to find out the fraction of the cumulative returns that was wagered each iteration of the game, the absolute value of $F(i)$ can be taken in Equation A.9. In the simple case of the unfair tossed coin, it is simply F, since $F(i)$ is either $+F$ or $-F$, ie., we simply remove the signs, and the equation reduces to[7]:

$$F = 2P - 1 \tag{A.11}$$

which is the same as Equation A.1. Although this is a generalization, this derivation has not shown that this is indeed an optimal solution. See Section 2.3.3 in Chapter 2 for a presentation on the optimal solution—it turns out that $F = 2P - 1$ is, indeed, the optimal solution.

[7]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

Appendix B

Computer Programs Used in the Analysis of Fractal Time Series

The "C" language sources to the following programs are available by sending an electronic mail to john-archive-request@johncon.johncon.com with a subject of "archive get fractal". The source distribution also contains the $\LaTeX$ sources to this document. The figures in this appendix were made from the regression tests for the programs, and can be reconstructed with the Unix make(1) utility in the ../simulation/test, and ../utilities/test directories.

B.1 Legal Restrictions

A license is hereby granted to reproduce this software source code and to create executable versions from this source code for personal, non—commercial use. The copyright notice included with the software must be maintained in all copies produced.

THESE PROGRAMS ARE PROVIDED "AS IS". THE AUTHOR PROVIDES NO WARRANTIES WHATSOEVER, EXPRESSED OR IMPLIED, INCLUDING WARRANTIES OF MERCHANTABILITY, TITLE, OR FITNESS FOR ANY PARTICULAR PURPOSE. THE AUTHOR DOES NOT WARRANT THAT USE OF THIS PROGRAM DOES NOT INFRINGE THE INTELLECTUAL PROPERTY RIGHTS OF ANY THIRD PARTY IN ANY COUNTRY.

B.2 Fractal Time Series Analytical Utilities

B.2.1 tsderivative

Source tsderivative.c, for taking the derivative of a time series. The value of a sample in the time series is subtracted from the previous sample in the time series. The derivative time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsderivative* program appears in Figure B.1.

B.2.2 tsintegrate

Source tsintegrate.c, for taking the integral of a time series. The value of a sample in the time series is added to the previous samples in the time series. The integral time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsintegrate* program appears in Figure B.2.

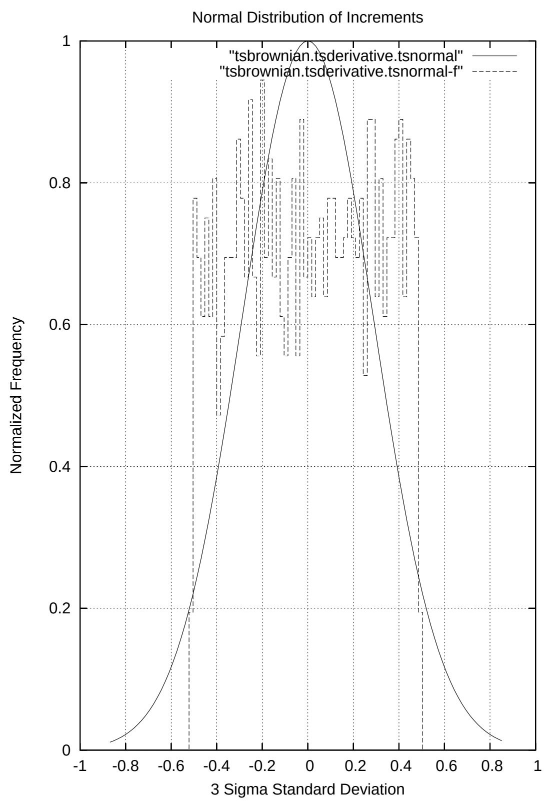

Figure B.1: Example output of the *tsderivative* program, using the output of the *tsbrownian* program with 1500 records as input. The frequency histogram should have the same distribution as that produced by the *tswhite* program in Figure B.43.

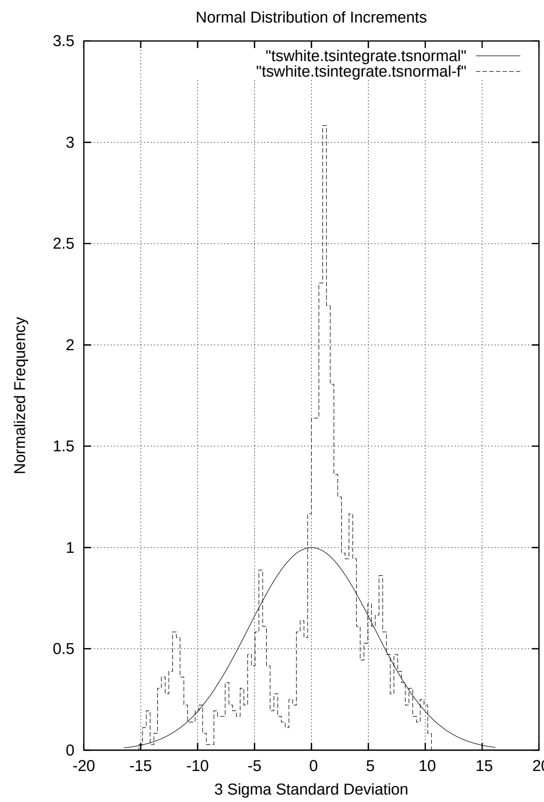

Figure B.2: Example output of the *tsintegrate* program, using the output of the *tswhite* program with 1500 records as input. The frequency histogram should have the same distribution as that produced by the *tsbrownian* program in Figure B.39.

B.2.3 tshcalc

Source tshcalc.c, for calculating the H parameter for a one variable fractional Brownian motion time series. The algorithm is from [Cro95, pp. 249].

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tshcalc* program appears in Figure B.3.

B.2.4 tshurst

Source tshurst.c, for calculating the Hurst coefficient for a time series. The method used is from [Cas94, pp. 253], [Pet91, pp. 63], [Sch91, pp. 129], or [Ç93, pp. 172]. The time series is broken into variable length intervals, which are assumed to be independent of each other, and the R/S value is computed for each interval based on the deviation from the average over the interval. These R/S values are then averaged for all of the intervals, then printed to stdout. The -r flag sets operation as described in "Chaos and Order in the Capital Markets," by Edgar E. Peters, pp 81, and should only be used for time series from market data since logarithmic returns sum to cumulative return—negative numbers in the time series file are not permitted with this option. The $\ln(\frac{R}{S})$ vs $\ln(time)$ plot is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tshurst* program appears in Figure B.4.

B.2.5 tslogreturns

Source tslogreturns.c, is for taking the logarithmic returns of of a time series. The value of a sample in the time series is divided by the value of the previous sample in the time series, and the logarithm of the quotient is printed to stdout.

The form of the best fit is e^{at} for exponential least squares fit, or x^t for power least squares fit, or 2^{bt} for binary fit.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.
Note: The derivation for exponential least squares fit is:

$$y(t) = e^{k1+k2t} \tag{B.1}$$

$$s(t) = \ln\left(\frac{e^{k1+k2t}}{e^{k1+k2(t-1)}}\right) \tag{B.2}$$

$$= \ln\left(e^{k1+k2t-k1-k2t+k2}\right) \tag{B.3}$$

$$= \ln\left(e^{k2}\right) \tag{B.4}$$

$$= k2 \tag{B.5}$$

Note: The derivation for power least squares fit is:

$$y(t) = e^{k1+k2t} \tag{B.6}$$

$$s(t) = \ln\left(\frac{e^{k1+k2t}}{e^{k1+k2(t-1)}}\right) \tag{B.7}$$

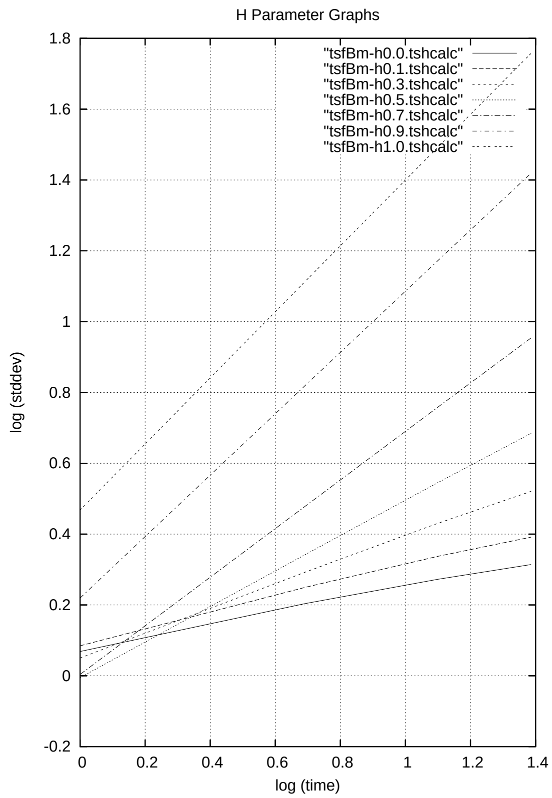

Figure B.3: Example output of the *tshcalc* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

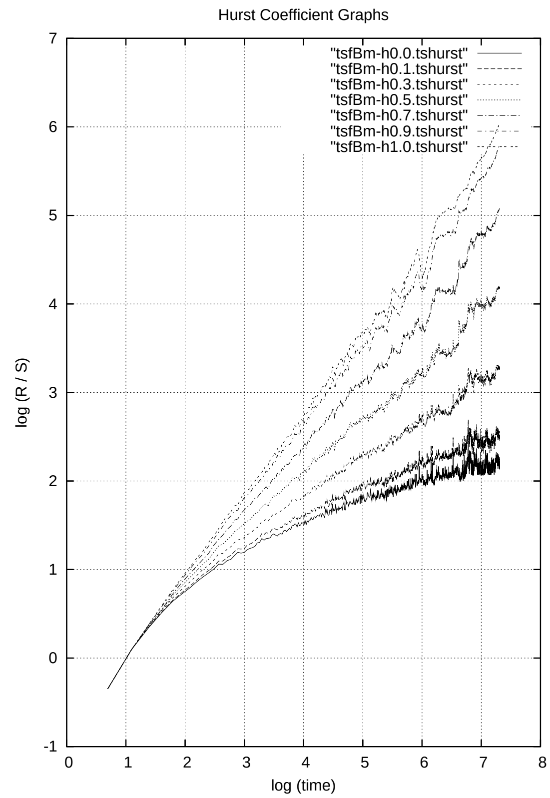

Figure B.4: Example output of the *tshurst* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

$$
\begin{aligned}
&= \ln\left(e^{k1+k2t-k1-k2t+k2}\right) && \text{(B.8)}\\
&= \ln\left(e^{k2}\right) && \text{(B.9)}\\
&= k2 && \text{(B.10)}\\
e^{xt} &= a^{t} && \text{(B.11)}\\
xt &= \ln\left(a^{t}\right) && \text{(B.12)}\\
&= t\ln(a) && \text{(B.13)}\\
x &= \ln(a) && \text{(B.14)}\\
a &= e^{x} && \text{(B.15)}
\end{aligned}
$$

Note: The derivation for the binary least squares fit is:

$$
\begin{aligned}
y(t) &= e^{k1+k2t} && \text{(B.16)}\\
s(t) &= \ln\left(\frac{e^{k1+k2t}}{e^{k1+k2(t-1)}}\right) && \text{(B.17)}
\end{aligned}
$$

$$
\begin{aligned}
&= \ln\left(e^{k1+k2t-k1-k2t+k2}\right) && \text{(B.18)}\\
&= \ln\left(e^{k2}\right) && \text{(B.19)}\\
&= k2 && \text{(B.20)}\\
e^{xt} &= 2^{kt} && \text{(B.21)}\\
a^{t} &= 2^{kt} && \text{(B.22)}\\
a &= 2^{k} && \text{(B.23)}\\
k\ln(2) &= \ln(a) && \text{(B.24)}\\
k &= \frac{\ln(a)}{\ln(2)} && \text{(B.25)}\\
& && \text{(B.26)}
\end{aligned}
$$

An example output from the *tslogreturns* program appears in Figure B.5.

B.2.6 tsshannon

Source tsshannon.c, for calculating the probability, given the Shannon information capacity. See [Sch91, pp. 128, 151]. Uses Newton—Raphson method for an iterative solution for the probability, p.

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

From [Sch91, pp. 151]:

$p = 0.55$, $2^{C(0.55)} = 0.005$, (probably a typo, meaning 1.005,) which by calculator, $C(0.55) = 0.0072$, (this program gives $C(0.549912) = 0.0072$).

Derivation, starting with [Sch91, pp. 151]:

$$
\begin{aligned}
C(p) &= 1 + p ln_2(p) + (1-p) ln_2(1-p) && \text{(B.27)}\\
C(p) &= 1 + p\left(\frac{ln(p)}{ln(2)}\right) + (1-p)\left(\frac{ln(1-p)}{ln(2)}\right) && \text{(B.28)}\\
C(p) &= \left(\frac{1}{ln(2)}\right)(ln(2) + p ln(p) + (1-p) ln(1-p)) && \text{(B.29)}\\
C(p) &= \left(\frac{1}{ln(2)}\right)(ln(2) + p ln(p) + ln(1-p) - p ln(1-p)) && \text{(B.30)}\\
\frac{dC(p)}{dp} &= \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - (\frac{1}{(1-p)}) - \left(ln(1-p) - (\frac{p}{(1-p)})\right)\right) && \text{(B.31)}\\
&= \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - (\frac{1}{(1-p)} - ln(1-p) + \left(\frac{p}{(1-p)}\right)\right) && \text{(B.32)}\\
&= \left(\frac{1}{ln(2)}\right)\left(ln(p) - ln(1-p) + \left(\frac{p}{(1-p)}\right) - \left(\frac{1}{(1-p)}\right)\right) && \text{(B.33)}\\
&= \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - ln(1-p) + \left(\frac{(p-1)}{(1-p)}\right)\right) && \text{(B.34)}\\
&= \left(\frac{1}{ln(2)}\right)(1 + ln(p) - ln(1-p) - 1) && \text{(B.35)}\\
&= \left(\frac{1}{ln(2)}\right)(ln(p) - ln(1-p)) && \text{(B.36)}
\end{aligned}
$$

An example output from the *tsshannon* program appears in Figure B.6.

$$\boxed{e^{(0.013797t)} = 1.013892^{(t)} = 2^{(0.019905t)}}$$

Figure B.5: Example output of the *tslogreturns* program, the input was produced by the *tscoin* program, with a Shannon probability of 0.6, as shown in Figure B.46 in Section B.3.8.

$$\boxed{C(0.582866) = 0.019905}$$

Figure B.6: Example output of the *tsshannon* program, the input was produced by the *tslogreturns* program, as shown in Figure B.5, which was derived from the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

B.2.7 tsshannonmax

Source tsshannonmax.c, for calculating unfair returns of a time series, as a function of Shannon probability. The input time series is presumed to have a Brownian distribution. The main function of this program is regression scenario verification—given an empirical time series, speculative market pro forma performance can be analyzed, as a function of Shannon probability. The cumulative sum process is Brownian in nature.

To find the maximum returns, the “golden” method of minimization is used. As a reference on the “golden” method of minimization, see [PFTV88, pp. 298].

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample’s time, and the last field as the sample’s value at that time.

An example output from the *tsshannonmax* program appears in Figure B.7.

B.2.8 tsfraction

Source tsfraction.c, for finding the fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. The fraction time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample’s time, and the last field as the sample’s value at that time.

An example output from the *tsfraction* program appears in Figure B.8.

B.2.9 tsrms

Source tsrms.c, for taking the root mean square of a time series. The value of a sample in the time series is squared and added to the cumulative sum of squares to make a new time series. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample’s time, and the last field as the sample’s value at that time.

An example output from the *tsrms* program appears in Figure B.9.

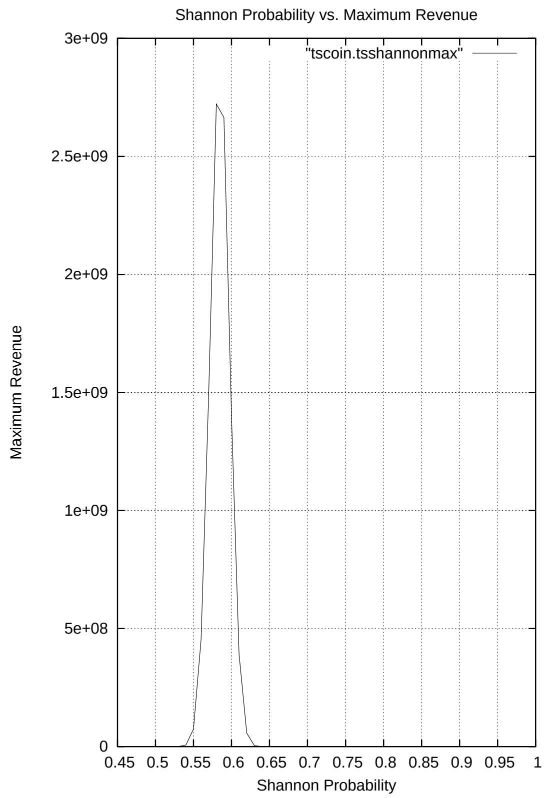

Figure B.7: Example output of the *tsshannonmax* program, using the file produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

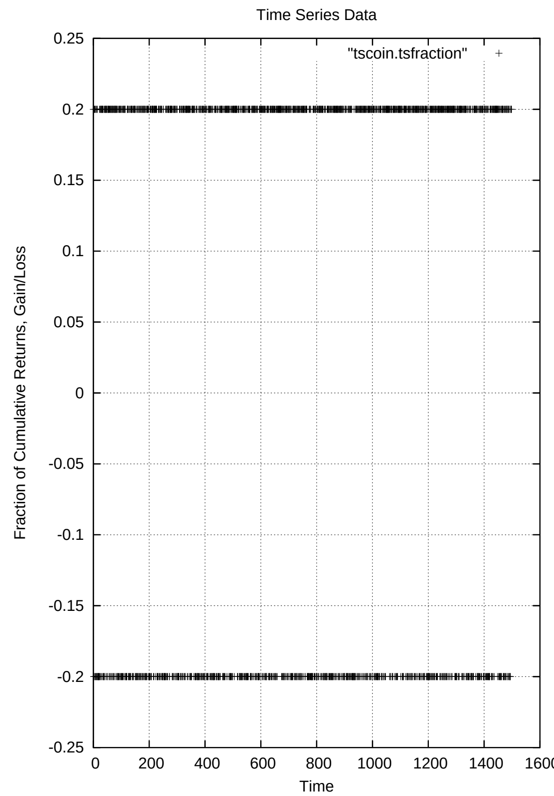

Figure B.8: Example output of the *tsfraction* program, the input was produced by the *tscoin* program, with a Shannon probability of 0.6, as shown in Figure B.46 in Section B.3.8. The *tsfraction* program produces a time series of the "wins" or "losses" in the game, which in this case is $\pm f = \pm(2P - 1) = \pm(2 \cdot 0.6 - 1) = \pm 0.2$.

B.2.10 tslsq

Source tslsq.c, for making a least squares fit time series from a time series.

The form of the best fit is $b + at$, for linear least squares fit, e^{b+at}, w^{x+t}, or 2^{y+zt} for exponential least squares fit, $c/(1 + e^{-(b+at)})$ for the logistic least squares fit, $\sqrt{b + at}$ for the square root fit, $\ln(b + at)$ for the natural logarithmic fit, and and $(b + at)^2$ for the square law fit.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time. Uses Newton—Raphson method for an iterative solution for the probability, p.

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

An example output from the *tslsq* program appears in Figure B.10.

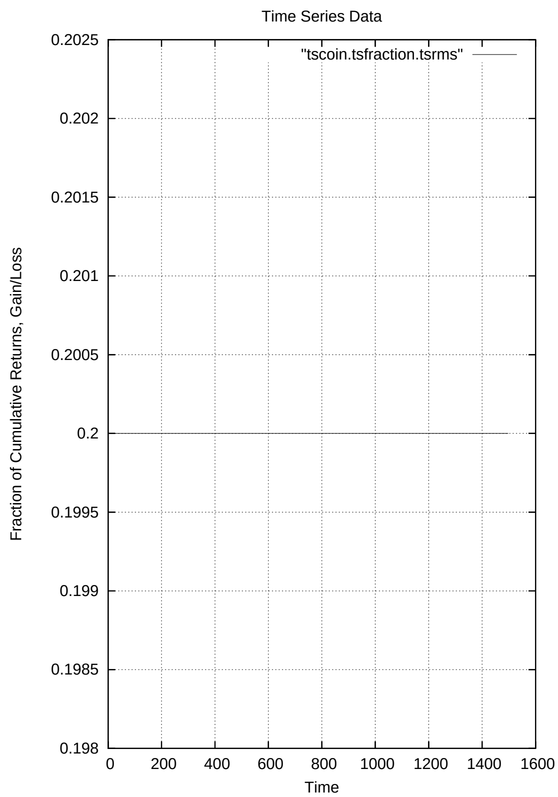

Figure B.9: Example output of the *tsrms* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

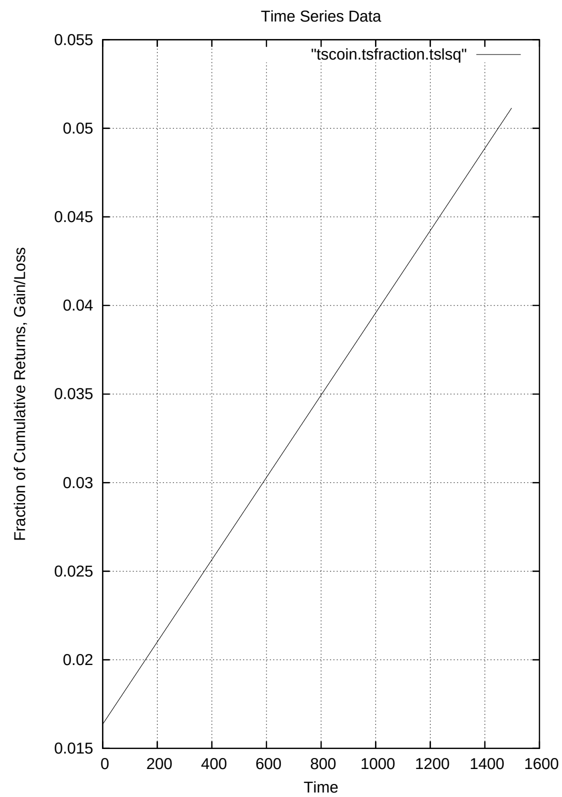

Figure B.10: Example output of the *tslsq* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

B.2.11 tsnormal

Source tsnormal.c, for making a histogram or frequency plot of a time series.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsnormal* program appears in Figure B.11.

B.2.12 tschangewager

Source tschangewager.c, for changing the unfair returns of a time series. The idea is to change the returns of a time series which is weighted unfairly, by changing the increments by a constant factor. The main function of this program is regression scenario verification—given an empirical time series, and a “wager” fraction, speculative market pro forma performance can be analyzed. The input time series is assumed to be cumulative sum with fractional or Brownian characteristics.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample’s time, and the last field as the sample’s value at that time.

An example output from the *tschangewager* program appears in Figure B.12.

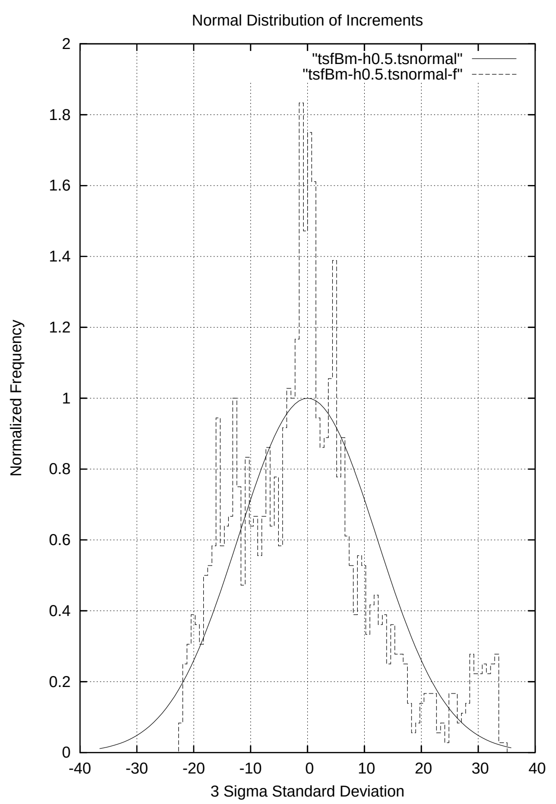

Figure B.11: Example output of the *tsnormal* program, using a simulated Hurst coefficient of 0.5 as simulated by the *tsfBm* program.

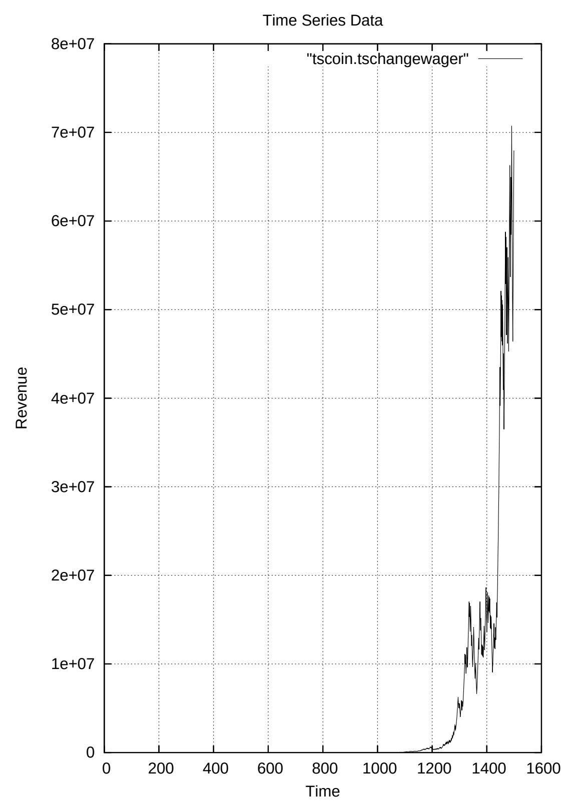

Figure B.12: Example output of the *tschangewager* program, using the file produced by the *tscoin* program, with a Shannon probability *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8. The wager was reduced by 50%.

B.2.13 tsavg

Source tsavg.c, for taking the average of a time series. The value of a sample in the time series is added to the cumulative sum of the samples to make a new time series by dividing the cumulative sum by the number of samples, for each sample. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsavg* program appears in Figure B.13.

B.2.14 tssample

Source tssample.c, for sampling a time series. The value of a sample in the time series is printed to stdio only if it is a multiple of the specified interval.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tssample* program appears in Figure B.14.

B.2.15 tsXsquared

Source tsXsquared.c, for taking the Chi—Square of two time series, the first file contains the observed values, the second contains the expected values.

The input file structures are text files consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsXsquared* program appears in Figure B.15.

B.2.16 tsavgwindow

Source tsavgwindow.c, for taking the average of a time series. The value of a sample in the time series added to the cumulative sum of the samples to make a new time series by dividing the cumulative sum by the number of samples, for each sample. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsavgwindow* program appears in Figure B.16.

B.2.17 tsrmswindow

Source tsrmswindow.c, is for taking the root mean square of a time series. The square of a value of a sample in the time series added to the cumulative sum of the square of the samples to make a new time series by dividing the cumulative

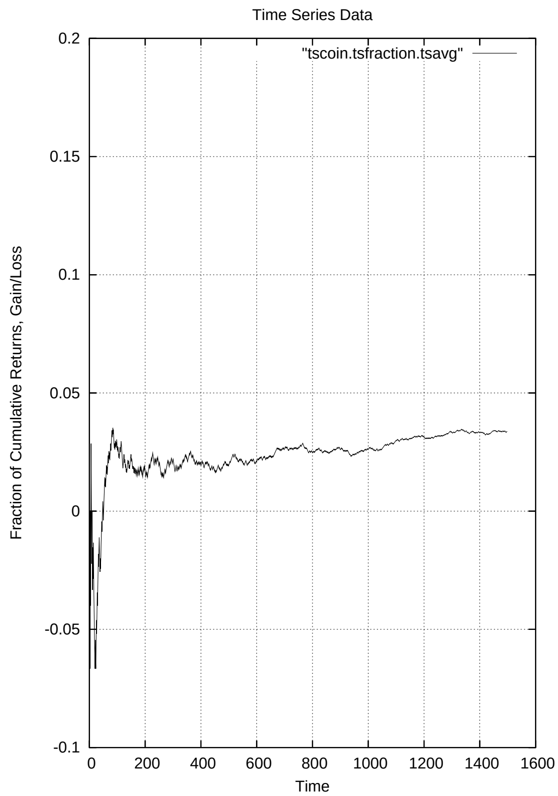

Figure B.13: Example output of the *tsavg* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

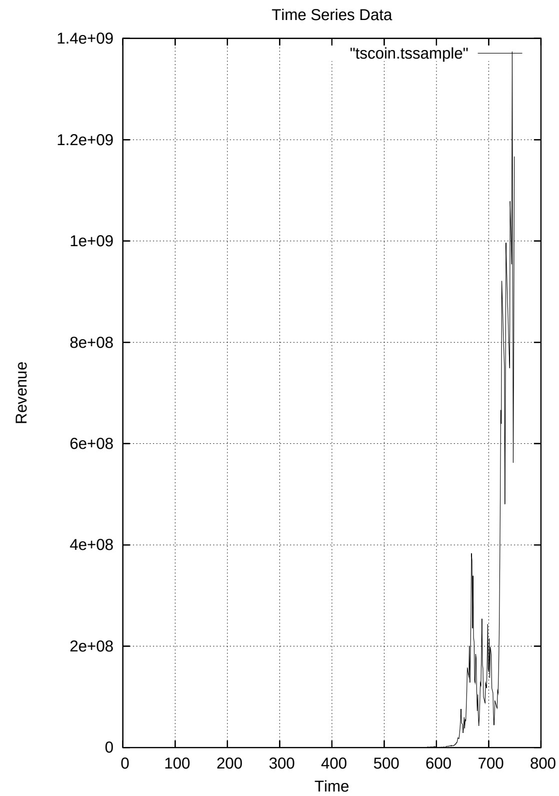

Figure B.14: Example output of the *tssample* program, sampling every other record. The input was produced by the *tscoin* program, shown in Figure B.46, in Section B.3.8, with a Shannon probability of 0.6.

$\boxed{\text{chi-squared value = 13.991, 5 percent critical value = 98.778, for 100 samples}}$

Figure B.15: Example output of the *tsXsquared* program, the input was produced by the *tsnormal* program, as shown in Figure B.1, which was derived from the output of the *tsbrownian* program with 1500 records as input.

sum of the square of the samples by the number of samples, for each sample. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample’s time, and the last field as the sample’s value at that time.

An example output from the *tsrmswindow* program appears in Figure B.17.

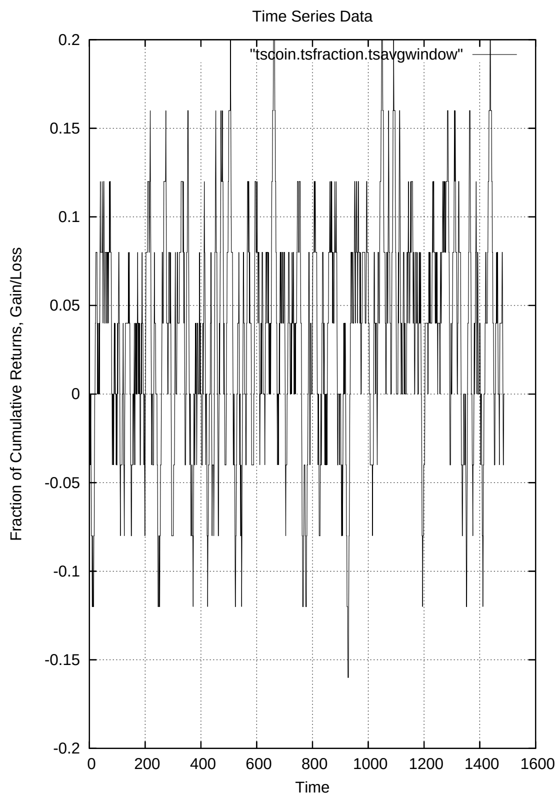

Figure B.16: Example output of the *tsavgwindow* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

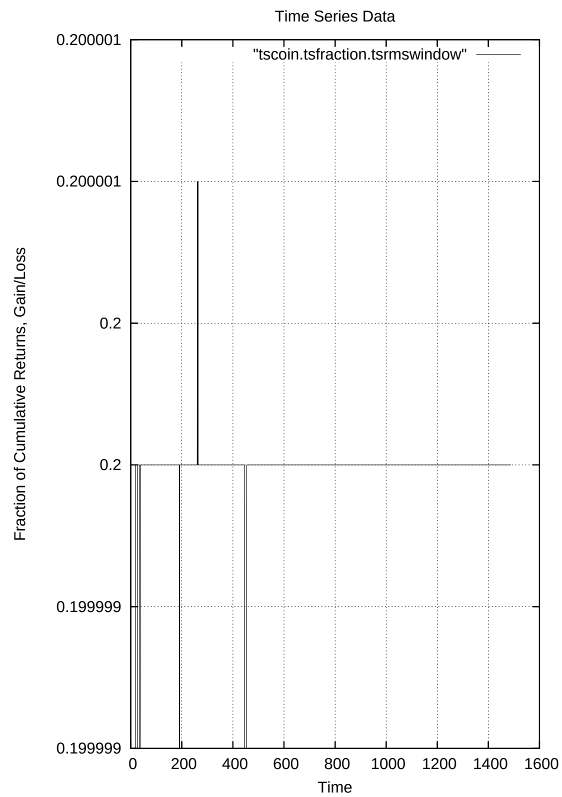

Figure B.17: Example output of the *tsrmswindow* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

B.2.18 tsshannonwindow

Source tsshannonwindow.c, for finding the windowed Shannon probability of a time series. The Shannon probability is calculated by the following method:

1. For each sample in the time series:

   (a) Find the value of the sample's normalized increment by subtracting the previous value of the time series from the current value of the time series, and then dividing this value of the increment by the previous value in the time series, (note that this is similar to the procedure used by the program *tsfraction*).

   (b) Find the running value of the root mean square of a window of the normalized increments, (note that this is similar to the procedure used by the program *tsrmswindow*).

(c) Find the running value of the average of a window of the normalized increments, (note that this is similar to the procedure used by the program *tsavgwindow*).

2. Compute the Shannon probability of the windows by eight methods:

(a) using the formula:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.37}$$

which is derived in Chapter 2, in Equation 2.58.

(b) using the formula:

$$P = \frac{rms + 1}{2} \tag{B.38}$$

which is derived in Chapter 2, by combining Equations 2.55 and 2.56.

(c) using the formula:

$$P = \frac{\sqrt{avg} + 1}{2} \tag{B.39}$$

which is derived in Chapter 2, by combining Equations 2.68, and, 2.55.

(d) by taking the absolute value of the normalized increments and using the formula:

$$P = \frac{abs + 1}{2} \tag{B.40}$$

which is derived in Appendix A, in Equation A.11.

(e) counting the up movements in the window of the time series, and considering adjacent elements from the time series with equal magnitude as an up movement.

(f) counting the up movements in the window of the time series, and considering adjacent elements from the time series with equal magnitude as a down movement.

(g) finding an exponential least squares fit of the values of the time series in a window, and iteratively calculating the Shannon probability from the least squares fit variable using Newton—Raphson method for finding the roots of a function.

(h) finding the logarithmic returns of the values of the time series in a window, and iteratively calculating the Shannon probability from the least squares fit variable using Newton—Raphson method for finding the roots of a function.

Where P is the Shannon probability, avg is the running average of a window of the normalized increments, and, rms is the running root mean square of a window of the increments. The Shannon probability of the windows of the increments is a time series that is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

Note: The derivation for exponential least squares fit is:

1. input the value of the time series for each time interval, $value(t)$, and store the logarithm of the value, ie.:

$$y(t) = \ln(value(t)) \tag{B.41}$$

2. compute the least squares fit to $y(t)$, $a + bt$, then:

$$\ln(y(t)) = b + at \tag{B.42}$$

3. exponentiate the values in $y(t)$:

$$\begin{aligned} fit(t) &= e^{b} \cdot e^{at} && \text{(B.43)} \\ &= e^{b+at} && \text{(B.44)} \end{aligned}$$

where $fit(t)$ is the least squares exponential fit.
Note: The derivation for exponential least squares fit is:

$$\begin{aligned} y(t) &= e^{k1+k2t} && \text{(B.45)} \\ s(t) &= \ln\left(\frac{e^{k1+k2t}}{e^{k1+k2(t-1)}}\right) && \text{(B.46)} \\ &= \ln\left(e^{k1+k2t-k1-k2t+k2}\right) && \text{(B.47)} \\ &= \ln\left(e^{k2}\right) && \text{(B.48)} \\ &= k2 && \text{(B.49)} \end{aligned}$$

And for the binary least squares fit, letting $k = k2$:

$$\begin{aligned} y(t) &= e^{k1+k2t} && \text{(B.50)} \\ s(t) &= \ln\left(\frac{e^{k1+k2t}}{e^{k1+k2(t-1)}}\right) && \text{(B.51)} \\ &= \ln\left(e^{k1+k2t-k1-k2t+k2}\right) && \text{(B.52)} \\ &= \ln\left(e^{k2}\right) && \text{(B.53)} \\ &= k2 && \text{(B.54)} \\ e^{xt} &= 2^{kt} && \text{(B.55)} \\ a^{t} &= 2^{kt} && \text{(B.56)} \\ a &= 2^{k} && \text{(B.57)} \\ k\ln(2) &= \ln(a) && \text{(B.58)} \\ k &= \frac{\ln(a)}{\ln(2)} && \text{(B.59)} \end{aligned}$$

Note: The derivation for calculating the Shannon probability, given the Shannon information capacity, where the information capacity is the exponent derived from the least squares fit to the values of the time series, divided by the natural logarithm of two. See [Sch91, pp. 128, 151]. Uses Newton-Raphson method for an iterative solution for the probability, p.

$$C(p) = 1 + p ln_2(p) + (1-p) ln_2(1-p) \tag{B.60}$$

$$C(p) = 1 + p\left(\frac{ln(p)}{ln(2)}\right) + (1-p)\left(\frac{ln(1-p)}{ln(2)}\right) \tag{B.61}$$

$$C(p) = \left(\frac{1}{ln(2)}\right)(ln(2) + p ln(p) + (1-p) ln(1-p)) \tag{B.62}$$

$$C(p) = \left(\frac{1}{ln(2)}\right)(ln(2) + p ln(p) + ln(1-p) - p ln(1-p)) \tag{B.63}$$

$$\frac{dC(p)}{dp} = \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - (\frac{1}{(1-p)}) - \left(ln(1-p) - (\frac{p}{(1-p)})\right)\right) \tag{B.64}$$

$$= \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - (\frac{1}{(1-p)} - ln(1-p) + \left(\frac{p}{(1-p)}\right)\right) \tag{B.65}$$

$$= \left(\frac{1}{ln(2)}\right)\left(ln(p) - ln(1-p) + \left(\frac{p}{(1-p)}\right) - \left(\frac{1}{(1-p)}\right)\right) \tag{B.66}$$

$$= \left(\frac{1}{ln(2)}\right)\left(1 + ln(p) - ln(1-p) + \left(\frac{(p-1)}{(1-p)}\right)\right) \tag{B.67}$$

$$= \left(\frac{1}{ln(2)}\right)(1 + ln(p) - ln(1-p) - 1) \tag{B.68}$$

$$= \left(\frac{1}{ln(2)}\right)(ln(p) - ln(1-p)) \tag{B.69}$$

An example output from the *tsshannonwindow* program appears in Figure B.18.

```
0.584390 0.600000 0.591864 0.600000 0.584390 0.584390 0.585330 0.582893
```

Figure B.18: Example output of the *tsshannonwindow* program, the input was produced by the the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

B.2.19 tspole

Source tspole.c, is for single pole low pass filtering of a time series. The single pole low pass filter is implemented from the following discrete time equation:

$$v_{n+1} = I \cdot k2 + v_n \cdot k1 \tag{B.70}$$

where I is the value of the current sample in the time series, v_n are the value of the output time series, and $k1$ and $k2$ are constants determined from the following equations:

$$k1 = e^{-2 \cdot p \cdot \pi} \tag{B.71}$$

and

$$k2 = 1 - k1 \tag{B.72}$$

where p is a constant that determines the frequency of the pole-a value of unity, the default, places the pole at the sample frequency of the time series.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

This program is based on [Con78, pp. 11].

An example output from the *tspole* program appears in Figure B.19.

B.2.20 tsdft

Source tsdft.c, is for taking the Discrete Fourier Transform (power spectrum) of a time series.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

Note: the algorithm used in this program is a modified version of the program dft.c, written and © 1985 Nicholas B. Tufillaro.

An example output from the *tsdft* program appears in Figure B.20.

B.2.21 tsmath

Source tsmath.c, for for performing arithmetic operations on each element in a time series. The resultant time series is printed to stdio.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsmath* program appears in Figure B.21.

B.2.22 tsdeterministic

Source tsdeterministic.c, is for determining if a time series was created by a deterministic mechanism. The idea is place each element of a time series in an array structure that contains the element and the next element in the time series, and then sort the array. The array is output and may be plotted. For example, using the program *tsdlogistic* to make a discrete time series of the logistic, (quadratic function,) with the command "tsdlogistic -a 4 -b -4 1000 > XXX" and then using this program on the output file, XXX, will result in a plot of a parabola. See [PJS92, pp. 745].

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsdeterministic* program appears in Figure B.22.

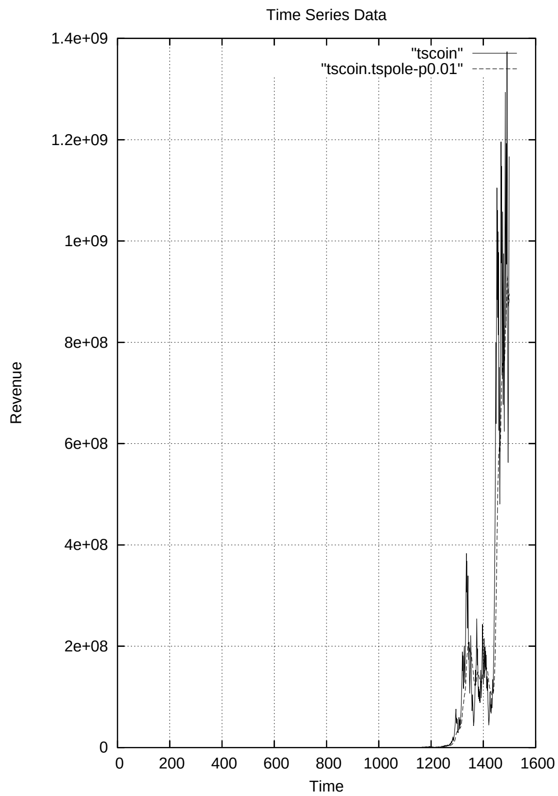

Figure B.19: Example output of the *tspole* program, the input was produced by the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8. The pole frequency was set at $\frac{samplefrequency}{100}$.

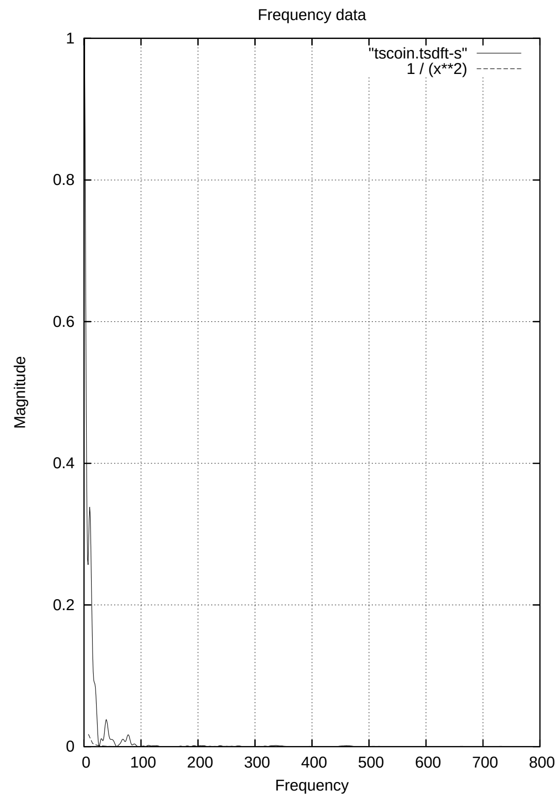

Figure B.20: Example output of the *tsdft* program, the input was produced by the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8. A plot of $\frac{1}{f^2}$ is superimposed on the plot.

B.2.23 tsstatest

Source tsstatest.c, for making a statistical estimation of a time series. The number of samples, given the maximum error estimate, and the confidence level required is computed for both the standard deviation, and the mean.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

Consider the following formula for determination of the Shannon Probability, P, of an equity market time series, using the average and root mean square of the normalized increments, avg, and, rms, respectively, by rearranging Equation 2.58:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.73}$$

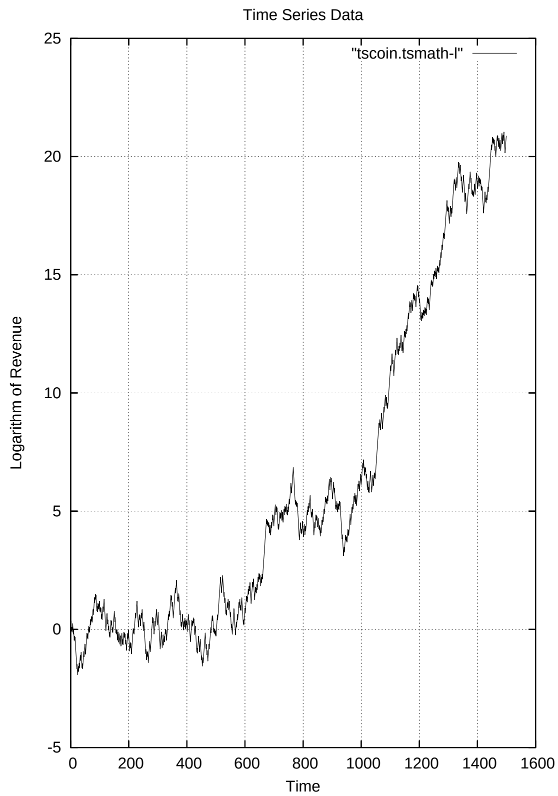

Figure B.21: Example output of the *tsmath* program, taking the logarithm of the file produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

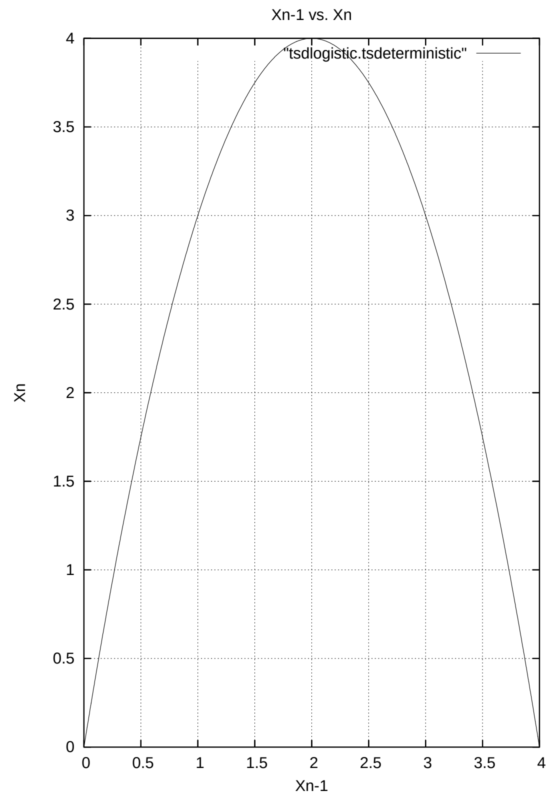

Figure B.22: Example output of the *tsdeterministic* program, the input was produced by the *tsdlogistic* program, using the command "tsdlogistic -a 4 -b -1 1000 > filename," which is shown in Figure B.50 in Section B.3.12.

which is useful in the determination of the optimal fraction of capital, f, to invest in a stock, from Equation 2.55:

$$f = 2P - 1 \tag{B.74}$$

The objective is to estimate how large the data set has to be for determining P to a given accuracy, possibly using statistical estimates of how many data points are required for a given confidence level that the error is less than a specific value.

Suppose we have a confidence level, $0 < c < 1$, that a value is within, plus or minus, an error level, e. What this means, for example if $c = 0.9$, and $e = 0.1$, is that for 90% of the cases, the value will be within the limits of $\pm e$, or, 5% of the time, on the average, it will be less than $-e$, and 5% of the time more than $+e$.

The error level for avg, e_{avg},for a given confidence level, will be:

$$e_{avg} = k\frac{rms}{\sqrt{n}} \tag{B.75}$$

where n is the number of records in the data set, and k is a function involving a normal distribution. The error level for rms, for the same given confidence level, will be:

$$e_{rms} = k \frac{rms}{\sqrt{2n}} \tag{B.76}$$

where k is identical in both cases. Also, the number of records required for a given error level would be:

$$n_{avg} = \left(\frac{(rms \cdot k)}{e_{rms}} \right)^2 \tag{B.77}$$

and

$$n_{rms} = \frac{1}{2} \left(\frac{(rms \cdot k)}{e_{rms}} \right)^2 \tag{B.78}$$

where k is the same as above.

For equity market indices, a typical value for rms would be 0.01, and 0.0003 for avg. This is probably typical for many stocks, however, high gain stocks, in a "bull" market can have an rms of 0.04, and an avg of 0.005.

The value of k can be determined from standard statistical tables, as shown in table B.1, where $k =$ sigma level, for a confidence level, c.

Table B.1: Confidence Level vs. σ Level.

Confidence Level, c (%)	σ level
50	0.67
68.27	1.00
80	1.28
90	1.64
95	1.96
95.45	2.00
99	2.58
99.73	3.00

Note that for a given confidence level:

$$\frac{avg}{rms} = \frac{avg \pm k\frac{rms}{\sqrt{n}}}{rms \pm k\frac{rms}{\sqrt{2n}}} \tag{B.79}$$

$$= \frac{\frac{avg}{rms} \pm k\frac{1}{\sqrt{n}}}{1 \pm k\frac{1}{4\sqrt{n}}} \tag{B.80}$$

Now, consider the specific example of avg and rms for an exponential function. In this specific case, $avg = rms$, and $\frac{avg}{rms} = 1$. Since k is assumed to be a function of a normally distributed random variable, the error in the ratio $\frac{avg}{rms}$ as a function of the data set size, n, can be found by superposition, and adding the contributing error values as a function of n for both rms and avg root mean square, or:

$$\sqrt{1^2 + \left(\frac{1}{4}\right)^2} = 1.030776406 \tag{B.81}$$

or:

$$\frac{avg}{rms} \sim \frac{avg}{rms} \pm 1.03 \frac{1}{\sqrt{n}} k \sim \frac{avg}{rms} \pm \frac{1}{\sqrt{n}} k \tag{B.82}$$

where k is determined from the table, above. In this specific case, where $avg = rms$:

$$\frac{avg}{rms} \sim \frac{avg}{rms} \left(1 \pm \frac{1}{\sqrt{n}} k \right) \tag{B.83}$$

An interpretation of what this means is that, given a data set size, n, and a confidence level of, say 90%, then 90% of the time, our measurements of $\frac{avg}{rms}$, would fall within an error level of $\pm 1.64 \frac{1}{\sqrt{n}}$, ie., 5% of the time it would be greater than the error value, and 5% of the time, it would be lower than the error value. In general, the concern is with the lower error value since from the equation:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.84}$$

(at least in this specific case where $avg = rms$,) that a 90% confidence level would imply that there is a 5% chance of the real value $\frac{avg}{rms}$ being zero is where:

$$\frac{k}{\sqrt{n}} = 1 \tag{B.85}$$

or:

$$\frac{1.64}{\sqrt{n}} = 1 \tag{B.86}$$

or $n = 2.6896 \sim 3$.

What this means is that, if we repeat the experiment of finding 3 records in a row that have $rms = avg$, with neither equal to zero, many times, that we would loose money in 5% of the cases, making the measured Shannon probability, P, unity, and the estimated Shannon probability, 0.95, eg., we should consider the Shannon probability as 0.95 in this specific case—ie., it would be ill advised to invest all of the capital in such a scenario, since, sooner or later, all of the capital would be lost, (on average, by the 20'th game.)

This implies a simple methodology. Measure avg and rms, and compute the Shannon probability. Decease that probability by a factor—ie., one minus the confidence level, divided by two—that the wager could be a loosing proposition, based on the estimates that avg could be zero, (which is a function of the confidence level, and the number of records in the data set.) This, conceivably, could provide a quantitative estimate on the number of records required in a data set.

Note that if $\frac{avg}{rms}$ is measured at 0.9, then:

$$\frac{1.64}{\sqrt{n}} = 0.9 \tag{B.87}$$

for the same confidence level of 0.9, or

$$n = 3.32 \tag{B.88}$$

and:

for the same confidence level 0.9. What the table means is that if you have a stock price time series of 67 records, then the minimum measured Shannon probability must be at least 0.6—and the wagering strategy should use the Shannon probability of 0.57—and the minimum number of records used to measure avg and rms is 67. Additionally, a stock time series with a Shannon probability of 0.53 should be measured using not less than 1076 records, and no wager should be made, unless the measurements involve substantially more than 1076 records. In general, the Shannon

Table B.2: Shannon Probability vs. Data Set Size.

$\frac{avg}{rms}$	n	$P_{measured}$	P
1.0	2.7	1.00	0.95
0.9	3.3	0.95	0.90
0.8	4.2	0.90	0.86
0.7	5.5	0.85	0.81
0.6	7.5	0.80	0.76
0.5	10.8	0.75	0.71
0.4	16.8	0.70	0.67
0.3	29.9	0.65	0.62
0.2	67.2	0.60	0.57
0.1	268.9	0.55	0.52
0.05	1075.8	0.53	0.50

probability of almost all stock time series fall, inclusively, in this range. 67 business days is, approximately, 13.4 weeks, or little more than a calendar quarter. 1076 business days is slightly longer than four calendar years.

Note that [Pet91, pp. 83] referencing [Fed88, pp. 179], the claim is made that 2500 records is the minimum size of the data set for using fractal analytical methodologies. Note that a data set of this size would have, with an $\frac{avg}{rms}$ of 0.5—which is "typical" for a stock time series, a Shannon probability error level that is approximately 1%, since it lies between 2 and 3 sigma, and c would be approximately 0.99. This would seem to be consistent with the empirical arguments of both Peters and Feder, although Peters implies that less could be used if the system being analyzed is "chaotic" in nature, and one "cycle" of the system's, apparently, "strange attractor" is less than 2500 time units. This analysis would seem to be consistent with the observations of these authors, provided that it is a requirement that the measured Shannon probability be used to calculate the optimum wager fraction.

What this analysis would tend to suggest is that, although Feder's and Peter's arguments seem to be confirmed, that there may, also, be other viable solutions for data sets, (or fragments thereof,) that are very much smaller, provided that the measured Shannon probability of the data set, or segment, is sufficiently large—for example, a stock that has a time series fragment that has 5 out of 6 upward movements may prove to be a viable investment opportunity at a measured Shannon probability that is greater than 0.85, ($\frac{5}{6}$ = a Shannon probability of 0.833 $\sim$ 0.85,) if played at a Shannon probability as high as 0.8, but no higher.

For example, using a Shannon probability, P, of 0.51 for the *tscoins* program, to provide an input fractal time series for the *tsstatest* program, and iterating, indicates that for a standard deviation of 0.020000, with a confidence level of 0.960784 that the error did not exceed 0.020000, 3 samples would be required.

Since the Shannon probability is calculated directly from the standard deviation, (ie., rms = root mean square of the normalized increments,) the maximum error can be calculated:

$$\frac{0.5}{0.51} = 0.980392157 \tag{B.89}$$

which means that a confidence level of 0.960784314 that the error level in the standard deviation is less than 0.02 because standard deviation $= rms = 0.02 - 0.02 = 0$, which would correspond to a Shannon probability, P, of 0.5, and since half the errors outside the range of 0.02 would be negative, (and the other half positive,) the confidence level required would be $1 - ((1 - 0.980392157) \cdot 2)$.

What this means is that $((1 - 0.960784314)/2) \cdot 100$ percent of the time, the actual rms value will be sufficiently small to make P equal to, or less than 0.5. This means that P must be decreased by 1.960784300 percent. The reasoning is that after many iterations, the measured P would be too small by 1.90784300% of the time, on average, making the measured P, over all of the iterations, 0.5.

This suggests a dynamic rule: do not wager unless the Shannon probability, P, is strictly greater than 0.51, as measured on strictly more than 3 time units. Interestingly, the Hurst Coefficient, as measured by the *tshurst* program, graph of a random walk, Brownian motion, or fractional Brownian motion fractals indicates that there is significant near term correlations for 4 or less time units. This suggests a dynamic trading methodology for equities.

Similar reasoning would indicate that using a value of $P = 0.6$ for the *tscoins* and *tsfraction* programs to provide input to the *tsstatest* program with a confidence level of 0.8, and an error of 0.12, (ie., 10% of the time the value of P would be less than $0.9 \cdot 0.6 = 0.54$, where $0.2 - 0.12 = 0.08$, and $0.54 = \frac{0.08+1}{2}$,) would require a minimum of 3 records. The fraction of capital wagered should be $2 \cdot 0.54 - 1 = 0.08$.

To review what has been presented so far, we really are not confident that we know the value of the Shannon probability, P, until we have sufficiently many records, n. One way of addressing this issue is to wait to make a wager until we do. But this strategy has an “opportunity cost,” since, approximately 50% of the time, we would not have made an investment when we should have. Note that since investing in equities is not a 100% assured proposition, we only invest a fraction of our capital, f, where $f = 2P - 1$. Since investing with a data set size that is insufficient, ie., n is too small, lowers the probability of the wins, the Shannon probability, P, will have to be lowered to maintain the optimum wager fraction of the capital. We can compute the value that the Shannon probability, P, must be lowered to account for this.

The relationship between the Shannon probability, P, and the root mean square of the normalized increments of a time series, rms, is:

$$P = \frac{rms + 1}{2} \tag{B.90}$$

Let the error, e, in rms created by an insufficient data set size be:

$$e = rms - rms' \tag{B.91}$$

where $0 \leq rms' \leq rms$. This means that although rms was measured it could be as low as rms'. The confidence level that rms is not less than rms' can be found by statistical estimate. The Shannon probability, P', associated with rms' is:

$$P' = \frac{rms' + 1}{2} \tag{B.92}$$

P' is the Shannon probability if the root mean square value of the normalized increments of the time series is rms'.

Since we want to alter the measured Shannon probability, P, to accommodate the error created by a insufficient data set size, we multiply P by the confidence level that the real value of P is not less than P', or the confidence level, C, is:

$$C = \frac{P'}{P} \tag{B.93}$$

The reasoning is that a value of C, say 0.9, means that the root mean square value of the increments could be below the measured value, rms, by an amount e for 5% of the time, and above rms by an amount e for 5% of the time, so that:

$$P' = CP \tag{B.94}$$

Substituting:

$$CP = \frac{rms' + 1}{2} \tag{B.95}$$

and solving for rms':

$$rms' = 2CP - 1 \tag{B.96}$$

or:

$$e = rms - (2CP - 1) = rms - 2CP + 1 \tag{B.97}$$

and substituting for rms, where $rms = 2P - 1$:

$$e = 2P - 1 - 2CP + 1 = 2P - 2CP = 2P(1 - C) \tag{B.98}$$

and substituting $P' = CP$:

$$e = 2P - 2P' = 2\left(P - P'\right) \tag{B.99}$$

C now has to be adjusted because we are only concerned with the values of rms' that are less than rms, where:

$$c = 1 - 2(1 - C) = 1 - 2 + 2C = 2C - 1 \tag{B.100}$$

but since $C = P'/P$:

$$c = \frac{2P'}{P} - 1 \tag{B.101}$$

or we have:

$$e = 2\left(P - P'\right) \tag{B.102}$$

and:

$$c = \frac{2P'}{P} - 1 \tag{B.103}$$

which are the two general equations for use of this program for trading equities.

Making a plot of these equations, of P' vs. n for various P presents an interesting conjecture. The graph can be crudely approximated by a single pole filter, with a pole at 0.033, ie., using the program *tscoins* with a -p 0.6 argument to simulate an equity value time series, and the program *tsinstant*, with the -s option, to calculate the instantaneous Shannon probability of the time series, followed by the program *tspole* with a -p 0.033 argument, would output, approximately, P'. The P' tends to under wager for $t \leq 7$, and over wager for $t \geq 0.7$. The approximation is simple, but crude. Interestingly, using the program *tshurst* on the same time series indicates that there is good correlation for $t \leq 5$, and if this temporal range is of interest, this simple solution may prove adequate for non-rigorous requirements. Additionally, perhaps using the *tsmath* program, the output of the *tspole* program could have 0.5 subtracted, multiplied by, say, 0.85, and then the 0.5 re-added to extend the usefulness to approximately 100 business days. The accuracy over this range is approximately ± 0.01 out of 0.55. Naturally, after very many days, for example, if $P = 0.6$, P' would still be 0.585, creating a long term error in rms of $0.2 - 0.17 = 0.03$. Note that the error created in the exponential growth of the capital would be $0.04 - 0.0289$. A substantial long term error. Alternately, perhaps a recursive feed-forward technique could be implemented that would allow the pole frequency to be selected for far term compatibility with the statistical estimate, while at the same time approximating the near term. Naturally, this, also, should not be considered a substitute for statistical estimates, but using statistical estimates would probably require a recursive procedure, and that is a formidable proposition.

This program will require finding the value of the normal function, given the standard deviation. The method used is to use Romberg/trapezoid integration to numerically solve for the value.

This program will require finding the functional inverse of the normal, ie., Gaussian, function. The method used is to use Romberg/trapezoid integration to numerically solve the equation:

$$F(x) = \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 \tag{B.104}$$

which has the derivative:

$$f(x) = \frac{1}{2\pi} e^{\frac{-x^2}{2}} \tag{B.105}$$

Since $F(x)$ is known, and it is desired to find x,

$$F(x) - \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 = P(x) = 0 \tag{B.106}$$

and the Newton-Raphson method of finding roots would be:

$$P_{n+1} = P_n - \frac{P(x)}{f(x)} \tag{B.107}$$

An example output from the *tsstatest* program appears in Figure B.23.

```
For a mean of 0.005574, with a confidence level of 0.900000
    that the error did not exceed 0.000557, 26949322 samples would be required.
    (With 1500 samples, the estimated error is 0.074707 = 1340.381071 percent.)
For a standard deviation of 1.759051, with a confidence level of 0.900000
    that the error did not exceed 0.175905, 136 samples would be required.
    (With 1500 samples, the estimated error is 0.052826 = 3.003078 percent.)
```

Figure B.23: Example output of the *tsstatest* program, the input was produced by the the *tsfBm* program, with a Hurst coefficient of 0.5. The *tsstatest* parameters are for a confidence level of 90%, with an error estimate of ± 10%, on the distribution of the normalized increments.

B.2.24 tsshannonaggregate

Source tsshannonaggregate.c, aggregate Shannon probability of many concurrent Shannon probabilities.

Consider gambling on two unfair coin tossing games, at the same time, one game having a Shannon probability of 0.55, and the other having a Shannon probability of 0.65. Assuming that the coins in both games are tossed concurrently for each iteration of the game, the combinatorics of the possible outcomes of wins and losses in each iteration are:

outcomes	probability	fraction	average
ll:	0.157500	−0.400000	−0.063000
wl:	0.192500	−0.200000	−0.038500
lw:	0.292500	0.200000	0.058500
ww:	0.357500	0.400000	0.143000

where l is a loss, and w is a win, and the probability is calculated by multiplying the individual probabilities of a loss or win for the respective coins, ie., for both coins to win, the probability would be $0.55 \cdot 0.65 = 0.3575$. ($1 - P$ is used for the probability of a loss for each coin.) The fraction is the fraction of capital waged on an individual game, and is computed as optimal, from the equation $2P - 1$, where P is the Shannon probability of the individual unfair coin and is either 0.55 or 0.65. The average is computed as the product of the probability and the fraction.

What this means is that 35.75% of the time, a win-win outcome will be observed in the iterated games, and 15.75% of the time, a lose-lose outcome will be observed. The amount won in the win-win scenario will be the sum of the fractions wagered on each coin, which is $(2 \cdot 0.55 - 1) + (2 \cdot 0.65 - 1) = 0.1 + 0.3 = 0.4$. The product of this fraction and probability is the contribution over many plays to the capital do to this outcome. Summing these averages for the different outcomes is the average over many plays of the capital growth by playing both games, and is numerically identical to the sum of the average of the normalized increments of both games.

Since the average and root mean square of the normalized increments are related by:

$$rms = sqrt(average) \tag{B.108}$$

squaring the average will be the root mean square of the normalized increments, or:

Average	rms	Shannon probability
0.100000	0.316228	0.658114

where the Shannon probability, P, is computed by:

$$P = \frac{rms + 1}{2} = \frac{1.316228}{2} = 0.658114 \tag{B.109}$$

The implication is that the two concurrent unfair coin tossing games could be "modeled" as a single game with a Shannon probability of 0.658114.

Although it is generally more expedient just to sum, root mean square, the individual root mean square of the normalized increments of each game, (where $f = rms = 2P - 1$,) and then compute the Shannon probability by:

$$\begin{aligned}
P &= \frac{\sqrt{((2 \cdot 0.55) - 1)^2 + ((2 \cdot 0.65) - 1)^2} + 1}{2} && \text{(B.110)}\\
&= \frac{\sqrt{0.1^2 + 0.3^2} + 1}{2} && \text{(B.111)}\\
&= \frac{\sqrt{0.01 + 0.09} + 1}{2} && \text{(B.112)}\\
&= \frac{\sqrt{0.1} + 1}{2} && \text{(B.113)}\\
&= \frac{0.316227766 + 1}{2} && \text{(B.114)}\\
&= \frac{1.316227766}{2} && \text{(B.115)}\\
&= 0.658113883 && \text{(B.116)}
\end{aligned}$$

this program does it with combinatorics.

An example output from the *tsshannonaggregate* program appears in Figure B.24.

B.2.25 tsunfraction

Source tsunfraction.c, is for making a cumulative sum of the fraction of change in a time series. The value of a sample in the time series is multiplied by the running cumulative sum of the time series, and added to the running sum of the time series. The resultant time series is printed to stdout. (This program is the inverse of the *tsfraction* program.) Note that:

tsfraction data | tsunfraction

```
ll:  probability   of  0.157500   *  fraction   of  -0.400000   =  average   of  -0.063000
wl:  probability   of  0.192500   *  fraction   of  -0.200000   =  average   of  -0.038500
lw:  probability   of  0.292500   *  fraction   of  0.200000   =  average   of  0.058500
ww:  probability   of  0.357500   *  fraction   of  0.400000   =  average   of  0.143000

Average  = 0.100000,   rms  = 0.316228,   Shannon  Probability   = 0.658114
```

Figure B.24: Example output of the *tsshannonaggregate* program, with verbose print option, and Shannon probabilities of 0.55 and 0.65.

does nothing. An interesting application of this program is:

tsgaussian -t 10000 | tsmath -t -m 0.01 | tsmath -t -a 0.0003 | tsunfraction

which would manufacture a data file with statistics that are similar to equity market indices.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsunfraction* program appears in Figure B.25.

B.2.26 tsinstant

Source tsinstant.c, for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. For Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[1]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change. The values are printed to stdout.

For fractional Brownian motion time series, substantial filtering will be required of the output time series. The programs *tspole* and *tsavgwindow* may be used, perhaps in a cascade fashion, to implement a filtering technique, which potentially could be used in an adaptive computational system. Markov techniques may also be applicable. Note that in fractal time series, the short term correlation, say less than three time units as a typical value, is quite high—this can be verified by the *tshurst* program, eg., filtering, to find the average value, over a few time units, may be an advantageous strategy in adaptive computational control systems.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsinstant* program appears in Figure B.26.

[1] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

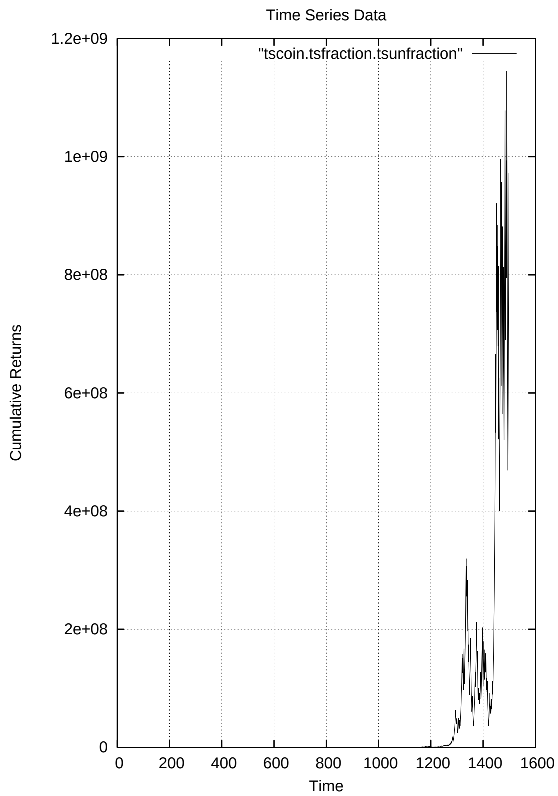

Figure B.25: Example output of the *tsunfraction* program, reconstructing the file produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8, after the program *tsfraction* was used to construct the normalized increments.

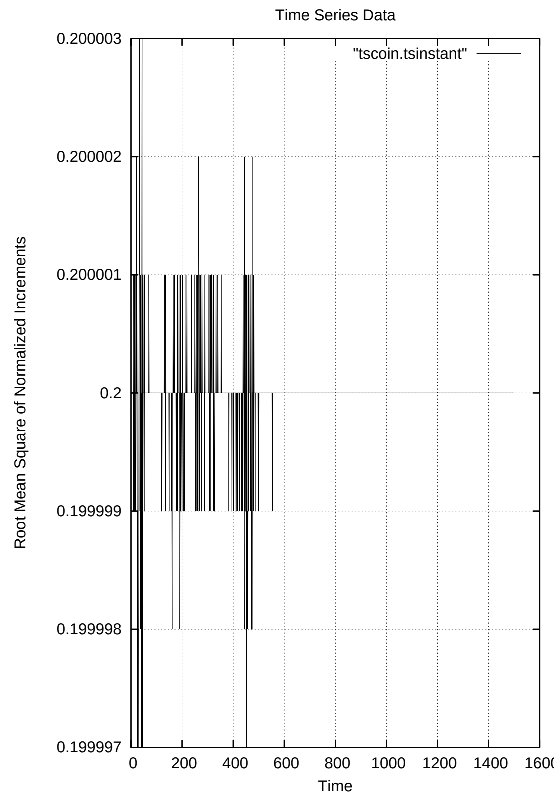

Figure B.26: Example output of the *tsinstant* program, the input file was produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

B.2.27 tsrunlength

Source tsrunlength.c, is for finding the run lengths of zero free intervals in a time series, which is assumed to be a Brownian fractal. The value of each sample in the time series is stored, and the run length to a like value in the time series is stored. A histogram of the number of run lengths of each run length value is printed to stdout as tab delimited columns of run length value, positive run lengths, negative run lengths, and the sum of both positive and negative run lengths, followed by the cumulative sum of the positive run lengths, the cumulative sum of negative run lengths, and the cumulative sum of both positive and negative run lengths.

The idea is to create a run length structure, that tallies how many time intervals a run length was either positive or negative, for each element in the time series. When a run length transition is made, (ie., when the value of the time series has *passed* through the value of the time series when the run length structure was created, from a positive or negative direction,) then the run length is tallied into histogram arrays, and the structure removed. See [Sch91, pp. 260]

As approximations for the probability, p, of the run lengths, for $t \gg 1$, $p = \frac{1}{2 \cdot x^{(\frac{3}{2})}}$, which can be integrated for the cumulative probability, P, for $t \gg 1$, $P = \frac{1}{\sqrt{t}}$. For $t \approx 1$, $P = erf(\frac{1}{\sqrt{t}})$.

Note: there is an issue with this methodology—a run length is not considered complete until the value is *passed*, so, for example, a square wave function input will never be tallied, ie., a 1 to -1 to 1 to -1 to 2 sequence is a negative run length of 3 time units.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsrunlength* program appears in Figures B.27 and B.28.

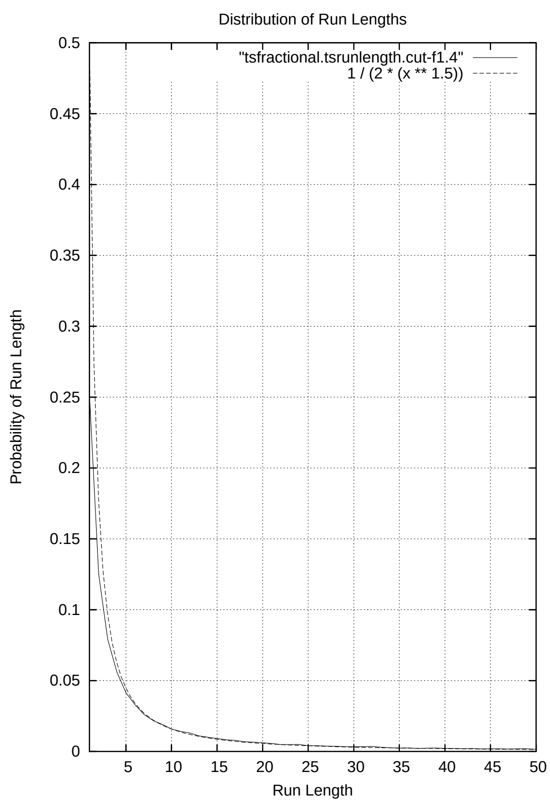

Figure B.27: Example output of the *tsrunlength* program, using 100,000 records produced by the *tsfractional*program. This is a plot of the run length distribution..

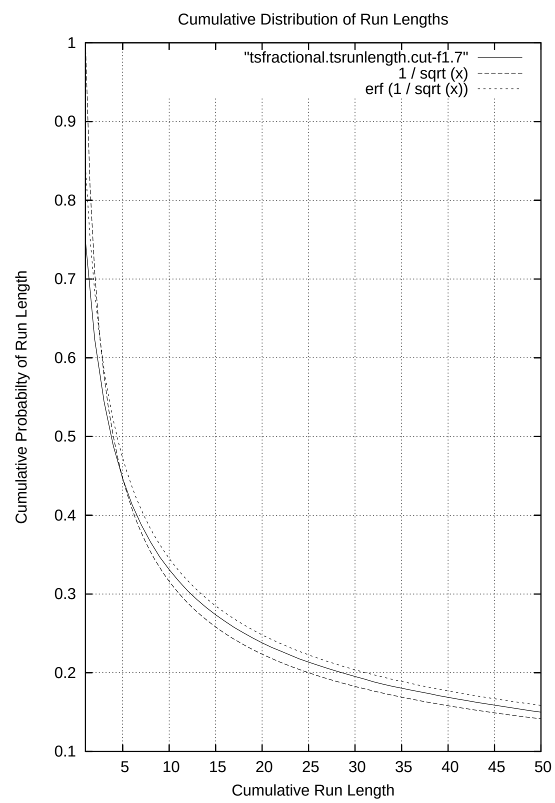

Figure B.28: Example output of the *tsrunlength* program, using 100,000 records produced by the *tsfractional* program.This is a plot of the cumulative run length distribution.

B.2.28 tsrootmean

Source tsrootmean.c for finding the root mean of a time series. The number of consecutive samples of like movements in the time series is tallied, and the resultant distribution is printed to stdout-a simple random walk fractal with a Gaussian/normal distributed increments would be the combinatorial probabilities, 0.5, 0.25, 0.125, 0.625, . . .

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsrootmean* program appears in Figure B.29.

B.2.29 tsrunmagnitude

Source tsrunmagnitude.c is for finding the magnitude of the run lengths in a time series. The value of each sample in the time series is stored, and subtracted from all other values in the time series, each point being tallied root mean square. The magnitude deviation is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsrunmagnitude* program appears in Figure B.30.

B.2.30 tsshannonvolume

Note: Conceptually, this program is used to "adjust" the Shannon probability of a stock by considering the volumes of trade in a time interval. Unfortunately, the results were not encouraging, and the concept was abandoned. It is left in the program inventory for future reference.

Tsshannonvolume.c, is for finding the fundamental Shannon probability of a time series, given a stocks value, and the number of shares traded, in each time interval. The value of a sample in the time series is divided by the volume, and added to the cumulative sum of the samples, and the square of the value, after dividing by the volume, is added to the sum of the squares to make a new time series by dividing both the cumulative sum and the square root of the sum of the squares by the number of samples for each sample. The new time series is printed to stdout. The time series printed to stdout is a tab delimited table of:

1. The average of a normalized increment, avg, and is computed by:

$$avg = \frac{v_t - v_{t-1}}{v_{t-1}} \cdot \frac{1}{N} \tag{B.117}$$

   where N is the trading volume, at time t.

2. The root mean square of the normalized increment, rms, and is computed by:

$$rms = \sqrt{\left(\frac{v_t - v_{t-1}}{v_{t-1}}\right)^2 \cdot \frac{1}{N}} \tag{B.118}$$

   where N is the trading volume, at time t.

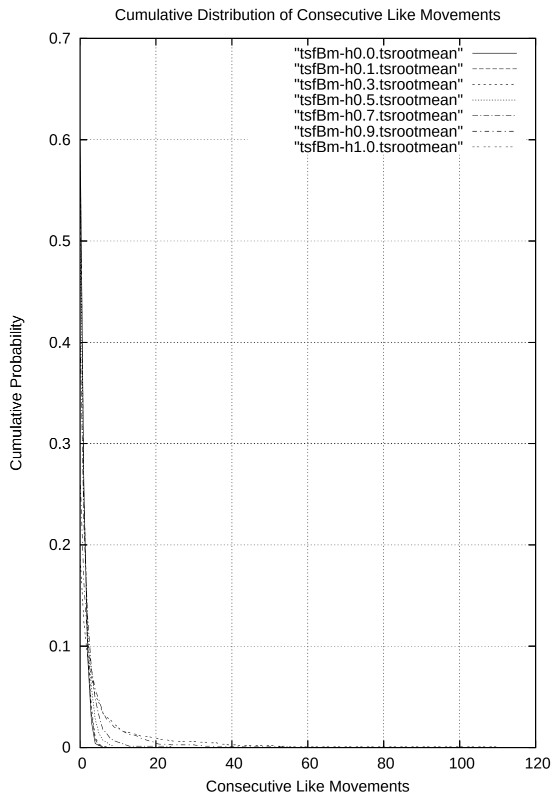

Figure B.29: Example output of the *tsrootmean* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

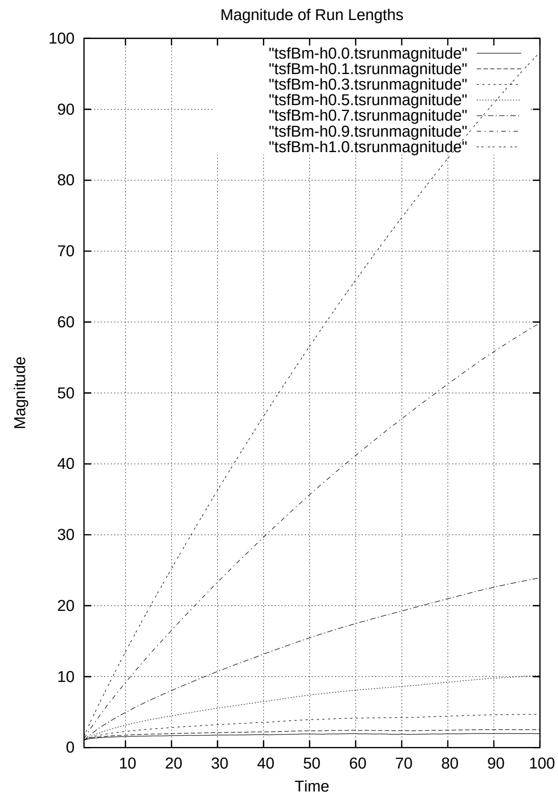

Figure B.30: Example output of the *tsrunmagnitude* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

3. The Shannon probability, P, as computed by:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.119}$$

4. The Shannon probability, P, as computed by:

$$P = \frac{\sqrt{avg} + 1}{2} \tag{B.120}$$

5. The Shannon probability, P, as computed by:

$$P = \frac{rms + 1}{2} \tag{B.121}$$

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a ‘#’ character as the first non white space character in the record. Data records must contain at least two fields, which is the data value of the sample, followed by the volume of the sample, but may contain many more fields-if the record contains many more fields, then the first field is regarded as the sample's time, and the next to the last field the value, with the last field as the sample's volume at that time.

Note that since the average of the normalized increments of a time sampled time series goes up linearly on the number of samples in a sampled interval, and the root mean square of the normalized increments goes up with the square root of the of the number of samples in a sampled interval, it would be reasonable to assume that that the average of the normalized increments would go up linearly with the trading volume of a stock, and the root mean square would to go up with the square root of the trading volume.

If we consider capital, V, invested in a savings account, and calculate the growth of the capital over time:

$$V_t = V_{t-1} \left(1 + a_t\right) \tag{B.122}$$

where a_t is the interest rate at time t, (usually a constant[2].) In equities, a_t is not constant, and varies—perhaps being negative at certain times, (meaning that the value of the equity decreased.) This fluctuation in an equity's value can be represented by modifying a_t in Equation B.122:

$$a_t = f_t F_t \tag{B.123}$$

where the product $f_t \cdot F_t$ is the fluctuation in the equity's value at time t.

An equity's value, over time, is similar to a simple tossed coin game [Sch91, pp. 128], where f_t is the fraction of a gambler's capital wagered on a toss of the coin, at time t, and F_t is a random variable[3], signifying whether the game was a win, or a loss, ie., whether the gambler's capital increased or decreased, and by how much. The amount the gambler's capital increased or decreased is $f_t \cdot F_t$. In general, F_t is a function of a random variable, with an average, over time, of avg_f, and a root mean square value, rms_f, of unity. Note that for simple, time invariant, compound interest, F_t has an average and root mean square, both being unity, and f_t is simply the interest rate, which is assumed to be constant. For a simple, single coin game, F_t is a fixed increment, (ie., either $+1$ or -1,) random generator. From an analytical perspective, it would be advantageous to measure the the statistical characteristics of the generator. Substituting Equation B.123 into Equation B.122[4]:

[2]For example, if $a = 0.06$, or 6%, then at the end of the first time interval the capital would have increased to 1.06 times its initial value. At the end of the second time interval it would be $(1.06)^2$, and so on. What Equation B.122 states is that the way to get the value, V in the next time interval is to multiply the current value by 1.06. Equation B.122 is nothing more than a "prescription," or a process to make an exponential, or "compound interest" mechanism. In general, exponentials can always be constructed by multiplying the current value of the exponential by a constant, to get the next value, which in turn, would be multiplied by the same constant to get the next value, and so on. Equation B.122 is nothing more than a construction of $V(t) = e^{kt}$ where $k = \ln(1 + a)$. The advantage of representing exponentials by the "prescription" defined in Equation B.122 is analytical expediency. For example, if you have data that is an exponential, the parameters, or constants, in Equation B.122 can be determined by simply reversing the "prescription," ie., subtracting the previous value, (at time $t - 1$,) from the current value, and dividing by the previous value would give the exponentiating constant, $(1 + a_t)$. This process of reversing the "prescription" is termed calculating the "normalized increments." (Increments are simply the difference between two values in the exponential, and normalized increments are this difference divided by the value of the exponential.) Naturally, since one usually has many data points over a time interval, the values can be averaged for better precision—there is a large mathematical infrastructure dedicated to precision enhancement, for example, least squares approximation to the normalized increments, and statistical estimation.

[3]"Random variable" means that the process, F_t, is random in nature, ie., there is no possibility of determining what the next value will be. However, F_t can be analyzed using statistical methods [Fed88, pp. 163], [Sch91, pp. 128]. For example, F_t typically has a Gaussian distribution for equity values [Cro95, pp. 249], in which case the it is termed a "fractional Brownian motion," or simply a "fractal" process. In the case of a single tossed coin, it is termed "fixed increment fractal," "Brownian," or "random walk" process. In any case, determination of the statistical characteristics of F_t are the essence of analysis. Fortunately, there is a large mathematical infrastructure dedicated to the subject. For example, F_t could be verified as having a Gaussian distribution using Chi—Square techniques. Frequently, it is convenient, from an analytical standpoint, to "model" F_t using a mathematically simpler process [Sch91, pp. 128]. For example, multiple iterations of tossing a coin can be used to approximate a Gaussian distribution, since the distribution of many tosses of a coin is binomial—which if the number of tosses is sufficient will represent a Gaussian distribution to within any required precision [Sch91, pp. 144], [Fed88, pp. 154].

[4]Equation B.124 is interesting in many other respects. For example, adding a single term, $m \cdot V_{t-1}$, to the equation results in $V_t = V_{t-1}\left(1 + f_t F_t + m \cdot V_{t-1}\right)$ which is the "logistic," or 'S' curve equation, (formally termed the "discreet time quadratic equation,") and has been used successfully in many unrelated fields such as manufacturing operations, market and economic forecasting, and analyzing disease

$$V_t = V_{t-1}(1 + f_t F_t) \tag{B.124}$$

and subtracting V_{t-1} from both sides:

$$V_t - V_{t-1} = V_{t-1}(1 + f_t F_t) - V_{t-1} \tag{B.125}$$

and dividing both sides by V_{t-1}:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_{t-1}(1 + f_t F_t) - V_{t-1}}{V_{t-1}} \tag{B.126}$$

and combining:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = (1 + f_t F_t) - 1 = f_t F_t \tag{B.127}$$

We now have a "prescription," or process, for calculating the characteristics of the random process that determines an equity's value. That process is, for each unit of time, subtract the value of the of the equity at the previous time from the value of the equity at the current time, and divide this by the value of the equity at the previous time. The root mean square[5] of these values are the root mean square of the random process. The average of these values are the average of the random process, avg_f. The root mean square of these values can be calculated by any convenient means, and will be represented by rms. The average of these values can be found by any convenient means, and will be represented by avg[6]. Therefore, if $f_t = f$, and does not vary over time:

$$rms = f \tag{B.128}$$

which, if there are sufficiently many samples, is a metric of the equity value's "volatility," and:

$$avg = f \cdot F_t \tag{B.129}$$

and if there are sufficiently many samples, the average of F_t is simply avg_f, or:

$$avg = f \cdot avg_f \tag{B.130}$$

which is a metric on the equity value's rate of "growth." Note that this is the "effective" compound interest rate from Equation B.122.

Equations B.128 and B.130 are important equations, since they can be used in portfolio management. For example, Equation B.128 states that the volatility of the capital invested in many equities, simultaneously, is calculated as the

epidemics [Mod92, pp. 131]. There is continuing research into the application of an additional "non-linear" term in Equation B.124 to model equity value non-linearities. Although there have been modest successes, to date, the successes have not proved to be exploitable in a systematic fashion [Pet91, pp. 133]. The reason for the interest is that the logistic equation can exhibit a wide variety of behaviors, among them, "chaotic." Interestingly, chaotic behavior is mechanistic, but not "long term" predictable into the future. A good example of such a system is the weather. It is an important concept that compound interest, the logistic function, and fractals are all closely related.

[5] In this section, "root mean square" is used to mean the variance of the normalized increments. In Brownian motion fractals, this is computed by $\sigma_t otal^2 = \sigma_1^2 + \sigma_2^2 + \cdots$ However, in many fractals, the variances are not calculated by adding the squares, (ie., a power of 2,) of the values—the power may be "fractional," ie., $3/2$ instead of 2, for example [Sch91, pp. 130], [Fed88, pp. 178]. However, as a first order approximation, the variances of the normalized increments of equity values can successfully be added root mean square [Cro95, kpp. 250]. The so called "Hurst" coefficient, which can be measured, determines the process to be used. The Hurst coefficient is range of the equity values over a time interval, divided by the standard deviation of the values over the interval, and its determination is commonly called "R/S" analysis. As pointed out in [Sch91, pp. 157] the errors committed in such simplified assumptions can be significant—however, for analysis of equities, squaring the variances seems to be a reasonable simplification.

[6] For example, many calculators have averaging and root mean square functionality, as do many spreadsheet programs—additionally, there are computer source codes available for both. See the programs *tsrms* and *tsavg*. The method used is not consequential.

root mean square of the individual volatility of the equities. Equation B.130 states that the growths in the same equity values add together linearly[7].

Dividing Equation B.130 by Equation B.128 results in the two f's canceling, or:

$$\frac{avg}{rms} = avg_f \tag{B.131}$$

There may be analytical advantages to "model" avg_f as a simple tossed coin game, (either played with a single coin, or multiple coins, ie., many coins played at one time, or a single coin played many times[8].) The number of wins minus the number of losses, in many iterations of a single coin tossing game would be:

$$P - (1 - P) = 2P - 1 \tag{B.132}$$

where P is the probability of a win for the tossed coin. (This probability is traditionally termed, the "Shannon probability" of a win.) Note that from the definition of F_t above, that $P = avg_f$. For a fair coin, (ie., one that comes up with a win 50% of the time,) $P = 0.5$, and there is no advantage, in the long run, to playing the game. However, if $P > 0.5$, then the optimal fraction of capital wagered on each iteration of the single coin tossing game, f, would be $2P - 1$. Note that if multiple coins were used for each iteration of the game, we would expect that the volatility of the gambler's capital to increase as the square root of the number of coins used, and the growth to increase linearly with the number of coins used, irregardless of whether many coins were tossed at once, or one coin was tossed many times, (ie., our random generator, F_t would assume a binomial distribution—and if the number of coins was very large, then F_t would assume, essentially, a Gaussian distribution.) Many equities have a Gaussian distribution for the random

[7]There are significant implications do to the fact that equity volatilities are calculated root mean square. For example, if capital is invested in N many equities, concurrently, then the volatility of the capital will be $\frac{1}{\sqrt{N}} \cdot rms$ of an individual equity's volatility, rms, provided all the equites have similar statistical characteristics. But the growth in the capital will be unaffected, ie., it would be statistically similar to investing all the capital in only one equity. What this means is that capital, or portfolio, volatility can be minimized without effecting portfolio growth—ie., volatility risk can addressed. Further, it does not make any difference, as far as portfolio value growth is concerned, whether the individual equities are invested in concurrently, or serially, ie., if one invested in 10 different equities for 100 days, concurrently, or one could invest in only one equity, for 10 days, and then the next equity for the next 10 days, and so on. The capital growth would have the same characteristics for both agendas. (Note that the concurrent agenda is superior since the volatility of the capital will be the root mean square of the individual equity volatilities divided by the square root of the number of equities. In the serial agenda, the volatility of the capital will be simply the root mean square of the individual equity volatilities.) Almost all equity wagering strategies will consist of optimizing variations on combinations of serial and concurrent agendas. There are further applications. For example, Equation B.127 could be modified by dividing both the normalized increments, and the square of the normalized increments by the daily trading volume. The quotient of the normalized increments divided by the trading volume is the instantaneous growth, avg_f, of the equity, on a per-share basis. Likewise, the square root of the square of the normalized increments divided by the daily trading volume is the instantaneous root mean square, rms_f, of the equity on a per-share basis, ie., its instantaneous volatility of the equity. (Note that these instantaneous values are the statistical characteristics of the equity on a per-share bases, similar to a coin toss, and not on time.) Additionally, it can be shown that the range—the maximum minus the minimum—of an equity's value over a time interval will increase with the square root of of the size of the interval of time [Fed88, pp. 178]. Also, it can be shown that the number of expected stock value "high and low" transitions scales with the square root of time, meaning that the probability of an equity value "high or low" exceeding a given time interval is proportional to the square root of the time interval [Sch91, pp. 153].

[8]Here the "model" is to consider two black boxes, one with a stock "ticker" in it, and the other with a casino game of a tossed coin in it. One could then either invest in the equity, or, alternatively, invest in the tossed coin game by buying many casino chips, which constitutes the starting capital for the tossed coin game. Later, either the equity is sold, or the chips "cashed in." If the statistics of the equity value over time is similar to the statistics of the coin game's capital, over time, then there is no way to determine which box has the equity, or the tossed coin game. The advantage of this model is that gambling games, such as the tossed coin, have a large analytical infrastructure, which, if the two black boxes are statistically the same, can be used in the analysis of equities. The concept is that if the value of the equity, over time, is statistically similar to the coin game's capital, over time, then the analysis of the coin game can be used on equity values. Note that in the case of the equity, the terms in $f_t \cdot F_t$ can not be separated. In this case, $f = rms$ is the fraction of the equity's value, at any time, that is "at risk," of being lost, ie., this is the portion of a equity's value that is to be "risk managed." This is usually addressed through probabilistic methods, as outlined below in the discussion of Shannon probabilities, where an optimal wagering strategy is determined. In the case of the tossed coin game, the optimal wagering strategy is to bet a fraction of the capital that is equal to $f = rms = 2P - 1$ [Sch91, pp. 128, 151], where P is the Shannon probability. In the case of the equity, since $f = rms$ is not subject to manipulation, the strategy is to select equities that closely approximate this optimization, and the equity's value, over time, on the average, would increase in a similar fashion to the coin game. The growth of either investment would be equal to $avg = rms^2$, on average, for each iteration of the coin game, or time unit of equity investment. This is an interesting concept from risk management since it maximizes the gain in the capital, while, simultaneously, minimizing risk exposure to the capital.

process, F_t. It may be advantageous to determine the Shannon probability to analyze equity investment strategies. From Equation B.131:

$$\frac{avg}{rms} = avg_f = 2P - 1 \tag{B.133}$$

or:

$$\frac{avg}{rms} + 1 = 2P \tag{B.134}$$

and:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.135}$$

where only the average and root mean square of the normalized increments need to be measured, using the "prescription" or process outlined above.

Interestingly, what Equation B.133 states is that the "best" equity investment is not, necessarily, the equity that has the largest average growth, avg_f. The best equity investment is the equity that has the largest growth, while simultaneously having the smallest volatility. In point of fact, the optimal decision criteria is to choose the equity that has the largest *ratio* of growth to volatility, where the volatility is measured by computing the root mean square of the normalized increments, and the growth is computed by averaging the normalized increments.

We now have a "first order prescription" that enables us to analyze fluctuations in equity values, although we have not explained why equity values fluctuate. For a formal presentation on the subject, see the bibliography in [Art95] which, also, offers non-mathematical insight into the explanation.

Consider a very simple equity market, with only two people holding equities. Equity value "arbitration" (ie., how equity values are determined,) is handled by one person posting (to a bulletin board,) a willingness to sell a given number of stocks at a given price, to the other person. There is no other communication between the two people. If the other person buys the stock, then that is the value of the stock at that time. Obviously, the other person will not buy the stock if the price posted is too high—even if ownership of the stock is desired. For example, the other person could simply decide to wait in hopes that a favorable price will be offered in the future. So the stock seller must not post a price that the other person would consider too high, and the other person would not buy at the price if it is reasoned that the seller's pricing strategy will be to lower the offering price in the future, which would be a reasonable deduction if the posted price is considered too high. What this means is that the seller must consider not only the behavior of the other person, but what the other person thinks the seller's behavior will be, ie., the seller must base the pricing strategy on the seller's pricing strategy. Such convoluted logical processes are termed "self referential," and the implication is that the market can never operate in a consistent fashion that can be the subject of deductive analysis [Pen89, pp. 101][9]. As pointed out by [Art95, Abstract], these types of indeterminacies pervade economics[10].

[9] Penrose, referencing Russell's paradox, presents a very good example of logical contradiction in a self-referential system. Consider a library of books. The librarian notes that some books in the library contain their titles, and some do not, and wants to add two index books to the library, labeled "A" and "B," respectively; the "A" book will contain the list of all of the titles of books in the library that contain their titles; and the "B" book will contain the list of all of the titles of the books in the library that do not contain their titles. Now, clearly, all book titles will go into either the "A" book, or the "B" book, respectively, depending on whether it contains its title, or not. Now, consider in which book, the "A" book or the "B" book, the title of the "B" book is going to be placed—no matter which book the title is placed, it will be contradictory with the rules. And, if you leave it out, the two books will be incomplete.)

[10] [Art95] cites the "El Farol Bar" problem as an example. Assume one hundred people must decide independently each week whether go to the bar. The rule is that if a person predicts that more than, say, 60 will attend, it will be too crowded, and the person will stay home; if less than 60 is predicted, the person will go to the bar. As trivial as this seems, it destroys the possibility of long-run shared, rational expectations. If all believe *few* will go, then *all* will go, thus invalidating the expectations. And, if all believe *many* will go, then *none* will go, likewise invalidating those expectations. Predictions of how many will attend depend on others' predictions, and others' predictions of others' predictions. Once again, there is no rational means to arrive at deduced *a-priori* predictions. The important concept is that expectation formation is a self-referential process in systems involving many agents with incomplete information about the future behavior of the other agents. The problem of logically forming expectations then becomes ill-defined, and rational deduction, can not be consistent or complete. This indeterminacy of expectation-formation is by no means an anomaly within the real economy. On the contrary, it pervades all of economics and game theory [Art95].

What the two players do, in absence of a deductively consistent and complete theory of the market, is to rely on inductive reasoning. They form subjective expectations or hypotheses about how the market operates. These expectations and hypothesis are constantly formulated and changed, in a world that forms from others' subjective expectations. What this means is that equity values will fluctuate as the expectations and hypothesis concerning the future of equity values change[11]. The fluctuations created by these indeterminacies in the equity market are represented by the term $f_t F_t$ in Equation B.124, and since there are many such indeterminacies, we would anticipate F_t to have a Gaussian distribution.

This is a rather interesting conclusion, since analyzing the actions of aggregately many "agents," each operating on subjective hypothesis in a market that is deductively indeterminate, can result in a system that can not only be analyzed, but optimized.

The only remaining derivation is to show that the optimal wagering strategy is, as cited above:

$$f = rms = 2P - 1 \tag{B.136}$$

where f is the fraction of a gambler's capital wagered on each toss of a coin that has a Shannon probability, P, of winning.

Following [Rez94, pp. 450], consider that the gambler has a private wire into the future who places wagers on the outcomes of a game of chance. We assume that the side information which he receives has a probability, P, of being true, and of $1 - P$, of being false. Let the original capital of gambler be $V(0)$, and $V(n)$ his capital after the n'th wager. Since the gambler is not certain that the side information is entirely reliable, he places only a fraction, f, of his capital on each wager. Thus, subsequent to n many wagers, assuming the independence of successive tips from the future, his capital is:

$$V(n) = (1 + f)^w (1 - f)^l V(0) \tag{B.137}$$

where w is the number of times he won, and $l = n - w$, the number of times he lost. These numbers are, in general, values taken by two random variables, denoted by W and L. According to the law of large numbers:

$$\lim_{n \to \infty} \frac{1}{n} W = P \tag{B.138}$$

and:

$$\lim_{n \to \infty} \frac{1}{n} L = q = 1 - P \tag{B.139}$$

The problem with which the gambler is faced is the determination of f leading to the maximum of the average exponential rate of growth of his capital. That is, he wishes to maximize the value of:

$$G = \lim_{n \to \infty} \frac{1}{n} \ln \frac{V(n)}{V(0)} \tag{B.140}$$

with respect to f, assuming a fixed original capital and specified P:

$$G = \lim_{n \to \infty} \frac{W}{n} \ln(1 + f) + \frac{L}{n} \ln(1 - f) \tag{B.141}$$

or:

$$G = P \ln(1 + f) + q \ln(1 - f) \tag{B.142}$$

[11] Interestingly, the system described is a stable system, ie., if the players have a hypothesis that changing equity positions may be of benefit, then the equity values will fluctuate—a self fulfilling prophecy. Not all such systems are stable, however. Suppose that one or both players suddenly discover that equity values can be "timed," ie., there are certain times when equities can be purchased, and chances are that the equity values will increase in the very near future. This means that at certain times, the equites would have more value, which would soon be arbitrated away. Such a scenario would not be stable.

which, by taking the derivative with respect to f, and equating to zero, can be shown to have a maxima when:

$$\frac{dG}{df} = P\left(1+f\right)^{P-1}\left(1-f\right)^{1-P} - \left(1-P\right)\left(1-f\right)^{1-P-1}\left(1+f\right)^{P} = 0 \tag{B.143}$$

combining terms:

$$P\left(1+f\right)^{P-1}\left(1-f\right)^{1-P} - \left(1-P\right)\left(1-f\right)^{P}\left(1+f\right)^{P} = 0 \tag{B.144}$$

and splitting:

$$P\left(1+f\right)^{P-1}\left(1-f\right)^{1-P} = \left(1-P\right)\left(1-f\right)^{P}\left(1+f\right)^{P} \tag{B.145}$$

then taking the logarithm of both sides:

$$\ln\left(P\right) + \left(P-1\right)\ln\left(1+f\right) + \left(1-P\right)\ln\left(1-f\right) = \ln\left(1-P\right) - P\ln\left(1-f\right) + P\ln\left(1+f\right) \tag{B.146}$$

and combining terms:

$$\left(P-1\right)\ln\left(1+f\right) - P\ln\left(1+f\right) + \left(1-P\right)\ln\left(1-f\right) + P\ln\left(1-f\right) = \ln\left(1-P\right) - \ln\left(P\right) \tag{B.147}$$

or:

$$\ln\left(1-f\right) - \ln\left(1+f\right) = \ln\left(1-P\right) - \ln\left(P\right) \tag{B.148}$$

and performing the logarithmic operations:

$$\ln\left(\frac{1-f}{1+f}\right) = \ln\left(\frac{1-P}{P}\right) \tag{B.149}$$

and exponentiating:

$$\frac{1-f}{1+f} = \frac{1-P}{P} \tag{B.150}$$

which reduces to:

$$P\left(1-f\right) = \left(1-P\right)\left(1+f\right) \tag{B.151}$$

and expanding:

$$P - Pf = 1 - Pf - P + f \tag{B.152}$$

or:

$$P = 1 - P + f \tag{B.153}$$

and, finally:

$$f = 2P - 1 \tag{B.154}$$

B.2.31 tsshannonfundamental

Source tsshannonfundamental.c, is for finding the fundamental Shannon probability of a time series, given a stocks value, and the number of shares traded. The value of a sample in the time series is added to the cumulative sum of the samples, and the square of the value is added to the sum of the squares to make a new time series by dividing the cumulative sum by the number of samples, and the square root of the sum of the squares divided by the number of samples for each sample. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least two fields, which is the data value of the sample, followed by the volume of the sample, but may contain many more fields—if the record contains many more fields, then the first field is regarded as the sample's time, and the next to the last field the value, with the last field as the sample's volume at that time.

Note that since the average of the normalized increments of a time sampled time series goes up linearly on the number of samples in a sampled interval, and the root mean square of the normalized increments go up with the square root of the of the number of samples in a sampled interval, it would be reasonable to assume that that the average of the normalized increments would go up linearly with the trading volume of a stock, and the root mean square to go up with the square root of the trading volume of a stock.

B.2.32 tsnumber

Source tsnumber.c, is for numbering the records of a time series. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

B.2.33 tsunshannon

Source tsunshannon.c, is for calculating the Shannon information capacity, (and optimal gain,) given the Shannon probability. See [Sch91, pp. 128, 151]

This program is the inverse of the *tsshannon* program, and solves the equation:

$$C\left(p\right) = 1 + p ln_2\left(p\right) + \left(1 - p\right) ln_2\left(1 - p\right) \tag{B.155}$$

where the optimal gain is calculated as $2^{C(p)}$, and f, the fraction of capital wagered, is $2p - 1$.
From [Sch91, pp. 151]:

$p = 0.55$, $2^{C(0.55)} = 0.005$, (probably a typo, meaning 1.005) by this program, $2^{C(0.550000)} = 2^{0.007226} = 1.005021$.

B.2.34 tskurtosis

Source tskurtosis.c is for finding the coefficient of excess kurtosis of a time series. The value of a sample in the time series is analyzed to find the running coefficient of excess kurtosis to make a new time series. The new time series is printed to stdout.

The method used is described in [She69, pp. 563]:

$$a_4 = \frac{m_4}{m_2^2} \tag{B.156}$$

where:

$$m_r = \frac{1}{n}\sum_{i=1}^{n} f_i(x_i)^r \tag{B.157}$$

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tskurtosis* program appears in Figure B.31.

B.2.35 tskurtosiswindow

Source tskurtosiswindow.c is for finding the coefficient of excess kurtosis of a time series. The value of a sample in the time series is analyzed to find the running coefficient of excess kurtosis to make a new time series. The new time series is printed to stdout.

The method used is described in [She69, pp. 563]:

$$a_4 = \frac{m_4}{m_2^2} \tag{B.158}$$

where:

$$m_r = \frac{1}{n}\sum_{i=1}^{n} f_i(x_i)^r \tag{B.159}$$

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tskurtosiswindow* program appears in Figure B.32.

B.2.36 tsgain

Source tsgain.c is for finding the gain of a time series. The value of a sample in the time series added to the cumulative sum of the samples, and is squared and added to the cumulative sum of squares, the Shannon probability, P, calculated using:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.160}$$

where rms is the root mean square of the marginal returns, and avg is the average of the marginal returns, and the gain, G, calculated using:

$$G = (1 + rms)^P \cdot (1 - rms)^{P-1} \tag{B.161}$$

to make a new time series. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many

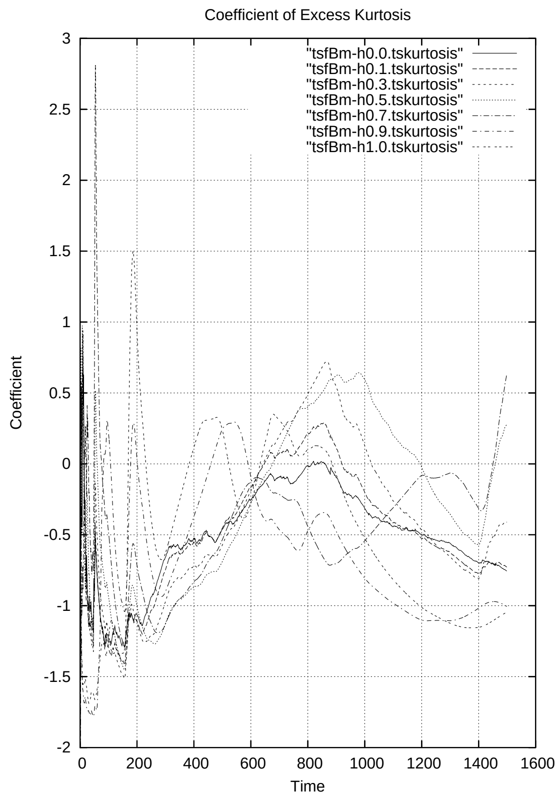

Figure B.31: Example output of the *tskurtosis* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

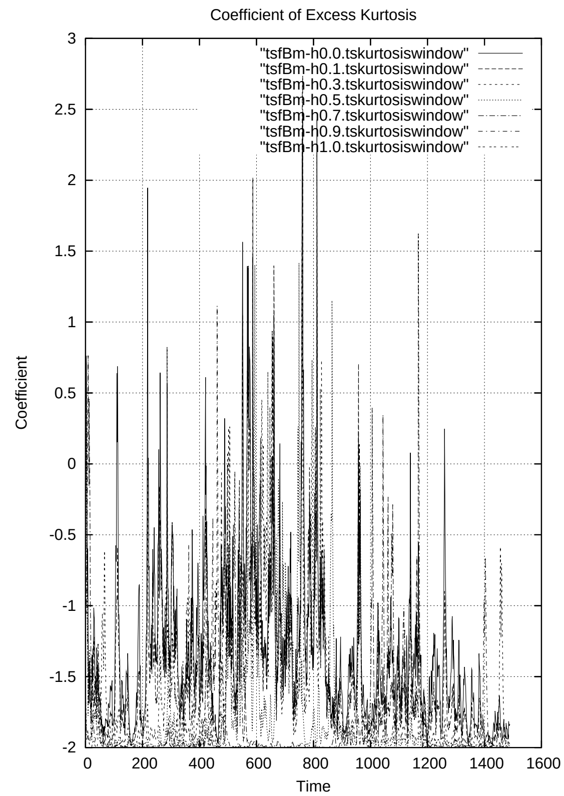

Figure B.32: Example output of the *tskurtosis* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsgain* program appears in Figure B.33.

B.2.37 tsgainwindow

Source tsgainwindow.c is for finding the windowed gain of a time series. The value of a sample in the time series added to the cumulative sum of the samples, and is squared and added to the cumulative sum of squares, the Shannon probability, P, calculated using:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.162}$$

where rms is the root mean square of the marginal returns, and avg is the average of the marginal returns, and the gain, G, calculated using:

$$G = (1 + rms)^P \cdot (1 - rms)^{P-1} \tag{B.163}$$

to make a new time series. The new time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsgainwindow* program appears in Figure B.34.

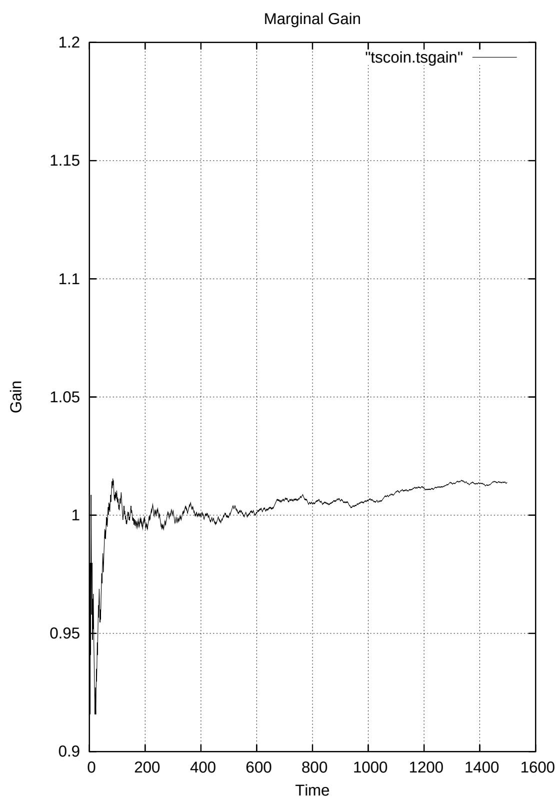

Figure B.33: Example output of the *tsgain* program, using the output of the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

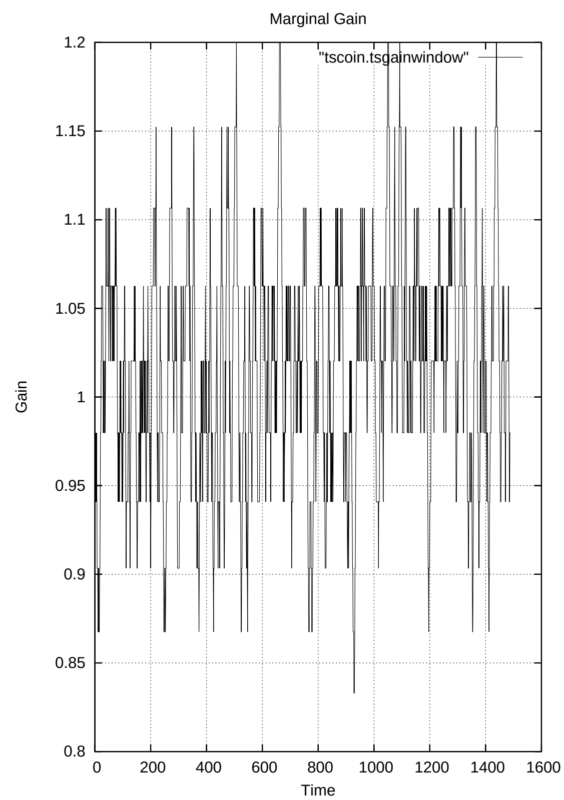

Figure B.34: Example output of the *tsgainwindow* program, using the output of the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

B.2.38 tsscalederivative

Source tsscalederivative.c, for taking the derivative of a time series. The value of a sample in the time series is subtracted from the previous sample in the time series. The derivative time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least two fields, which are the time followed by the data value of the sample at that time, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsscalederivative* program appears in Figure B.35.

B.2.39 tsrootmeanscale

Source tsrootmeanscale.c is for finding the root mean of a time series, at different scales. The number of consecutive samples of like movements in the time series is tallied, at different scales, and the resultant value of the distribution, as calculated by using the first value in the distribution, the running mean of the distribution, and the least squares fit of the distribution, is printed to stdout-a simple random walk fractal with a Gaussian/normal distributed increments would be the combinatorial probabilities, 0.5, 0.25, 0.125, 0.625, . . .

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsrootmeanscale* program appears in Figure B.36.

B.2.40 tskalman

Source tskalman.c for taking the Kalman filtered average of a time series. The n'th running Kalman filtered linear average, A, of a time series is calculated by:

$$A_n = \frac{n-1}{n} A_{n-1} + \frac{1}{n} a_n \tag{B.164}$$

where a is the n'th value in the time series. The new time series of the running Kalman filtered average is printed to stdout.

Note the similarity to the running average:

$$A_n = \frac{1}{n} \left(a_1 + a_2 + \cdot + a_n \right) \tag{B.165}$$

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tskalman* program appears in Figure B.37.

B.2.41 tsroot

Source tsroot.c is for finding the root of a time series. The range, as a function of time, is summed, for each and every point in the time series. For example, the output should be proportional to the $\sqrt{t}$ for a Brownian motion fractal.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many

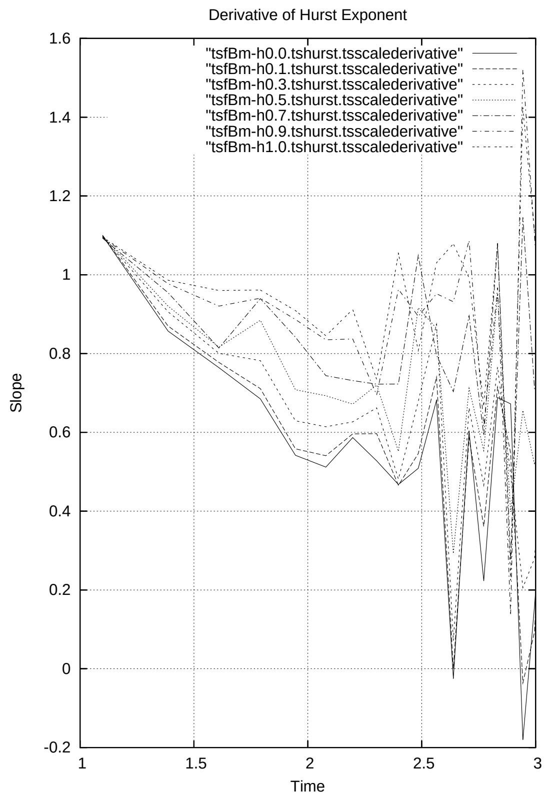

Figure B.35: Example output of the *tsscalederivative* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program, and the *tshurst* program in Figure B.4.

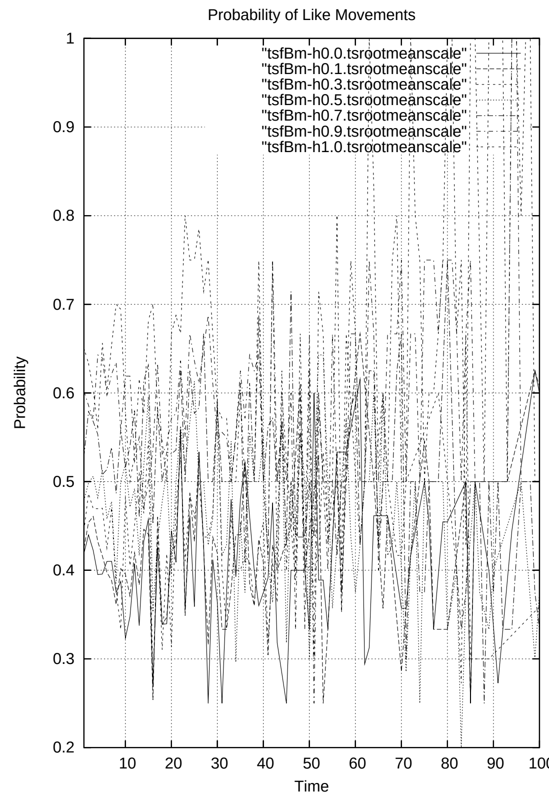

Figure B.36: Example output of the *tsrootmeanscale* program, using simulated Hurst coefficients of 0.0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, as simulated by the *tsfBm* program.

fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsroot* program appears in Figure B.38.

B.3 Fractal Time Series Simulation Utilities

Note: these programs use the following functions from other references:

ran1: which returns a uniform random deviate between 0.0 and 1.0. See [PFTV88, pp. 210], referencing Knuth.

gasdev: which returns a normally distributed deviate with zero mean and unit variance, using ran1 () as the source of uniform deviates. See [PFTV88, pp. 217].

gammln: which returns the log of the results of the gamma function. See [PFTV88, pp. 168].

romberg: which returns the integral of a function, using iterate (), and interpolate (). See [PFTV88, pp. 124].

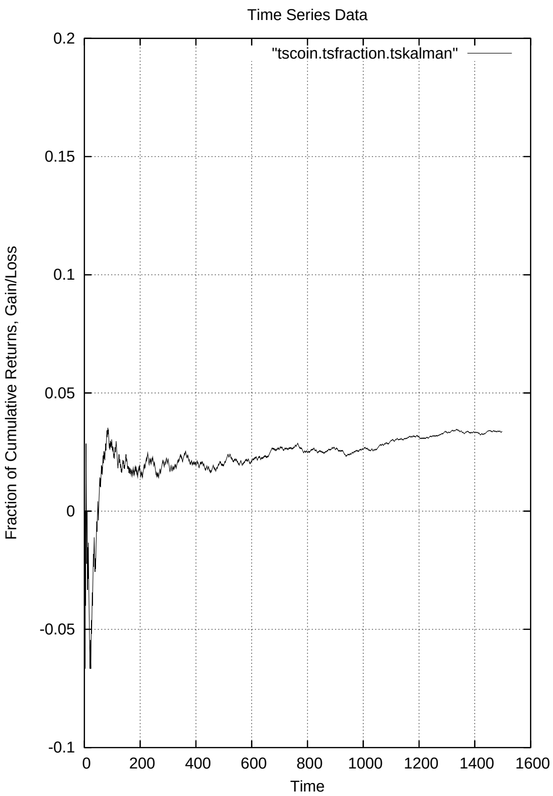

Figure B.37: Example output of the *tskalman* program, the input was produced by the *tsfraction* program, shown in Figure B.8, which used the output of the *tscoin* program, with a Shannon probability of 0.6, and is shown in Figure B.46 in Section B.3.8.

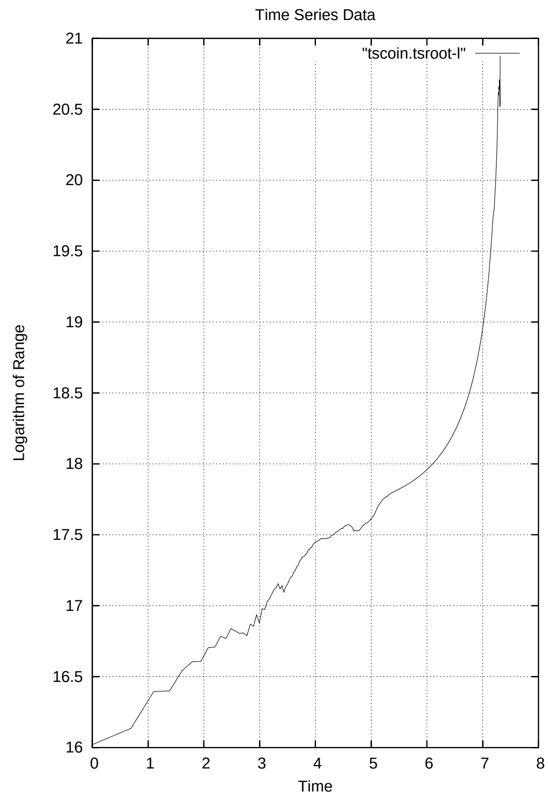

Figure B.38: Example output of the *tsroot* program, taking the logarithm of the range of the file produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46 in Section B.3.8.

iterate: which computes the n'th stage of refinement of an extended iterate rule using trapezoid iteration. See [PFTV88, pp. 120].

interpolate: which interpolates the y value for point x using polynomial interpolation. See [PFTV88, pp. 90].

B.3.1 tsbrownian

Source tsbrownian.c, brownian noise generator—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution by running a cumulative sum on white noise. See [Sch91, pp. 128].

An example output from the *tsbrownian* program appears in Figure B.39.

B.3.2 tsblack

Source tsblack.c, black noise generator—generates a time series. The idea is to produce a $1/f$ cubed power spectrum distribution by running a cumulative sum on pink noise which is made by running a cumulative sum on relaxation processes which are generated by a white noise generator. See [Sch91, pp. 126].

An example output from the *tsblack* program appears in Figure B.40.

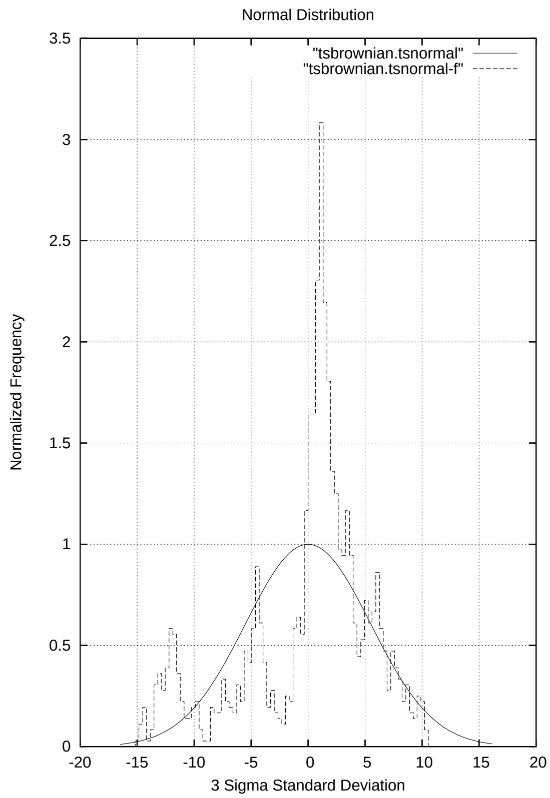

Figure B.39: Example output of the *tsbrownian* program, using 1500 records. This is a plot of the frequency histogram.

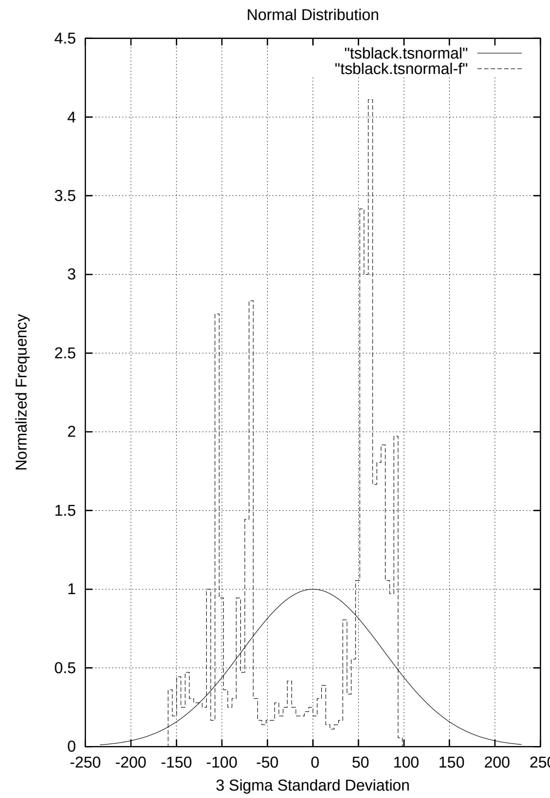

Figure B.40: Example output of the *tsblack* program, using 1500 records. This is a plot of the frequency histogram.

B.3.3 tsfractional

Source tsfractional.c, fractional brownian noise generator—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution by running a cumulative sum on a Gaussian power spectrum distribution. See [Fed88, pp. 172].

An example output from the *tsfractional* program appears in Figure B.41.

B.3.4 tsgaussian

Source tsgaussian.c, Gaussian noise generator—generates a time series. The idea is to produce a Gaussian power spectrum distribution.

An example output from the *tsgaussian* program appears in Figure B.42.

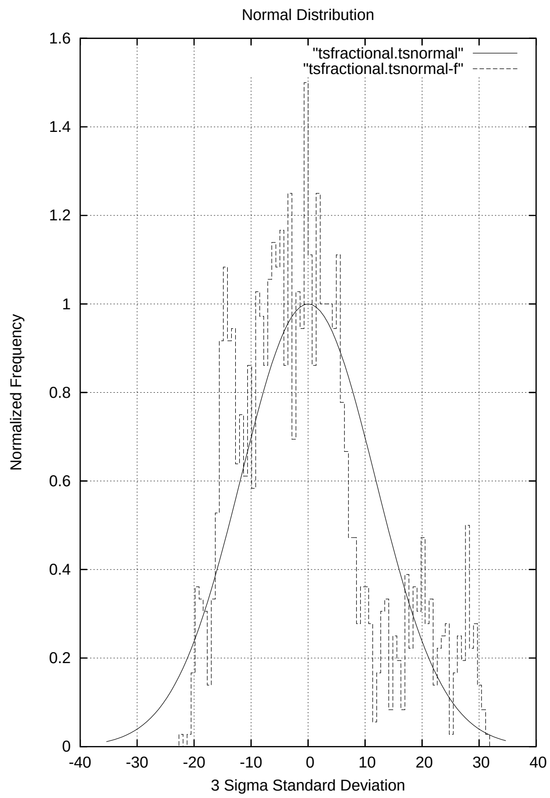

Figure B.41: Example output of the *tsfractional* program, using 1500 records. This is a plot of the frequency histogram.

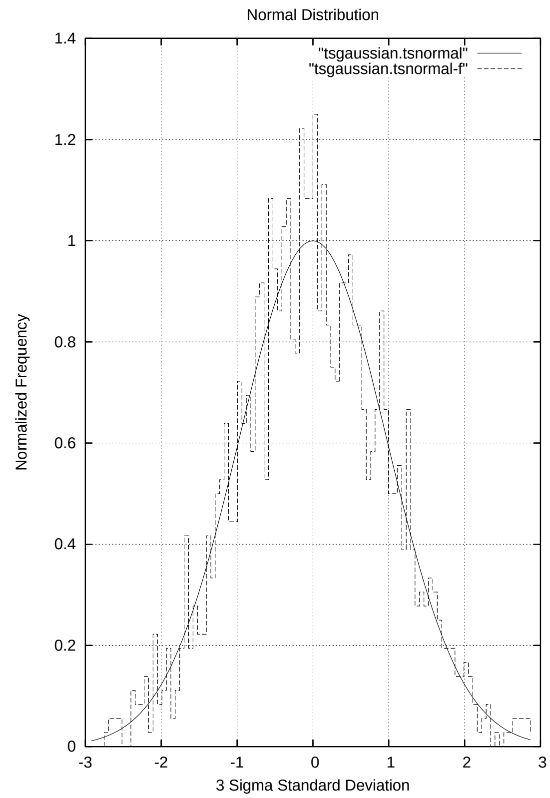

Figure B.42: Example output of the *tsgaussian* program, using 1500 records. This is a plot of the frequency histogram.

B.3.5 tswhite

Source tswhite.c, white noise generator—generates a time series. The idea is to produce a flat power spectrum distribution.

An example output from the *tswhite* program appears in Figure B.43.

B.3.6 tspink

Source tspink.c, pink noise generator—generates a time series. The idea is to produce a $1/f$ power spectrum distribution by running a cumulative sum on relaxation processes which are generated by a white noise generator. See [Sch91, pp. 122].

An example output from the *tspink* program appears in Figure B.44.

B.3.7 tsunfairbrownian

Source tsunfairbrownian.c, unfair returns of a time series. The idea is to produce the returns of a time series which is weighted unfairly, by a Shannon probability, p, or alternately, a fraction of reserves to be wagered on each time

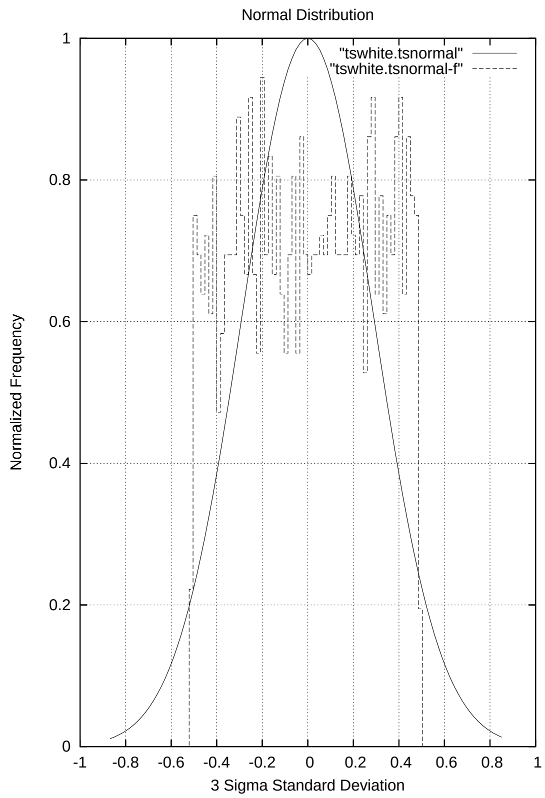

Figure B.43: Example output of the *tswhite* program, using 1500 records. This is a plot of the frequency histogram.

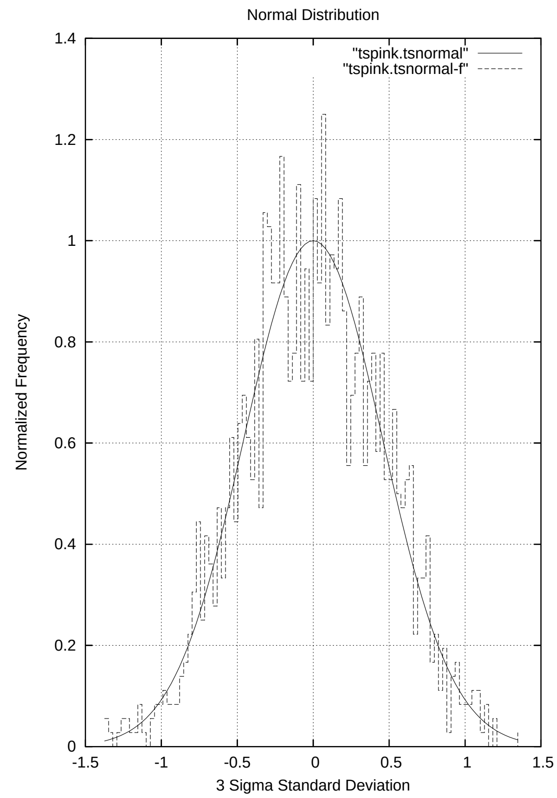

Figure B.44: Example output of the *tspink* program, using 1500 records. This is a plot of the frequency histogram.

increment. The input time series is presumed to have a Brownian distribution. The main function of this program is regression scenario verification—given an empirical time series, a Shannon probability, or a "wager" fraction, (which were probably derived from the program *tsshannon*,) speculative market pro forma performance can be analyzed. The cumulative sum process is Brownian in nature.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsunfairbrownian* program appears in Figure B.45.

B.3.8 tscoin

Source tscoin.c, brownian noise generator, with unfair bias, and cumulative sum—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution by running a cumulative sum on white noise. The program accepts an unfair bias and a wager factor. See [Sch91, pp. 128].

The discreet time formula is:

$$x_t = x_{t-1} + f \cdot R \cdot x_{t-1} \tag{B.166}$$

where f is the fraction of the capital to be wagered, and R is a uniform random deviate between 0.0 and 1.0, with the mean offset appropriately to provide a Shannon probability, P. For the logistic, function, the discreet time formula is:

$$x_t = x_{t-1} + f \cdot R \cdot x_{t-1} + n \cdot x_{t-1}^2 \tag{B.167}$$

An example output from the *tscoin* program appears in Figure B.46.

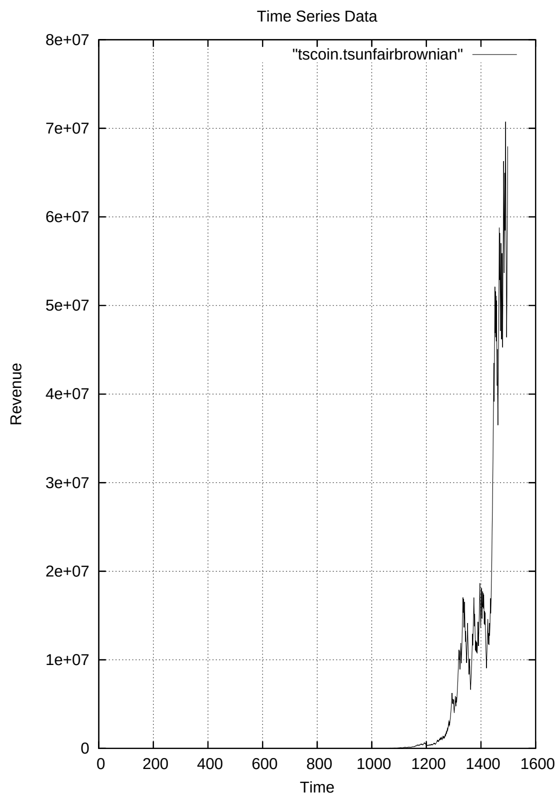

Figure B.45: Example output of the *tsunfairbrownian* program, changing the wager fraction from 0.2 to 0.1. The original time series data set was produced by the *tscoin* program using a Shannon probability of 0.6, and is shown in Figure B.46, corresponding to a wager fraction, f, of $f = 2P - 1 = 2 \cdot 0.6 - 1 = 0.2$.

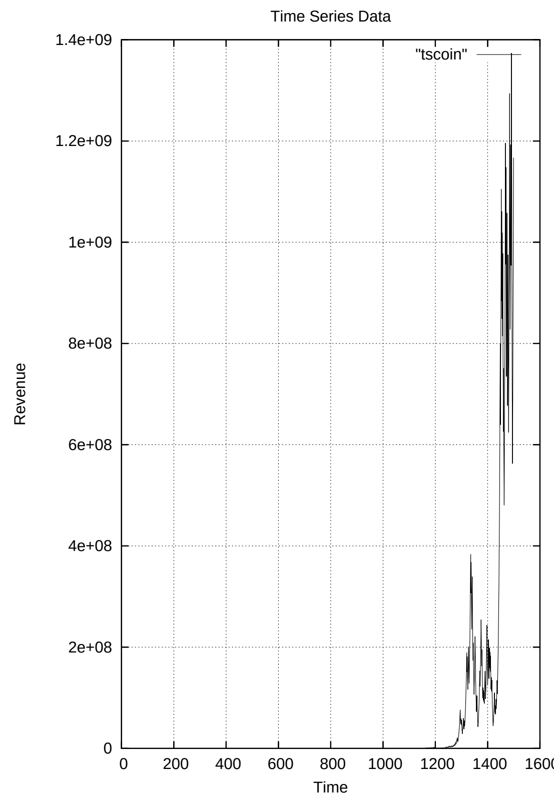

Figure B.46: Example output of the *tscoin* program, using a Shannon probability of 0.6.

B.3.9 tscoins

Source tscoins.c, fractional brownian noise generator, with unfair bias, and cumulative sum—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution by running a cumulative sum on a Gaussian

power spectrum distribution. The program accepts an unfair bias and a wager factor. See [Fed88, pp. 172]. Uses Newton—Raphson method for an iterative solution for the probability, p.
The discreet time formula is:

$$x_t = x_{t-1} + f \cdot R \cdot x_{t-1} \tag{B.168}$$

where f is the fraction of the capital to be wagered, and R is a Gaussian function, with the mean offset appropriately to provide a Shannon probability, P. For the logistic function, the discreet time formula is:

$$x_t = x_{t-1} + f \cdot R \cdot x_{t-1} + n \cdot x_{t-1}^2 \tag{B.169}$$

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

The name, tscoins, was chosen since pitching many coins, at once, and counting the number of heads, many times, will approach a gaussian distribution, if the number of coins is large, and the number of times is large. [Fed88, pp. 154]
The general outline of this program is:

1. given the Shannon probability, compute the abscissa value that divides the area under the normal curve, into two sections, such that the area to the left of the value, divided by the total area under the normal curve is the Shannon probability—a Newton-Raphson iterated approach using Romberg integration to find the area is used for this
2. for each record:
   (a) compute a gaussian distributed random number
   (b) add the computed abscissa value to the gaussian distributed number
   (c) multiply this number by the fraction of cumulative sum to be wagered
   (d) multiply this number by the cumulative sum
   (e) add this number to the cumulative sum

This program will require finding the value of the normal function, given the standard deviation. The method used is to use Romberg/trapezoid integration to numerically solve for the value.

This program will require finding the functional inverse of the normal, ie., Gaussian, function. The method used is to use Romberg/trapezoid integration to numerically solve the equation:

$$F(x) = \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 \tag{B.170}$$

which has the derivative:

$$f(x) = \frac{1}{2\pi} e^{\frac{-x^2}{2}} \tag{B.171}$$

Since $F(x)$ is known, and it is desired to find x,

$$F(x) - \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 = P(x) = 0 \tag{B.172}$$

and the Newton-Raphson method of finding roots would be:

$$P_{n+1} = P_n - \frac{P(x)}{f(x)} \tag{B.173}$$

An example output from the *tscoins* program appears in Figure B.47. The distribution of the normalized increments appears in Figure B.48.

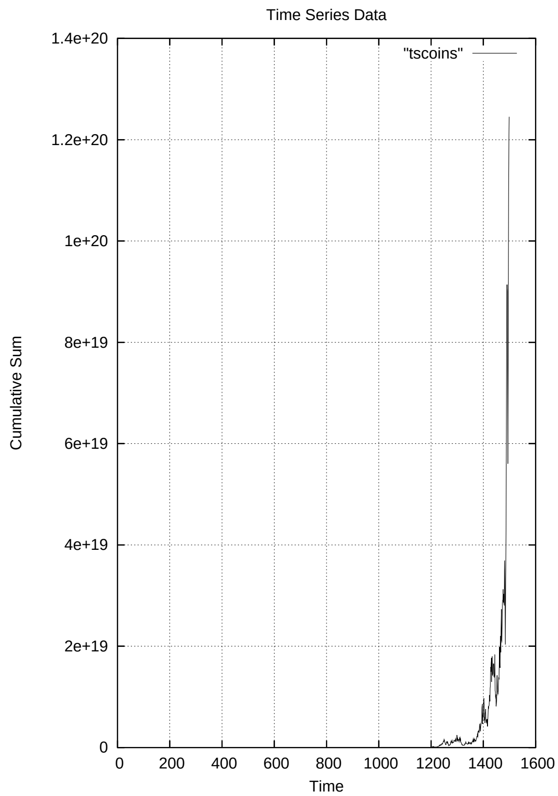

Figure B.47: Example output of the *tscoins* program, using a Shannon probability of 0.6.

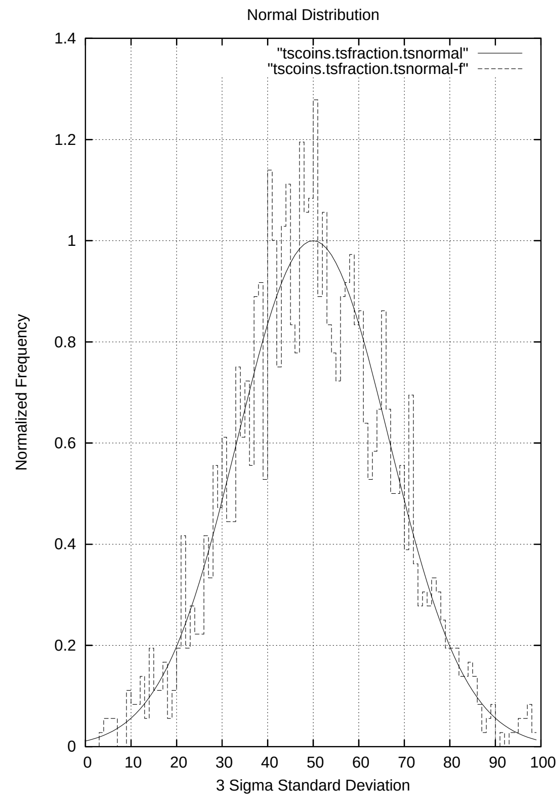

Figure B.48: Example output of the *tscoins* program, normalized histogram of the normalized increments of the time series data shown in Figure B.47. The area under the two curves is identical.

B.3.10 tsfBm

Source tsfBm.c, fractional brownian noise generator—generates a time series. The idea is to produce a programmable power spectrum distribution. See [Pet91, pp. 211], or [Fed88, pp. 173], referencing Mandelbrot and Wallis, 1969.

Example outputs from the *tsfBm* program appear in figures B.11, B.3, and B.4.

B.3.11 tslogistic

Source tslogistic.c, logistic function generator—generates a time series. The idea is to produce a function of the form $y(t) = c/(1 + e^{-(at+b)})$. See [Mor91, pp. 100], or [Mod92, pp. 230].

An example output from the *tslogistic* program appears in Figure B.49.

B.3.12 tsdlogistic

Source tsdlogistic.c, discreet logistic function generator—generates a time series. The idea is to iterate the function $x(t) = x(t-1) \cdot (a + b \cdot x(t-1))$. See [Pet91, pp. 121].

As a simple set of examples:

tsdlogistic -a 2 -b -2 100

tsdlogistic -a 2.4 -b -2.4 100

tsdlogistic -a 3 -b -3 100

tsdlogistic -a 3.4495 -b -3.4495 100

tsdlogistic -a 3.544 -b -3.544 100

tsdlogistic -a 3.5688 -b -3.5688 100

tsdlogistic -a 3.5696 -b -3.5696 100

tsdlogistic -a 3.5699456 -b -3.5699456 100

An example output from the *tsdlogistic* program appears in Figure B.50.

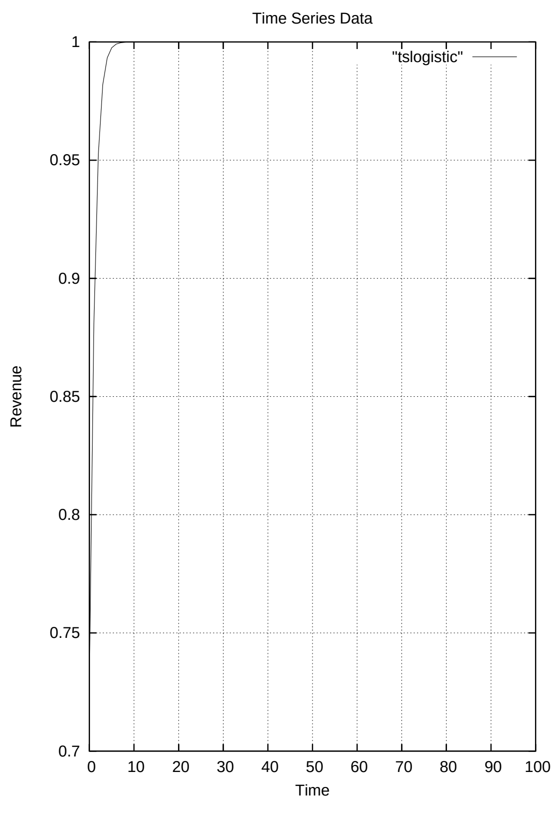

Figure B.49: Example output of the *tslogistic* program, using the command “tslogistic 1 1 1 100 > filename.”

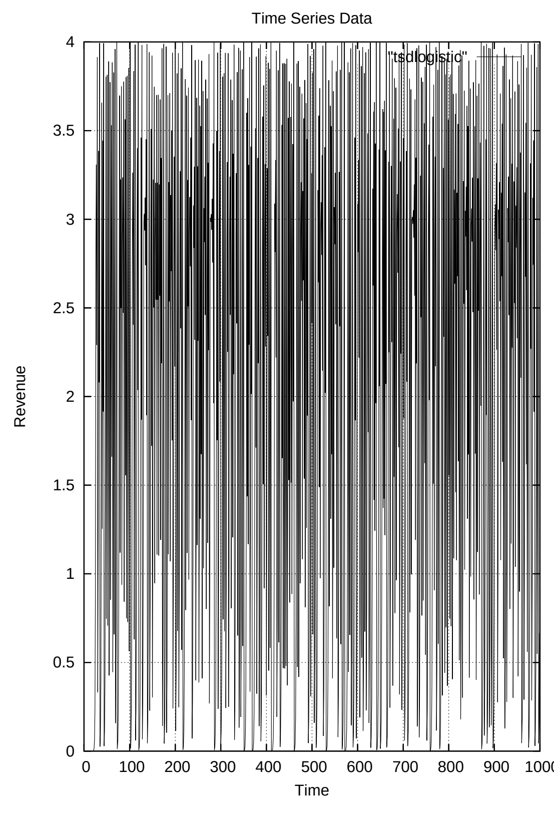

Figure B.50: Example output of the *tsdlogistic* program, using the command “tsdlogistic -a 4 -b -1 1000 > filename.”

B.3.13 tsstockwager

Source tsstockwager.c, stock capital investment simulation. The idea is to simulate an optimal wagering strategy, dynamically determining the Shannon probability by counting the up movements in a stock's value in a window from the stock's value time series, and using this to compute the fraction of the total capital to be invested in the stock for the next iteration of the time series, which is $2P - 1$, where P is the Shannon probability. See, [Sch91, pp. 129, 151]. The assumption is that a stock's price time series could be modeled as a fixed increment fractal.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsstockwager* program appears in Figure B.51.

B.3.14 tsbinomial

Source tsbinomial.c, is for generating binomial distribution noise, with unfair bias, and cumulative sum—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution by running a cumulative sum on a binomial distribution. The program accepts a an unfair bias and a wager factor. See [Fed88, pp. 154].

This program is a modification of the program *tscoin*. The wager fraction is computed by first calculating the optimal wager fraction, $f = 2P - 1$, where P is the Shannon probability, and f is the optimal wager fraction, (which is the root mean square = standard deviation of the normalized increments of the time series,) and then reducing this value by the standard deviation of the binomial distribution, which is the square root of the number of elements in the distribution, ie., the root mean square of the normalized increments of the cumulative sum is the same as the standard deviation of the binomial distribution. See [Fed88, pp. 155].

An example output from the *tsbinomial* program appears in Figure B.52.

B.3.15 tsunfairfractional

Note: Conceptually, this program is used to "weight" the returns of a time series with a Gaussian distribution, ie., produce a fractional Brownian motion time series, as opposed to a Brownian distribution which would produce a Brownian time series, as produced by the program *tsunfairbrownian*. Unfortunately, the precision of the results were not encouraging, and the concept was abandoned. It is left in the program inventory for future reference.

Source tsunfairfractional.c, unfair returns of a time series. The idea is to produce the returns of a time series which is weighted unfairly, by a Shannon probability, p. The input time series is presumed to have a Gaussian distribution. The main function of this program is regression scenario verification—given an empirical time series, a Shannon probability, and a "wager" fraction, (which were probably derived from the program *tsshannon*,) speculative market pro forma performance can be analyzed. Uses Newton—Raphson method for an iterative solution for the inverse function of the normal function. Also iterates, using Romberg integration, to calculate the cumulative interval value of the normal function.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

As a reference on Romberg Integration, see [PFTV88, pp. 124].

As a reference on trapezoid iteration, see [PFTV88, pp. 120].

As a reference on polynomial interpolation, see [PFTV88, pp. 90].

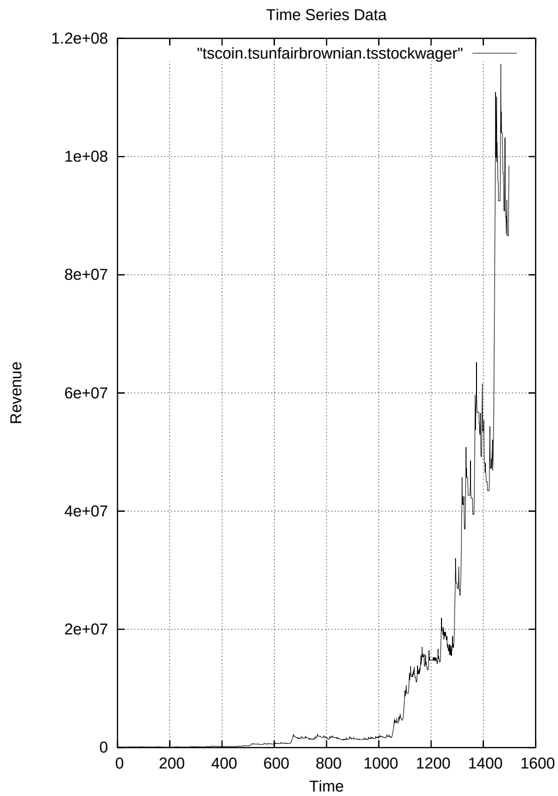

Figure B.51: Example output of the *tsstockwager* program, using the *tscoin* program with a Shannon probability of 0.6, which is shown in Figure B.46. The program *tsunfairbrownian* was used to change the wager of the output of the *tscoin* program from 0.2 to 0.1. This file was used as the input to the *tsstockwager* program, and is shown in Figure B.45.

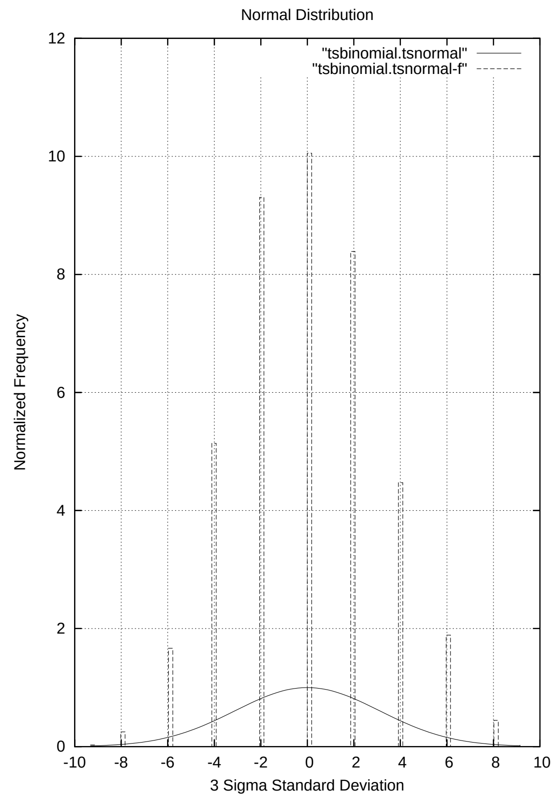

Figure B.52: Example output of the *tsbinomial* program, using 1500 records. This is a plot of the frequency histogram.

An example output from the *tsunfairfractional* program appears in Figure B.53.

B.3.16 tsintegers

Source tsintegers.c, integers function generator—generates a time series.

An example output from the *tsintegers* program appears in Figure B.54.

B.3.17 tsshannonstock

Source tsshannonstock.c, is for simulating the gains of a stock investment using Shannon probability. See [Sch91, pp. 128, 151].

the algorithm used is:

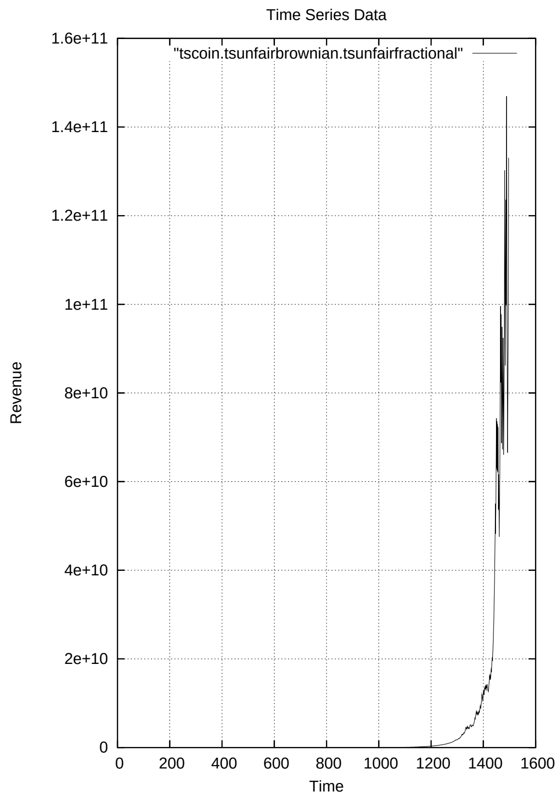

Figure B.53: Example output of the *tsunfairfractional* program, using the *tscoin* program with a Shannon probability of 0.6, which is shown in Figure B.46. The program *tsunfairbrownian* was used to change the wager of the output of the *tscoin* program from 0.2 to 0.1. This file was used as the input to the *tsunfairfractional* program, and is shown in Figure B.45.

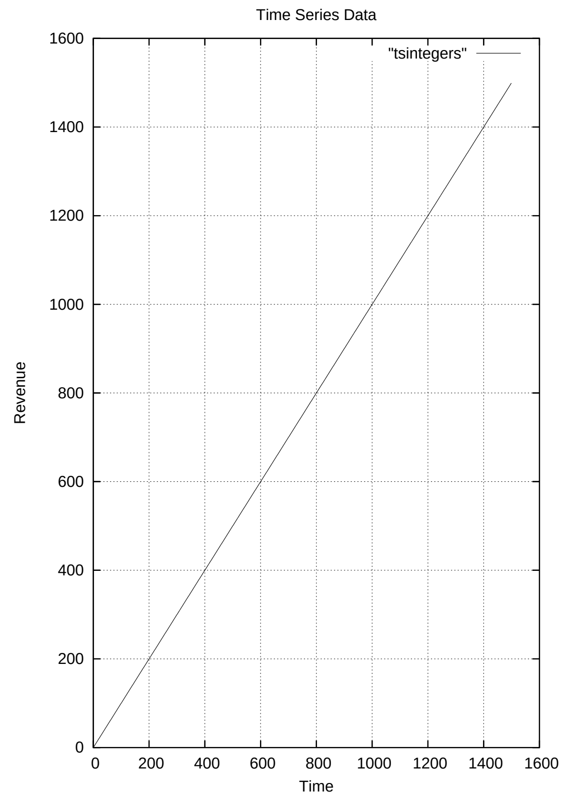

Figure B.54: Example output of the *tsintegers* program, using 1500 records.

- Let $I(t)$ be the amount of capital at time t.
- Let $W(t)$ be the amount of the capital wagered at time t.
- Let $V(t)$ be the value of the stock at time t.

Let f be the fraction of the capital, wagered at any time, and assumed not to be a function of time.

$$W(t) = fI(t-1) \tag{B.174}$$

$$I(t) = I(t-1) + W(t)\frac{V(t) - V(t-1)}{V(t-1)} \tag{B.175}$$

$$I(t) = I(t-1) + fI(t-1)\frac{V(t) - V(t-1)}{V(t-1)} \tag{B.176}$$

$$\frac{I(t)}{I(t-1)} = 1 + f\frac{V(t) - V(t-1)}{V(t-1)} \tag{B.177}$$

If it is assumed that the stock's price time series can be represented as a Brownian noise fractal, then the optimum value of f would be:

$$f = 2P - 1 \tag{B.178}$$

where P is the Shannon probability of the time series, found by:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.179}$$

where avg is the average, and rms is the root mean square, of the normalized increments of the stock's price time series, which can be calculated by

$$\frac{V(t) - V(1-1)}{V(t-1)} \tag{B.180}$$

for each data point in the time series.
Represented in pseudo code:

1. for each data point in the stock's price time series, find the normalized increment from the following equation:

$$\frac{V(t) - V(1-1)}{V(t-1)} \tag{B.181}$$

2. calculate the average of all normalized increments in the stock's price time series by the following equation:

$$avg = \frac{1}{n}\sum_{i=0}^{n}\frac{V(t) - V(1-1)}{V(t-1)} \tag{B.182}$$

3. calculate the root mean square of all normalized increments in the stock's price time series by the following equation:

$$rms^2 = \frac{1}{n}\sum_{i=0}^{n}\left(\frac{V(t) - V(1-1)}{V(t-1)}\right)^2 \tag{B.183}$$

4. calculate the Shannon probability, P, by the following equation:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.184}$$

5. calculate the optimal fraction of the capital to be wagered, f , by the following equation:

$$f = 2P - 1 \tag{B.185}$$

6. since the stock's price time series already has a value rms as the root mean square of the normalized increments, for the optimal wagering strategy, the fraction should be divided by rms to provide a multiplier:

$$multiplier = \frac{f}{rms} \tag{B.186}$$

so that:

$$\frac{I(t)}{I(t-1)} = 1 + multiplier \cdot \frac{V(t) - V(1-1)}{V(t-1)} \tag{B.187}$$

What this means is that if you have capital, (ie, a portfolio,) $I(t)$, the fraction of $I(t)$ that should be wagered with each iteration of the game, (ie., time unit,) would be twice the Shannon probability minus unity, where the capital, (or portfolio,) is the sum total of cash on hand, $C(t)$, and the current value of stocks held, $V(t) \cdot N$, where N is the number of stocks held, or:

$$I(t) = C(t) + V(t) \cdot N \tag{B.188}$$

It would be convenient, from a comparative standpoint, to let $I(0)$, the beginning capital, be the same as $V(0)$, the price of the stock at the beginning of the simulation, so that the wagering strategy can be compared to the price of the stock over time.
N will be adjusted for the next game, (time unit,) such that:

$$N(t+1) = \frac{I(t) \cdot f}{V(t)} \tag{B.189}$$

where, as above, f is the fraction of capital, (portfolio,) to be wagered:

$$f = fraction = 2P - 1 = (2 \cdot shannon) - 1 \tag{B.190}$$

It would, additionally, for the simulation, be convenient, from an information-theoretic standpoint, to let f be a fraction, (either larger or smaller,) of the root mean square value of the normalized increments of the stock's price time series, ie., let $f = F \cdot rms$, where F is a constant value, (usually around unity,) and rms is the average of the root mean square value of the normalized increments of the stock's price time series. This would allow a comparison of the stock's price, over time, to the capital, over time, with a wagering strategy that is optimal for a stock price that is characterized as a Brownian motion fractal over time.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

An example output from the *tsshannonstock* program appears in Figure B.55.

B.3.18 tsmarket

Source tsmarket.c, is for market simulation by fractional brownian noise generation, with unfair bias, and cumulative sum—generates a time series. The idea is to produce a $1/f$ squared power spectrum distribution for each company in an industrial market by running a cumulative sum on a Gaussian power spectrum distribution. The aggregate of all companies participating in the market is obtained by summing the production of the individual companies. The program accepts an unfair bias and a wager factor, and the number of companies in the market. See [Fed88, pp. 172]. Uses Newton—Raphson method for an iterative solution for the probability, p.

As an example, consider the Semiconductor Industry Association (SIA,) historical time series, (see the directory ../markets/ic.namerica,) data for the integrated circuit marketplace in North America:

- From the program *tsshannonwindow*, the Shannon probability, $P = 0.758207$.
- From the programs *tsfraction* and *tsrms*, the root mean square value of the normalized increments, $rms = 0.087396$.
- From the programs *tsfraction* and *tsavg*, the average of the normalized increments, $avg = 0.045132$.

Interestingly, the optimal rms value would be $rms = 2P - 1 = 0.516414$, if the SIA time series could be represented a Brownian fractal, (ie., represented as a gambler's capital time series in an unfair coin toss game. See [Sch91, pp. 128].)
For this analysis, it is assumed that:

1. Each company acts independently, and will receive cash flow from the market.
2. Some of this cash flow will be diverted into new product manufacturing, development, etc., which in turn will go back into the market, which in turn will create cash flow, and so on-but there is a random element in this process.
3. Analysis of the SIA graph yields that it is probably a fractal, (fractional Brownian variety,) with a fairly accurate distribution of the normalized increments that appears to be Gaussian in nature, a range that appears to increase with the square root of time, and an exponential curvature. These are indicative of system that can be modeled by as a gambler's capital in an unfair coin toss game, or Brownian fractal.

To analyze the SIA time series, it is interesting to note that the avg is 0.045132, which would be the sum total of the average of all companies in the market. If the individual companies are assumed to be operating optimally, (and all identical,) then the rms would be the square root of the avg, which is 0.212442934. This would be the amount "wagered" in each iteration of the unfair coin game, (which is a Brownian fractal,) and the Shannon probability would be $0.212442934 = 2P - 1$, or $P = 0.606221467$.
Using the program *tsmarket*:
tsmarket -p 0.6 -c n 2500 > data
The variable n was altered to approximate the statistical data of the SIA numbers. The best seems to be with $n = 5$:

from tsshannonwindow, $P = 0.744495$

from tsfraction and tsrms, $rms = 0.102880$

from tsfraction and tsavg, $avg = 0.050307$

which compares favorably, to about ±5%, with the original SIA numbers:

from tsshannonwindow, $P = 0.758207$

from tsfraction and tsrms, $rms = 0.087396$

from tsfraction and tsavg, $avg = 0.045132$

which would tend to indicate that the constituent companies in the aggregate are operating optimally, and that the measurements on the aggregate sum of the market, ie., the SIA numbers, would indicate a higher Shannon probability, P, and a smaller root mean square value of the normalized increments, rms.
The reason is as follows:

1. Consider a market that is supplied by a single company. The time series for the market could be represented, at least statistically, as an unfair coin tossing game, (see the *tscoins* program,) with each time unit of manufacturing going into the marketplace, the marketplace returning cash to the company's P & L, which is distributed to the company's operations to manufacture more product, and so on. But there is an element of randomness in this process that represents the aggregate of customer desires and market forces-this is assumed be a central limit phenomena, ie., it can be represented as a random variable with a normal, (Gaussian,) distribution. Note, that like the gambler, the company's operations managers are continually wagering on the future-and each wager may, or may not prove to be a successful. It is further assumed that the company will commit capital to enhancing its market position, (ie., increase manufacturing capacity, develop new products, etc.,) and, as above, the decision to do so will contain an element of risk, and will sometimes work out, and sometimes not.

2. Now consider that another company decides to participate in the marketplace-under the same scenario, above. If everything else is equal, we would expect the market, eventually, to be divided equally between the two companies, or each company would have half the market. When the second company was added to the market, the first company's contribution to the marketplace was cut in half-and its root mean square value of its normalized increments contribution to the marketplace was also cut in half. The second company's contribution to the marketplace is the remaining one half, and its contribution to the root mean square value of its normalized increments is the same as the first company's. (The point is that the contributions to the marketplace add linearly, but the contribution of to the normalized increments of the marketplace add root mean square-so we would expect the root mean square value of the normalized increments to decrease when the number of participants in the marketplace changes from one to two-since the value of the normalized increments for each company is proportional to the contribution to its the market.) Think of it as a Gaussian noise generator. If we cut the root mean square value (amplitude,) of the noise generator in one half, and add an identical noise generator, the resulting noise output of both generators will be the square root of two, divided by two.

3. Or in general, the root mean square value of the normalized increments of a marketplace time series will be proportional to one over the square root of the number of companies in the market.

The general outline of this program is:

1. given the Shannon probability, compute the abscissa value that divides the area under the normal curve, into two sections, such that the area to the left of the value, divided by the total area under the normal curve is the Shannon probability—a Newton-Raphson iterated approach using Romberg integration to find the area is used for this

2. for each record:

   (a) compute a gaussian distributed random number

   (b) add the computed abscissa value to the gaussian distributed number

   (c) multiply this number by the fraction of cumulative sum to be wagered

   (d) add this number to the cumulative sum for the company

   (e) add this number to the temporary aggregate sum of the market

3. add the temporary aggregate sum of the market to the aggregate sum of the market

This program will require finding the value of the normal function, given the standard deviation. The method used is to use Romberg/trapezoid integration to numerically solve for the value.

This program will require finding the functional inverse of the normal, ie., Gaussian, function. The method used is to use Romberg/trapezoid integration to numerically solve the equation:

$$F(x) = \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 \tag{B.191}$$

which has the derivative:

$$f(x) = \frac{1}{2\pi} e^{\frac{-x^2}{2}} \tag{B.192}$$

Since $F(x)$ is known, and it is desired to find x,

$$F(x) - \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 = P(x) = 0 \tag{B.193}$$

and the Newton-Raphson method of finding roots would be:

$$P_{n+1} = P_n - \frac{P(x)}{f(x)} \tag{B.194}$$

An example output from the *tsmarket* program appears in Figure B.56.

B.3.19 tsstock

Source tsstock.c, is for simulating the gains of a stock investment using Shannon probability.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

A large mathematical infrastructure of analytical techniques and methodologies have been dedicated to the analysis of equity market time series. See, [Cro95, pp. 249], [Sch91, pp. 126], [Pet91], [Lew92, pp. 196, 269, 273, 329], [Cas90, pp. 195, 214], [Cas94, pp. 82, 106, 102, 255, 269], [Mod92, pp. 155].

In addition, a large infrastructure of information—theoretic techniques have been suggested for optimal speculative wagering strategies in the equity markets, based, generally, on the suggested interpretation in the Kelly reference[12]. See, [Pie80, pp. 270], [Rez94, pp. 450], [Ash65, pp. 9], [SW49, pp. 39], [KF88, pp. 155], [Sch91, pp. 151].

This program is an investigation into whether a stock price time series could be modeled as a fractal Brownian motion time series, and, further, whether, a mechanical wagering strategy could be devised to optimize portfolio growth in the equity markets.

Specifically, the paradigm is to establish an isomorphism between the fluctuations in a gambler's capital in the speculative unfair tossed coin game, as suggested in [Sch91], and speculative investment in the equity markets. The advantage in doing this is that there is a large infrastructure in mathematics dedicated to the analysis and optimization of parlor games, specifically, the unfair tossed coin game. See Schroeder reference.

Currently, there is a repository of historical price time series for stocks available at:

http://www.ai.mit.edu/stocks.html

that contains the historical price time series of many hundreds of stocks. The stock's prices are by close of business day, and are updated daily.

The stock price history files in the repository are available via anonymous ftp, (ftp.ai.mit.edu,) and the programs *tsfraction*, *tsrms*, *tsavg*, and *tsnormal* can be used to verify that, as a reasonable first approximation, stock prices can be represented as a fractional Brownian motion fractal, as suggested by Schroeder and Crownover. (Note the assumption that, as a first approximation, a stock's price time series can be generated by independent increments.)

[12] "New Interpretation of Information Rate," J. L. Kelly, Jr., Bell System Technical Journal, Vol. 35, (July, 1956,) pp. 917.

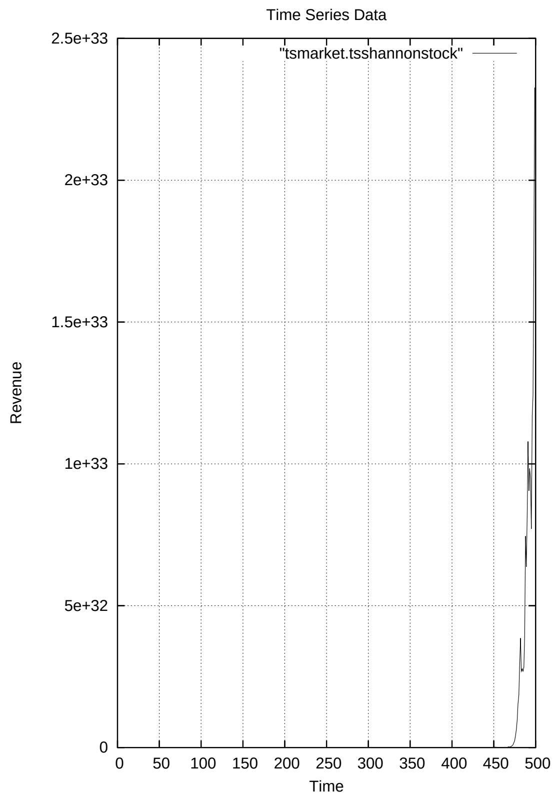

Figure B.55: Example output of the *tsshannonstock* program, the input was produced by the *tsmarket* program, with a Shannon probability of 0.6, and 5 companies participating in the market, and is shown in Figure B.56.

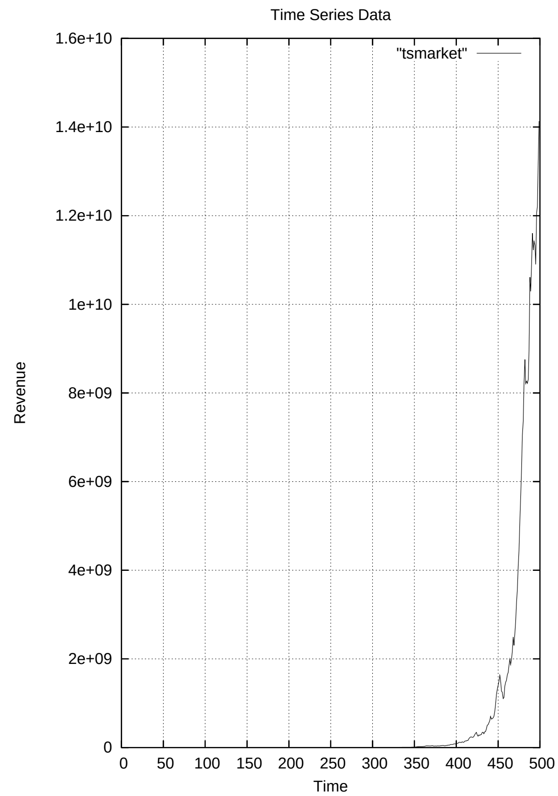

Figure B.56: Example output of the *tsmarket* program with a Shannon probability of 0.6 and 5 companies participating in the market, using 500 records.

This would tend to imply that there is an isomorphism between the underlying mechanism that produces the fluctuations in speculative stock prices and the the mechanism that produces the fluctuations in a gambler's capital that is speculating on iterations of an unfair tossed coin.

If this is a reasonably accurate approximation, then the underlying mechanism of a stock's price time series can be analyzed, (by "disassembling" the time series,) and a wagering strategy, similar to that of the optimal wagering strategy in the iterated unfair coin tossing game, can be formulated to optimize equity market portfolio growth.

As a note in passing, it is an important and subtile point, that there are "operational" differences in wagering on the iterated unfair coin game, and wagering on a stock. Specifically, in the coin game, a fraction of the gambler's capital is wagered on the speculative outcome of the toss of the coin, and, depending on whether the toss of the coin resulted in a win, (or a loss,) the wager is added to the gambler's capital, (or subtracted from it,) respectively. However, in the speculative stock game, the gambler wagers on the anticipated *fluctuations* of the stock's price, by purchasing the stock. The important difference is that the stock gambler does not win or loose an amount that was equal to the stock's price, (which was equivalent to the wager in the iterated unfair coin game,) but only the fluctuations of the stock's price, ie., it is an important concept that a portfolio's value (which has an investment in a stock,) and the stock's price do not, necessarily, “track” each other.

In some sense, wagering on a stock is *not* like a gambler wagering on the outcome of the toss of an unfair coin, but like wagering on the capital of the gambler that wagered on the outcome of the toss of an unfair coin. A very subtile difference, indeed.

Note that the paradigm of the isomorphism between wagering on a stock and wagering in an unfair tossed coin game is that the graph, (ie., time series,) of the gambler’s capital, who is wagering on the iterated outcomes of an unfair tossed coin, and the graph of a stock’s price over time are statistically similar.

If this is the case, at least in principle, it should be possible to “dissect” the time series of both “games,” and determine the underlying statistical mechanism of both. Further, it should be, at least in principle, possible to optimize portfolio growth of speculative investments in the equity markets using information—theoretic entropic techniques. See, [Pie80], [Rez94], and [Sch91].

Under these assumptions, the amount of capital won or lost in each iteration of the unfair tossed coin game would be:

$$V(t) - V(t-1) \tag{B.195}$$

for all data points in the gambler’s capital time series. This would correspond to the amount of money won or lost on each share of stock at each interval in the stock price time series.

Likewise, the normalized increments of the gambler’s capital time series can be obtained by subtracting the value of the gambler’s capital in the last interval from the value of the gambler’s capital in the current interval, and dividing by the value of the gambler’s capital in the last interval:

$$\frac{V(t) - V(t-1)}{V(t-1)} \tag{B.196}$$

for all data points in the gambler’s capital time series. This would correspond to the fraction of the gambler’s capital that was won or lost on each iteration of the game, or, alternatively, the fraction that the stock price increased or decreased in each interval.

The normalized increments are a very useful “tool” in analyzing time series data. In the case of the unfair coin tossing game, the normalized increments are a “graph,” (or time series,) of the fraction of the capital that was won or lost, every iteration of the game. Obviously, in the unfair coin game, to win or lose, a wager had to be made, and the graph of the absolute value, or more appropriately, the root mean square[13], of the normalized increments is the fraction of the capital that was wagered on each iteration of the game. As suggested in Schroeder, if an unfair coin has a chance, P, of coming up heads, (winning) and a chance $1 - P$, of coming up tails, (loosing,) then the optimal wagering strategy would be to wager a fraction, f, of the gambler’s capital, on every iteration of the game, that is:

$$f = 2P - 1 \tag{B.197}$$

This would optimize the exponential growth of the gambler’s capital. Wagering more than this value would result in less capital growth, and wagering less than this value would result in less capital growth, over time. The variable f is also equal to the root mean square of the normalized increments, rms, and the average, avg, of the normalized increments is the constant of the average exponential growth of the gambler’s capital:

$$C(t) = (1 + avg)^t \tag{B.198}$$

where $C(t)$ is the gambler’s capital. It can be shown that the formula for the probability, P, as a function of avg and rms is:

[13]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.199}$$

where the empirical measurement of avg and rms are:

$$avg = \frac{1}{n} \sum_{i=0}^{n} \frac{V(t) - V(t-1)}{V(t-1)} \tag{B.200}$$

and,

$$rms^2 = \frac{1}{n} \sum_{i=0}^{n} \left(\frac{V(t) - V(t-1)}{V(t-1)} \right)^2 \tag{B.201}$$

respectively, (additionally, note that these formulas can be used to produce the running average and running root mean square, ie., they will work "on the fly.")

The formula for the probability, P, will be true whether the game is played optimally, or not, ie., the game we are "dissecting," may not be played with $f = 2P - 1$. However, the formula for the probability, P:

$$P' = \frac{rms + 1}{2} \tag{B.202}$$

will be the same as P, only if the game is played optimally, (which, also, is applicable in "on the fly" methodologies.)

Interestingly, the measurement, perhaps dynamically, (ie., "on the fly,") of the average and root mean square of the normalized increments is all that is necessary to optimize the "play of the game." Note that if P' is smaller than P, then we need to increase rms, by increasing f, and, likewise, if P' is larger than P, we need to decrease f. Thus, without knowing any of the underlying mechanism of the game, we can formulate a methodology for an optimal wagering strategy. (The only assumption being that the capital can be represented as an independent increment fractal—and, this too can, and should, be verified with meticulous application of fractal analysis using the programs *tsfraction*, *tsrms*, *tsavg*, and *tsnormal*.)

At this point, it would seem that the optimal wagering strategy and analytical methodology used to optimize the growth of the gambler's capital in the the unfair tossed coin gain is well in hand. Unfortunately, when applying the methodology to the equity markets, one finds that, for almost all stocks, P is greater than P', perhaps tending to imply that in the equity markets, stocks are over priced.

To illustrate a simple stock wagering strategy, suppose that analytical measurements are made on a stock's price time series, and it is found, conveniently, that $P = P'$, implying that $f = rms$, (after computing the normalized increments of the stock's price time series and calculating avg, rms, P, and P'.) Note that in the optimized unfair coin tossing game, that wagering a fraction, $f = rms$, of the gambler's capital would optimize the exponential growth of the gambler's capital, and that the fluctuations, over time, of the gambler's capital would simply be the normalized increments of the gambler's capital. The root mean square of the fluctuations, over time, are the fraction of that the gambler's capital wagered, over time. To achieve an optimal strategy when wagering on a stock, the objective would be that the normalized increments in the value of the portfolio, and the root mean square value of the normalized increments of the portfolio, also, satisfy the criteria, $f = rms$. Note that the fraction of the portfolio that is invested in the stock will have normalized increments that have a root mean square value that are the same as the root mean square value of the normalized increments of the stock.

The issue is to determine the fraction of the stock portfolio that should be invested in the stock such that that fraction of the portfolio would be equivalent to the gambler wagering a fraction of the capital on a coin toss. It is important to note that the optimized wagering strategy used by the gambler, when wagering on the outcome of a coin toss, is to never wager the entire capital, but to hold some capital in reserve, and wager only a fraction of the capital—and in the optimum case this wager fraction is $f = rms$. In a stock portfolio, even though the investment is totally in stocks, it could be considered that some of this value is wagered, and the rest held in reserve. The amount wagered would be the root mean square of the normalized increments of the stocks price, and the amount held in reserve would be the remainder of the portfolio's value. (Note the paradigm—there is an isomorphism between the fluctuating gambler's capital in the unfair coin tossing game, and the fluctuating value of a stock portfolio.) In the simple case where $P = P'$, the fraction of the portfolio value that should be invested in the stock is $f =$ root mean square of the stock's normalized increments, which would be the same as $f = 2P - 1$, where $P = \frac{\frac{avg}{rms}+1}{2}$ or $P = \frac{rms+1}{2}$. Note that the fluctuations in the value of the portfolio do to the fluctuations in the stocks price would be statistically similar to the fluctuations in the gambler's capital when playing the unfair coin tossing game.

This also leads to a generality, where P and P' are not equal. If the root mean square of the normalized increments of the stock price time series are too small, say by a factor of 2, then the fraction of the portfolio invested in the stock should be increased, by a factor of 2 (in this example.) This would make the root mean square of the fluctuations in the value of the portfolio the same as the the root mean square of the fluctuations in the gambler's capital under similar statistical circumstances, (albeit with twice as much of the portfolio's equivalent "cash reserves" tied up in the investment in the stock.

To calculate the ratio by which the fraction of the portfolio invested in a stock must be increased:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.203}$$

and,

$$f = 2P - 1 = rms \tag{B.204}$$

and letting the measured rms by rms_m,

$$f = 2P - 1 = 2\frac{\frac{avg_m}{rms_m} + 1}{2} - 1 = \frac{avg_m}{rms_m} = rms \tag{B.205}$$

(Note that both of the values, avg and rms, are functions of the probability, P, but their ratio is not.)
and letting F be the ratio by which the fraction of the portfolio invested in a stock must be increased to accommodate P not being equal to P':

$$F = \frac{rms}{rms_m} = \frac{avg_m}{rms_m^2} \tag{B.206}$$

and multiplying both sides of the equation by f, to get the fraction of the portfolio that should be invested in the stock while accommodating P not being equal to P':

$$F \cdot f = \frac{avg_m}{rms_m^2} \cdot \frac{avg_m}{rms_m} = \frac{avg_m^2}{rms_m^3} \tag{B.207}$$

which can be computed, dynamically, or "on the fly," and where avg and rms are the average and root mean square of the normalized increments of the stock's price time series, and assuming that the stock's price time series is composed of independent increments, and can be represented as a fractional Brownian motion fractal.

Representing such an algorithm in pseudo code:

1. for each data point in the stock's price time series, find the, possibly running, normalized increment from the following equation:

$$\frac{V(t) - V(t-1)}{V(t-1)} \tag{B.208}$$

2. calculate the, possibly running, average of all normalized increments in the stock's price time series by the following equation:

$$avg = \frac{1}{n} \sum_{i=0}^{n} \frac{V(t) - V(t-1)}{V(t-1)} \tag{B.209}$$

3. calculate the, possibly running, root mean square of all normalized increments in the stock's price time series by the following equation:

$$rms^2 = \frac{1}{n} \sum_{i=0}^{n} \left(\frac{V(t) - V(t-1)}{V(t-1)} \right)^2 \tag{B.210}$$

4. calculate the, possibly running, fraction of the portfolio to be invested in the stock, $F \cdot f$:

$$F \cdot f = \frac{avg_m^2}{rms_m 3} \tag{B.211}$$

To reiterate what we have so far, consider a gambler, iterating a tossed unfair coin. The gambler's capital, over time, could be a represented as a Brownian fractal, on which measurements could be performed to optimize the gambler's wagering strategy. There is supporting evidence that stock prices can be "modeled" as a Brownian fractal, and it would seem reasonable that the optimization techniques that the gambler uses could be applied to stock portfolios. As an example, suppose that it is desired to invest in a stock. We would measure the average and root mean square of the normalized increments of the stock's price time series to determine a wagering strategy for investing in the stock. Suppose that the measurement yielded that the the the fraction of the capital to be invested, f, was 0.2, (ie., a Shannon probability of 0.6,) then we might invest the entire portfolio in the stock, and our portfolio would be modeled as 20% of the portfolio would be wagered at any time, and 80% would be considered as "cash reserves," even though the 80% is actually invested in the stock. Additionally, we have a metric methodology, requiring only the measurement of the average and root mean square of the increments of the stock price time series, to formulate optimal wagering strategies for investment in the stocks. The assumption is that the stock's price time series is composed of independent increments, and can be represented as a fractional Brownian motion fractal, both of which can be verified through a metric methodology.

Note the isomorphism. Consider a gambler that goes to a casino, buys some chips, then plays many iterations of an unfair coin tossing game, and then cashes in the chips. Then consider investing in a stock, and some time later, selling the stock. If the Shannon probability of the time series of the unfair coin tossing game is the same as the time series of the stock's value, then both "games" would be statistically similar. In point of fact, if the toss of the unfair coin was replaced with whether the stock price movement was up or down, then the two time series would be identical. The implication is that stock values can be modeled by an unfair tossed coin. In point of fact, stock values are, generally, fractional Brownian motion in nature, implying that the day to day fluctuations in price can be modeled with a time sampled unfair tossed coin game.

There is an implication with the model. It would appear that the "best" portfolio strategy would be to continually search the stock market exchanges for the stock that has the largest value of the quotient of the average and root mean square of the normalized increments of the stock's price time series, (ie., $\frac{avg}{rms}$,) and invest 100% of the portfolio in that single stock. This is in contention with the concept that a stock portfolio should be "diversified," although it is not clear that the prevailing concept of diversification has any scientific merit.

To address the issue of diversification of stocks in a stock portfolio, consider the example where a gambler, tossing an unfair coin, makes a wager. If the coin has a 60% chance of coming up heads, then the gambler should wager 20% of the capital on hand on the next toss of the coin. The remaining 80% is kept as "cash reserves." It can be argued that the cash reserves are not being used to enhance the capital, so the gambler should play multiple games at once, investing all of the capital, investing 20% of the capital, in each of 5 games at once, (assuming that the coins used in each game have a probability of coming up heads 60% of the time—note that the fraction of capital invested in each game would be different for each game if the probabilities of the coins were different, but could be measured by calculating the $\frac{avg}{rms}$ of each game.)

Likewise, with the same reasoning, we would expect that stock portfolio management would entail measuring the quotient of the average and root mean square of the normalized increments of every stock's price time series, (ie., $\frac{avg}{rms}$,) choosing those stocks with the largest quotient, and investing a fraction of the portfolio that is equal to the this quotient. Note that with an $\frac{avg}{rms} = 0.1$, (corresponding to a Shannon probability of 0.55—which is "typical" for the better performing stocks on the New York Stock Exchange,) we would expect the portfolio to be diversified into 10 stocks, which seems consistent with the recommendations of those purporting diversification of portfolios. In reality, since most stocks in the United States exchanges, (at least,) seem to be "over priced," (ie., P larger than P',) it will take more capital than is available in the value of the portfolio to invest, optimally, in all of the stocks in the portfolio, (ie., the fraction of the portfolio that has to be invested in each stock, for optimal portfolio performance, will sum to greater than 100%.) The interpretation, I suppose, in the model, is that at least a portion of the investment in each stock would be on "margin," which is a relatively low risk investment, and, possibly, could be extended into a formal optimization of "buying stocks on the margin."

The astute reader would note that the fractions of the portfolio invested in each stock was added linearly, when these values are really the root mean square of the normalized increments, implying that they should be added root mean square. The rationale in linear addition is that the Hurst Coefficient in the near term is near unity, and for the far term 0.5. (By definition, this is the characteristic of a Brownian motion fractal process.) Letting the Hurst Coefficient be H, then the method of summing multiple processes would be:

$$V_{tot}^{H} = V_1^{H} + V_2^{H} + \cdots \tag{B.212}$$

so in the far term, the values would be added root mean square, and in the near term, linearly. Note that this is also a quantitative definition of the terms "near term" and "far term." Since the Hurst Coefficient plot is on a log-log scale, the demarcation between the two terms is where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ The important point is that the "root mean square formula" used varies with time. For the near term, $H \approx 1$, and linear addition is used. For the far term, a root mean square summation process is used. (Note, also, that a far term H of 0.5 is unique to Brownian motion fractals. In general, it can be different than 0.5. If it is larger than 0.5, then it is termed fractional Brownian motion, depending on who is doing the defining.)

There are some interesting implications to this near term/far term interpretation. First, the "forecastability" is better in the near term than far term-which could be interpreted as meaning that short term strategies would yield better portfolio performance than long term strategies—see [Pet91, pp. 83-84]. Secondly, it can be used to optimize portfolio long term strategy. For example, suppose that a stock's Shannon probability is 0.52, and all stocks in the portfolio have the same Shannon probability. This means that the portfolio should consist of 25 stocks. However, in the long run, the portfolio would have a root mean square value of the square root of 25 times 0.04, or 0.2. This would tend to imply that, on the average, over the long run, the stock portfolio would be one fifth of the total investments. Naturally, this ratio could be adjusted, over time, depending on the instantaneous value of the Shannon probabilities of all different investments, like bonds, metals, etc.

This would imply that "timing of the market" would have to be initiated to adjust the ratio of investment in stocks. One of the implications of entropic theory is that this is impossible. However, as the Shannon probability of the various investments change, statistical estimation can be used to asses the statistical accuracy of these movements, and the ratios adjusted accordingly. This would tend to suggest that adaptive computational control system methodology would be an applicable alternative.

As a note in passing, the average and root mean square of the normalized increments of a stock's price time series, avg and rms, respectively, represent a qualitative metric of the stock. The average, avg, is an expression of the stock's growth in price, and the root mean square, rms, is a expression of the stock's price volatility. It would seem, incorrectly, at first glance that stocks should be selected that have high price growth, and low price volatility—however, it is a more complicated issue since avg and rms are interrelated, and not independent of each other.

In the diversified portfolio, the "volatilities" of the individual stocks add root mean square to the volatility of the portfolio value, so, everything else being equal, we would expect that the volatility of the portfolio value to be about $\frac{1}{3}$ the volatility of the stocks that make up the portfolio. (The ratio $\frac{1}{3}$ came from the $\sqrt{\frac{1}{10}}$, which is about $\frac{1}{3}$.) (There is a qualification here, it is assumed that all stock price time series are made up of independent increments, and can be represented as a fractional Brownian motion fractal—note that this statement is not true if the time series is characterized as simple Brownian motion, like the gambler's capital in the unfair coin toss game—see [Sch91, pp. 157], for details.) So, it can be supposed, if one desires maximum performance in a stock portfolio, then one should search the stock market exchanges for the stock that has the highest quotient of the average and root mean square of the normalized increments of stock price time series, and invest 100% of the portfolio in that stock. As an alternative strategy, one could diversify the portfolio, investing in multiple stocks, and lower the portfolio volatility at the expense of lower portfolio performance. Arguments can probably be made for both strategies.

As a note in passing, I have made the statement that, at least in the United States exchanges, stocks tend to be over priced. The rationale behind the statement is as follows. If the stock's price time series represents an independent increment, fractional Brownian fractal, and if the stock's price performance is optimal, then the equation:

$$f = 2P - 1 \tag{B.213}$$

where P is the Shannon probability for the stock's price time series, and f is the fraction of the capital wagered per game, (or unit time, and where the capital is the stock's price,) will represent fluctuations in the stock's price, since the symbol f is also the root mean square value of the normalized increments of the stock's time series. Also, the absolute value of the time derivative of the stock's price time series is the fluctuations in the stocks price, ie., at any instant, if V is the stock's price, then fV will be the fluctuation in price, which is the derivative, $D = \frac{dV}{dt}$, or, $V = D/f$. In other words, the fair market value of the stock, in relation to the normalized increments of the stock's value, will be the derivative of the stock's price, divided by the root mean square of the normalized increments of the stock's price, which is also f[14]. If the argument has merit, then, at least the stocks available from http://www.ai.mit.edu/stocks.html would seem to be over priced. (It is a straight forward shell programming exercise, using the programs *tsderivative*, *tsfraction*, *tsmath*, and *tsrms*, to verify this.)

A final derivation, following [Rez94]. Consider the case of a gambler with a private wire into the future who places wagers on the outcomes of a game of chance. We assume that the side information which he receives has a probability, P, of being true, and of $1 - P$, of being false. Let the original capital of gambler be $V(0)$, and $V(n)$ his capital after the n'th wager. Since the gambler is not certain that the side information is entirely reliable, he places only a fraction, f, of his capital on each wager. Thus, subsequent to n many wagers, assuming the independence of successive tips from the future, his capital is:

$$V(n) = (1 + f)^w (1 - f)^l V(0) \tag{B.214}$$

where w is the number of times he won, and $l = n - w$, the number of times he lost. These numbers are, in general, values taken by two random variables, denoted by W and L. According to the law of large numbers:

$$\lim_{n \to \infty} \frac{1}{n} W = P \tag{B.215}$$

and:

$$\lim_{n \to \infty} \frac{1}{n} L = q = 1 - P \tag{B.216}$$

[14] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

The problem with which the gambler is faced is the determination of f leading to the maximum of the average exponential rate of growth of his capital. That is, he wishes to maximize the value of:

$$G = \lim_{n \to \infty} \frac{1}{n} \ln \frac{V(n)}{V(0)} \tag{B.217}$$

with respect to f, assuming a fixed original capital and specified P:

$$G = \lim_{n \to \infty} \frac{W}{n} \ln(1+f) + \frac{L}{n} \ln(1-f) \tag{B.218}$$

or:

$$G = P \ln(1+f) + q \ln(1-f) \tag{B.219}$$

which, by taking the derivative with respect to f, and equating to zero, can be shown to have a maxima when:

$$\frac{dG}{df} = P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{1-P-1}(1+f)^{P} = 0 \tag{B.220}$$

combining terms:

$$P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{P}(1+f)^{P} = 0 \tag{B.221}$$

and splitting:

$$P(1+f)^{P-1}(1-f)^{1-P} = (1-P)(1-f)^{P}(1+f)^{P} \tag{B.222}$$

then taking the logarithm of both sides:

$$\ln(P) + (P-1)\ln(1+f) + (1-P)\ln(1-f) = \ln(1-P) - P\ln(1-f) + P\ln(1+f) \tag{B.223}$$

and combining terms:

$$(P-1)\ln(1+f) - P\ln(1+f) + (1-P)\ln(1-f) + P\ln(1-f) = \ln(1-P) - \ln(P) \tag{B.224}$$

or:

$$\ln(1-f) - \ln(1+f) = \ln(1-P) - \ln(P) \tag{B.225}$$

and performing the logarithmic operations:

$$\ln\left(\frac{1-f}{1+f}\right) = \ln\left(\frac{1-P}{P}\right) \tag{B.226}$$

and exponentiating:

$$\frac{1-f}{1+f} = \frac{1-P}{P} \tag{B.227}$$

which reduces to:

$$P(1-f) = (1-P)(1+f) \tag{B.228}$$

and expanding:

$$P - Pf = 1 - Pf - P + f \tag{B.229}$$

or:

$$P = 1 - P + f \tag{B.230}$$

and, finally:

$$f = 2P - 1 \tag{B.231}$$

As a passing note, the methodology used in this derivation comes from information—theoretic concepts, formally called entropic principles, and is firmly entrenched branch of market and economic analysis.

Continuing with the derivation of the methodology used herein, consider a gambler, wagering on the iterated outcomes of an unfair tossed coin game. A fraction, f, of the gambler's capital will be wagered on the outcome of each iteration of the unfair tossed coin, and if the coin comes up heads, with a probability, P, then the gambler wins the iteration, (and an amount equal to the wager is added to the gambler's capital,) and if the coin comes up tails, with a probability of $1 - P$, then the gambler looses the iteration, (and an amount of the wager is subtracted from the gambler's capital.)

As a passing note, the iterations of a random variable, a flipped coin in this case, that are added together (ie., to a cumulative sum,) the gambler's capital, in this case, are called "fractal" processes. The origins of the name are recent and obscure, but there are different varieties of fractal processes. In this case, since the distribution of the increments is either plus or minus one, it is called a Brownian motion fractal. If the distribution of the increments had a Gaussian, or normal distribution, it would be called a fractional Brownian motion fractal. (Typically distribution of the increments in a stock price time series fall someplace in between the two.) The analytical methodology of investigation into such matters is called "fractal analysis," and that is what is going to be done here, in general, for the gambler's capital, which is a Brownian motion fractal.

If we let the outcome of the first coin toss, (ie., whether it came up as a win or a loss,) be $c(1)$ and the outcome of the second toss be $c(2)$, and so on, then the outcome of the n'th toss, $c(n)$, would be:

$$C(n) = \begin{cases} win, & \text{with a probability of P} \\ loose, & \text{with a probability of 1 - P} \end{cases} \tag{B.232}$$

for convenience, let a win to be represented by $+1$, and a loss by -1:

$$C(n) = \begin{cases} +1, & \text{with a probability of P} \\ -1, & \text{with a probability of 1 - P} \end{cases} \tag{B.233}$$

for the reason that when we multiply the wager, f, by $c(n)$, it will be a positive number, (ie., the wager will be added to the capital,) and for a loss, it will be a negative number, (ie., the wager will be subtracted from the capital.) This is convenient, since the increment, by with the gambler's capital increased or decreased in the n'th iteration of the game is $f \cdot c(n)$.

If we let $C(0)$ be the initial value of the gambler's capital, $C(1)$ be the value of the gambler's capital after the first iteration of the game, then:

$$C(1) = C(0) \cdot (1 + c(1) \cdot f(1)) \tag{B.234}$$

after the first iteration of the game, and:

$$C(2) = C(0) \cdot ((1 + c(1) \cdot f(1)) \cdot (1 + c(2) \cdot f(2))) \tag{B.235}$$

after the second iteration of the game, and, in general, after the n'th iteration of the game:

$$C(n) = C(0) \cdot ((1 + c(1) \cdot f(1)) \cdot (1 + c(2) \cdot f(2)) \cdot \quad \cdots \quad \cdot (1 + c(n) \cdot f(n)) \cdot (1 + c(n+1) \cdot f(n+1))) \tag{B.236}$$

For the normalized increments of the time series of the gambler's capital, it would be convenient to rearrange these formulas. For the first iteration of the game:

$$C(1) - C(0) = C(0) \cdot (1 + c(1) \cdot f(1)) - C(0) \tag{B.237}$$

or:

$$\frac{C(1) - C(0)}{C(0)} = \frac{C(0) \cdot (1 + c(1) \cdot f(1)) - C(0)}{C(0)} \tag{B.238}$$

and after reducing, the first normalized increment of the gambler's capital time series is:

$$\frac{C(1) - C(0)}{C(0)} = (1 + c(1) \cdot f(1)) - 1 = c(1) \cdot f(1) \tag{B.239}$$

and for the second iteration of the game:

$$C(2) = C(0) \cdot ((1 + c(1) \cdot f(1)) \cdot (1 + c(2) \cdot f(2))) \tag{B.240}$$

but $C(0) \cdot ((1 + c(1) \cdot f(1))$ is simply $C(1)$:

$$C(2) = C(1) \cdot (1 + c(2) \cdot f(2)) \tag{B.241}$$

or:

$$C(2) - C(1) = C(1) \cdot (1 + c(2) \cdot f(2)) - C(1) \tag{B.242}$$

which is:

$$\frac{C(2) - C(1)}{C(1)} = \frac{C(1) \cdot (1 + c(2) \cdot f(2)) - C(1)}{C(1)} \tag{B.243}$$

and after reducing, the second normalized increment of the gambler's capital time series is:

$$\frac{C(2) - C(1)}{C(1)} = 1 + c(2) \cdot f(2) - 1 = c(2) \cdot f(2) \tag{B.244}$$

and it should be obvious that the process can be repeated indefinitely, so, the n'th normalized increment of the gambler's capital time series is:

$$\frac{C(n) - C(n-1)}{C(n)} = c(n) \cdot f(n) \tag{B.245}$$

Note that we can tell the fraction of the capital that the gambler wagered in the n'th iteration, it is simply the absolute value of the normalized increment for the iteration, $|c(n) \cdot f(n)|$, ie., $c(n) \cdot f(n)$ is what was won or lost in the n'th iteration, and removing $c(n) = \pm 1$, is the fraction of the wager. Another, more formal alternative, is to square the n'th normalized increment, (which, also, removes any negative sign,) and then take the square root of the square. Which leads to the formalization for the root mean square of the normalized increments, rms, (provided that n is sufficiently large[15]):

[15] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

$$rms^2 = \frac{1}{n} \sum_{i=0}^{n} \left(\frac{C(t) - C(t-1)}{C(t-1)} \right)^2 \tag{B.246}$$

This is an important concept, since it shows that $rms = f$, or:

$$rms^2 = \frac{1}{n} \sum_{i=0}^{n} f^2 = \frac{1}{n} n \cdot f^2 = f^2 \tag{B.247}$$

or, importantly:

$$rms = f \tag{B.248}$$

For the average, avg, of the normalized increments of the gambler's capital, consider that in an interval of n many iterations of the game, (provided that n is sufficiently large,) there will be P many wins, and $1 - P$ many losses, and since the gambler's capital increased by $+f$ for the wins, and $-f$ for the losses, or:

$$avg = f \cdot [P - (1 - P)] = f \cdot (2P - 1) \tag{B.249}$$

but since f = rms:

$$avg = rms \cdot (2P - 1) \tag{B.250}$$

or:

$$\frac{avg}{rms} = 2P - 1 \tag{B.251}$$

and rearranging:

$$2P = \frac{avg}{rms} + 1 \tag{B.252}$$

and solving for P:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.253}$$

Which is the formula for the Shannon probability, P, as a function of the average and root mean square of the normalized increments of the gambler's capital, avg and rms, respectively. It is an important concept that with the measurement of these two quantities, (and the metrics on these two quantities can be deduced dynamically, or "on the fly,") that an optimal wagering strategy, (or cash flow optimization,) can be formulated.

It should be noted that this derivation is for analyzing a time series that is characterized as a Brownian motion fractal. A similar derivation can be used for time series that are characterized by fractional Brownian motion. However, the derivation is much more formidable, mathematically.

As a matter of practical interest, the term "provided that n is sufficiently large" needs to be qualified. Note that when the term "running average" or "running root mean square" is used, we really need to know how many iterations of coin tosses, n, are necessary to be considered "sufficiently large." If we consider the formula:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.254}$$

and noting that the Shannon probability, P, has a range $0 \leq P \leq 1$, and we are using a summing process for both the average, and root mean square of the normalized increments, then n would have to be 100 to achieve a somewhat less than 1% error in P. The reasoning is that if we sum 100 ones, then the resultant sum would be 100, and the next iteration that is to be added to the sum could create at most a 1% error. The implication of this is that one should use a window of at least 100 time units. (hours, days, weeks, or whatever is being used as a unit time in the time series being analyzed,) to achieve a 1%, or better uncertainty in P. In stock price performance analysis, this is a marginal accuracy, so a larger window size would be recommended. A more formal methodology would use the program *tsstatest* to determine, precisely, the size of the data set required.

As a few examples of using very simple programs to perform fractal metric analysis on stock time series:

tscoin -p 0.6 2500

would generate a fractal time series characterized by optimal Brownian motion consisting of 2500 records, and a Shannon probability, P, of 0.6.

tscoins -p 0.6 2500

would generate a fractal time series characterized by optimal fractional Brownian motion consisting of 2500 records, and a Shannon probability, P, of 0.6.

tscoins -p 0.6 -f 0.55 2500

would generate a fractal time series characterized by non—optimal fractional Brownian motion consisting of 2500 records, and a Shannon probability, P, of 0.6, with a wagering fraction of 0.1.

tscoins -p 0.6 -f 0.55 2500 | tsfraction

would generate the normalized increments of a fractal time series characterized by non—optimal fractional Brownian motion consisting of 2500 records, and a Shannon probability, P, of 0.6, with a wagering fraction of 0.1.

tscoins -p 0.6 -f 0.55 2500 | tsfraction | tsavg -p
tscoins -p 0.6 -f 0.55 2500 | tsfraction | tsrms -p

would generate average and the root mean square of the normalized increments of a fractal time series characterized by non—optimal fractional Brownian motion consisting of 2500 records, and a Shannon probability, P, of 0.6, with a wagering fraction of 0.1.

tsfraction my.stock | tsavg -p
tsfraction my.stock | tsrms -p

would measure the average and the root mean square of the normalized increments of the stock time series, my.stock.

It would be convenient to consolidate the various programs into a single monolithic architecture for the analysis and simulation of wagering strategies of stock market time series. It would, further, be convenient, from a comparative standpoint, to let value of the portfolio, at time zero, be the same as the price of a single stock at the beginning of the simulation, so that the portfolio value using the wagering strategy to invest in a single stock can be compared to the price of the stock, over time. To reiterate the previous concepts, suppose that the measurement yielded that the the the fraction of the capital to be invested, f, was 0.2, (ie., a Shannon probability of 0.6,) then we might invest the entire portfolio in the stock, and our portfolio would be modeled as 20% of the portfolio would be wagered at any time, and 80% would be considered as "cash reserves," even though the 80% is actually invested in the stock. Assume the following pseudo code:

calculate the average and root mean square of the normalized increments, avg **and** rms**, respectively**

$capital =$ **value of stock at time** 0**, (ie., the portfolio value at time zero, is one share of stock)**

multiplier $= \frac{avg^2}{rms^3} \cdot \frac{1}{f}$ **(ie., the value of the multiplier,** $F \cdot f$ **in the derivations, by which the fraction of the capital that is to be wagered must be increased, ie.,** $F = m$ **multiplier)**

for each time interval, (ie., for each increment in the time series)

if not the first interval?, (ie., we need to calculate the normalized increments, so the first interval can not be used)

$capital = lastcapital \cdot multiplier \cdot (1 + increment)$**, (ie., this is the new capital for today)**

$lastcapital = capital$**, (ie., this is yesterday's capital, tomorrow)**

where the increment is calculated by subtracting todays stock value from yesterday's stock value, and dividing by yesterday's stock value:

$$increment = \frac{V(t) - V(t-1)}{V(t-1)} \tag{B.255}$$

Note that:

$$capital = lastcapital \cdot multiplier \cdot (1 + increment) \tag{B.256}$$

$$capital = lastcapital \cdot multiplier \cdot \left(1 + \frac{V(t) - V(t-1)}{V(t-1)}\right) \tag{B.257}$$

$$capital = lastcapital \cdot multiplier \cdot \left(1 + \frac{V(t)}{V(t-1)} - 1\right) \tag{B.258}$$

$$capital = lastcapital \cdot multiplier \cdot \frac{V(t)}{V(t-1)} \tag{B.259}$$

which, not surprisingly, if $multiplier = 1$, (ie., $P = P'$):

$$capital = lastcapital \cdot \frac{V(t)}{V(t-1)} \tag{B.260}$$

meaning that the portfolio value would track the stock's value, as we would expect. Likewise, if $multiplier$ is greater than 1, the portfolio value would linearly track the stock value, by a constant of proportionality, and the amount of the portfolio invested in the stock would be greater than the value of the portfolio, possibly indicating that the the remainder of the stock investment was purchased on margin. If the program is used to determine the fraction of the portfolio that is to be invested in a specific stock, then the fraction can be calculated from:

$$f = 2P - 1 \tag{B.261}$$

and:

$$fraction = multiplier \cdot f \tag{B.262}$$

It would also be desirable to be able to automatically determine the number of stocks that should be held. The total capital invested in a stock is:

$$capital \cdot multiplier \tag{B.263}$$

and dividing this value by the current value of the stock will give the number of stocks that should be invested in.

Note that the portfolio investment simulation model is very simple, and assumes perfect liquidity of the stock, (ie., as many as necessary can be bought or sold at exactly the day's closing price of the stock,) and that there are no transaction commissions.

An example output from the *tsstock* program appears in Figure B.57.

```
(4.925045   @ 374.353743   = 1843.708895   ) + 341.049570   = 2184.758465
(4.796908   @ 449.224492   = 2154.888603   ) + 398.611643   = 2553.500246
(4.672105   @ 539.069390   = 2518.588969   ) + 465.888996   = 2984.477965
(4.854439   @ 431.255512   = 2093.503546   ) + 387.256625   = 2480.760171
(5.043888   @ 345.004410   = 1740.163700   ) + 321.895764   = 2062.059464
(4.912660   @ 414.005292   = 2033.867132   ) + 376.225072   = 2410.092204
(4.784845   @ 496.806350   = 2377.141591   ) + 439.724037   = 2816.865628
(4.660356   @ 596.167620   = 2778.353640   ) + 513.940307   = 3292.293946
(4.539106   @ 715.401144   = 3247.281935   ) + 600.682739   = 3847.964674
(4.716250   @ 572.320915   = 2699.208298   ) + 499.299989   = 3198.508286
```

Figure B.57: Example output of the *tsstock* program, the input file was produced by the *tscoin* program, with a Shannon probability of 0.6, which is shown in Figure B.46.

B.3.20 tsstocks

Source tsstocks.c, is for simulating the optimal gains of multiple stock investments. The program decides which of all available stocks to invest in at any single time, by calculating the instantaneous Shannon probability of all stocks, and using an approximation to statistical estimation techniques to estimate the accuracy of the calculated Shannon probability.

One of the implications of considering stock prices to have fractal characteristics, ie., random walk or Brownian motion, is that future prices can not be predicted from past stock price performance. The Shannon probability of a stock price time series is the likelihood that a stocks price will increase in the next time interval. It is typically 0.51, on a day to day bases, (although, occasionally, it will be as high as 0.6) What this means, for a typical stock, is that 51% of the time, a stock's price will increase, and 49% of the time it will decrease—and there is no possibility of determining which will occur—only the probability.

However, another implication of considering stock prices to have fractal characteristics is that there are statistical optimizations to maximize portfolio performance. The Shannon probability, P, is related to the volatility of a stock's price, (measured as the root mean square of the normalized increments of the stock's price time series,) rms, by $rms = 2P - 1$. Also, the average of the normalized increments is the growth in the stock's price, and is equal to the square of the rms. Unfortunately, the measurements of avg and rms must be made over a long period of time, to construct a very large data set for analytical purposes do to the necessary accuracy requirements. Statistical estimation techniques are usually employed to quantitatively determine the size of the data set for a given analytical accuracy.

There are several techniques used to optimize stock portfolio performance. Since the volatility of an individual stock price, rms, is considered to have a Gaussian distribution, the volatilities add root mean square. What this means is that if the portfolio consists of 10 stocks, concurrently, with each stock representing 10% of the portfolio, then the volatility of the portfolio will be decreased by a factor of the square root of 10, (assuming all stocks are statistically identical.) Further, since it is assumed that the stocks are statistically identical, the average growth of the stocks adds linearly in the portfolio, ie., it would not make any difference, from a portfolio growth standpoint, whether the portfolio consisted of 1 stock, or 10 stocks. This indicates that control of stock portfolio volatility can be an "engineered solution." (In reality, of course, the stocks are not statistically identical, but the volatilities still add root mean square. The growth of the portfolio would be less, since it was not totally invested in the stock with the highest growth rate—this would be the cost of managing the volatility risk.)

Now consider "timing the market." If a stock's price has fractal characteristics, this is impossible, (at least more than 51% of the time, on average, for most stocks.) Attempting to do so, say by selling a stock for the speculative reason that the stocks price will decrease in the future, will result in selling a stock that 51% of the time would increase in value in the future, and 49% of the time would decrease in value. Of course, holding a stock would have the same probabilities, also.

If a stock's price is fractal, it will, over time, exhibit price increases, and decreases, that have a range that is proportional to the square root of time, and a probable duration that is proportional to the reciprocal of the square root of time. In point of fact, measurements on these characteristics in stock pro forma for the past century offer compelling evidence that stock prices exhibit fractal characteristics. These increases and decreases in stock price over time would lead to the intuitive presumption that a "buy low and sell high" strategy could be implemented. Unfortunately, if stock prices are indeed fractal in nature, that is not the case, because no matter what time scale you use, the characteristics are invariant, (ie., on a time scale—be it by the tick, by the day, by the month, or by the year—the range and duration phenomena is still the same, ie., made up of "long term" increases and decreases, that have no predictive qualities, other than probabilistic.)

The issue with attempting to "time the market" is that if you sell a stock to avoid an intuitively expected price decrease, (which will be correct, 49% of the time, typically,) then you will, also, give up the chance of the stock price increasing, (which will happen 51% of the time.) However, there is an alternative, and that would be to sell the stock, and invest in another stock, (which would also have a 51% chance of increasing in price, on the average—a kind of "hedging" strategy.)

To implement such a strategy, one would never sell a stock for a stock with a smaller Shannon probability, without compelling reasons. In point of fact, it would probably be, at least heuristically, the best strategy to always be invested in the stocks with the most recent largest Shannon probability, the assumption being that during the periods when a stock's price is increasing, the short term "instantaneous" average Shannon probability will be larger than the long term average Shannon probability. (Not that this will always be true—only 51% of the time, for an average stock, will it succeed in the next time interval.) This will require specialized filtering, (to "weight" the most recent instantaneous Shannon probability more than the least recent,) and statistical estimation (to determine the accuracy of the measurement of the Shannon probability, upon which the decision will be made as to which stocks are in the portfolio at any instant in time.)

This decision would be based on the normalized increments,

$$\frac{V_t - V_{t-1}}{V_{t-1}} \tag{B.264}$$

of the time series, which, when averaged over a "sufficiently large" number of increments, is the mean of the normalized increments, avg. The term "sufficiently large" must be analyzed quantitatively. For example, the following table is the statistical estimate for a Shannon probability, P, of a time series, vs, the number of records required, based on a mean of the normalized increments = 0.04, as shown in table B.3.

where avg is the average of the normalized increments, e is the error estimate in avg, c is the confidence level of the error estimate, and n is the number of records required for that confidence level in that error estimate. What this table means is that if a step function, from zero to 0.04, (corresponding to a Shannon probability of 0.6,) is applied to the system, then after 27 records, we would be 70% confident that the error level was not greater than 0.0396, or avg was not lower than 0.0004, which corresponds to an effective Shannon probability of 0.51. Note that if many iterations of this example of 27 records were performed, then 30% of the time, the average of the time series, avg, would be less than 0.0004, and 70% greater than 0.0004. This means that the the Shannon probability, 0.6, would have to be reduced by a factor of 0.85 to accommodate the error created by an insufficient data set size to get the effective

Table B.3: Shannon Probability vs. Number of Records.

P	avg	e	c	n
0.51	0.0004	0.0396	0.7000	27
0.52	0.0016	0.0384	0.7333	33
0.53	0.0036	0.0364	0.7667	42
0.54	0.0064	0.0336	0.8000	57
0.55	0.0100	0.0300	0.8333	84
0.56	0.0144	0.0256	0.8667	135
0.57	0.0196	0.0204	0.9000	255
0.58	0.0256	0.0144	0.9333	635
0.59	0.0324	0.0076	0.9667	3067
0.60	0.0400	0.0000	1.0000	inf

Shannon probability of 0.51. Since half the time the error would be greater than 0.0004, and half less, the confidence level would be $1 - ((1 - 0.85) \cdot 2) = 0.7$, meaning that if we measured a Shannon probability of 0.6 on only 27 records, we would have to use an effective Shannon probability of 0.51, corresponding to an avg of 0.0004. For 33 records, we would use an avg of 0.0016, corresponding to a Shannon probability of 0.52, and so on.

The table above was made by iterating the *tsstatest* program, and can be approximated by a single pole low pass recursive discreet time filter [Con78, pp. 11], with the pole frequency at 0.00045 times the time series sampling frequency. The accuracy of the approximation is about $\pm$10% for the first 260 samples, with the approximation accuracy prediction becoming optimistic thereafter, ie., about $+$30%.

A pole frequency of 0.033 seems a good approximation for working with the root mean square of the normalized increments, with a reasonable approximation to about $5 - - - 10$ time units.

The "instantaneous," weighted, and statistically estimated Shannon probability, P, can be determined by dividing the filtered rms by the filtered avg, adding unity, and dividing by two.

(Note: there is some possibility of operating on the absolute value of the normalized increments, which is a close approximation to the root mean square of the normalized increments. Another possibility is to use trading volumes to calculate the instantaneous value for the average and root mean square of the increments as in the *tsshannonvolume* program. Also, another reasonable statistical estimate approximation is $P_{est} = 0.5 + (1 - 1/sqrt(n)) \cdot ((2 \cdot P_{meas}) - 1) \cdot 0.5$, where P_{meas} is the measured Shannon probability over n many records, and P_{est} is the Shannon probability that should be used do to the uncertainty created by an inadequate data set size.)

The advantage of the discreet time recursive single pole filter approximation is that it requires only 3 lines of code in the implementation—two for initialization, and one in the calculation construct.

The single pole low pass filter is implemented from the following discrete time equation:

$$v_{n+1} = I \cdot k2 + v_n \cdot k1 \tag{B.265}$$

where I is the value of the current sample in the time series, v are the value of the output time series, and $k1$ and $k2$ are constants determined from the following equations:

$$k1 = e^{-2 \cdot p \cdot \pi} \tag{B.266}$$

and

$$k2 = 1 - k1 \tag{B.267}$$

where p is a constant that determines the frequency of the pole—a value of unity places the pole at the sample frequency of the time series.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

B.3.21 tstrade

Source tstrade.c is for simulating the optimal gains of multiple equity investments. The program decides which of all available equities to invest in at any single time, by calculating the instantaneous Shannon probability of all equities, and using an approximation to statistical estimation techniques to estimate the accuracy of the calculated Shannon probability.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Each data record represents an equity transaction, consisting of a minium of six fields, separated by white space. The fields are ordered by time stamp, equity ticker identifier, maximum price in time unit, minimum price in time unit, closing price in time unit, and trade volume. The existence of a record with more than 6 fields is used to suspend transactions on the equity, concluding with the record, for example:

```
930830    AA      38.125   37.875   37.938   333.6
930830    AALR    3.250    2.875    3.250    7.2      Suspend
930830    AHP     64.375   63.625   64.375   335.9
```

Note: this program uses the following functions from other references:

ran1: which returns a uniform random deviate between 0.0 and 1.0. See [PFTV88, pp. 210], referencing Knuth.

gasdev: which returns a normally distributed deviate with zero mean and unit variance, using ran1 () as the source of uniform deviates. See [PFTV88, pp. 217].

Introduction

One of the prevailing concepts in financial quantitative analysis, (eg., "financial engineering,") is that equity prices exhibit "random walk," (eg., Brownian motion, or fractal,) characteristics. The presentation by Brian Arthur [Art95] offers a compelling theoretical framework for the random walk model. William A. Brock and Pedro J. F. de Lima [BdL95] among others, have published empirical evidence supporting Arthur's theoretical arguments.

There is a large mathematical infrastructure available for applications of fractal analysis to equity markets. For example, the publications authored by Richard M. Crownover [Cro95], Edgar E. Peters [Pet91], and Manfred Schroeder [Sch91] offer formal methodologies, while the books by John L. Casti [Cas90], [Cas94] offer a less formal approach for the popular press.

There are interesting implications that can be exploited if equity prices exhibit fractal characteristics:

1. It would be expected that equity portfolio volatility would be equal to the root mean square of the individual equity volatilities in the portfolio.

2. It would be expected that equity portfolio growth would be equal to the linear addition of the growths of the individual equities in the portfolio.

3. It would be expected that an equity's price would fluctuate, over time, and the range, of these fluctuations (ie., the maximum price minus the minimum price,) would increase with the square root of time.

4. It would be expected that the number of equity price fluctuations in a time interval, (ie., the number of times an equity's price reaches a local maximum, then reverse direction and decreases to a local minimum,) would increase with the square root of time.

5. It would be expected that the time between fluctuations in an equity's price, (ie., the interval between an equity's price reaching a local maximum and then a local minimum,) would decrease with the reciprocal of the square root of time.

6. It would be expected that an equity's price, over time, would be mean reverting, (ie., if an equity's price is below its average, there would be a propensity for the equity's price to increase, and vice versa.)

Note that 1 and 2 above can be exploited to formulate an optimal hedging strategy; 3, 4, and 5 would tend to imply that "market timing" is not attainable; and 6 can be exploited to formulate an optimal buy-sell strategy.

Derivation

As a tutorial, the derivation will start with a simple compound interest equation. This equation will be extended to a first order random walk model of equity prices. Finally, optimizations will derived based on the random walk model that are useful in optimizing equity portfolio performance.

If we consider capital, V, invested in a savings account, and calculate the growth of the capital over time:

$$V_t = V_{t-1}(1 + a_t) \tag{B.268}$$

where a_t is the interest rate at time t, (usually a constant[16].) In equities, a_t is not constant, and varies—perhaps being negative at certain times, (meaning that the value of the equity decreased.) This fluctuation in an equity's value can be represented by modifying a_t in Equation B.268:

$$a_t = f_t F_t \tag{B.269}$$

where the product $f_t \cdot F_t$ is the fluctuation in the equity's value at time t.

An equity's value, over time, is similar to a simple tossed coin game [Sch91, pp. 128], where f_t is the fraction of a gambler's capital wagered on a toss of the coin, at time t, and F_t is a random variable[17], signifying whether the

[16]For example, if a = 0.06, or 6%, then at the end of the first time interval the capital would have increased to 1.06 times its initial value. At the end of the second time interval it would be $(1.06)^2$, and so on. What Equation B.268 states is that the way to get the value, V in the next time interval is to multiply the current value by 1.06. Equation B.268 is nothing more than a "prescription," or a process to make an exponential, or "compound interest" mechanism. In general, exponentials can always be constructed by multiplying the current value of the exponential by a constant, to get the next value, which in turn, would be multiplied by the same constant to get the next value, and so on. Equation B.268 is nothing more than a construction of $V(t) = e^{kt}$ where $k = \ln(1 + a)$. The advantage of representing exponentials by the "prescription" defined in Equation B.268 is analytical expediency. For example, if you have data that is an exponential, the parameters, or constants, in Equation B.268 can be determined by simply reversing the "prescription," ie., subtracting the previous value, (at time $t - 1$,) from the current value, and dividing by the previous value would give the exponentiating constant, $(1 + a_t)$. This process of reversing the "prescription" is termed calculating the "normalized increments." (Increments are simply the difference between two values in the exponential, and normalized increments are this difference divided by the value of the exponential.) Naturally, since one usually has many data points over a time interval, the values can be averaged for better precision—there is a large mathematical infrastructure dedicated to precision enhancement, for example, least squares approximation to the normalized increments, and statistical estimation.

[17]"Random variable" means that the process, F_t, is random in nature, ie., there is no possibility of determining what the next value will be. However, F_t can be analyzed using statistical methods [Fed88, pp. 163], [Sch91, pp. 128]. For example, F_t typically has a Gaussian distribution for equity values [Cro95, pp. 249], in which case the it is termed a "fractional Brownian motion," or simply a "fractal" process. In the case of a single tossed coin, it is termed "fixed increment fractal," "Brownian," or "random walk" process. In any case, determination of the statistical characteristics of F_t are the essence of analysis. Fortunately, there is a large mathematical infrastructure dedicated to the subject. For example, F_t could be verified as having a Gaussian distribution using Chi—Square techniques. Frequently, it is convenient, from an analytical standpoint, to

game was a win, or a loss, ie., whether the gambler's capital increased or decreased, and by how much. The amount the gambler's capital increased or decreased is $f_t \cdot F_t$.

In general, F_t is a function of a random variable, with an average, over time, of avg_f, and a root mean square value, rms_f, of unity. Note that for simple, time invariant, compound interest, F_t has an average and root mean square, both being unity, and f_t is simply the interest rate, which is assumed to be constant. For a simple, single coin game, F_t is a fixed increment, (ie., either $+1$ or -1,) random generator. From an analytical perspective, it would be advantageous to measure the the statistical characteristics of the generator. Substituting Equation B.269 into Equation B.268[18]:

$$V_t = V_{t-1} \left(1 + f_t F_t\right) \tag{B.270}$$

and subtracting V_{t-1} from both sides:

$$V_t - V_{t-1} = V_{t-1} \left(1 + f_t F_t\right) - V_{t-1} \tag{B.271}$$

and dividing both sides by V_{t-1}:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_{t-1} \left(1 + f_t F_t\right) - V_{t-1}}{V_{t-1}} \tag{B.272}$$

and combining:

$$\frac{V_t - V_{t-1}}{V_{t-1}} = \left(1 + f_t F_t\right) - 1 = f_t F_t \tag{B.273}$$

We now have a "prescription," or process, for calculating the characteristics of the random process that determines an equity's price, over time. That process is, for each unit of time, subtract the value of the of the equity at the previous time from the value of the equity at the current time, and divide this by the value of the equity at the previous time. The root mean square[19] of these values are the root mean square of the random process. The average of these values are the average of the random process, avg_f. The root mean square of these values can be calculated by any convenient means, and will be represented by rms. The average of these values can be found by any convenient means, and will be represented by avg[20]. Therefore, if $f_t = f$, and assuming that it does not vary over time:

$$rms = f \tag{B.274}$$

which, if there are sufficiently many samples, is a metric of the equity value's "volatility," and:

"model" F_t using a mathematically simpler process [Sch91, pp. 128]. For example, multiple iterations of tossing a coin can be used to approximate a Gaussian distribution, since the distribution of many tosses of a coin is binomial—which if the number of tosses is sufficient will represent a Gaussian distribution to within any required precision [Sch91, pp. 144], [Fed88, pp. 154].

[18]Equation B.270 is interesting in many other respects. For example, adding a single term, $m \cdot V_{t-1}$, to the equation results in $V_t = V_{t-1} \left(1 + f_t F_t + m \cdot V_{t-1}\right)$ which is the "logistic," or 'S' curve equation, (formally termed the "discreet time quadratic equation,") and has been used successfully in many unrelated fields such as manufacturing operations, market and economic forecasting, and analyzing disease epidemics [Mod92, pp. 131]. There is continuing research into the application of an additional "non-linear" term in Equation B.270 to model equity value non-linearities. Although there have been modest successes, to date, the successes have not proved to be exploitable in a systematic fashion [Pet91, pp. 133]. The reason for the interest is that the logistic equation can exhibit a wide variety of behaviors, among them, "chaotic." Interestingly, chaotic behavior is mechanistic, but not "long term" predictable into the future. A good example of such a system is the weather. It is an important concept that compound interest, the logistic function, and fractals are all closely related.

[19]In this section, "root mean square" is used to mean the variance of the normalized increments. In Brownian motion fractals, this is computed by $\sigma_t otal^2 = \sigma_1^2 + \sigma_2^2 + \cdots$ However, in many fractals, the variances are not calculated by adding the squares, (ie., a power of 2,) of the values—the power may be "fractional," ie., $3/2$ instead of 2, for example [Sch91, pp. 130], [Fed88, pp. 178]. However, as a first order approximation, the variances of the normalized increments of equity values can successfully be added root mean square [Cro95, kpp. 250]. The so called "Hurst" coefficient, which can be measured, determines the process to be used. The Hurst coefficient is range of the equity values over a time interval, divided by the standard deviation of the values over the interval, and its determination is commonly called "R/S" analysis. As pointed out in [Sch91, pp. 157] the errors committed in such simplified assumptions can be significant—however, for analysis of equities, squaring the variances seems to be a reasonably accurate simplification.

[20]For example, many calculators have averaging and root mean square functionality, as do many spreadsheet programs—additionally, there are computer source codes available for both. See the programs *tsrms* and *tsavg*. The method used is not consequential.

$$avg = f \cdot F_t \tag{B.275}$$

and if there are sufficiently many samples, the average of F_t is simply avg_f, or:

$$avg = f \cdot avg_f \tag{B.276}$$

which is a metric on the equity value's rate of "growth." Note that this is the "effective" compound interest rate from Equation B.268. Equations B.274 and B.276 are important equations, since they can be used in portfolio management. For example, Equation B.274 states that the volatility of the capital invested in many equities, simultaneously, is calculated as the root mean square of the individual volatility of the equities. Equation B.276 states that the growths in the same equity values add together linearly[21]. Dividing Equation B.276 by Equation B.274 results in the two f's canceling, or:

$$\frac{avg}{rms} = avg_f \tag{B.277}$$

There may be analytical advantages to "model" F_t as a simple tossed coin game, (either played with a single coin, or multiple coins, ie., many coins played at one time, or a single coin played many times[22].) The number of wins minus the number of losses, in many iterations of a single coin tossing game would be:

$$P - (1 - P) = 2P - 1 \tag{B.278}$$

where P is the probability of a win for the tossed coin. (This probability is traditionally termed, the "Shannon probability" of a win.) Note that from the definition of F_t above, that $P = avg_f$. For a fair coin, (ie., one that comes up with a win 50% of the time,) $P = 0.5$, and there is no advantage, in the long run, to playing the game. However, if $P > 0.5$, then the optimal fraction of capital wagered on each iteration of the single coin tossing game, f, would be $2P - 1$. Note that if multiple coins were used for each iteration of the game, we would expect that the volatility of the gambler's capital to increase as the square root of the number of coins used, and the growth to increase linearly with

[21] There are significant implications do to the fact that equity volatilities are calculated root mean square. For example, if capital is invested in N many equities, concurrently, then the volatility of the capital will be $\frac{1}{\sqrt{N}} \cdot rms$ of an individual equity's volatility, rms, provided all the equites have similar statistical characteristics. But the growth in the capital will be unaffected, ie., it would be statistically similar to investing all the capital in only one equity. What this means is that capital, or portfolio, volatility can be minimized without effecting portfolio growth—ie., volatility risk can addressed. There are further applications. For example, Equation B.273 could be modified by dividing both the normalized increments, and the square of the normalized increments by the daily trading volume. The quotient of the normalized increments divided by the trading volume is the instantaneous growth, avg_f, of the equity, on a per-share basis. Likewise, the square root of the square of the normalized increments divided by the daily trading volume is the instantaneous root mean square, rms_f, of the equity on a per-share basis, ie., its instantaneous volatility of the equity. (Note that these instantaneous values are the statistical characteristics of the equity on a per-share bases, similar to a coin toss, and not on time.) Additionally, it can be shown that the range—the maximum minus the minimum—of an equity's value over a time interval will increase with the square root of of the size of the interval of time [Fed88, pp. 178]. Also, it can be shown that the number of expected equity value "high and low" transitions scales with the square root of time, meaning that the probability of an equity value "high or low" exceeding a given time interval is proportional to the square root of the time interval [Sch91, pp. 153].

[22] Here the "model" is to consider two black boxes, one with a equity "ticker" in it, and the other with a casino game of a tossed coin in it. One could then either invest in the equity, or, alternatively, invest in the tossed coin game by buying many casino chips, which constitutes the starting capital for the tossed coin game. Later, either the equity is sold, or the chips "cashed in." If the statistics of the equity value over time is similar to the statistics of the coin game's capital, over time, then there is no way to determine which box has the equity, or the tossed coin game. The advantage of this model is that gambling games, such as the tossed coin, have a large analytical infrastructure, which, if the two black boxes are statistically the same, can be used in the analysis of equities. The concept is that if the value of the equity, over time, is statistically similar to the coin game's capital, over time, then the analysis of the coin game can be used on equity values. Note that in the case of the equity, the terms in $f_t \cdot F_t$ can not be separated. In this case, $f = rms$ is the fraction of the equity's value, at any time, that is "at risk," of being lost, ie., this is the portion of a equity's value that is to be "risk managed." This is usually addressed through probabilistic methods, as outlined below in the discussion of Shannon probabilities, where an optimal wagering strategy is determined. In the case of the tossed coin game, the optimal wagering strategy is to bet a fraction of the capital that is equal to $f = rms = 2P - 1$ [Sch91, pp. 128, 151], where P is the Shannon probability. In the case of the equity, since $f = rms$ is not subject to manipulation, the strategy is to select equities that closely approximate this optimization, and the equity's value, over time, on the average, would increase in a similar fashion to the coin game. The growth of either investment would be equal to $avg = rms^2$, on average, for each iteration of the coin game, or time unit of equity investment. This is an interesting concept from risk management since it maximizes the gain in the capital, while, simultaneously, minimizing risk exposure to the capital.

the number of coins used, irregardless of whether many coins were tossed at once, or one coin was tossed many times, (ie., our random generator, F_t would assume a binomial distribution—and if the number of coins was very large, then F_t would assume, essentially, a Gaussian distribution.) Many equities have a Gaussian distribution for the random process, F_t. It may be advantageous to determine the Shannon probability to analyze equity investment strategies. From Equation B.277:

$$\frac{avg}{rms} = avg_f = 2P - 1 \tag{B.279}$$

or:

$$\frac{avg}{rms} + 1 = 2P \tag{B.280}$$

and:

$$P = \frac{\frac{avg}{rms} + 1}{2} \tag{B.281}$$

where only the average and root mean square of the normalized increments need to be measured, using the "prescription" or process outlined above.

Interestingly, what Equation B.279 states is that the "best" equity investment is not, necessarily, the equity that has the largest average growth, avg_f. The best equity investment is the equity that has the largest growth, while simultaneously having the smallest volatility. In point of fact, the optimal decision criteria is to choose the equity that has the largest *ratio* of growth to volatility, where the volatility is measured by computing the root mean square of the normalized increments, and the growth is computed by averaging the normalized increments.

Market

We now have a "first order prescription" that enables us to analyze fluctuations in equity values, although we have not explained why equity values fluctuate. For a formal presentation on the subject, see the bibliography in [Art95] which, also, offers non-mathematical insight into the subject.

Consider a very simple equity market, with only two people holding equities. Equity value "arbitration" (ie., how equity values are determined,) is handled by one person posting (to a bulletin board,) a willingness to sell a given number of equities at a given price, to the other person. There is no other communication between the two people. If the other person buys the equity, then that is the value of the equity at that time. Obviously, the other person will not buy the equity if the price posted is too high—even if ownership of the equity is desired. For example, the other person could simply decide to wait in hopes that a favorable price will be offered in the future. What this means is that the seller must consider not only the behavior of the other person, but what the other person thinks the seller's behavior will be, ie., the seller must base the pricing strategy on the seller's pricing strategy. Such convoluted logical processes are termed "self referential," and the implication is that the market can never operate in a consistent fashion that can be the subject of deductive analysis [Pen89, pp. 101][23]. As pointed out by [Art95, Abstract], these types of indeterminacies pervade economics[24]. What the two players do, in absence of a deductively consistent and complete

[23]Penrose, referencing Russell's paradox, presents a very good example of logical contradiction in a self-referential system. Consider a library of books. The librarian notes that some books in the library contain their titles, and some do not, and wants to add two index books to the library, labeled "A" and "B," respectively; the "A" book will contain the list of all of the titles of books in the library that contain their titles; and the "B" book will contain the list of all of the titles of the books in the library that do not contain their titles. Now, clearly, all book titles will go into either the "A" book, or the "B" book, respectively, depending on whether it contains its title, or not. Now, consider in which book, the "A" book or the "B" book, the title of the "B" book is going to be placed—no matter which book the title is placed, it will be contradictory with the rules. And, if you leave it out, the two books will be incomplete.)

[24][Art95] cites the "El Farol Bar" problem as an example. Assume one hundred people must decide independently each week whether go to the bar. The rule is that if a person predicts that more than, say, 60 will attend, it will be too crowded, and the person will stay home; if less than 60 is predicted, the person will go to the bar. As trivial as this seems, it destroys the possibility of long-run shared, rational expectations. If all believe *few* will go, then *all* will go, thus invalidating the expectations. And, if all believe *many* will go, then *none* will go, likewise invalidating

theory of the market, is to rely on inductive reasoning. They form subjective expectations or hypotheses about how the market operates. These expectations and hypothesis are constantly formulated and changed, in a world that forms from others' subjective expectations. What this means is that equity values will fluctuate as the expectations and hypothesis concerning the future of equity values change[25]. The fluctuations created by these indeterminacies in the equity market are represented by the term $f_t F_t$ in Equation B.270, and since there are many such indeterminacies, we would anticipate F_t to have a Gaussian distribution. This is a rather interesting conclusion, since analyzing the actions of aggregately many "agents," each operating on subjective hypothesis in a market that is deductively indeterminate, can result in a system that can not only be analyzed, but optimized.

Optimization

The only remaining derivation is to show that the optimal wagering strategy is, as cited above:

$$f = rms = 2P - 1 \tag{B.282}$$

where f is the fraction of a gambler's capital wagered on each toss of a coin that has a Shannon probability, P, of winning.

Following [Rez94, pp. 450], consider that the gambler has a private wire into the future who places wagers on the outcomes of a game of chance. We assume that the side information which he receives has a probability, P, of being true, and of $1 - P$, of being false. Let the original capital of gambler be $V(0)$, and $V(n)$ his capital after the n'th wager. Since the gambler is not certain that the side information is entirely reliable, he places only a fraction, f, of his capital on each wager. Thus, subsequent to n many wagers, assuming the independence of successive tips from the future, his capital is:

$$V(n) = (1 + f)^w (1 - f)^l V(0) \tag{B.283}$$

where w is the number of times he won, and $l = n - w$, the number of times he lost. These numbers are, in general, values taken by two random variables, denoted by W and L. According to the law of large numbers:

$$\lim_{n \to \infty} \frac{1}{n} W = P \tag{B.284}$$

and:

$$\lim_{n \to \infty} \frac{1}{n} L = q = 1 - P \tag{B.285}$$

The problem with which the gambler is faced is the determination of f leading to the maximum of the average exponential rate of growth of his capital. That is, he wishes to maximize the value of:

$$G = \lim_{n \to \infty} \frac{1}{n} \ln \frac{V(n)}{V(0)} \tag{B.286}$$

with respect to f, assuming a fixed original capital and specified P:

those expectations. Predictions of how many will attend depend on others' predictions, and others' predictions of others' predictions. Once again, there is no rational means to arrive at deduced *a-priori* predictions. The important concept is that expectation formation is a self-referential process in systems involving many agents with incomplete information about the future behavior of the other agents. The problem of logically forming expectations then becomes ill-defined, and rational deduction, can not be consistent or complete. This indeterminacy of expectation-formation is by no means an anomaly within the real economy. On the contrary, it pervades all of economics and game theory [Art95].

[25] Interestingly, the system described is a stable system, ie., if the players have a hypothesis that changing equity positions may be of benefit, then the equity values will fluctuate—a self fulfilling prophecy. Not all such systems are stable, however. Suppose that one or both players suddenly discover that equity values can be "timed," ie., there are certain times when equities can be purchased, and chances are that the equity values will increase in the very near future. This means that at certain times, the equites would have more value, which would soon be arbitrated away. Such a scenario would not be stable.

$$G = \lim_{n \to \infty} \frac{W}{n} \ln(1+f) + \frac{L}{n} \ln(1-f) \tag{B.287}$$

or:

$$G = P \ln(1+f) + q \ln(1-f) \tag{B.288}$$

which, by taking the derivative with respect to f, and equating to zero, can be shown to have a maxima when:

$$\frac{dG}{df} = P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{1-P-1}(1+f)^{P} = 0 \tag{B.289}$$

combining terms:

$$P(1+f)^{P-1}(1-f)^{1-P} - (1-P)(1-f)^{P}(1+f)^{P} = 0 \tag{B.290}$$

and splitting:

$$P(1+f)^{P-1}(1-f)^{1-P} = (1-P)(1-f)^{P}(1+f)^{P} \tag{B.291}$$

then taking the logarithm of both sides:

$$\ln(P) + (P-1)\ln(1+f) + (1-P)\ln(1-f) = \ln(1-P) - P\ln(1-f) + P\ln(1+f) \tag{B.292}$$

and combining terms:

$$(P-1)\ln(1+f) - P\ln(1+f) + (1-P)\ln(1-f) + P\ln(1-f) = \ln(1-P) - \ln(P) \tag{B.293}$$

or:

$$\ln(1-f) - \ln(1+f) = \ln(1-P) - \ln(P) \tag{B.294}$$

and performing the logarithmic operations:

$$\ln\left(\frac{1-f}{1+f}\right) = \ln\left(\frac{1-P}{P}\right) \tag{B.295}$$

and exponentiating:

$$\frac{1-f}{1+f} = \frac{1-P}{P} \tag{B.296}$$

which reduces to:

$$P(1-f) = (1-P)(1+f) \tag{B.297}$$

and expanding:

$$P - Pf = 1 - Pf - P + f \tag{B.298}$$

or:

$$P = 1 - P + f \tag{B.299}$$

and, finally:

$$f = 2P - 1 \tag{B.300}$$

Fixed Increment Fractal

It was mentioned that it would be useful to model equity prices as a fixed increment fractal, ie., an unfair tossed coin game.

As above, consider a gambler, wagering on the iterated outcomes of an unfair tossed coin game. A fraction, f, of the gambler's capital will be wagered on the outcome of each iteration of the unfair tossed coin, and if the coin comes up heads, with a probability, P, then the gambler wins the iteration, (and an amount equal to the wager is added to the gambler's capital,) and if the coin comes up tails, with a probability of $1 - P$, then the gambler looses the iteration, (and an amount of the wager is subtracted from the gambler's capital.)

If we let the outcome of the first coin toss, (ie., whether it came up as a win or a loss,) be $c(1)$ and the outcome of the second toss be $c(2)$, and so on, then the outcome of the n'th toss, $c(n)$, would be:

$$C\left(n\right) = \begin{cases} win, & \text{with a probability of P} \\ loose, & \text{with a probability of 1 - P} \end{cases} \tag{B.301}$$

for convenience, let a win to be represented by $+1$, and a loss by -1:

$$C\left(n\right) = \begin{cases} +1, & \text{with a probability of P} \\ -1, & \text{with a probability of 1 - P} \end{cases} \tag{B.302}$$

for the reason that when we multiply the wager, f, by $c(n)$, it will be a positive number, (ie., the wager will be added to the capital,) and for a loss, it will be a negative number, (ie., the wager will be subtracted from the capital.) This is convenient, since the increment, by with the gambler's capital increased or decreased in the n'th iteration of the game is $f \cdot c(n)$.

If we let $C(0)$ be the initial value of the gambler's capital, $C(1)$ be the value of the gambler's capital after the first iteration of the game, then:

$$C\left(1\right) = C\left(0\right) \cdot \left(1 + c\left(1\right) \cdot f\left(1\right)\right) \tag{B.303}$$

after the first iteration of the game, and:

$$C\left(2\right) = C\left(0\right) \cdot \left(\left(1 + c\left(1\right) \cdot f\left(1\right)\right) \cdot \left(1 + c\left(2\right) \cdot f\left(2\right)\right)\right) \tag{B.304}$$

after the second iteration of the game, and, in general, after the n'th iteration of the game:

$$C\left(n\right) = C\left(0\right) \cdot \left(\left(1 + c\left(1\right) \cdot f\left(1\right)\right) \cdot \left(1 + c\left(2\right) \cdot f\left(2\right)\right) \cdot \quad \cdots \quad \cdot \left(1 + c\left(n\right) \cdot f\left(n\right)\right) \cdot \left(1 + c\left(n+1\right) \cdot f\left(n+1\right)\right)\right) \tag{B.305}$$

For the normalized increments of the time series of the gambler's capital, it would be convenient to rearrange these formulas. For the first iteration of the game:

$$C\left(1\right) - C\left(0\right) = C\left(0\right) \cdot \left(1 + c\left(1\right) \cdot f\left(1\right)\right) - C\left(0\right) \tag{B.306}$$

or:

$$\frac{C\left(1\right) - C\left(0\right)}{C\left(0\right)} = \frac{C\left(0\right) \cdot \left(1 + c\left(1\right) \cdot f\left(1\right)\right) - C\left(0\right)}{C\left(0\right)} \tag{B.307}$$

and after reducing, the first normalized increment of the gambler's capital time series is:

$$\frac{C\left(1\right) - C\left(0\right)}{C\left(0\right)} = \left(1 + c\left(1\right) \cdot f\left(1\right)\right) - 1 = c\left(1\right) \cdot f\left(1\right) \tag{B.308}$$

and for the second iteration of the game:

$$C(2) = C(0) \cdot ((1 + c(1) \cdot f(1)) \cdot (1 + c(2) \cdot f(2))) \tag{B.309}$$

but $C(0) \cdot ((1 + c(1) \cdot f(1))$ is simply $C(1)$:

$$C(2) = C(1) \cdot (1 + c(2) \cdot f(2)) \tag{B.310}$$

or:

$$C(2) - C(1) = C(1) \cdot (1 + c(2) \cdot f(2)) - C(1) \tag{B.311}$$

which is:

$$\frac{C(2) - C(1)}{C(1)} = \frac{C(1) \cdot (1 + c(2) \cdot f(2)) - C(1)}{C(1)} \tag{B.312}$$

and after reducing, the second normalized increment of the gambler's capital time series is:

$$\frac{C(2) - C(1)}{C(1)} = 1 + c(2) \cdot f(2) - 1 = c(2) \cdot f(2) \tag{B.313}$$

and it should be obvious that the process can be repeated indefinitely, so, the n'th normalized increment of the gambler's capital time series is:

$$\frac{C(n) - C(n-1)}{C(n)} = c(n) \cdot f(n) \tag{B.314}$$

which is Equation B.273.

Data Set Requirements

One of the implications of considering equity prices to have fractal characteristics, ie., random walk or Brownian motion, is that future prices can not be predicted from past equity price performance. The Shannon probability of a equity price time series is the likelihood that a equities price will increase in the next time interval. It is typically 0.51, on a day to day bases, (although, occasionally, it will be as high as 0.6) What this means, for a typical equity, is that 51% of the time, a equity's price will increase, and 49% of the time it will decrease—and there is no possibility of determining which will occur—only the probability.

However, another implication of considering equity prices to have fractal characteristics is that there are statistical optimizations to maximize portfolio performance. The Shannon probability, P, is related to the volatility of a equity's price, (measured as the root mean square of the normalized increments of the equity's price time series,) rms, by $rms = 2P - 1$. Also, the average of the normalized increments is the growth in the equity's price, and is equal to the square of the rms. Unfortunately, the measurements of avg and rms must be made over a long period of time, to construct a very large data set for analytical purposes do to the necessary accuracy requirements. Statistical estimation techniques are usually employed to quantitatively determine the size of the data set for a given analytical accuracy.

The calculation of the Shannon probability, P, from the average and root mean square of the normalized increments, avg and rms, respectively, will require require specialized filtering, (to "weight" the most recent instantaneous Shannon probability more than the least recent,) and statistical estimation (to determine the accuracy of the measurement of the Shannon probability.)

This measurement would be based on the normalized increments, as derived in Equation B.273:

$$\frac{V_t - V_{t-1}}{V_{t-1}} \tag{B.315}$$

which, when averaged over a "sufficiently large" number of increments, is the mean of the normalized increments, avg. The term "sufficiently large" must be analyzed quantitatively. For example, the following table is the statistical estimate for a Shannon probability, P, of a time series, vs, the number of records required, based on a mean of the normalized increments $=$ 0.04, as shown in table B.4.

Table B.4: Shannon Probability vs. Number of Records.

P	avg	e	c	n
0.51	0.0004	0.0396	0.7000	27
0.52	0.0016	0.0384	0.7333	33
0.53	0.0036	0.0364	0.7667	42
0.54	0.0064	0.0336	0.8000	57
0.55	0.0100	0.0300	0.8333	84
0.56	0.0144	0.0256	0.8667	135
0.57	0.0196	0.0204	0.9000	255
0.58	0.0256	0.0144	0.9333	635
0.59	0.0324	0.0076	0.9667	3067
0.60	0.0400	0.0000	1.0000	inf

where avg is the average of the normalized increments, e is the error estimate in avg, c is the confidence level of the error estimate, and n is the number of records required for that confidence level in that error estimate. What this table means is that if a step function, from zero to 0.04, (corresponding to a Shannon probability of 0.6,) is applied to the system, then after 27 records, we would be 70% confident that the error level was not greater than 0.0396, or avg was not lower than 0.0004, which corresponds to an effective Shannon probability of 0.51. Note that if many iterations of this example of 27 records were performed, then 30% of the time, the average of the time series, avg, would be less than 0.0004, and 70% greater than 0.0004. This means that the the Shannon probability, 0.6, would have to be reduced by a factor of 0.85 to accommodate the error created by an insufficient data set size to get the effective Shannon probability of 0.51. Since half the time the error would be greater than 0.0004, and half less, the confidence level would be $1 - ((1 - 0.85) \cdot 2) = 0.7$, meaning that if we measured a Shannon probability of 0.6 on only 27 records, we would have to use an effective Shannon probability of 0.51, corresponding to an avg of 0.0004. For 33 records, we would use an avg of 0.0016, corresponding to a Shannon probability of 0.52, and so on.

The table above was made by iterating the *tsstatest* program, and can be approximated by a single pole low pass recursive discreet time filter [Con78, pp. 11], with the pole frequency at 0.00045 times the time series sampling frequency. The accuracy of the approximation is about $\pm$10% for the first 260 samples, with the approximation accuracy prediction becoming optimistic thereafter, ie., about $+$30%.

A pole frequency of 0.033 seems a good approximation for working with the root mean square of the normalized increments, with a reasonable approximation to about $5 - - - 10$ time units.

The "instantaneous," weighted, and statistically estimated Shannon probability, P, can be determined by dividing the filtered rms by the filtered avg, adding unity, and dividing by two, as in Equation B.281.

The advantage of the discreet time recursive single pole filter approximation is that it requires only 3 lines of code in the implementation—two for initialization, and one in the calculation construct.

The single pole low pass filter is implemented from the following discrete time equation:

$$v_{n+1} = I \cdot k2 + v_n \cdot k1 \tag{B.316}$$

where I is the value of the current sample in the time series, v are the value of the output time series, and $k1$ and $k2$ are constants determined from the following equations:

$$k1 = e^{-2 \cdot p \cdot \pi} \tag{B.317}$$

and

$$k2 = 1 - k1 \tag{B.318}$$

where p is a constant that determines the frequency of the pole—a value of unity places the pole at the sample frequency of the time series.

B.3.22 tstradesim

Source tstradesim.c is for generating a time series for the tstrade program. Generates a fractal time series, of many stocks, concurrently.

The input file is organized, one stock per record, with each record having up to five fields, of which only the Shannon probability need be specified. The fields are sequential, in any order, with field the type specified by a single letter—P for Shannon probability, F for wager fraction, N for trading volume, and I for initial value. Any field that is not one of these letters is assumed to be the stock's name. For example:

```
ABC,  P = 0.51,  F = 0.01,  N = 1000,  I = 31
DEF,  P = 0.52,  F = 0.02,  N = 500,  I = 4
GHI,  P = 0.53,  F = 0.03,  N = 300,  I = 65
```

Naturally, single letter stock names should be avoided, (since P, F, N, and I, are reserved tokens.) Any punctuation is for clarity, and ignored. Upper or lower case characters may be used. The fields are delimited by whitespace, or punctuation. Comment records are are signified by a '#' character as the first non whitespace character in a record. Blank records are ignored.

The output file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Each data record represents an equity transaction, consisting of a minium of six fields, separated by white space. The fields are ordered by time stamp, equity ticker identifier, maximum price in time unit, minimum price in time unit, closing price in time unit, and trade volume. The existence of a record with more than 6 fields is used to suspend transactions on the equity, concluding with the record, for example:

```
1       ABC       38.125    37.875    37.938    333.6
2       DEF       3.250     2.875     3.250     7.2
3       GHI       64.375    63.625    64.375    335.9
```

American markets, since 1950, can be emulated with 300 stocks, each having p = 0.505, and f = 0.03; p = 0.52, f = 0.03 for 300 stocks seems to emulate recent markets.

Note: this program uses the following functions from other references:

ran1: which returns a uniform random deviate between 0.0 and 1.0. See [PFTV88, pp. 210], referencing Knuth.

gasdev: which returns a normally distributed deviate with zero mean and unit variance, using ran1 () as the source of uniform deviates. See [PFTV88, pp. 217].

gammln: which returns the log of the results of the gamma function. See [PFTV88, pp. 168].

The general outline of this program is:

1. given the Shannon probability, compute the abscissa value that divides the area under the normal curve, into two sections, such that the area to the left of the value, divided by the total area under the normal curve is the Shannon probability—a Newton-Raphson iterated approach using Romberg integration to find the area is used for this

2. for each record:

   (a) compute a gaussian distributed random number

   (b) add the computed abscissa value to the gaussian distributed number

   (c) multiply this number by the fraction of cumulative sum to be wagered

   (d) multiply this number by the cumulative sum

   (e) add this number to the cumulative sum

This program will require finding the value of the normal function, given the standard deviation. The method used is to use Romberg/trapezoid integration to numerically solve for the value.

This program will require finding the functional inverse of the normal, ie., Gaussian, function. The method used is to use Romberg/trapezoid integration to numerically solve the equation:

$$F(x) = \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 \tag{B.319}$$

which has the derivative:

$$f(x) = \frac{1}{2\pi} e^{\frac{-x^2}{2}} \tag{B.320}$$

Since $F(x)$ is known, and it is desired to find x,

$$F(x) - \int_0^x \frac{1}{2\pi} e^{\frac{-t^2}{2}} dt + 0.5 = P(x) = 0 \tag{B.321}$$

and the Newton-Raphson method of finding roots would be:

$$P_{n+1} = P_n - \frac{P(x)}{f(x)} \tag{B.322}$$

As a reference on Newton—Raphson Method of root finding, see [PFTV88, pp. 270].

As a reference on Romberg Integration, see [PFTV88, pp. 124].

As a reference on trapezoid iteration, see [PFTV88, pp. 120].

As a reference on polynomial interpolation, see [PFTV88, pp. 90].

B.3.23 tscauchy

Source tscauchy.c, Cauchy distributed noise generator—generates a time series. The idea is to produce a 1 / f power spectrum distribution.

The particular method used is from [Sch91, pp. 159].

An example output from the *tscauchy* program appears in Figure B.58.

B.3.24 tslognormal

Source tslognormal.c is for changing the distribution of a time series to a log-normal distribution. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. This value is multiplied by its exponentiation, (i.e., e-to-the-power,) and the log-normal fractional time series is printed to stdout.

The input file structure is a text file consisting of records, in temporal order, one record per time series sample. Blank records are ignored, and comment records are signified by a '#' character as the first non white space character in the record. Data records must contain at least one field, which is the data value of the sample, but may contain many fields—if the record contains many fields, then the first field is regarded as the sample's time, and the last field as the sample's value at that time.

Example usage:

tscoins -t -p 0.6 20000 | tslognormal | tsunfraction

An example output from the *tslognormal* program appears in Figure B.59.

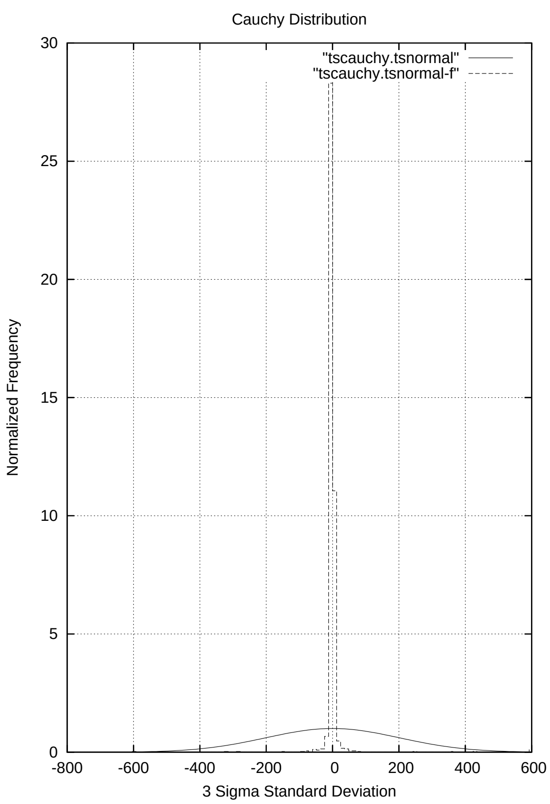

Figure B.58: Example output of the *tscauchy* program, using 1500 records. This is a plot of the frequency histogram.

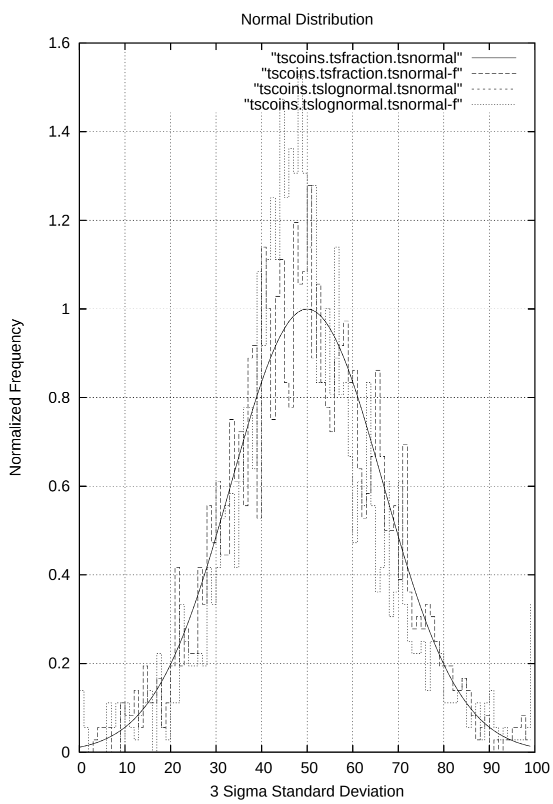

Figure B.59: Example output of the *tslognormal* program, using 2500 records of the *tscoins* and *tsnormal* programs. The data is the same as in Figure B.47, overlayed with the distribution in Figure B.48.

B.3.25 tslaplacian

Source tslaplacian.c, Laplacian noise generator—generates a time series. The idea is to produce a a time series with a Laplacian distribution. The elements, e, of the time series are made from elements from the uniform distribution, n, and:

$$e = \begin{cases} \sqrt{0.5} \cdot \ln(2 \cdot n) & : \quad n < 0.5 \\ -\sqrt{0.5} \cdot \ln(2 \cdot (1 - n)) & : \quad n >= 0.5 \end{cases} \tag{B.323}$$

where the $\sqrt{0.5}$ is the required scaling for a variance of unity.

An example output from the *tslaplacian* program appears in Figure B.60.

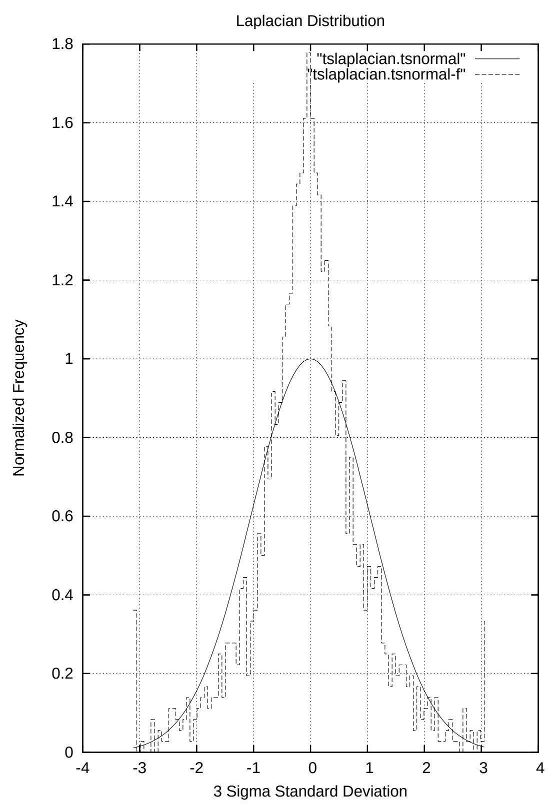

Figure B.60: Example output of the *tslaplacian* program, using the *tsnormal* program for analysis with 1500 records as input.

Appendix C

Fractal Analysis of Various Market Segments in the North American Electronics Industry

This appendix presents a remedial analysis on the optimization of fiscal strategies in various market segments in the North American electronics industry. It is offered in academic perspective, and under no circumstances would it be appropriate to consider it financial advice. It can serve, however, as an illustrative method for comparative analysis of various market segments. Rigorous and sophisticated approaches that address the issues of financial strategies in industrial markets are contained in the bibliography. The analysis of the Dow Jones Average, United States Gross Domestic Product, United States M2, United States Leading Economic Indicators, and United States Employment Figures, and United States Treasury Bill Returns, are presented for comparative purposes[1]—although the optimum fiscal strategies were derived, these optimums may have no real meaning, interpretation, or significance for other than comparative purposes with the rather large research already done by others on these time series. The coin tossing games are presented for "theoretical" comparison of the characteristics of Brownian motion, and regression testing, as are the constructions using the program *tsunfairbrownian*, etc., and are useful in evaluating software system correctness. Additionally, note that the fiscal strategies that are derived in each case, are the financial strategies that will do at least as well as the rest of the industry, in the long run, and may not, necessarily, be the maximal strategy if the rest of the industry is not maximally optimum—ie., it is commensurate with the industry as a whole. Additionally, it should be noted that the amount of data, from various sources, that was analyzed in each market section was very sparce, see [Fed88, pp. 179], [Pet91, pp. 83]. The reader is urged to use caution when judging the accuracy of these presentations.

For the analysis, the data for the various market segments was in the directory ../market, the simulation programs where in the directory ../simulation, and the utility programs in the directory ../utilities. A brief description of the programs appears in Appendix B, and the methodology used is described in Chapter 3. To add a new market segment to the analysis, make a new directory in ../market, and copy all of the files from any other directory into the new directory. The file, named "data," should contain the market time series, with a syntax that is consistent with the program *tsfraction*, which is described briefly in appendix B. Several simulation files are created during the analysis,

[1] One of the reasons that these are included in the analysis is for reasons of scientific induction. The reasoning is as follows. Since the electronics industry is one of the major industries in the United States, fluctuations in the rate of revenue returns of the industry should have correlations in the total production of the United States, flow of money, which can be related by the GNP and M2, leading indicators, and employment figures. Of course, bonds should have an anti-correlation. Additionally, it would seem that a company's equity value, represented by its stock evaluation would rise exponentially as the industry's rate of revenue returns increased exponentially—and this should be reflected in the aggregate industry stock index. The intent was to investigate the correlations in the normalized increments in the decomposition of the time series for each of the macro economic entities. Whether such a correlation can be induced remains conjecture.

for example "data.tsshannonmax-p.tsunfairbrownian-p," which may be re-analyzed by the same method.

The data presented in this appendix is presented in in condensed tabular form in appendix D.

C.1 North American Integrated Circuit Market

For the analysis, the data was in the directory ../markets/ic.namerica[2].

The data in this section is presented in tabular form in Section D.1.

C.1.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.1.1. Figure C.1 is a graph of the time series data for the North American Integrated Circuit Market.

Figure C.2 is a graph of the normalized increments of the time series data presented in Figure C.1. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.3 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.2. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[3].

Figure C.4 is the normalized histogram of the normalized increments of the time series data shown in Figure C.2. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.4.

Figure C.5 is the statistical estimate for the data presented in Figure C.2, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.641843$, as derived in Section C.1.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.6 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.2. In principle, if the distribution of the normalized increments presented in Figure C.4 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.7 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.2. In principle, if the distribution of the normalized increments presented in Figure C.4 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.8 is the range of values of the time series shown in Figure C.1. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.8 would be a

[2]Data from the Semiconductor Industry Association, 1979—1994, by quarters, in millions of dollars, US.

[3]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

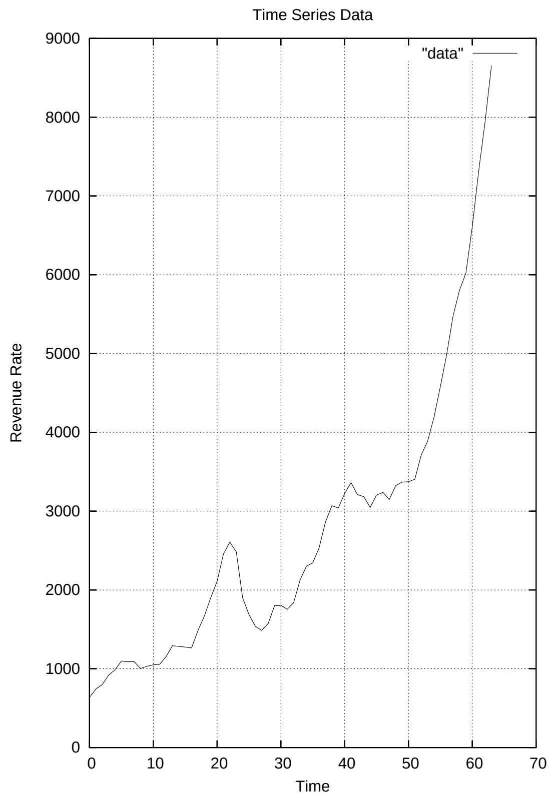

Figure C.1: North American Integrated Circuit Market, time series data.

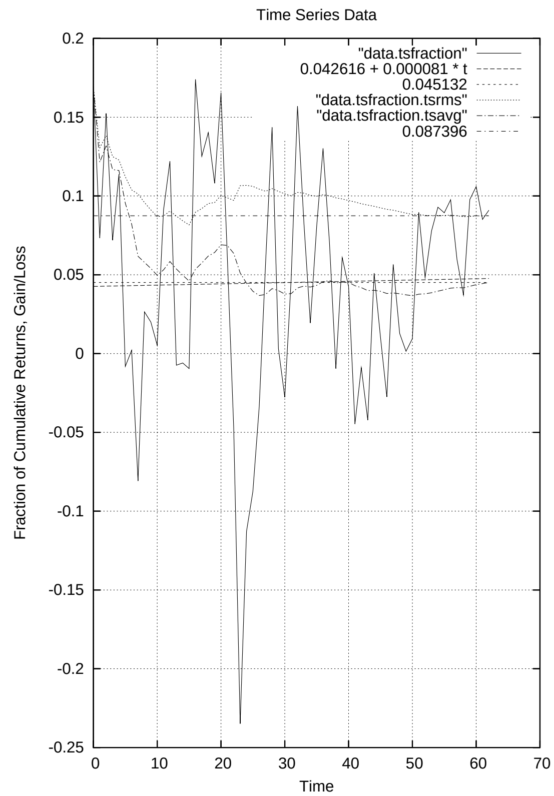

Figure C.2: North American Integrated Circuit Market, normalized increments of the time series data presented in Figure C.1. The mean is 0.045132 with a standard deviation of 0.075442. The formula for the least squares approximation is $0.042616 + 0.000081t$, and the root mean squared value is 0.087396. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000081, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

square root function[4]. Figure C.9 is the deterministic map of the normalized increments of the time series data shown in Figure C.2. The deterministic map is useful for determining if a time series was created by a deterministic mechanism.

[4]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.8 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

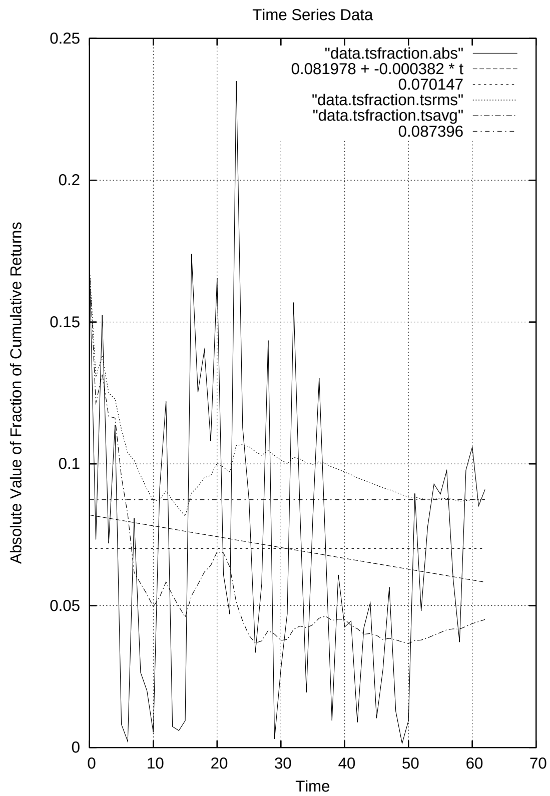

Figure C.3: North American Integrated Circuit Market, absolute value of the normalized increments of the time series data presented in Figure C.2. The mean is 0.070147 with a standard deviation of 0.052548. The formula for the least squares approximation is $0.081978 + -0.000382t$, and the root mean square value, from Figure C.2, is 0.087396. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.2, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

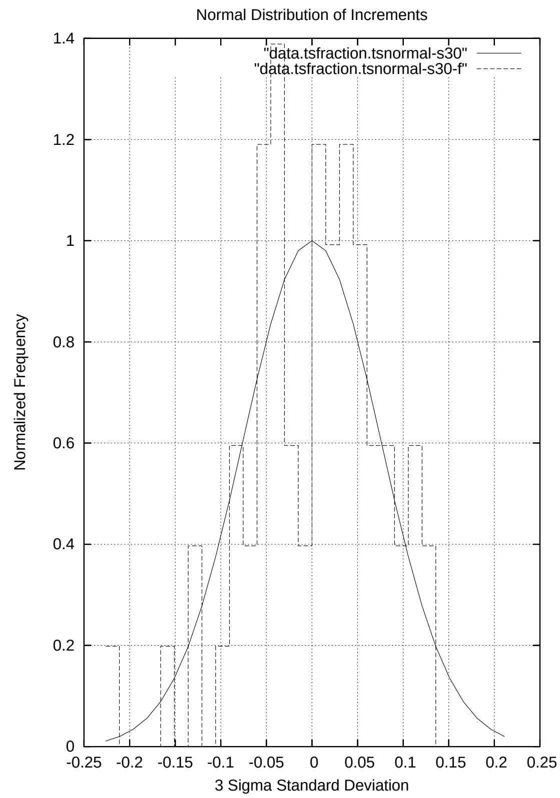

Figure C.4: North American Integrated Circuit Market, normalized histogram of the normalized increments of the time series data shown in Figure C.2. The data has a mean of 0.045132, with a standard deviation of 0.075442. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 6.219000, with a critical value of 42.557000.

This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

```
For a mean of 0.044427, with a confidence level of 0.900000
    that the error did not exceed 0.004443, 1047 samples would be required.
    (With 64 samples, the estimated error is 0.017969 = 40.446295 percent.)
For a standard deviation of 0.087396, with a confidence level of 0.900000
    that the error did not exceed 0.008740, 136 samples would be required.
    (With 64 samples, the estimated error is 0.012706 = 14.538589 percent.)
```

Figure C.5: North American Integrated Circuit Market, statistical estimates of the normalized increments of the time series shown in Figure C.2. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.2.

Observations on the Time Series Increments Analysis

Figure C.4 would seem to indicate that the time series data for the North American Integrated Circuit Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.1.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[5]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.10 is the instantaneous value of the root mean square of the normalized increments for the North American Integrated Circuit Market, and Figure C.11 is the instantaneous Shannon probability for the normalized increments.

C.1.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.1.4. Figure C.12 is a graph of the logistic function estimates of the time series data for the North American Integrated Circuit Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[6]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.12 is a graph of the logistic function for the time series data presented in Figure C.1. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.2. The program *tslsq* was used to derive the constant and the slope

[5] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[6] For example, in Figures C.12 and C.13, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.1.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

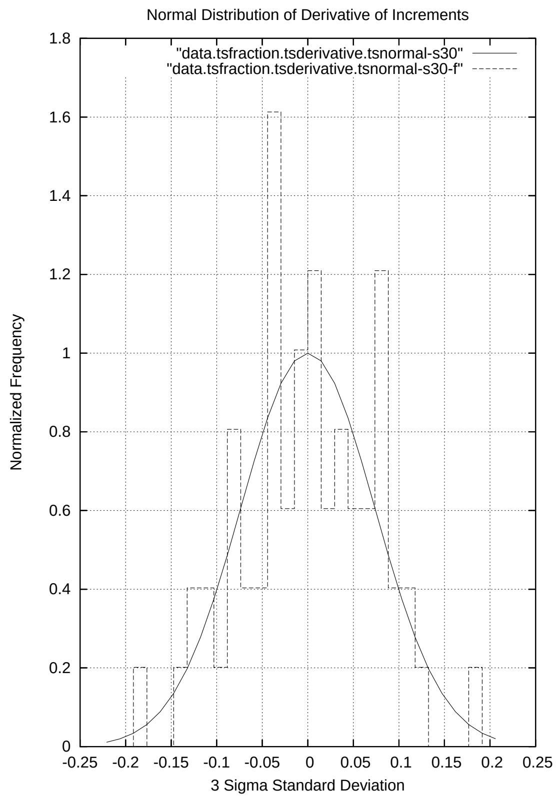

Figure C.6: North American Integrated Circuit Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.2.

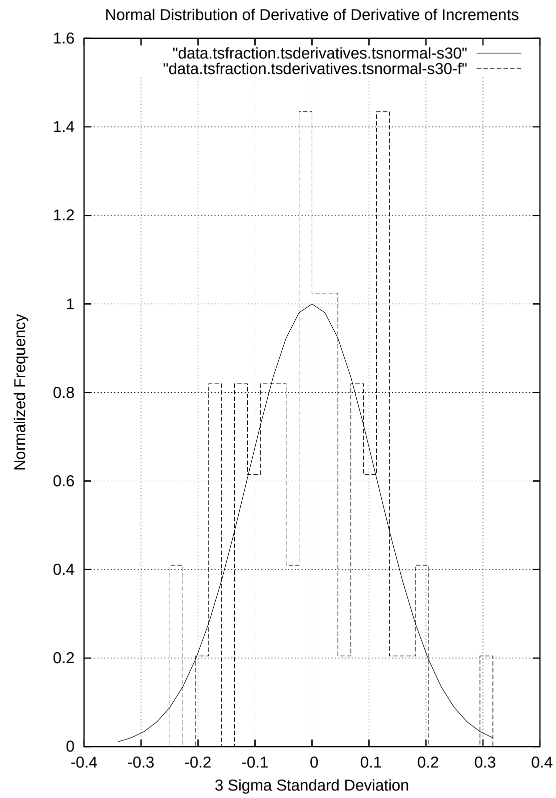

Figure C.7: North American Integrated Circuit Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.2.

of the normalized increments of the data presented in Figure C.2. Figure C.13 is the same graph, but with the time scale expanded by a factor of two.

C.1.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.1.5. Figure C.14 is a graph of the Hurst coefficient data time series data shown in Figure C.1. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.15 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.2. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.14 implies that the variance of the rate of revenue returns, (per quarter,) in the North American Integrated Circuit Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability

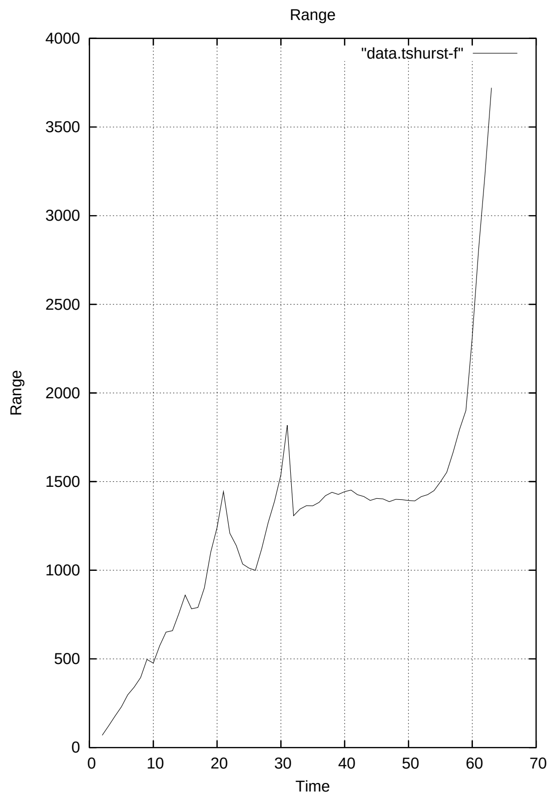

Figure C.8: North American Integrated Circuit Market, range of the time series data shown in Figure C.1.

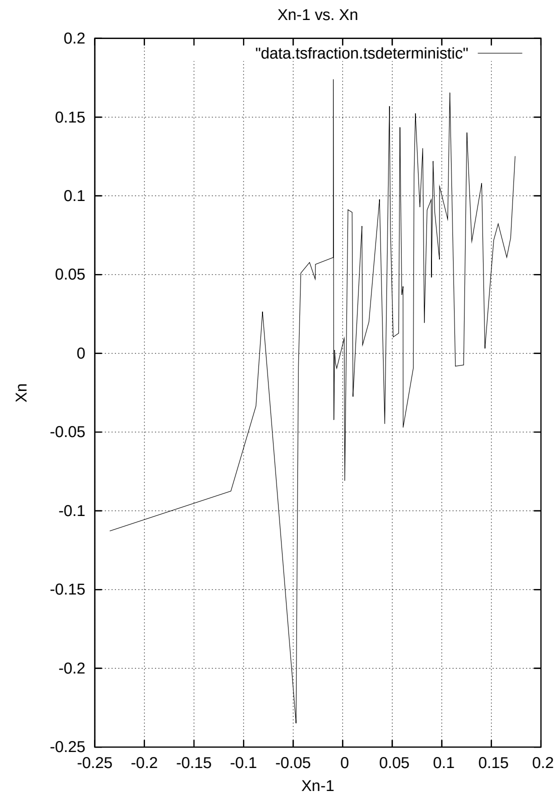

Figure C.9: North American Integrated Circuit Market, deterministic map of the normalized increments of the time series data shown in Figure C.2.

of the state of affairs repeating sometime in the future goes down with increasing time[7], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.18, and, C.19 compare methods of approximation of the "forecastability" of the rate of revenue returns in the North American Integrated Circuit Market for the near term and far term, respectively [Pet91, pp. 83-84][8]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per quarter.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.14, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.997635, so that:

[7] It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[8] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

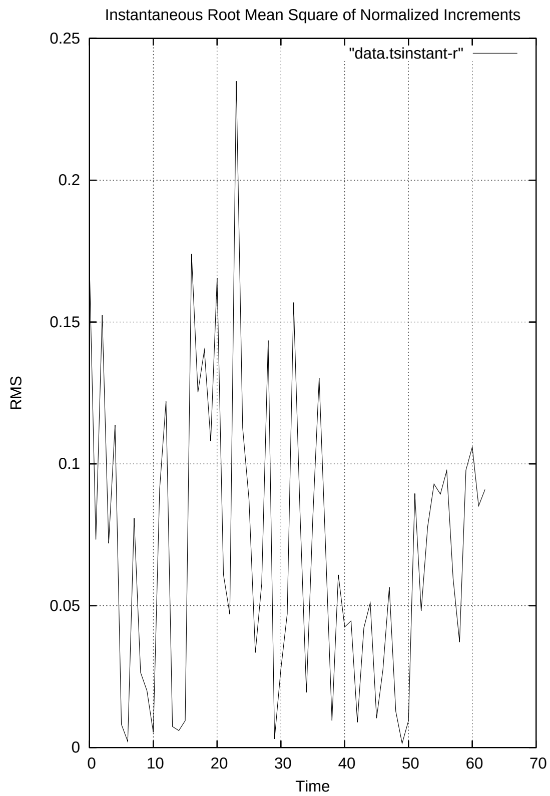

Figure C.10: North American Integrated Circuit Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.1.

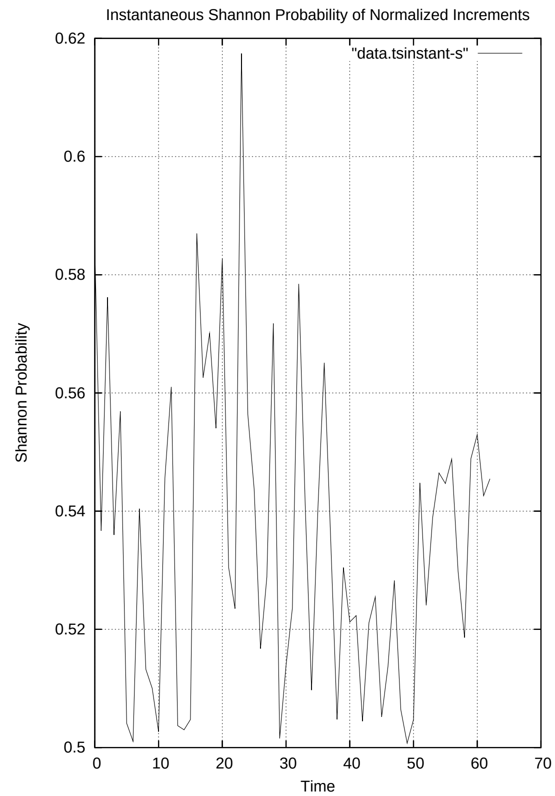

Figure C.11: North American Integrated Circuit Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.1.

$$V\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^{2 \cdot H} \tag{C.1}$$

$$V\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^{2 \cdot 0.997635} \tag{C.2}$$

$$\propto \quad \left(t_2 - t_1\right)^{1.995270} \tag{C.3}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per quarter,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[9]. A Hurst coefficient of 0.997635, (for the near future, and 0.720515 for the distant future.) implies

[9]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^{H} = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For

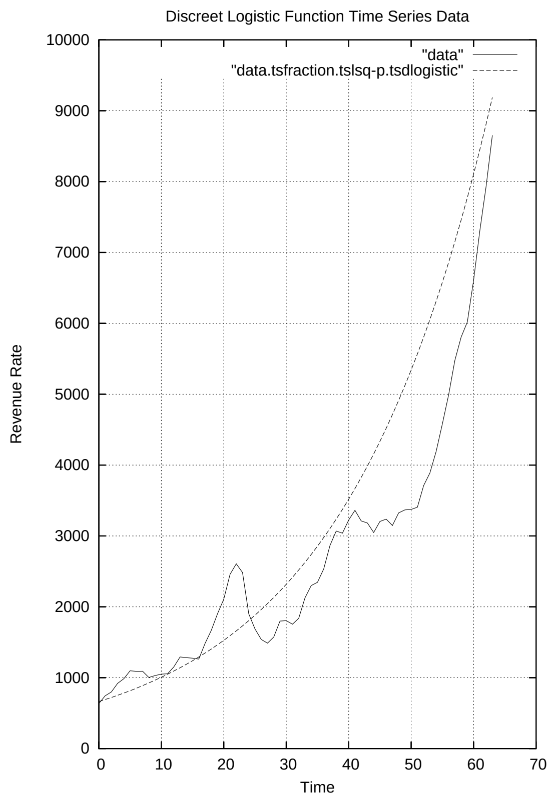

Figure C.12: North American Integrated Circuit Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.2 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

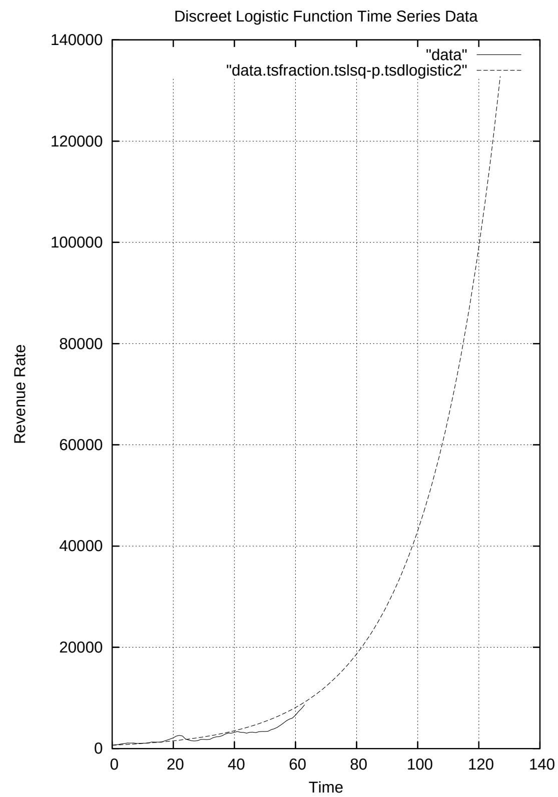

Figure C.13: North American Integrated Circuit Market, logistic function estimates of Figure C.12 with the time scale expanded by a factor of two.

that the likelihood of the rate of revenue returns, (per quarter,) for any two consecutive quarters being the same is 99.763500% [Pet91, pp. 66] for the near future, and 0.720515 for the distant future. Likewise, there is a 99.763500% chance of the rate of revenue returns, (per quarter,) movements being the same in consecutive time periods—ie., if, in a given quarter, the rate of revenue returns, (per quarter,) is increasing, there is a 99.763500% that the rate of revenue returns, (per quarter,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per quarter,) for the North American Integrated Circuit Market are over time, since the probability of having n many consecutive quarters of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many quarters of the same market agenda, p_a,

the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.1.5 for the particulars on using Hurst Coefficient to sum fractal process' for the North American Integrated Circuit Market. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

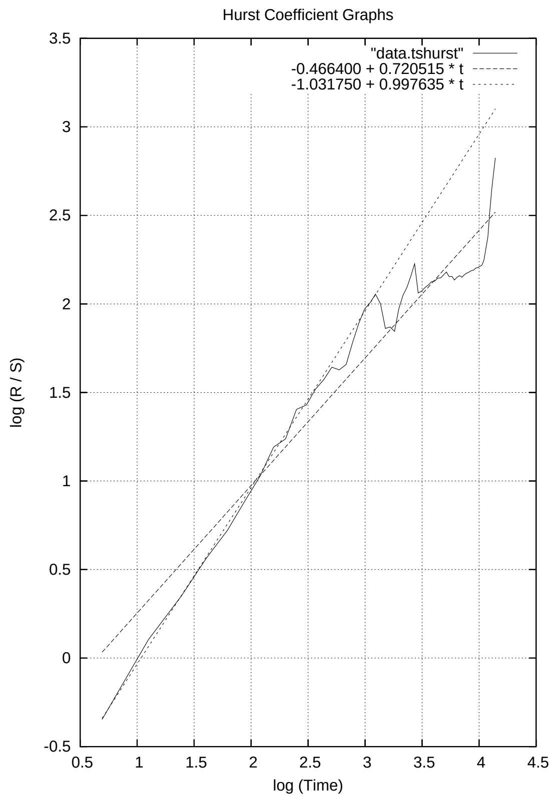

Figure C.14: North American Integrated Circuit Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.2. The slope of the graph is the Hurst coefficient.

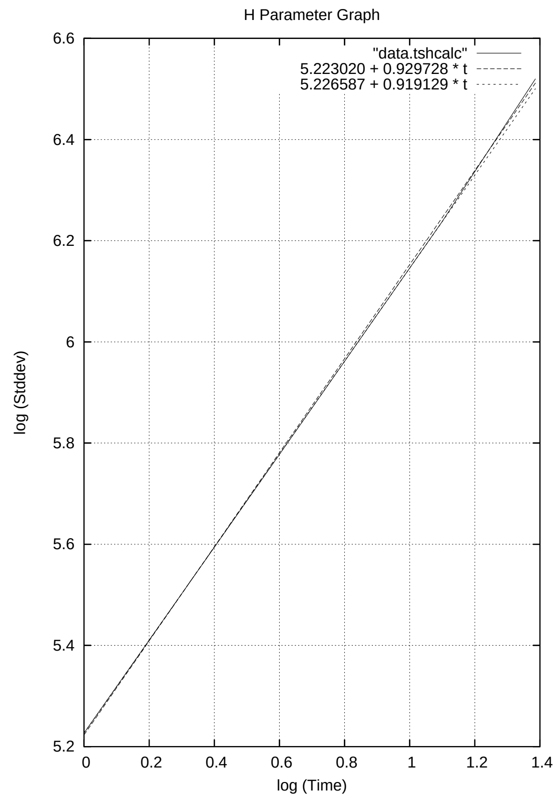

Figure C.15: North American Integrated Circuit Market, H parameter data for the normalized increments of the time series data shown in Figure C.2 The slope of the graph is the H parameter.

is:

$$p_a(n) = H^n \tag{C.4}$$

$$= 0.997635^n \tag{C.5}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.2, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next quarter's rate of revenue returns would be the same as the current quarter's revenue rate. Interestingly, it is $0.045132 \cdot 100$ percent, on the average, with a standard deviation of $0.075442 \cdot 100$ percent, and a root mean square error value of $0.087396 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per quarter,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.6}$$

$$\propto \quad (t_2 - t_1)^{0.997635} \tag{C.7}$$

where R is the range of values in the increments of the rate of revenue returns, (per quarter.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per quarter,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per quarter) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.7 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.8}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.9}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per quarter,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.10}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per quarter,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[10].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \quad \propto \quad \frac{X\,(rt)}{r^H} \tag{C.11}$$

$$\propto \quad \frac{X\,(rt)}{r^{0.997635}} \tag{C.12}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.15, to provide a least squares approximation to the H parameter for the North American Integrated Circuit Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.919129 for the near future, and 0.929728 for the distant future.

Figures C.14 and C.15 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.2. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.2, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.16 and C.17 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Many North American Integrated Circuit Market industry analyst speculate that there is "periodic" behavior in the market place, at approximately 5 year intervals. Both the Hurst coefficient and H parameter graphs would tend to support the intuition. Notice that the slope of the graphs, in figures C.14 and C.15, tend to decrease abruptly at $t \approx \ln(3) \approx 20$ quarters, which is approximately 60 months, or 5 years [Pet91, pp. 96]. Whether this is "periodic" behavior, or an indication of more complex system dynamics, perhaps "chaotic," remains to be seen. If that is the case, it could provide an exploitive venue.

[10] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

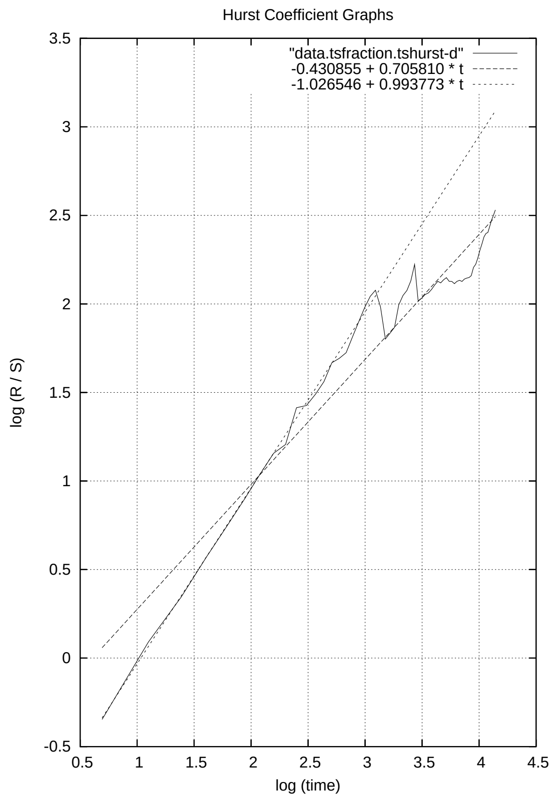

Figure C.16: North American Integrated Circuit Market, traditional Hurst coefficient data for the time series data shown in Figure C.1. The slope of the graph is the Hurst coefficient, and is 0.993773 for the near term, and 0.705810 for the far term.

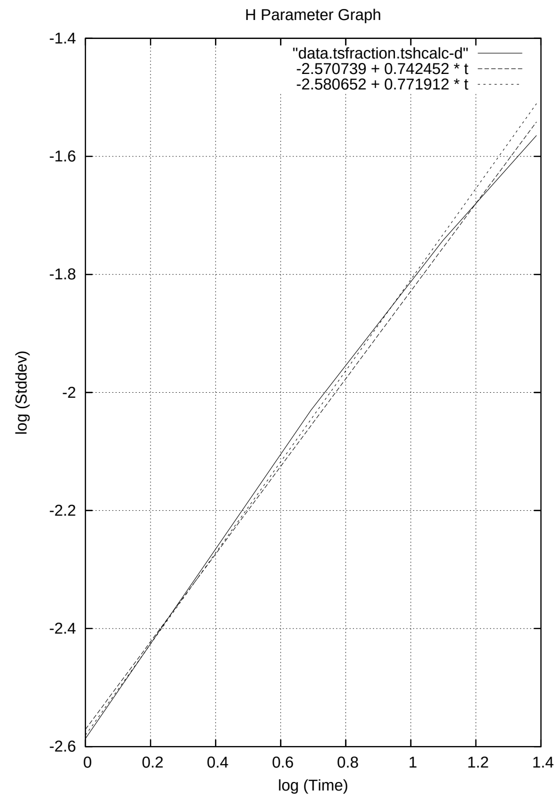

Figure C.17: North American Integrated Circuit Market, traditional H parameter data for the time series data shown in Figure C.1 The slope of the graph is the H parameter, and is 0.771912 for the near term, and 0.742452 for the far term.

C.1.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.1.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.3. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.2. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Integrated Circuit Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the North American Integrated Circuit Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.2, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.045132 + 1)}{\ln(2)} = 0.063685 \tag{C.13}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.2, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.042616 + 1)}{\ln(2)} = 0.060208 \tag{C.14}$$

Note that if the mean is not constant in Figure C.2, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.1:

$$bits = 0.046835 \tag{C.15}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.1:

$$bits = 0.058857 \tag{C.16}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.1.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.058857t} \tag{C.17}$$

therefore:

$$C(p) = 0.058857 \tag{C.18}$$

and, *tsshannon* 0.058857 gives:

$$C(0.641843) = 0.058857 \tag{C.19}$$

therefore:

$$\begin{aligned} 2^{C(0.641843)} &= 2^{0.058857} && \text{(C.20)} \\ &= 1.041640 && \text{(C.21)} \\ &= 4.164018\% && \text{(C.22)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.641843) - 1 && \text{(C.23)} \\ &= 0.283686 && \text{(C.24)} \\ &= 28.368600\% && \text{(C.25)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the North American Integrated Circuit Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every quarter, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 28.368600% of its rate of revenue returns, (per quarter.) As a conceptual model, the remaining 71.631400% will be held in “reserve” with a 64.184300% chance of making twice the 28.368600% back, (and a 35.815700% chance of making 0.0,) in one quarter, on the average, for an average growth in its rate of revenue returns, (per quarter,) of 4.164018%, or a doubling of its rate of revenue returns, (per quarter,) in 16.990333 quarters.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 28.368600% per quarter of the rate of revenue returns, (per quarter,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 4.164018%, per quarter, on average.

Note that the metrics presented in this section are representative of the North American Integrated Circuit Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 28.368600% of its rate of revenue returns, (per quarter,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some quarters, depending on the North American Integrated Circuit Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other quarters, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 4.164018% per quarter.

As another simple example, a company re-invests 28.368600% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 28.368600% per quarter investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 4.164018% per quarter.

As an example of "product portfolio" management, suppose a company re-invests 28.368600% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 28.368600$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 28.368600$ percent for the second product, implying that the company should diversify its product line[11]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 28.368600%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the North American Integrated Circuit Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.283686}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.2, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex

[11] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 28.368600% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 28.368600% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 28.368600 of the rate of revenue returns per quarter does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 28.368600% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.283686}$ products seems consistent with the industry, also.

C.1.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the North American Integrated Circuit Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the North American Integrated Circuit Market time series is 0.045132, and 0.087396respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 5.908830.

If this value seems consistent number of companies in the North American Integrated Circuit Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the North American Integrated Circuit Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.606221, which would be the value which should be used in Section C.1.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.1.5 is greater than the average Shannon probability for the companies participating in the North American Integrated Circuit Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.1.5. The maximum exploitability for the North American Integrated Circuit Market is derived in Section C.1.10, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the North American Integrated Circuit Market is 0.606221, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.758204 in the North American Integrated Circuit Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.26}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the North American Integrated Circuit Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.1.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.3. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.2. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Integrated Circuit Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.1.5, is derived from the North American Integrated Circuit Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.1.10.

An additional exploitable strategy may be time itself. Equations C.3, C.7, and, C.5, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the North American Integrated Circuit Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.18, and, C.19 compare methods of approximation of the "forecastability" of rate of revenue returns in the North American Integrated Circuit Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the North American Integrated Circuit Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[12]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.18, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.5, $0.997635^n = 0.5$ quarters of operations. Since the optimal amount of inventory and, from Equation C.3, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.19, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[13]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with

[12] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[13] For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

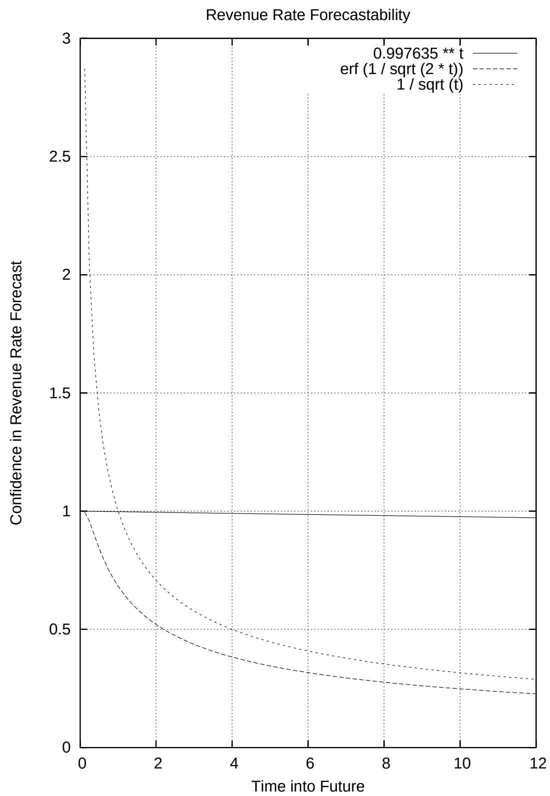

Figure C.18: North American Integrated Circuit Market, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.997635^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

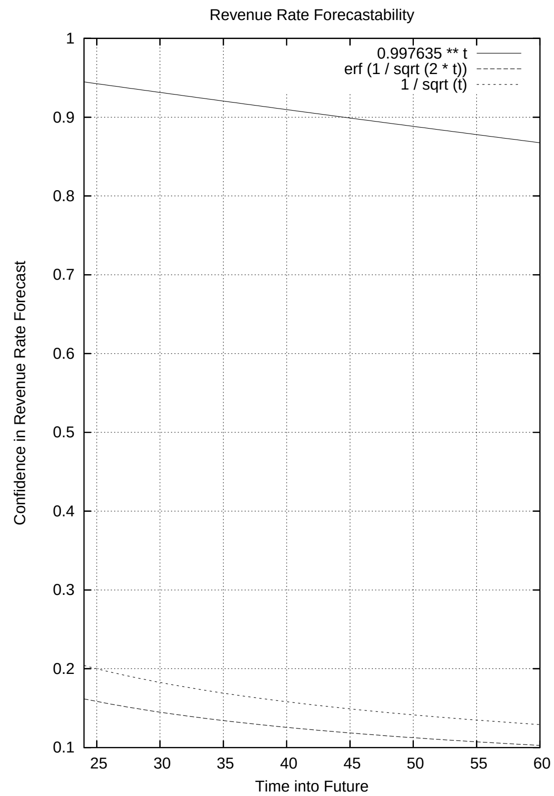

Figure C.19: North American Integrated Circuit Market, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

$\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.1.4, Equation C.5, and the preceeding section, approximately 3 times the value where $0.997635^n = 0.5$ could be interpreted as an approximation to the “average” product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.997635}$ many quarters seems consistent with the author’s experience in the industry.

For convenience of comparison, converting from quarters to months by dividing the logarithmic returns by 3:

C.1.8 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.1.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.3. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.2. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Integrated Circuit Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the North American Integrated Circuit Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.2, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.015044 + 1)}{\ln(2)} = 0.021542 \tag{C.27}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.2, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.014205 + 1)}{\ln(2)} = 0.020350 \tag{C.28}$$

Note that if the mean is not constant in Figure C.2, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.1:

$$bits = 0.015612 \tag{C.29}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.1:

$$bits = 0.019619 \tag{C.30}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.1.8 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.019619t} \tag{C.31}$$

therefore:

$$C(p) = 0.019619 \tag{C.32}$$

and, *tsshannon* 0.019619 gives:

$$C(0.582271) = 0.019619 \tag{C.33}$$

therefore:

$$2^{C(0.582271)} = 2^{0.019619} \tag{C.34}$$

$$= 1.013692 \tag{C.35}$$

$$= 1.369174\% \tag{C.36}$$

and:

$$2p - 1 = (2 \cdot 0.582271) - 1 \tag{C.37}$$
$$= 0.164542 \tag{C.38}$$
$$= 16.454200\% \tag{C.39}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the North American Integrated Circuit Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 16.454200% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 83.545800% will be held in "reserve" with a 58.227100% chance of making twice the 16.454200% back, (and a 41.772900% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 1.369174%, or a doubling of its rate of revenue returns, (per month,) in 50.970998 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 16.454200% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.369174%, per month, on average.

Note that the metrics presented in this section are representative of the North American Integrated Circuit Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 16.454200% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the North American Integrated Circuit Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.369174% per month.

As another simple example, a company re-invests 16.454200% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 16.454200% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.369174% per month.

As an example of "product portfolio" management, suppose a company re-invests 16.454200% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 16.454200$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 16.454200$ percent for the second product, implying that the company should diversify its product line[14]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated

[14]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 16.454200%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65-1)}{(2 \cdot 0.55-1)} = 3:1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the North American Integrated Circuit Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.164542}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.2, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 16.454200% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 16.454200% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.1.9 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.1.9. Figure C.20 represents a constructional simulation of the time series data presented in Figure C.1. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.2. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.2 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.21 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.4.

C.1.10 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.1.3. One of the issues of analysis, as mentioned in Section C.1.7, is to determine the maximum Shannon probability for the time series presented in Figure C.1. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.22 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.1. Figure C.23 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.1. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.641843, as derived in Section C.1.5 to 0.750000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.1, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability,

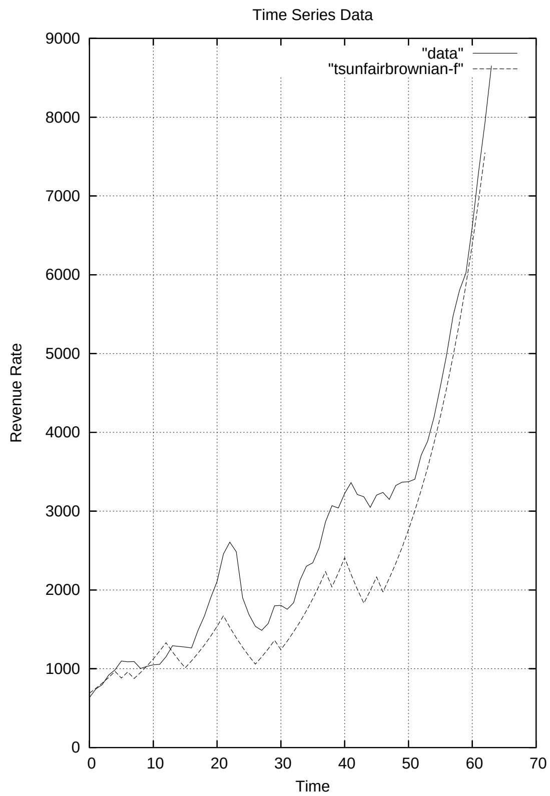

Figure C.20: North American Integrated Circuit Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.087396. This data is superimposed on the data presented in Figure C.1.

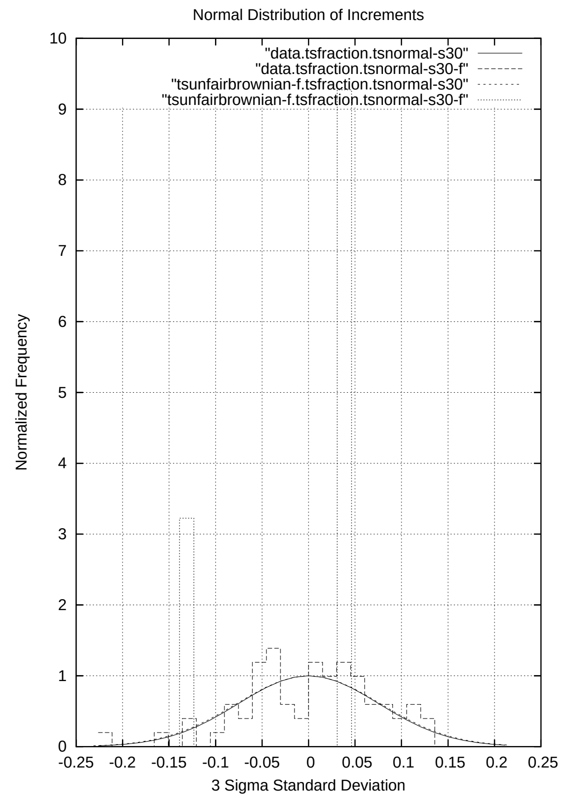

Figure C.21: North American Integrated Circuit Market, normalized histogram of the normalized increments of the time series data shown in Figure C.20, empirical and simulated. The empirical data has a mean of 0.045132, with a standard deviation of 0.075442. By comparison, the simulated data has a mean of 0.042288 with a standard deviation of 0.077108. This data is superimposed on the data presented in Figure C.4. The area under the four curves is identical.

the increments in the original data represent a relatively complex process, that may not be “modeled” with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.1, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of quarters that the North American Integrated Circuit Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.746032, as compared with the predicted value from the program *tsshannonmax* of 0.750000.

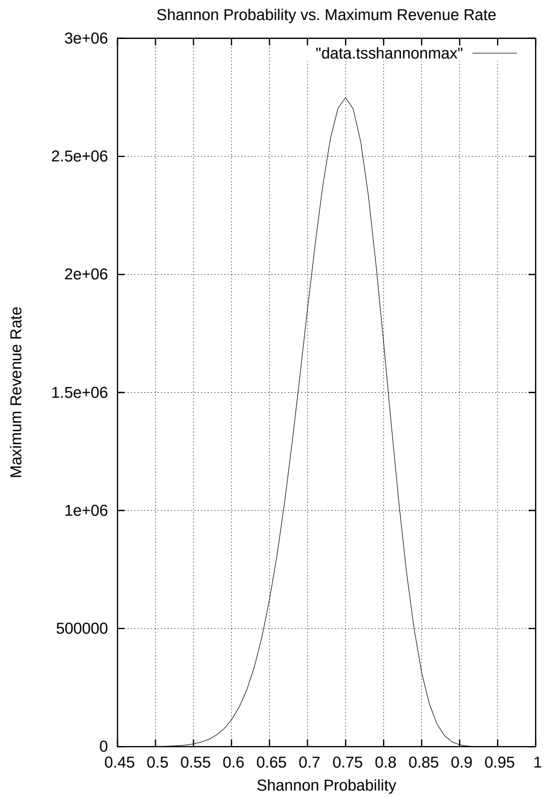

Figure C.22: North American Integrated Circuit Market, maximum rate of revenue returns, per quarter, vs. Shannon probability. The maximum rate of revenue returns, per quarter, occurs at a Shannon probability of 0.750000.

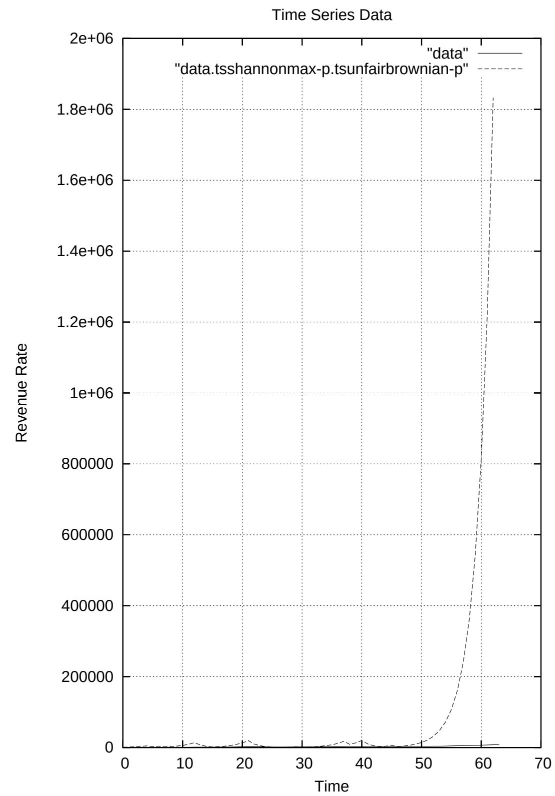

Figure C.23: North American Integrated Circuit Market, maximum rate of revenue returns, per quarter, at a Shannon probability, of 0.750000, corresponding to a "wager" fraction of 0.500000.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.1.1, Figure C.4, it would appear that the North American Integrated Circuit Market's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.500000 of the rate of returns every quarter is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any quarter, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.40}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.2 of 0.087396, vs. an "theoretical optimal" value of 0.500000 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per quarter, which is unacceptably high. However, it is not clear why the North American Integrated Circuit Market is running at a value of 0.087396, which seems very conservative. However, a re-investment strategy of 0.087396 per quarter does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the North American Integrated Circuit Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx$ 0.5, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.500000}{2.5}$, compared with an operational value, from Figure C.4 of 0.087396.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the North American Integrated Circuit Market are not running near the optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, far below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run too close, or even above, the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.1. It would seem that the natural selection process of the competitive environment would allow only those companies that run sufficiently near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.750000 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.641843 is not just marginally lower than the maximum Shannon probability of 0.750000. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.1.11 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.3. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.2. These values will be used in a fixed increment Brownian fractal analysis of the North American Integrated Circuit Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.1.6 and D.1.7. As a subjective evaluation of the "quality" of the analysis of the North American Integrated Circuit Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.1 from Figure C.2, and the Shannon probability as calculated by counting the total number of quarters that the North American Integrated Circuit Market movement was positive, as presented in Section C.1.10:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.41}$$

$$0.746032 \approx \frac{\frac{0.045132}{0.087396} + 1}{2} \tag{C.42}$$

$$0.746032 \approx 0.758204 \tag{C.43}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.746032 \approx 0.758204 \approx 0.750000 \tag{C.44}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.1.5, should be compared. The four methods used were the mean of Figure C.2, the constant in the least squares approximation to Figure C.2, the least squares exponential approximation to Figure C.1, and the logarithmic returns of Figure C.1, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.063685 \approx 0.060208 \approx 0.046835 \approx 0.058857 \tag{C.45}$$

It is implied in Section C.1.5, Subsection C.1.5 and in Section C.1.9 that, a Brownian motion with fixed increments fractal may "model" the North American Integrated Circuit Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P - 1) \approx \frac{\sigma\,(2P - 1)}{2\sqrt{P\,(1 - P)}} \tag{C.46}$$

$$0.087396\,(2 \cdot 0.746032 - 1) \approx \frac{0.075442\,(2 \cdot 0.746032 - 1)}{2\sqrt{0.746032\,(1 - 0.746032)}} \tag{C.47}$$

$$0.087396 \cdot 0.492063 \approx 0.075442 \cdot 0.565227 \tag{C.48}$$

$$0.043004 \approx 0.042642 \tag{C.49}$$

and, equating to the mean:

$$0.045132 \approx 0.043004 \approx 0.042642 \tag{C.50}$$

where, as in Equation C.43 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.1 from Figure C.2, and the Shannon probability as calculated by counting the total number of quarters that the North American Integrated Circuit Market movement was positive, as presented in Section C.1.10.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[15], where the absolute value is presented in Figure C.3, and the root mean square value is presented in Figure C.2:

[15]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

$$0.070147 \approx 0.087396 \tag{C.51}$$

Note, that if the North American Integrated Circuit Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.1 from Figure C.2 should be zero. It is 0.052548.

C.2 World Semiconductor Market

For the analysis, the data was in the directory ../markets/semiconductors.world[16].

The data in this section is presented in tabular form in Section D.2.

C.2.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.2.1. Figure C.24 is a graph of the time series data for the World Semiconductor Market.

Figure C.25 is a graph of the normalized increments of the time series data presented in Figure C.24. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.26 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.25. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[17].

Figure C.27 is the normalized histogram of the normalized increments of the time series data shown in Figure C.25. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.27.

Figure C.28 is the statistical estimate for the data presented in Figure C.25, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.641794$, as derived in Section C.2.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.29 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.25. In principle, if the distribution of the normalized increments presented in Figure C.27 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.30 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.25. In principle, if the distribution of the normalized increments presented in Figure C.27 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

[16]Data from the Semiconductor Industry Association, 1982—1994, by quarters, in millions of dollars, US.

[17]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

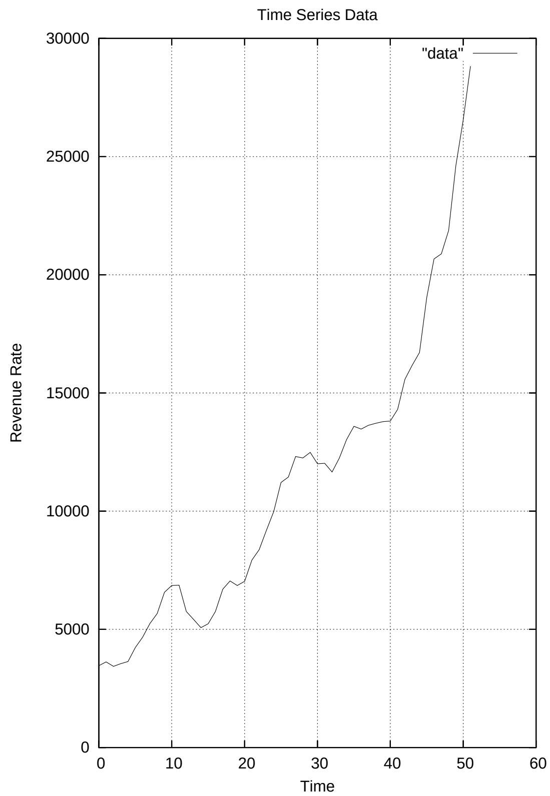

Figure C.24: World Semiconductor Market, time series data.

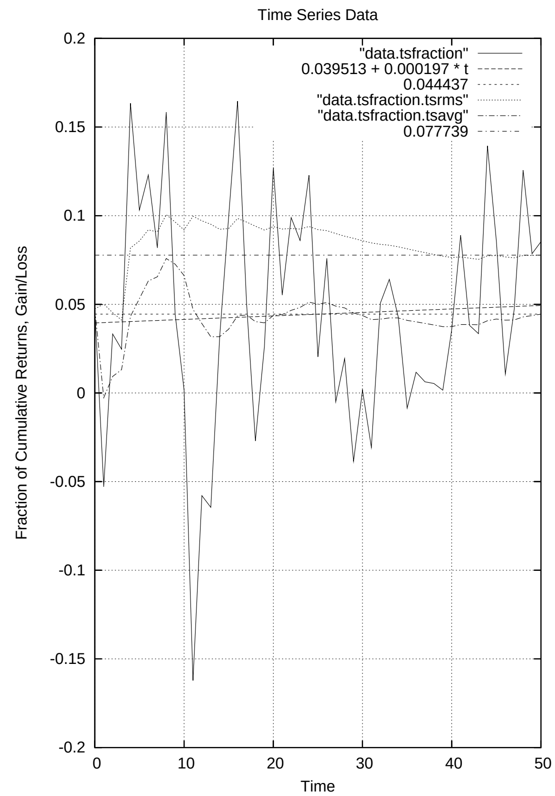

Figure C.25: World Semiconductor Market, normalized increments of the time series data presented in Figure C.24. The mean is 0.044437 with a standard deviation of 0.064421. The formula for the least squares approximation is $0.039513 + 0.000197t$, and the root mean squared value is 0.077739. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000197, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.31 is the range of values of the time series shown in Figure C.24. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.31 would be a square root function[18]. Figure C.32 is the deterministic map of the normalized increments of the time series data

[18]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.31 are a computational artifact—caused by not using the -m

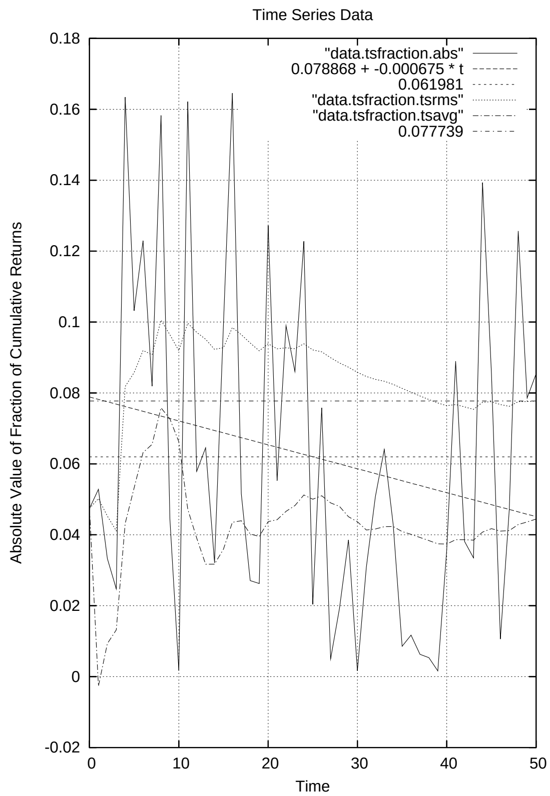

Figure C.26: World Semiconductor Market, absolute value of the normalized increments of the time series data presented in Figure C.25. The mean is 0.061981 with a standard deviation of 0.047389. The formula for the least squares approximation is $0.078868 + -0.000675t$, and the root mean square value, from Figure C.25, is 0.077739. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.25, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

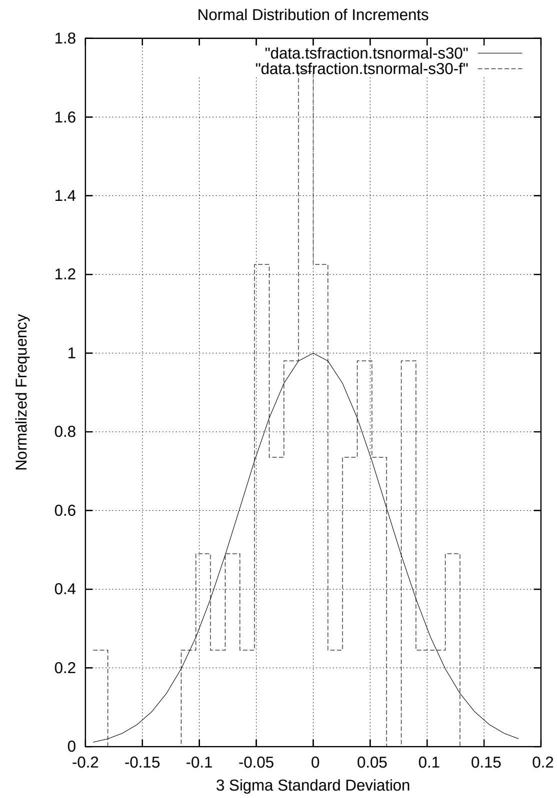

Figure C.27: World Semiconductor Market, normalized histogram of the normalized increments of the time series data shown in Figure C.25. The data has a mean of 0.044437, with a standard deviation of 0.064421. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 9.194000, with a critical value of 42.557000.

shown in Figure C.25. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

option to the program *tshurst*, which is computationally inefficient.

```
For  a mean  of 0.043582,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.004358,   861 samples  would  be  required.
     (With  52 samples,   the  estimated   error  is 0.017732   = 40.686694   percent.)
For  a standard   deviation   of 0.077739,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.007774,   136 samples  would  be  required.
     (With  52 samples,   the  estimated   error  is 0.012539   = 16.129117   percent.)
```

Figure C.28: World Semiconductor Market, statistical estimates of the normalized increments of the time series shown in Figure C.25. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.25.

Observations on the Time Series Increments Analysis

Figure C.27 would seem to indicate that the time series data for the World Semiconductor Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.2.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[19]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.33 is the instantaneous value of the root mean square of the normalized increments for the World Semiconductor Market, and Figure C.34 is the instantaneous Shannon probability for the normalized increments.

C.2.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.2.4. Figure C.35 is a graph of the logistic function estimates of the time series data for the World Semiconductor Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[20]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.35 is a graph of the logistic function for the time series data presented in Figure C.24. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.25. The program *tslsq* was used to derive the constant and the slope

[19]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is $\approx$ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[20]For example, in Figures C.35 and C.36, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.2.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

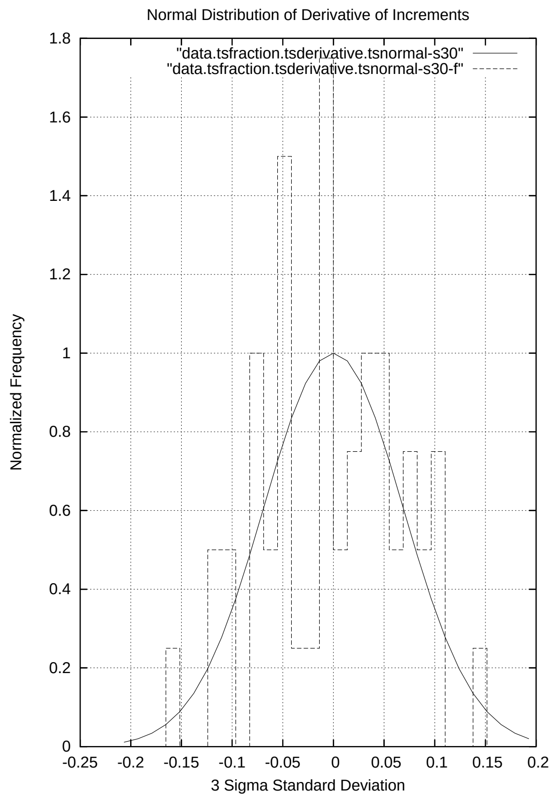

Figure C.29: World Semiconductor Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.25.

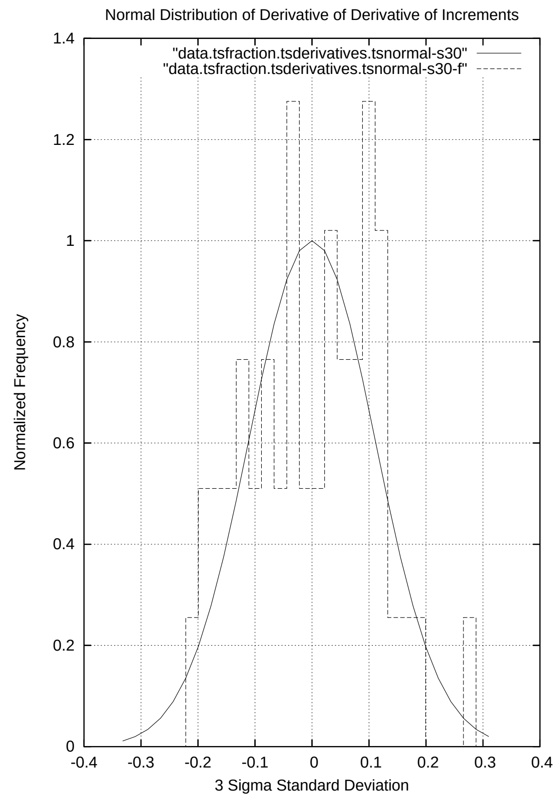

Figure C.30: World Semiconductor Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.25.

of the normalized increments of the data presented in Figure C.25. Figure C.36 is the same graph, but with the time scale expanded by a factor of two.

C.2.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.2.5. Figure C.37 is a graph of the Hurst coefficient data time series data shown in Figure C.24. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.38 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.25. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.37 implies that the variance of the rate of revenue returns, (per quarter,) in the World Semiconductor Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs

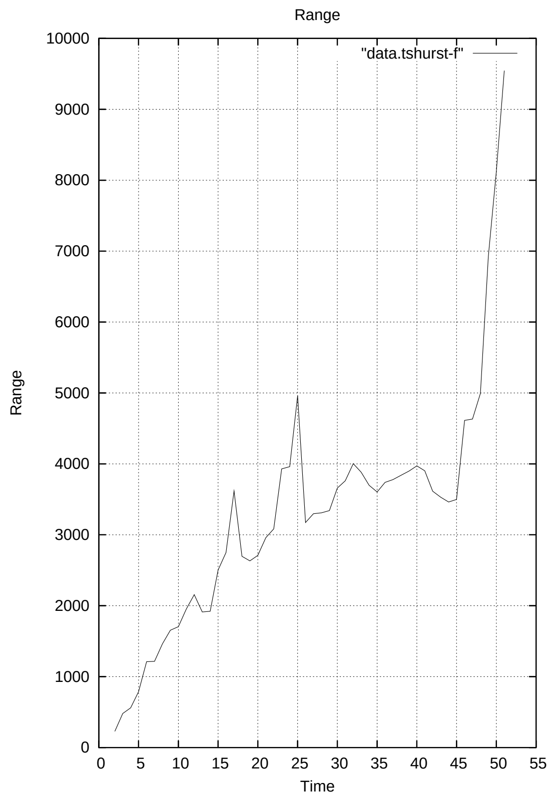

Figure C.31: World Semiconductor Market, range of the time series data shown in Figure C.24.

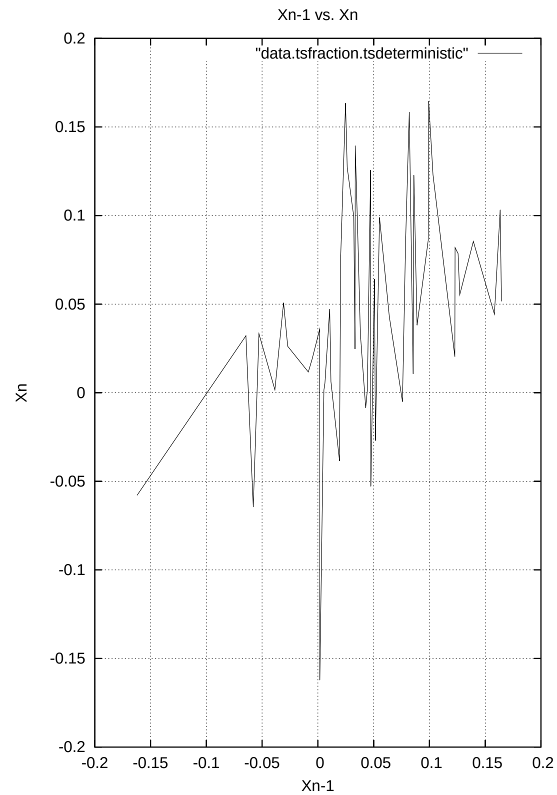

Figure C.32: World Semiconductor Market, deterministic map of the normalized increments of the time series data shown in Figure C.25.

repeating sometime in the future goes down with increasing time[21], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.41, and, C.42 compare methods of approximation of the "forecastability" of the rate of revenue returns in the World Semiconductor Market for the near term and far term, respectively [Pet91, pp. 83-84][22]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per quarter.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.37, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 1.025249, so that:

[21]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[22]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

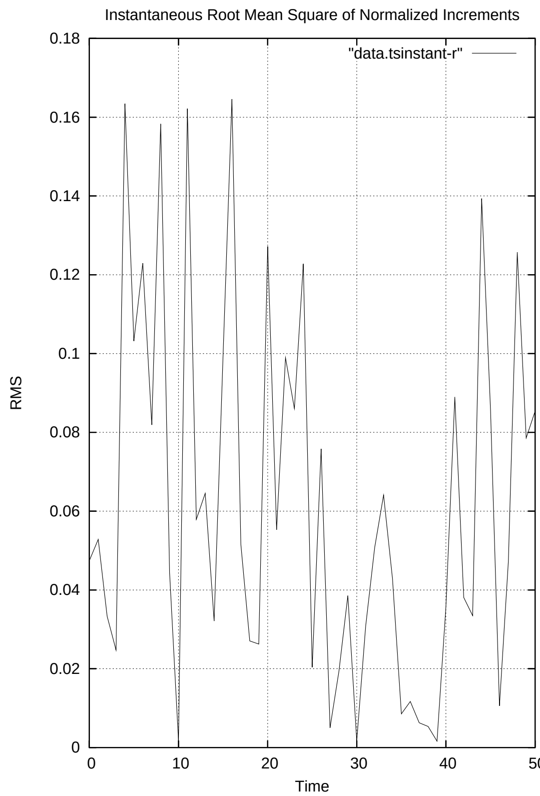

Figure C.33: World Semiconductor Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.24.

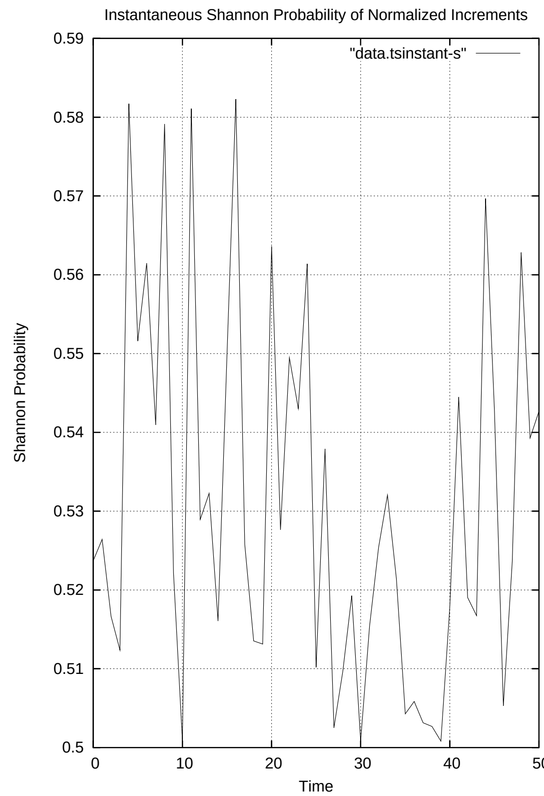

Figure C.34: World Semiconductor Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.24.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.52}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 1.025249} \tag{C.53}$$

$$\propto (t_2 - t_1)^{2.050498} \tag{C.54}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per quarter,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[23]. A Hurst coefficient of 1.025249, (for the near future, and 0.725956 for the distant future.) implies

[23] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For

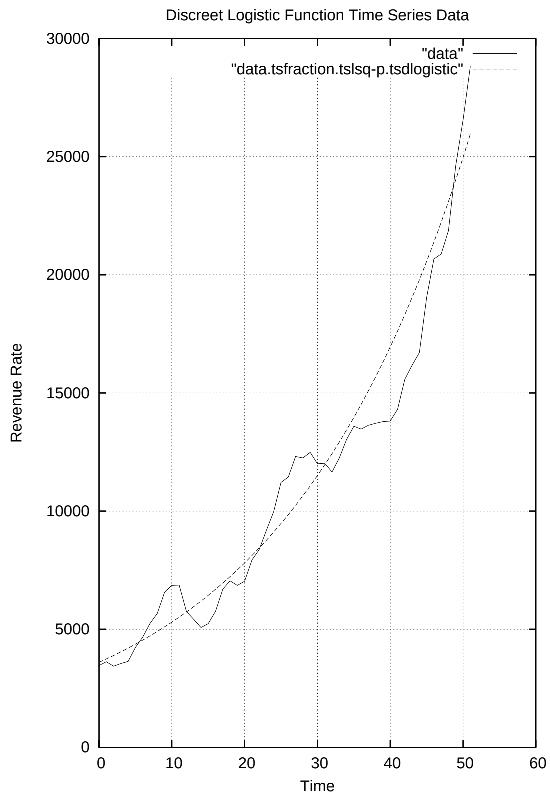

Figure C.35: World Semiconductor Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.25 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

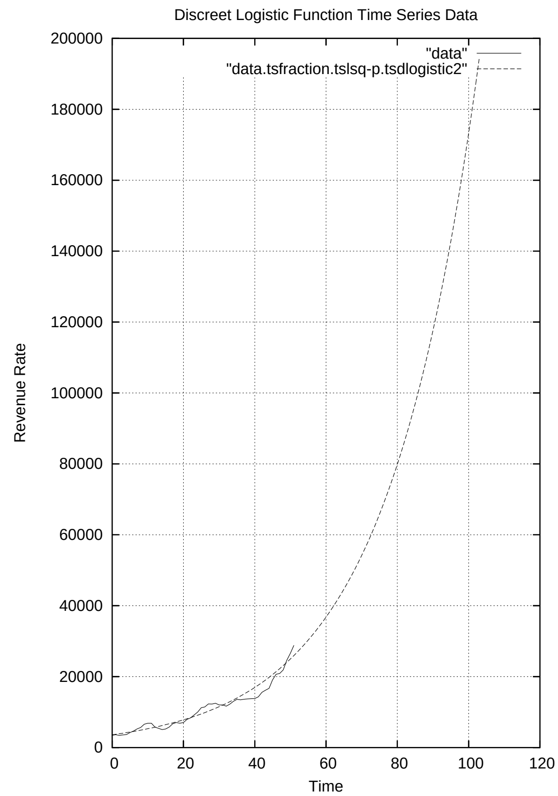

Figure C.36: World Semiconductor Market, logistic function estimates of Figure C.35 with the time scale expanded by a factor of two.

that the likelihood of the rate of revenue returns, (per quarter,) for any two consecutive quarters being the same is 102.524900% [Pet91, pp. 66] for the near future, and 0.725956 for the distant future. Likewise, there is a 102.524900% chance of the rate of revenue returns, (per quarter,) movements being the same in consecutive time periods—ie., if, in a given quarter, the rate of revenue returns, (per quarter,) is increasing, there is a 102.524900% that the rate of revenue returns, (per quarter,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per quarter,) for the World Semiconductor Market are over time, since the probability of having n many consecutive quarters of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many quarters of the same market agenda, p_a, is:

the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.2.5 for the particulars on using Hurst Coefficient to sum fractal process' for the World Semiconductor Market. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

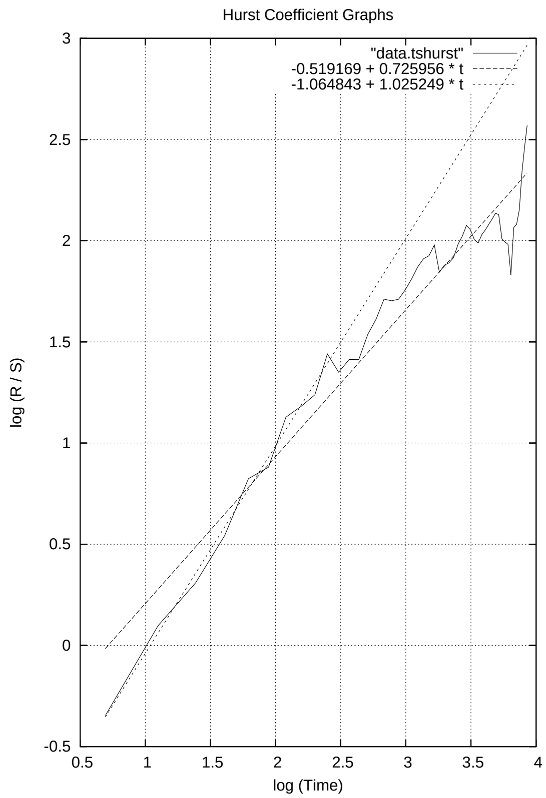

Figure C.37: World Semiconductor Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.25. The slope of the graph is the Hurst coefficient.

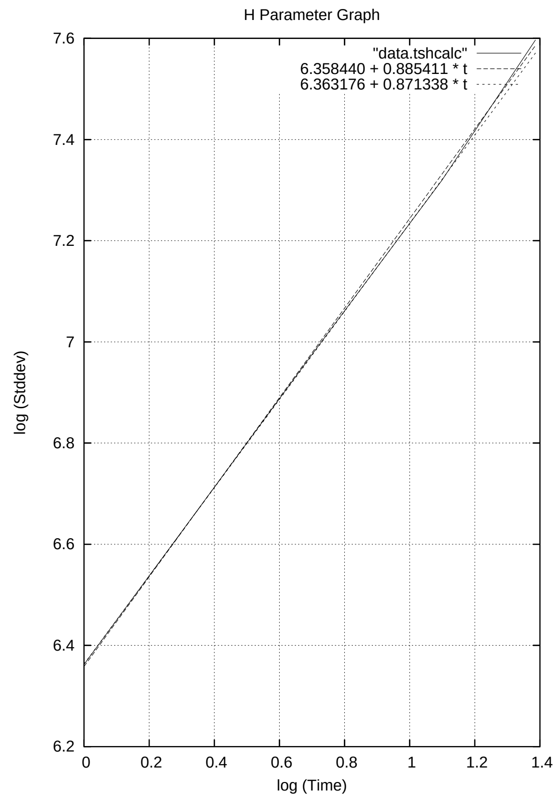

Figure C.38: World Semiconductor Market, H parameter data for the normalized increments of the time series data shown in Figure C.25 The slope of the graph is the H parameter.

$$p_a(n) = H^n \tag{C.55}$$
$$= 1.025249^n \tag{C.56}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.25, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next quarter's rate of revenue returns would be the same as the current quarter's revenue rate. Interestingly, it is $0.044437 \cdot 100$ percent, on the average, with a standard deviation of $0.064421 \cdot 100$ percent, and a root mean square error value of $0.077739 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per quarter,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.57}$$

$$\propto \quad (t_2 - t_1)^{1.025249} \tag{C.58}$$

where R is the range of values in the increments of the rate of revenue returns, (per quarter.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per quarter,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per quarter) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.58 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.59}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.60}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per quarter,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.61}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per quarter,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[24].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \quad \propto \quad \frac{X\,(rt)}{r^H} \tag{C.62}$$

$$\propto \quad \frac{X\,(rt)}{r^{1.025249}} \tag{C.63}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.38, to provide a least squares approximation to the H parameter for the World Semiconductor Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.871338 for the near future, and 0.885411 for the distant future.

Figures C.37 and C.38 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.25. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.25, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.39 and C.40 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Many World Semiconductor Market industry analyst speculate that there is "periodic" behavior in the market place, at approximately 5 year intervals. Both the Hurst coefficient and H parameter graphs would tend to support the intuition. Notice that the slope of the graphs, in figures C.37 and C.38, tend to decrease abruptly at $t \approx \ln(3) \approx 20$ quarters, which is approximately 60 months, or 5 years [Pet91, pp. 96]. Whether this is "periodic" behavior, or an indication of more complex system dynamics, perhaps "chaotic," remains to be seen. If that is the case, it could provide an exploitive venue.

[24] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

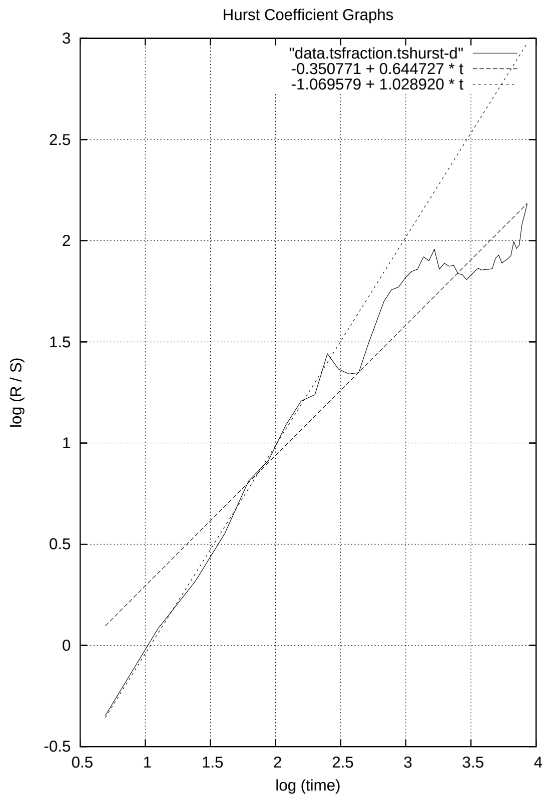

Figure C.39: World Semiconductor Market, traditional Hurst coefficient data for the time series data shown in Figure C.24. The slope of the graph is the Hurst coefficient, and is 1.028920 for the near term, and 0.644727 for the far term.

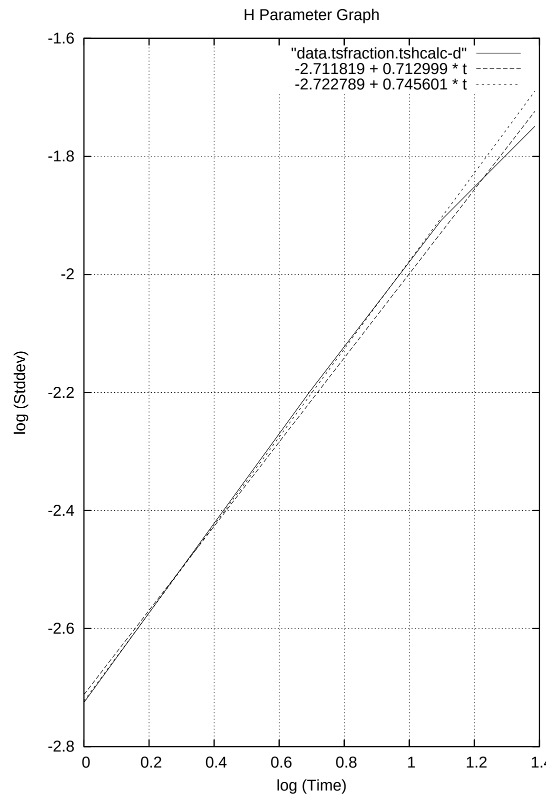

Figure C.40: World Semiconductor Market, traditional H parameter data for the time series data shown in Figure C.24 The slope of the graph is the H parameter, and is 0.745601 for the near term, and 0.712999 for the far term.

C.2.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.2.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.26. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.25. These values will be used in a fixed increment Brownian fractal analysis and simulation of the World Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the World Semiconductor Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in

Chapter B, and is presented in Figure C.25, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln (0.044437 + 1)}{\ln (2)} = 0.062725 \tag{C.64}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.25, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln (0.039513 + 1)}{\ln (2)} = 0.055908 \tag{C.65}$$

Note that if the mean is not constant in Figure C.25, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.24:

$$bits = 0.053777 \tag{C.66}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.24:

$$bits = 0.058816 \tag{C.67}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.2.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.058816t} \tag{C.68}$$

therefore:

$$C(p) = 0.058816 \tag{C.69}$$

and, *tsshannon* 0.058816 gives:

$$C(0.641794) = 0.058816 \tag{C.70}$$

therefore:

$$\begin{aligned} 2^{C(0.641794)} &= 2^{0.058816} && \text{(C.71)} \\ &= 1.041611 && \text{(C.72)} \\ &= 4.161057\% && \text{(C.73)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.641794) - 1 && \text{(C.74)} \\ &= 0.283588 && \text{(C.75)} \\ &= 28.358800\% && \text{(C.76)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the World Semiconductor Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every quarter, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 28.358800% of its rate of revenue returns, (per quarter.) As a conceptual model, the remaining 71.641200% will be held in “reserve” with a 64.179400% chance of making twice the 28.358800% back, (and a 35.820600% chance of making 0.0,) in one quarter, on the average, for an average growth in its rate of revenue returns, (per quarter,) of 4.161057%, or a doubling of its rate of revenue returns, (per quarter,) in 17.002176 quarters.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 28.358800% per quarter of the rate of revenue returns, (per quarter,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 4.161057%, per quarter, on average.

Note that the metrics presented in this section are representative of the World Semiconductor Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 28.358800% of its rate of revenue returns, (per quarter,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some quarters, depending on the World Semiconductor Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other quarters, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 4.161057% per quarter.

As another simple example, a company re-invests 28.358800% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 28.358800% per quarter investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 4.161057% per quarter.

As an example of "product portfolio" management, suppose a company re-invests 28.358800% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 28.358800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 28.358800$ percent for the second product, implying that the company should diversify its product line[25]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 28.358800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the World Semiconductor Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.283588}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.25, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of

[25]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

"product portfolio" management, consider the issue of product mix. In this interpretation, 28.358800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 28.358800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 28.358800 of the rate of revenue returns per quarter does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 28.358800% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.283588}$ products seems consistent with the industry, also.

C.2.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the World Semiconductor Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the World Semiconductor Market time series is 0.044437, and 0.077739respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 7.353038.

If this value seems consistent number of companies in the World Semiconductor Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the World Semiconductor Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.605400, which would be the value which should be used in Section C.2.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.2.5 is greater than the average Shannon probability for the companies participating in the World Semiconductor Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.2.5. The maximum exploitability for the World Semiconductor Market is derived in Section C.2.10, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the World Semiconductor Market is 0.605400, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.785809 in the World Semiconductor Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.77}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the World Semiconductor Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.2.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.26. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.25. These values will be used in a fixed increment Brownian fractal analysis and simulation of the World Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.2.5, is derived from the World Semiconductor Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.2.10.

An additional exploitable strategy may be time itself. Equations C.54, C.58, and, C.56, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the World Semiconductor Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.41, and, C.42 compare methods of approximation of the "forecastability" of rate of revenue returns in the World Semiconductor Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the World Semiconductor Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[26]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.41, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.56, $1.025249^n = 0.5$ quarters of operations. Since the optimal amount of inventory and, from Equation C.54, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.42, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[27]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with $\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.2.4, Equation C.56, and the preceeding section, approximately 3 times the value where $1.025249^n = 0.5$ could be interpreted as an approximation to the "average" product life cycle. This seems consistent

[26]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[27]For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

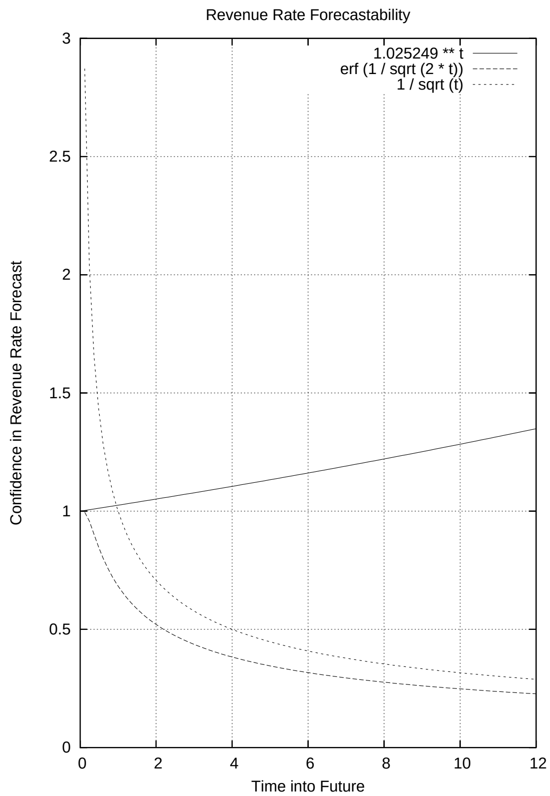

Figure C.41: World Semiconductor Market, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 1.025249^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

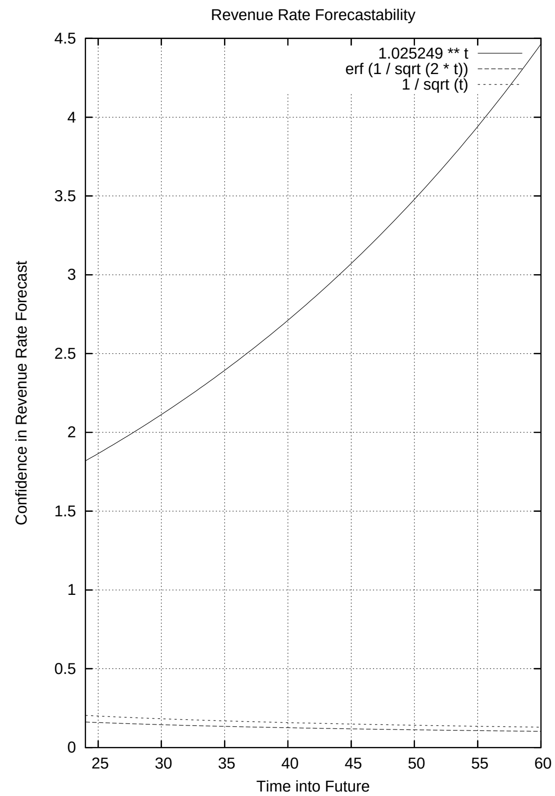

Figure C.42: World Semiconductor Market, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 1.025249}$ many quarters seems consistent with the author's experience in the industry.

For convenience of comparison, converting from quarters to months by dividing the logarithmic returns by 3:

C.2.8 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.2.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.26. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.25. These values will be used in a fixed increment Brownian fractal analysis and simulation of the World Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the World Semiconductor Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.25, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.014812 + 1)}{\ln(2)} = 0.021213 \tag{C.78}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.25, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.013171 + 1)}{\ln(2)} = 0.018878 \tag{C.79}$$

Note that if the mean is not constant in Figure C.25, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.24:

$$bits = 0.017926 \tag{C.80}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.24:

$$bits = 0.019605 \tag{C.81}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.2.8 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.019605t} \tag{C.82}$$

therefore:

$$C(p) = 0.019605 \tag{C.83}$$

and, *tsshannon* 0.019605 gives:

$$C(0.582242) = 0.019605 \tag{C.84}$$

therefore:

$$\begin{aligned} 2^{C(0.582242)} &= 2^{0.019605} && \text{(C.85)} \\ &= 1.013682 && \text{(C.86)} \\ &= 1.368190\% && \text{(C.87)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.582242) - 1 && \text{(C.88)} \\ &= 0.164484 && \text{(C.89)} \\ &= 16.448400\% && \text{(C.90)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the World Semiconductor Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 16.448400% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 83.551600% will be held in "reserve" with a 58.224200% chance of making twice the 16.448400% back, (and a 41.775800% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 1.368190%, or a doubling of its rate of revenue returns, (per month,) in 51.007396 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 16.448400% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.368190%, per month, on average.

Note that the metrics presented in this section are representative of the World Semiconductor Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 16.448400% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the World Semiconductor Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.368190% per month.

As another simple example, a company re-invests 16.448400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 16.448400% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.368190% per month.

As an example of "product portfolio" management, suppose a company re-invests 16.448400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 16.448400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 16.448400$ percent for the second product, implying that the company should diversify its product line[28]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 16.448400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3:1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the World Semiconductor Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.164484}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue

[28]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.25, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 16.448400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 16.448400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.2.9 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.2.9. Figure C.43 represents a constructional simulation of the time series data presented in Figure C.24. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.25. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.25 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.44 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.27.

C.2.10 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.2.3. One of the issues of analysis, as mentioned in Section C.2.7, is to determine the maximum Shannon probability for the time series presented in Figure C.24. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.45 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.24. Figure C.46 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.24. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.641794, as derived in Section C.2.5 to 0.826923. This process, essentially, extracts the random statistical data from the time series presented in Figure C.24, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.24, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of quarters that the World Semiconductor Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.823529, as compared with the predicted value from the program *tsshannonmax* of 0.826923.

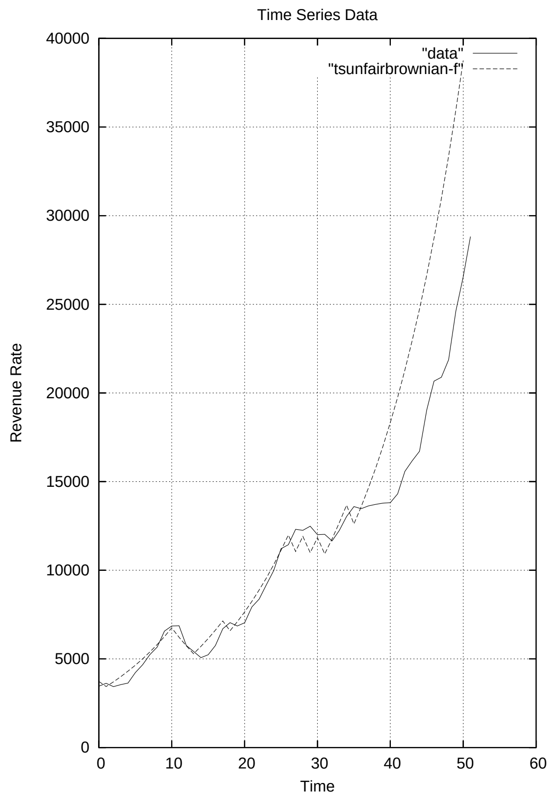

Figure C.43: World Semiconductor Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.077739. This data is superimposed on the data presented in Figure C.24.

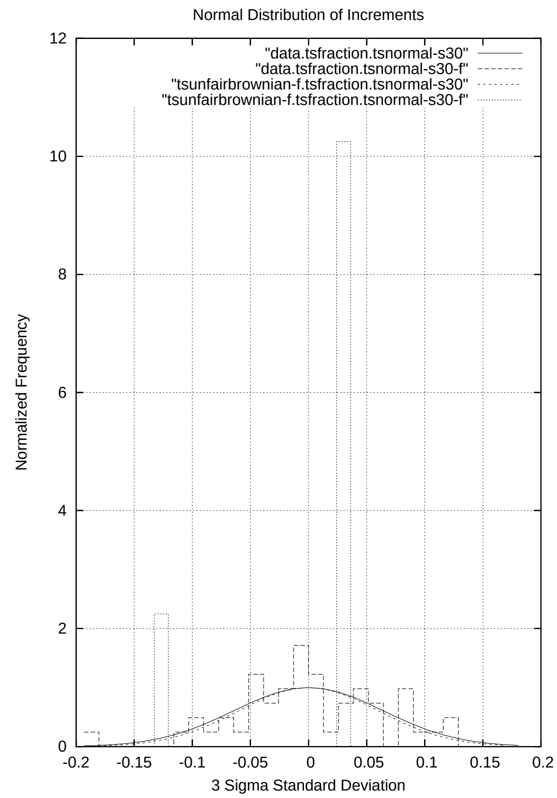

Figure C.44: World Semiconductor Market, normalized histogram of the normalized increments of the time series data shown in Figure C.43, empirical and simulated. The empirical data has a mean of 0.044437, with a standard deviation of 0.064421. By comparison, the simulated data has a mean of 0.049753 with a standard deviation of 0.060339. This data is superimposed on the data presented in Figure C.27. The area under the four curves is identical.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.2.1, Figure C.27, it would appear that the World Semiconductor Market's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.653846 of the rate of returns every quarter is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any quarter, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.91}$$

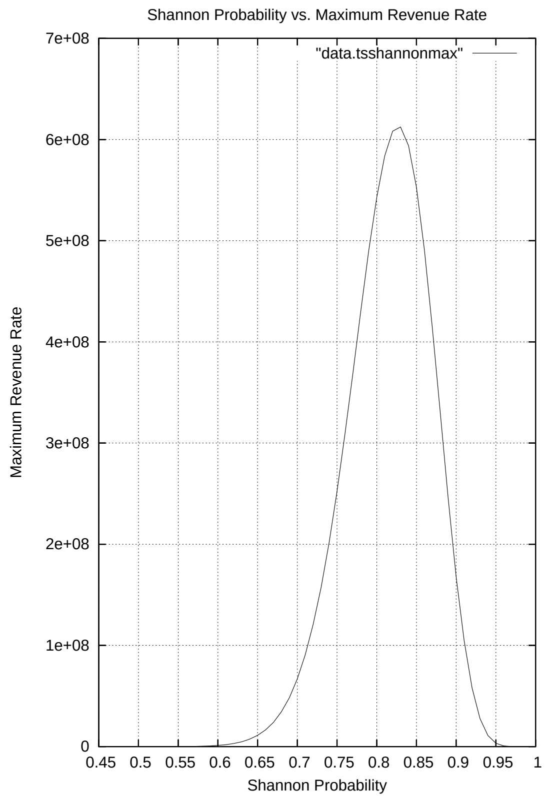

Figure C.45: World Semiconductor Market, maximum rate of revenue returns, per quarter, vs. Shannon probability. The maximum rate of revenue returns, per quarter, occurs at a Shannon probability of 0.826923.

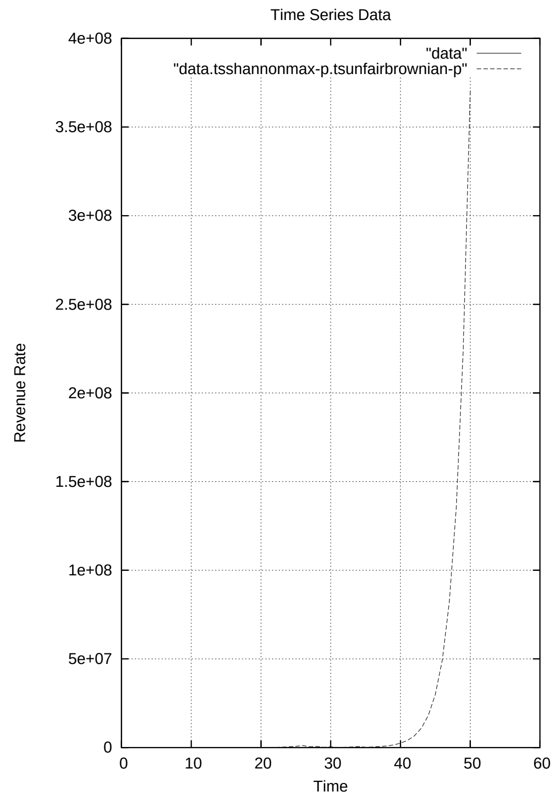

Figure C.46: World Semiconductor Market, maximum rate of revenue returns, per quarter, at a Shannon probability, of 0.826923, corresponding to a "wager" fraction of 0.653846.

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.25 of 0.077739, vs. an "theoretical optimal" value of 0.653846 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per quarter, which is unacceptably high. However, it is not clear why the World Semiconductor Market is running at a value of 0.077739, which seems very conservative. However, a re-investment strategy of 0.077739 per quarter does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the World Semiconductor Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx 0.5$, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.653846}{2.5}$, compared with an operational value, from Figure C.27 of 0.077739.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the World Semiconductor Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.24. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.826923 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.641794 is not just marginally lower than the maximum Shannon probability of 0.826923. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.2.11 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.26. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.25. These values will be used in a fixed increment Brownian fractal analysis of the World Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.2.6 and D.2.7. As a subjective evaluation of the "quality" of the analysis of the World Semiconductor Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.24 from Figure C.25, and the Shannon probability as calculated by counting the total number of quarters that the World Semiconductor Market movement was positive, as presented in Section C.2.10:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.92}$$

$$0.823529 \approx \frac{\frac{0.044437}{0.077739} + 1}{2} \tag{C.93}$$

$$0.823529 \approx 0.785809 \tag{C.94}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.823529 \approx 0.785809 \approx 0.826923 \tag{C.95}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.2.5, should be compared. The four methods used were the mean of Figure C.25, the constant in the least squares approximation to Figure C.25, the least squares exponential approximation to Figure C.24, and the logarithmic returns of Figure C.24, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.062725 \approx 0.055908 \approx 0.053777 \approx 0.058816 \tag{C.96}$$

It is implied in Section C.2.5, Subsection C.2.5 and in Section C.2.9 that, a Brownian motion with fixed increments fractal may "model" the World Semiconductor Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.97}$$

$$0.077739\,(2 \cdot 0.823529 - 1) \approx \frac{0.064421\,(2 \cdot 0.823529 - 1)}{2\sqrt{0.823529\,(1 - 0.823529)}} \tag{C.98}$$

$$0.077739 \cdot 0.647059 \approx 0.064421 \cdot 0.848668 \tag{C.99}$$

$$0.050302 \approx 0.054672 \tag{C.100}$$

and, equating to the mean:

$$0.044437 \approx 0.050302 \approx 0.054672 \tag{C.101}$$

where, as in Equation C.94 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.24 from Figure C.25, and the Shannon probability as calculated by counting the total number of quarters that the World Semiconductor Market movement was positive, as presented in Section C.2.10.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[29], where the absolute value is presented in Figure C.26, and the root mean square value is presented in Figure C.25:

$$0.061981 \approx 0.077739 \tag{C.102}$$

Note, that if the World Semiconductor Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.24 from Figure C.25 should be zero. It is 0.047389.

C.3 North American Semiconductor Market

For the analysis, the data was in the directory ../markets/semiconductors.namerica[30].

The data in this section is presented in tabular form in Section D.3.

[29] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[30] Data from the Semiconductor Industry Association, 1979—1994, by quarters, in millions of dollars, US.

C.3.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.3.1. Figure C.47 is a graph of the time series data for the North American Semiconductor Market.

Figure C.48 is a graph of the normalized increments of the time series data presented in Figure C.47. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.49 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.48. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[31].

Figure C.50 is the normalized histogram of the normalized increments of the time series data shown in Figure C.48. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.50.

Figure C.51 is the statistical estimate for the data presented in Figure C.48, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.634320$, as derived in Section C.3.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.52 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.48. In principle, if the distribution of the normalized increments presented in Figure C.50 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.53 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.48. In principle, if the distribution of the normalized increments presented in Figure C.50 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.54 is the range of values of the time series shown in Figure C.47. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.54 would be a square root function[32]. Figure C.55 is the deterministic map of the normalized increments of the time series data shown in Figure C.48. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.50 would seem to indicate that the time series data for the North American Semiconductor Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments

[31] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[32] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.54 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

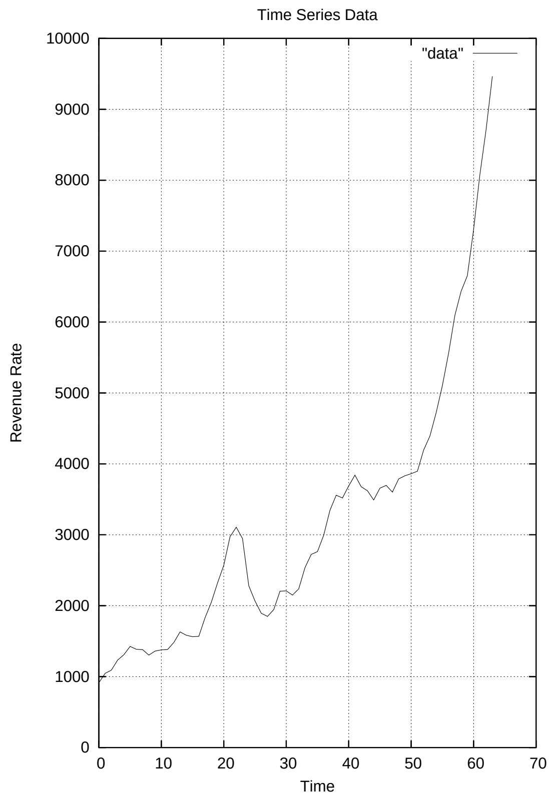

Figure C.47: North American Semiconductor Market, time series data.

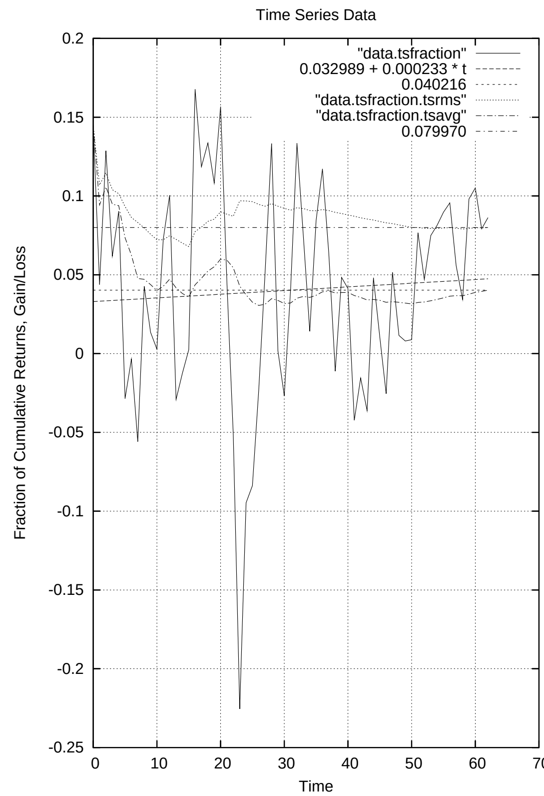

Figure C.48: North American Semiconductor Market, normalized increments of the time series data presented in Figure C.47. The mean is 0.040216 with a standard deviation of 0.069677. The formula for the least squares approximation is $0.032989 + 0.000233t$, and the root mean squared value is 0.079970. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000233, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

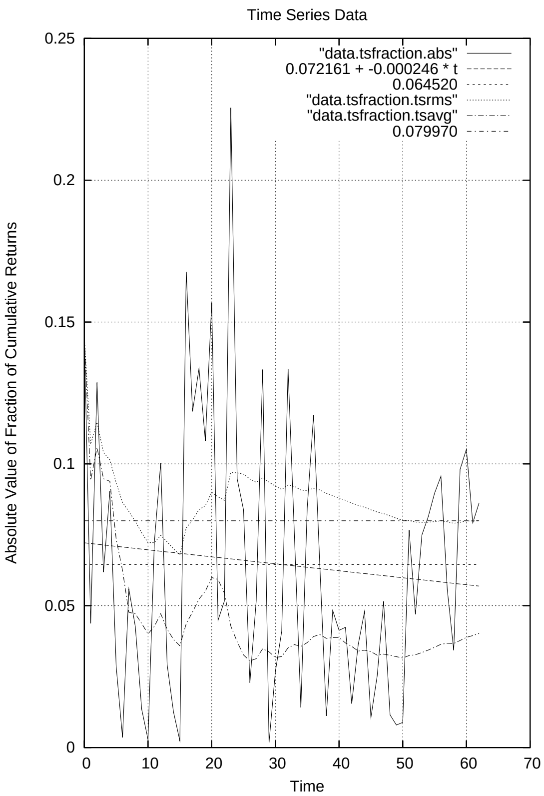

Figure C.49: North American Semiconductor Market, absolute value of the normalized increments of the time series data presented in Figure C.48. The mean is 0.064520 with a standard deviation of 0.047627. The formula for the least squares approximation is $0.072161 + -0.000246t$, and the root mean square value, from Figure C.48, is 0.079970. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments presented in Figure C.48, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

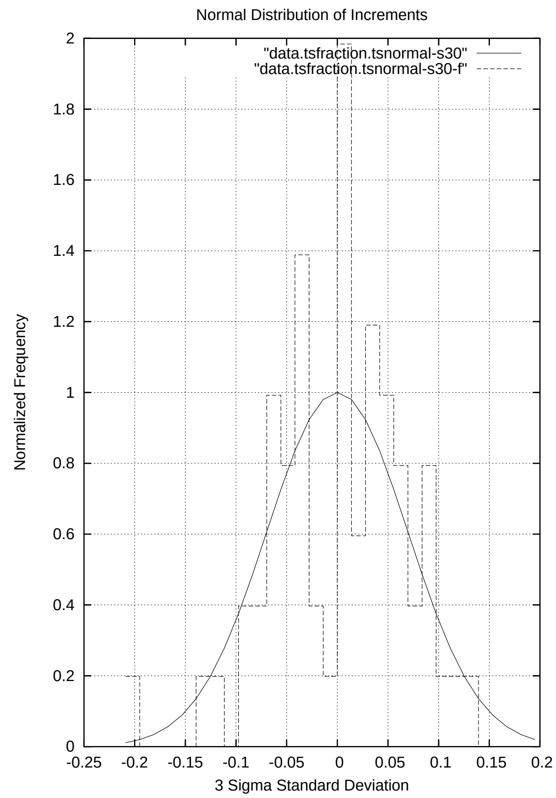

Figure C.50: North American Semiconductor Market, normalized histogram of the normalized increments of the time series data shown in Figure C.48. The data has a mean of 0.040216, with a standard deviation of 0.069677. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 7.163000, with a critical value of 42.557000.

C.3.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

```
For a mean of 0.039587, with a confidence level of 0.900000
    that the error did not exceed 0.003959, 1105 samples would be required.
    (With 64 samples, the estimated error is 0.016442 = 41.534296 percent.)
For a standard deviation of 0.079970, with a confidence level of 0.900000
    that the error did not exceed 0.007997, 136 samples would be required.
    (With 64 samples, the estimated error is 0.011626 = 14.538589 percent.)
```

Figure C.51: North American Semiconductor Market, statistical estimates of the normalized increments of the time series shown in Figure C.48. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.48.

mean square of the instantaneous fraction of change[33]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.56 is the instantaneous value of the root mean square of the normalized increments for the North American Semiconductor Market, and Figure C.57 is the instantaneous Shannon probability for the normalized increments.

C.3.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.3.4. Figure C.58 is a graph of the logistic function estimates of the time series data for the North American Semiconductor Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[34]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.58 is a graph of the logistic function for the time series data presented in Figure C.47. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.48. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.48. Figure C.59 is the same graph, but with the time scale expanded by a factor of two.

C.3.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.3.5. Figure C.60 is a graph of the Hurst coefficient data time series data shown in Figure C.47. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.61 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.48. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.60 implies that the variance of the rate of revenue returns, (per quarter,) in the North American Semiconductor Market, $V(t_2 - t_1)$, over a period of time is proportional to the

[33]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[34]For example, in Figures C.58 and C.59, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.3.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

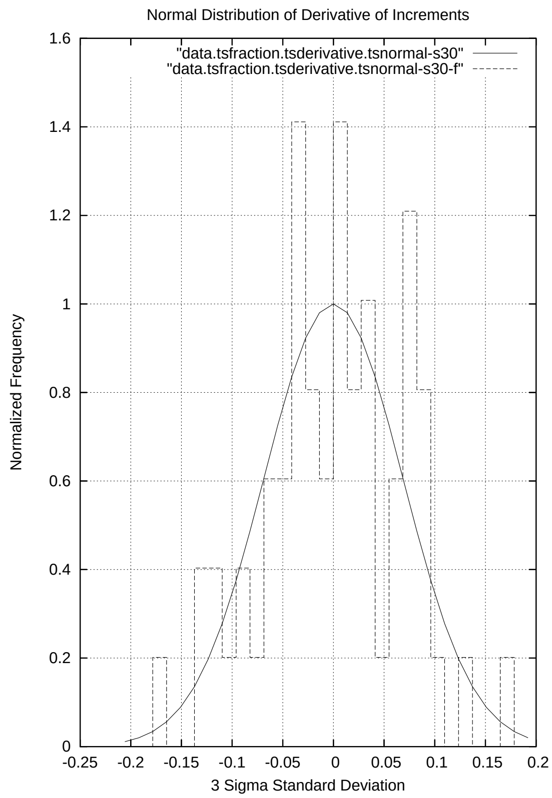

Figure C.52: North American Semiconductor Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.48.

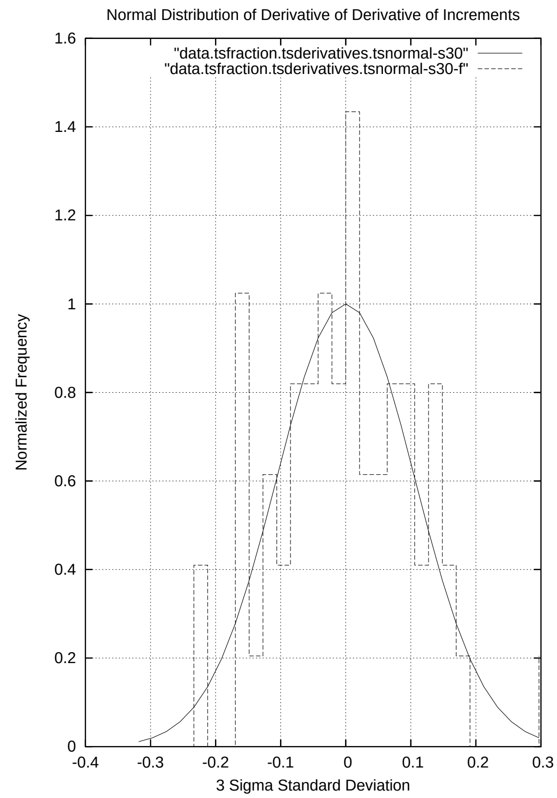

Figure C.53: North American Semiconductor Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.48.

period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[35], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.64, and, C.65 compare methods of approximation of the "forecastability" of the rate of revenue returns in the North American Semiconductor Market for the near term and far term, respectively [Pet91, pp. 83-84][36]. This seems to be a quantitative statement concerning "windows of

[35]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[36]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

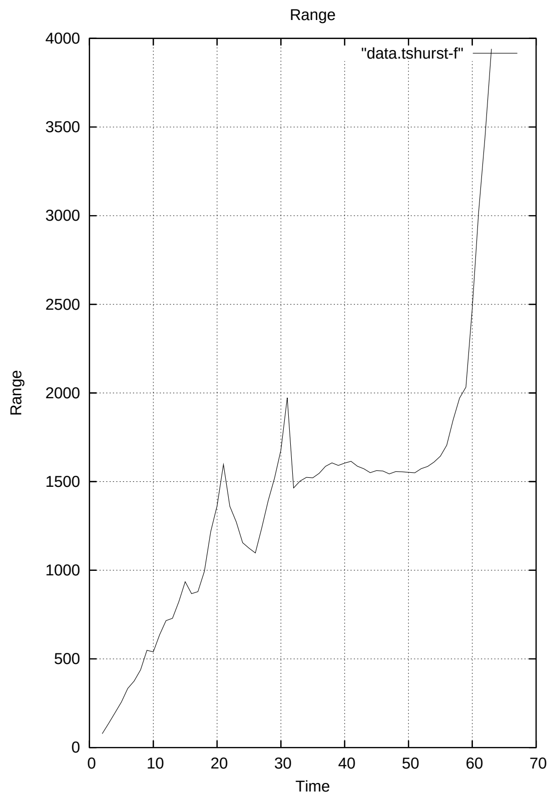

Figure C.54: North American Semiconductor Market, range of the time series data shown in Figure C.47.

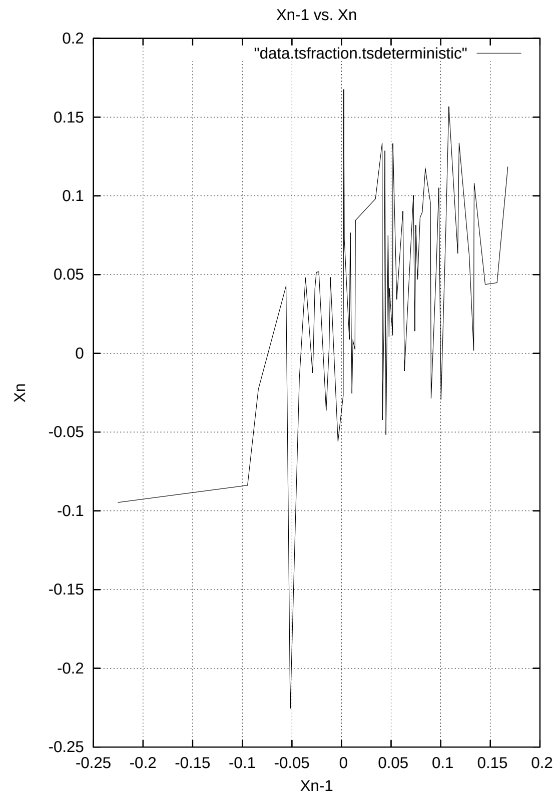

Figure C.55: North American Semiconductor Market, deterministic map of the normalized increments of the time series data shown in Figure C.48.

opportunity" in the rate of revenue returns, (per quarter.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.60, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.998014, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.103}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.998014} \tag{C.104}$$

$$\propto (t_2 - t_1)^{1.996028} \tag{C.105}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per quarter,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[37]. A Hurst coefficient of 0.998014, (for the near future, and 0.714241 for the distant future.) implies

[37] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient,

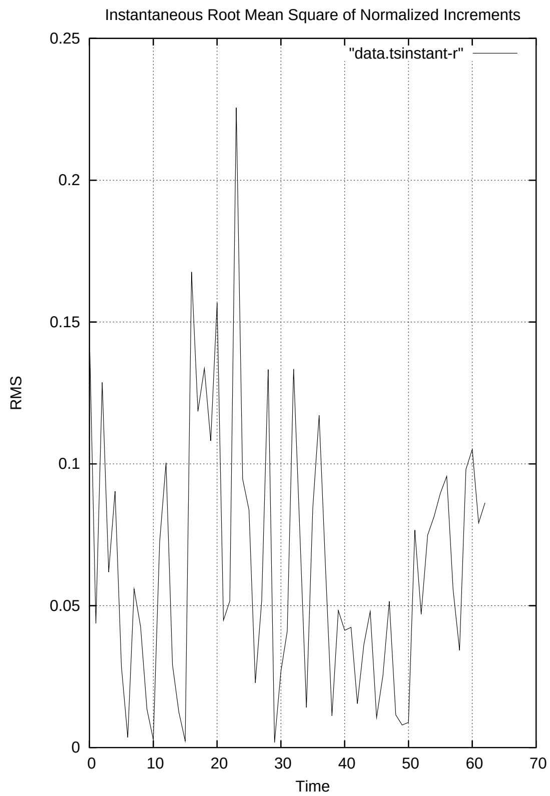

Figure C.56: North American Semiconductor Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.47.

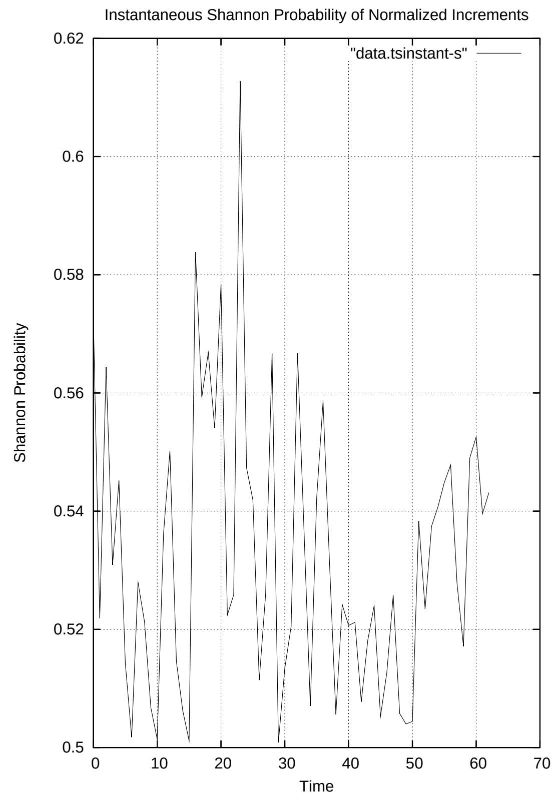

Figure C.57: North American Semiconductor Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.47.

that the likelihood of the rate of revenue returns, (per quarter,) for any two consecutive quarters being the same is 99.801400% [Pet91, pp. 66] for the near future, and 0.714241 for the distant future. Likewise, there is a 99.801400% chance of the rate of revenue returns, (per quarter,) movements being the same in consecutive time periods—ie., if, in a given quarter, the rate of revenue returns, (per quarter,) is increasing, there is a 99.801400% that the rate of revenue returns, (per quarter,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per quarter,) for the North American Semiconductor Market are over time, since the probability of having n many consecutive quarters of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many quarters of the same market agenda, p_a,

H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.3.5 for the particulars on using Hurst Coefficient to sum fractal process' for the North American Semiconductor Market. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

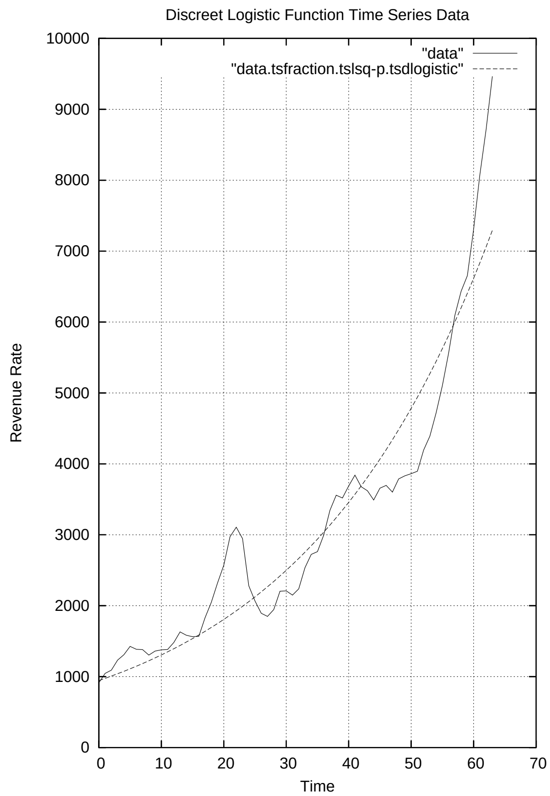

Figure C.58: North American Semiconductor Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.48 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

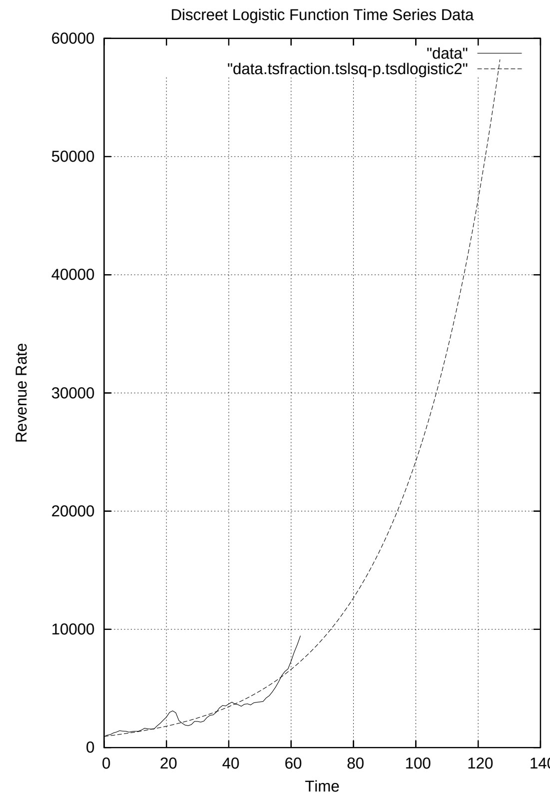

Figure C.59: North American Semiconductor Market, logistic function estimates of Figure C.58 with the time scale expanded by a factor of two.

is:

$$p_a(n) = H^n \tag{C.106}$$
$$= 0.998014^n \tag{C.107}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.48, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next quarter's rate of revenue returns would be the same as the current quarter's revenue rate. Interestingly, it is $0.040216 \cdot 100$ percent, on the average, with a standard deviation of $0.069677 \cdot 100$ percent, and a root mean square error value of $0.079970 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per quarter,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

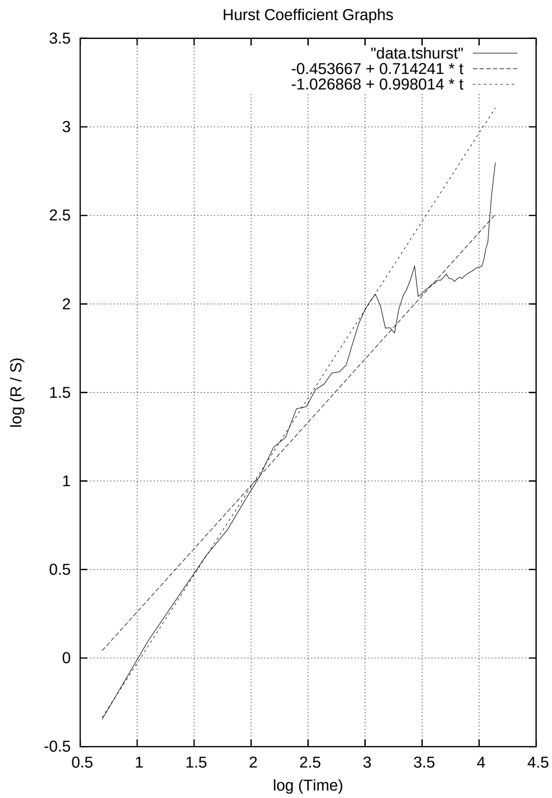

Figure C.60: North American Semiconductor Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.48. The slope of the graph is the Hurst coefficient.

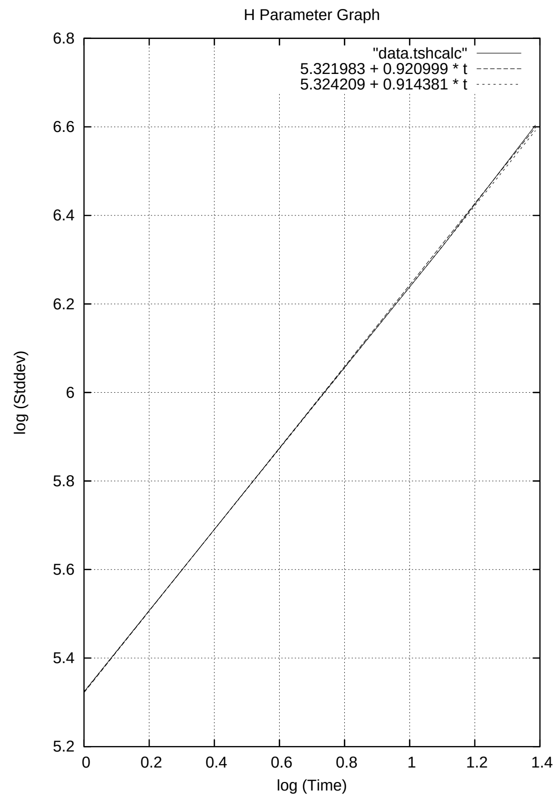

Figure C.61: North American Semiconductor Market, H parameter data for the normalized increments of the time series data shown in Figure C.48 The slope of the graph is the H parameter.

$$R_v\,(t_2 - t_1) \propto (t_2 - t_1)^H \quad \text{(C.108)}$$

$$\propto (t_2 - t_1)^{0.998014} \quad \text{(C.109)}$$

where R is the range of values in the increments of the rate of revenue returns, (per quarter.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per quarter,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per quarter) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.109 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \quad \text{(C.110)}$$

$$\propto \sqrt{(t_2 - t_1)} \quad \text{(C.111)}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per quarter,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.112}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per quarter,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[38].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \propto \frac{X\,(rt)}{r^H} \tag{C.113}$$

$$\propto \frac{X\,(rt)}{r^{0.998014}} \tag{C.114}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.61, to provide a least squares approximation to the H parameter for the North American Semiconductor Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.914381 for the near future, and 0.920999 for the distant future.

Figures C.60 and C.61 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.48. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.48, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.62 and C.63 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Many North American Semiconductor Market industry analyst speculate that there is “periodic” behavior in the market place, at approximately 5 year intervals. Both the Hurst coefficient and H parameter graphs would tend to support the intuition. Notice that the slope of the graphs, in figures C.60 and C.61, tend to decrease abruptly at $t \approx \ln(3) \approx 20$ quarters, which is approximately 60 months, or 5 years [Pet91, pp. 96]. Whether this is “periodic” behavior, or an indication of more complex system dynamics, perhaps “chaotic,” remains to be seen. If that is the case, it could provide an exploitive venue.

C.3.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.3.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.49. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.48. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the North American Semiconductor Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

[38] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

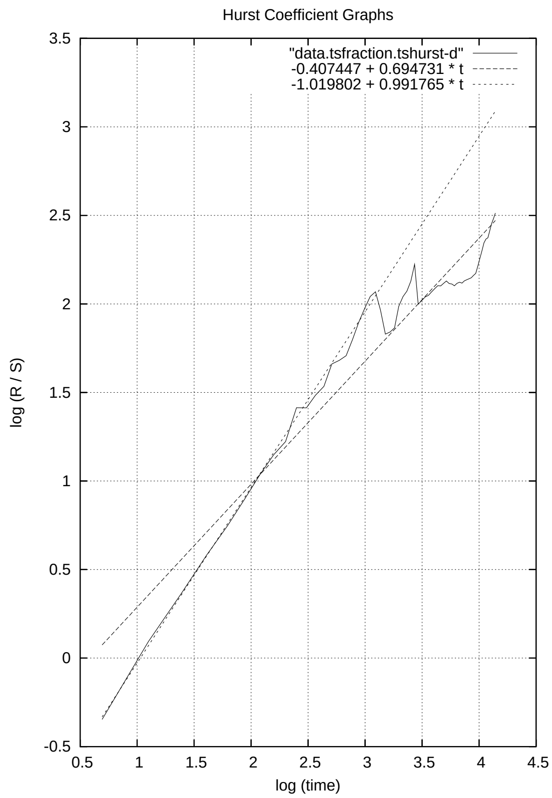

Figure C.62: North American Semiconductor Market, traditional Hurst coefficient data for the time series data shown in Figure C.47. The slope of the graph is the Hurst coefficient, and is 0.991765 for the near term, and 0.694731 for the far term.

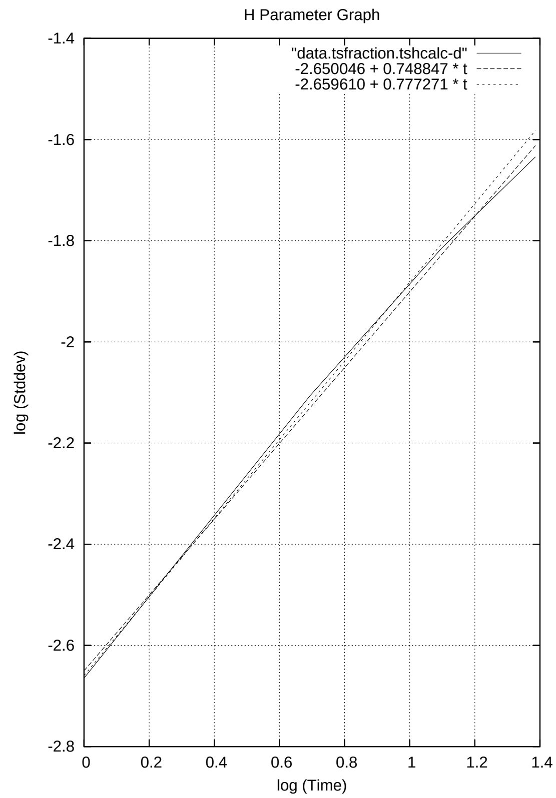

Figure C.63: North American Semiconductor Market, traditional H parameter data for the time series data shown in Figure C.47 The slope of the graph is the H parameter, and is 0.777271 for the near term, and 0.748847 for the far term.

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.48, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.040216 + 1)}{\ln(2)} = 0.056883 \tag{C.115}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.48, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.032989 + 1)}{\ln(2)} = 0.046825 \tag{C.116}$$

Note that if the mean is not constant in Figure C.48, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.47:

$$bits = 0.042107 \tag{C.117}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.47:

$$bits = 0.052703 \tag{C.118}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.3.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.052703t} \tag{C.119}$$

therefore:

$$C(p) = 0.052703 \tag{C.120}$$

and, *tsshannon* 0.052703 gives:

$$C(0.634320) = 0.052703 \tag{C.121}$$

therefore:

$$\begin{aligned} 2^{C(0.634320)} &= 2^{0.052703} && \text{(C.122)} \\ &= 1.037206 && \text{(C.123)} \\ &= 3.720639\% && \text{(C.124)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.634320) - 1 && \text{(C.125)} \\ &= 0.268640 && \text{(C.126)} \\ &= 26.864000\% && \text{(C.127)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the North American Semiconductor Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every quarter, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 26.864000% of its rate of revenue returns, (per quarter.) As a conceptual model, the remaining 73.136000% will be held in "reserve" with a 63.432000% chance of making twice the 26.864000% back, (and a 36.568000% chance of making 0.0,) in one quarter, on the average, for an average growth in its rate of revenue returns, (per quarter,) of 3.720639%, or a doubling of its rate of revenue returns, (per quarter,) in 18.974252 quarters.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 26.864000% per quarter of the rate of revenue returns, (per quarter,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 3.720639%, per quarter, on average.

Note that the metrics presented in this section are representative of the North American Semiconductor Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 26.864000% of its rate of revenue returns, (per quarter,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some quarters, depending on the North American Semiconductor Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other quarters, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 3.720639% per quarter.

As another simple example, a company re-invests 26.864000% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 26.864000% per quarter investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 3.720639% per quarter.

As an example of "product portfolio" management, suppose a company re-invests 26.864000% of its rate of revenue returns, (per quarter,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 26.864000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 26.864000$ percent for the second product, implying that the company should diversify its product line[39]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 26.864000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the North American Semiconductor Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.268640}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.48, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 26.864000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 26.864000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

[39]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 26.864000 of the rate of revenue returns per quarter does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 26.864000% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.268640}$ products seems consistent with the industry, also.

C.3.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the North American Semiconductor Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the North American Semiconductor Market time series is 0.040216, and 0.079970respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 6.288465.

If this value seems consistent number of companies in the North American Semiconductor Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the North American Semiconductor Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.600270, which would be the value which should be used in Section C.3.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.3.5 is greater than the average Shannon probability for the companies participating in the North American Semiconductor Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.3.5. The maximum exploitability for the North American Semiconductor Market is derived in Section C.3.10, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the North American Semiconductor Market is 0.600270, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.751444 in the North American Semiconductor Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.128}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the North American Semiconductor Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.3.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.49. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.48. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.3.5, is derived from the North American Semiconductor Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.3.10.

An additional exploitable strategy may be time itself. Equations C.105, C.109, and, C.107, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the North American Semiconductor Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.64, and, C.65 compare methods of approximation of the "forecastability" of rate of revenue returns in the North American Semiconductor Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the North American Semiconductor Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[40]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.64, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.107, $0.998014^n = 0.5$ quarters of operations. Since the optimal amount of inventory and, from Equation C.105, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.65, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[41]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with $\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.3.4, Equation C.107, and the preceeding section, approximately 3 times the value where $0.998014^n = 0.5$ could be interpreted as an approximation to the "average" product life cycle. This seems consistent

[40] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[41] For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

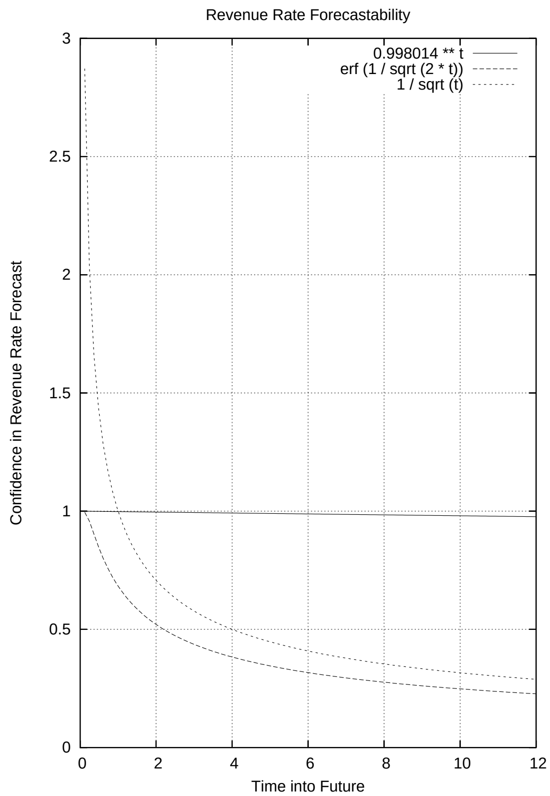

Figure C.64: North American Semiconductor Market, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.998014^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

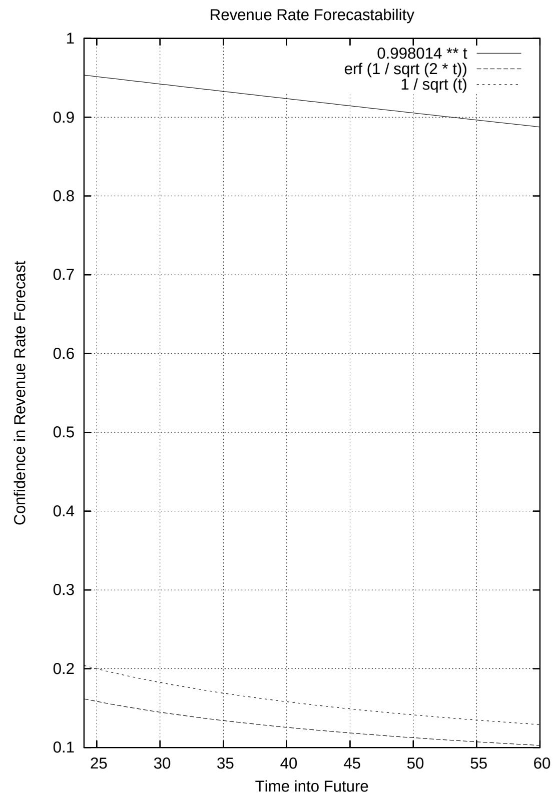

Figure C.65: North American Semiconductor Market, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.998014}$ many quarters seems consistent with the author's experience in the industry.

For convenience of comparison, converting from quarters to months by dividing the logarithmic returns by 3:

C.3.8 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.3.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.49. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.48. These values will be used in a fixed increment Brownian fractal analysis and simulation of the North American Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the North American Semiconductor Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.48, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.013405 + 1)}{\ln(2)} = 0.019211 \tag{C.129}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.48, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.010996 + 1)}{\ln(2)} = 0.015778 \tag{C.130}$$

Note that if the mean is not constant in Figure C.48, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.47:

$$bits = 0.014036 \tag{C.131}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.47:

$$bits = 0.017568 \tag{C.132}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.3.8 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.017568t} \tag{C.133}$$

therefore:

$$C(p) = 0.017568 \tag{C.134}$$

and, *tsshannon* 0.017568 gives:

$$C(0.577871) = 0.017568 \tag{C.135}$$

therefore:

$$\begin{aligned} 2^{C(0.577871)} &= 2^{0.017568} && \text{(C.136)} \\ &= 1.012252 && \text{(C.137)} \\ &= 1.225165\% && \text{(C.138)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.577871) - 1 && \text{(C.139)} \\ &= 0.155742 && \text{(C.140)} \\ &= 15.574200\% && \text{(C.141)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the North American Semiconductor Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 15.574200% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 84.425800% will be held in "reserve" with a 57.787100% chance of making twice the 15.574200% back, (and a 42.212900% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 1.225165%, or a doubling of its rate of revenue returns, (per month,) in 56.921676 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 15.574200% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.225165%, per month, on average.

Note that the metrics presented in this section are representative of the North American Semiconductor Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 15.574200% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the North American Semiconductor Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.225165% per month.

As another simple example, a company re-invests 15.574200% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 15.574200% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.225165% per month.

As an example of "product portfolio" management, suppose a company re-invests 15.574200% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 15.574200$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 15.574200$ percent for the second product, implying that the company should diversify its product line[42]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 15.574200%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the North American Semiconductor Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.155742}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example

[42]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a “down market” concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal “hedging of bets,” in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.48, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of “product portfolio” management, consider the issue of product mix. In this interpretation, 15.574200% of the product manufactured should be “proprietary,” while the rest is “industry standard.” As yet another possibility, 15.574200% of the product manufactured should be predatory into new markets, and the remainder in markets that are “traditional” for the company.

C.3.9 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.3.9. Figure C.66 represents a constructional simulation of the time series data presented in Figure C.47. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.48. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.48 represent a relatively complex process, that may not be “modeled” with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.67 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.50.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.3.1, Figure C.50, it would appear that the North American Semiconductor Market’s normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to “wager” a fraction of 0.500000 of the rate of returns every quarter is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any quarter, was no more that what was “wagered.” However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.142}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational “wager” fraction, from Figure C.48 of 0.079970, vs. an “theoretical optimal” value of 0.500000 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per quarter, which is unacceptably high. However, it is not clear why the North American Semiconductor Market is running at a value of 0.079970, which seems very conservative. However, a re-investment strategy of 0.079970 per quarter does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the North American Semiconductor Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx$ 0.5, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard

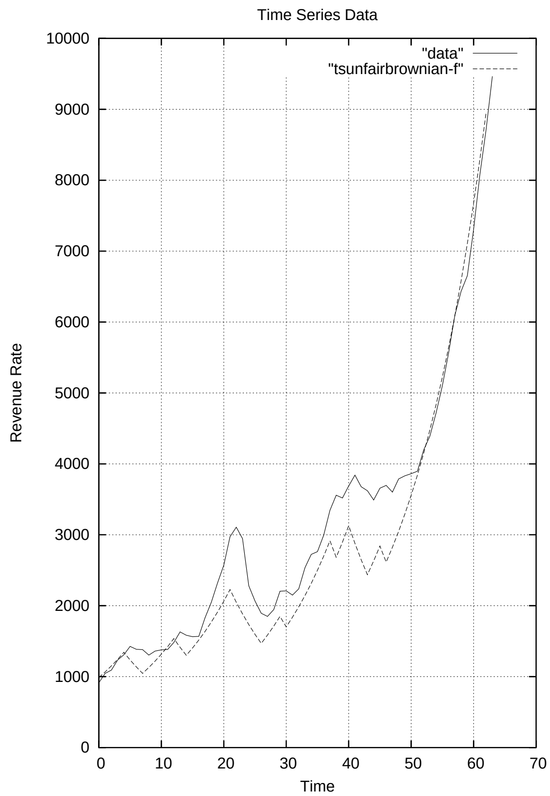

Figure C.66: North American Semiconductor Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.079970. This data is superimposed on the data presented in Figure C.47.

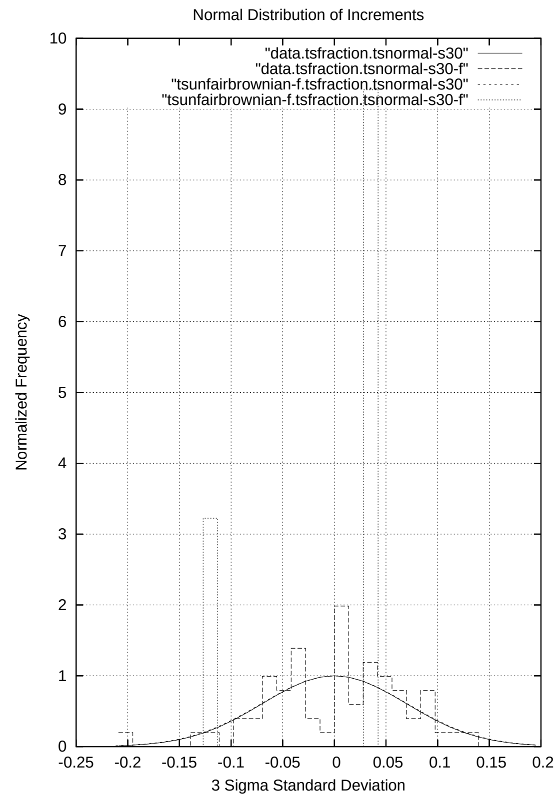

Figure C.67: North American Semiconductor Market, normalized histogram of the normalized increments of the time series data shown in Figure C.66, empirical and simulated. The empirical data has a mean of 0.040216, with a standard deviation of 0.069677. By comparison, the simulated data has a mean of 0.038695 with a standard deviation of 0.070556. This data is superimposed on the data presented in Figure C.50. The area under the four curves is identical.

deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.500000}{2.5}$, compared with an operational value, from Figure C.50 of 0.079970.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the North American Semiconductor Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.47. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.750000 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. But an operational Shannon probability of 0.634320 is not just marginally lower than the maximum Shannon probability of 0.750000. There is a significant disparity. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be optimized because the ordering of priorities is intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

C.3.10 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.3.3. One of the issues of analysis, as mentioned in Section C.3.7, is to determine the maximum Shannon probability for the time series presented in Figure C.47. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.68 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.47. Figure C.69 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.47. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.634320, as derived in Section C.3.5 to 0.750000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.47, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.47, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of quarters that the North American Semiconductor Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.746032, as compared with the predicted value from the program *tsshannonmax* of 0.750000.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.3.1, Figure C.50, it would appear that the North American Semiconductor Market's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.500000 of the rate of returns every quarter is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any quarter, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

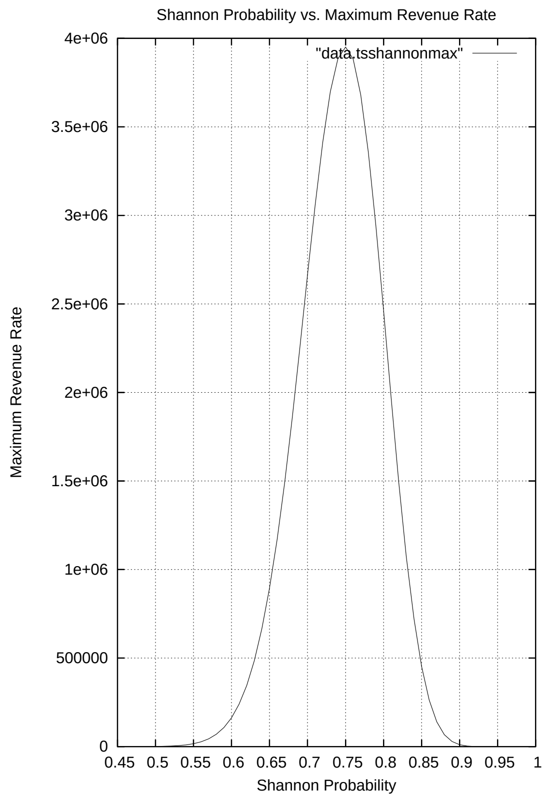

Figure C.68: North American Semiconductor Market, maximum rate of revenue returns, per quarter, vs. Shannon probability. The maximum rate of revenue returns, per quarter, occurs at a Shannon probability of 0.750000.

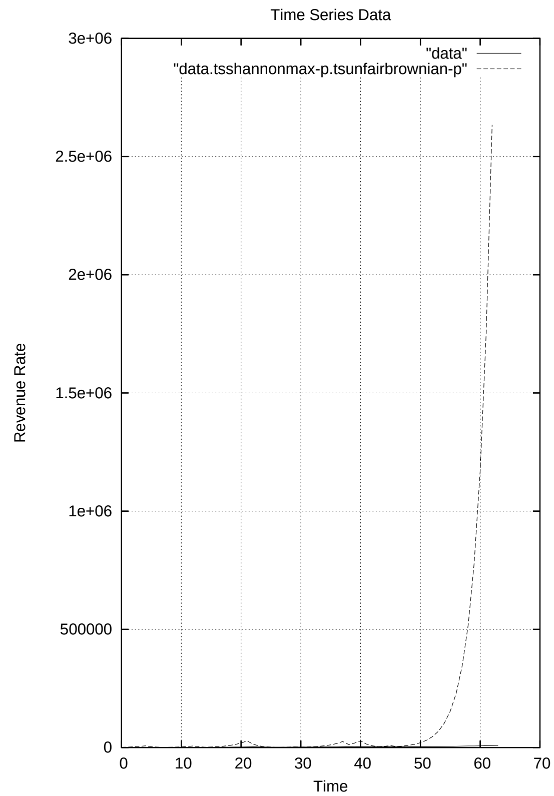

Figure C.69: North American Semiconductor Market, maximum rate of revenue returns, per quarter, at a Shannon probability, of 0.750000, corresponding to a “wager” fraction of 0.500000.

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.143}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational “wager” fraction, from Figure C.48 of 0.079970, vs. an “theoretical optimal” value of 0.500000 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per quarter, which is unacceptably high. However, it is not clear why the North American Semiconductor Market is running at a value of 0.079970, which seems very conservative. However, a re-investment strategy of 0.079970 per quarter does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the North American Semiconductor Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx$ 0.5, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.500000}{2.5}$, compared with an operational value, from Figure C.50 of 0.079970.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the North American Semiconductor Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.47. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.750000 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.634320 is not just marginally lower than the maximum Shannon probability of 0.750000. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.3.11 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.49. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.48. These values will be used in a fixed increment Brownian fractal analysis of the North American Semiconductor Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.3.6 and D.3.7. As a subjective evaluation of the "quality" of the analysis of the North American Semiconductor Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.47 from Figure C.48, and the Shannon probability as calculated by counting the total number of quarters that the North American Semiconductor Market movement was positive, as presented in Section C.3.10:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.144}$$

$$0.746032 \approx \frac{\frac{0.040216}{0.079970} + 1}{2} \tag{C.145}$$

$$0.746032 \approx 0.751444 \tag{C.146}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.746032 \approx 0.751444 \approx 0.750000 \tag{C.147}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.3.5, should be compared. The four methods used were the mean of Figure C.48, the constant in the least squares approximation to Figure C.48, the least squares exponential approximation to Figure C.47, and the logarithmic returns of Figure C.47, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.056883 \approx 0.046825 \approx 0.042107 \approx 0.052703 \tag{C.148}$$

It is implied in Section C.3.5, Subsection C.3.5 and in Section C.3.9 that, a Brownian motion with fixed increments fractal may "model" the North American Semiconductor Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.149}$$

$$0.079970\,(2 \cdot 0.746032 - 1) \approx \frac{0.069677\,(2 \cdot 0.746032 - 1)}{2\sqrt{0.746032\,(1 - 0.746032)}} \tag{C.150}$$

$$0.079970 \cdot 0.492063 \approx 0.069677 \cdot 0.565227 \tag{C.151}$$

$$0.039350 \approx 0.039383 \tag{C.152}$$

and, equating to the mean:

$$0.040216 \approx 0.039350 \approx 0.039383 \tag{C.153}$$

where, as in Equation C.146 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.47 from Figure C.48, and the Shannon probability as calculated by counting the total number of quarters that the North American Semiconductor Market movement was positive, as presented in Section C.3.10.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[43], where the absolute value is presented in Figure C.49, and the root mean square value is presented in Figure C.48:

$$0.064520 \approx 0.079970 \tag{C.154}$$

Note, that if the North American Semiconductor Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.47 from Figure C.48 should be zero. It is 0.047627.

[43] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

C.4 United States Electronic Component Shipments

For the analysis, the data was in the directory ../markets/electronic.components.shipments[44].

The data in this section is presented in tabular form in Section D.4.

C.4.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.4.1. Figure C.70 is a graph of the time series data for the United States Electronic Component Shipments.

Figure C.71 is a graph of the normalized increments of the time series data presented in Figure C.70. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.72 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.71. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[45].

Figure C.73 is the normalized histogram of the normalized increments of the time series data shown in Figure C.71. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.73.

Figure C.74 is the statistical estimate for the data presented in Figure C.71, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.566532$, as derived in Section C.4.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.75 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.71. In principle, if the distribution of the normalized increments presented in Figure C.73 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.76 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.71. In principle, if the distribution of the normalized increments presented in Figure C.73 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.77 is the range of values of the time series shown in Figure C.70. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.77 would be a square root function[46]. Figure C.78 is the deterministic map of the normalized increments of the time series data shown in Figure C.71. The deterministic map is useful for determining if a time series was created by a deterministic

[44] Data from the United States Department of Commerce, 1979—1994, by months, in millions of dollars, US.

[45] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[46] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.77 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

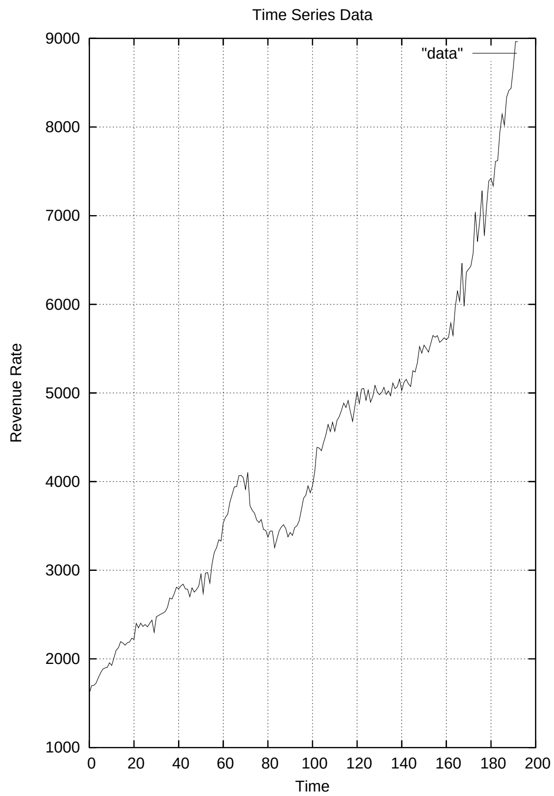

Figure C.70: United States Electronic Component Shipments, time series data.

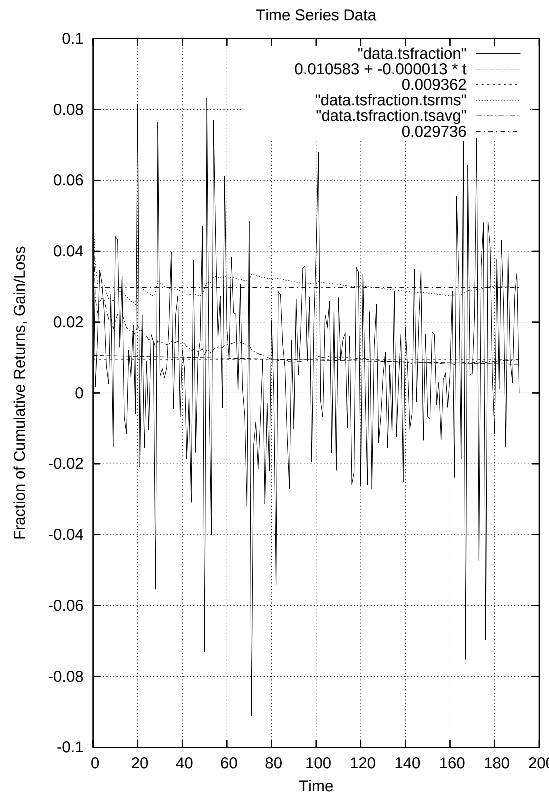

Figure C.71: United States Electronic Component Shipments, normalized increments of the time series data presented in Figure C.70. The mean is 0.009362 with a standard deviation of 0.028297. The formula for the least squares approximation is $0.010583 + -0.000013t$, and the root mean squared value is 0.029736. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000013, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

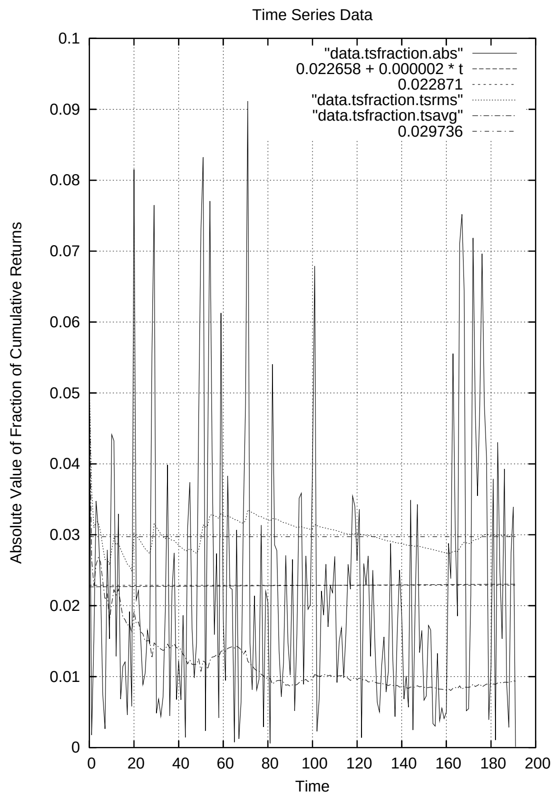

Figure C.72: United States Electronic Component Shipments, absolute value of the normalized increments of the time series data presented in Figure C.71. The mean is 0.022871 with a standard deviation of 0.019053. The formula for the least squares approximation is $0.022658 + 0.000002t$, and the root mean square value, from Figure C.71, is 0.029736. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.71, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

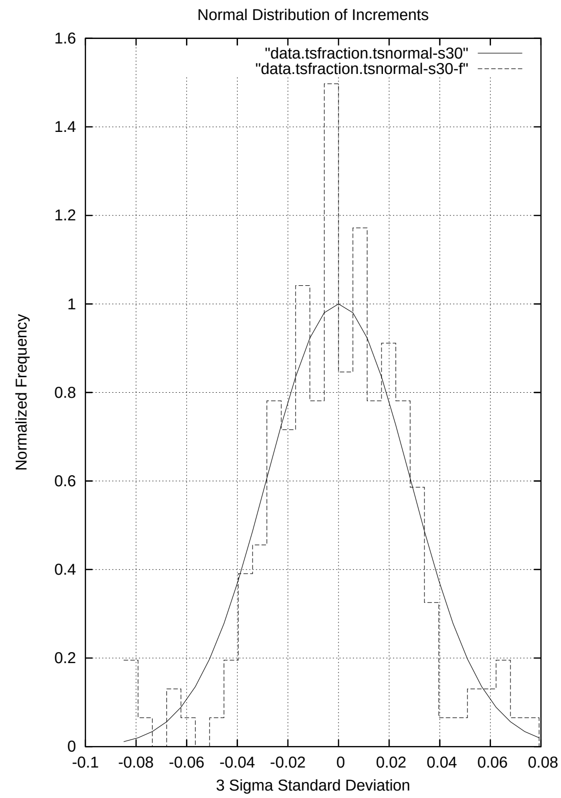

Figure C.73: United States Electronic Component Shipments, normalized histogram of the normalized increments of the time series data shown in Figure C.71. The data has a mean of 0.009362, with a standard deviation of 0.028297. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 4.711000, with a critical value of 42.557000.

Observations on the Time Series Increments Analysis

Figure C.73 would seem to indicate that the time series data for the United States Electronic Component Shipments represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

```
For  a mean  of 0.009314,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000931,   2758  samples  would  be required.
     (With  193  samples,   the  estimated   error  is 0.003521  = 37.801077   percent.)
For  a standard  deviation  of 0.029736,  with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.002974,   136  samples  would  be required.
     (With  193  samples,   the  estimated   error  is 0.002490  = 8.372085   percent.)
```

Figure C.74: United States Electronic Component Shipments, statistical estimates of the normalized increments of the time series shown in Figure C.71. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.71.

C.4.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[47]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.79 is the instantaneous value of the root mean square of the normalized increments for the United States Electronic Component Shipments, and Figure C.80 is the instantaneous Shannon probability for the normalized increments.

C.4.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.4.4. Figure C.81 is a graph of the logistic function estimates of the time series data for the United States Electronic Component Shipments. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[48]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.81 is a graph of the logistic function for the time series data presented in Figure C.70. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.71. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.71. Figure C.82 is the same graph, but with the time scale expanded by a factor of two.

[47]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[48]For example, in Figures C.81 and C.82, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.4.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

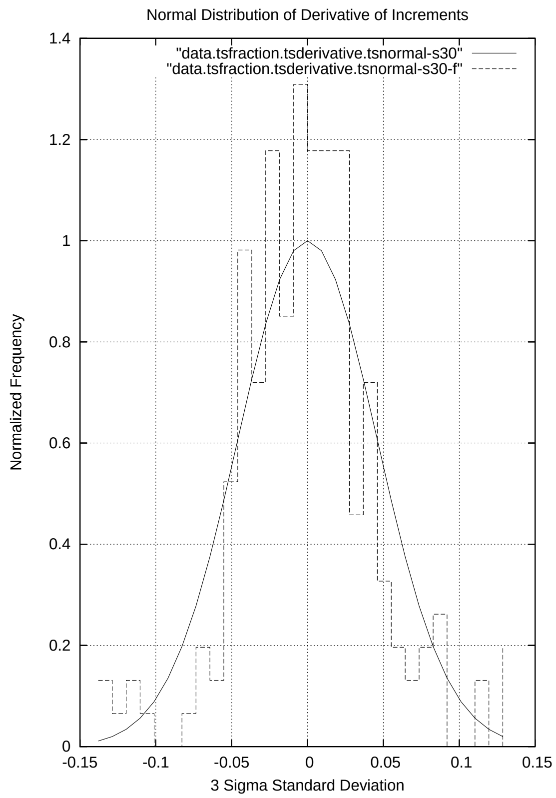

Figure C.75: United States Electronic Component Shipments, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.71.

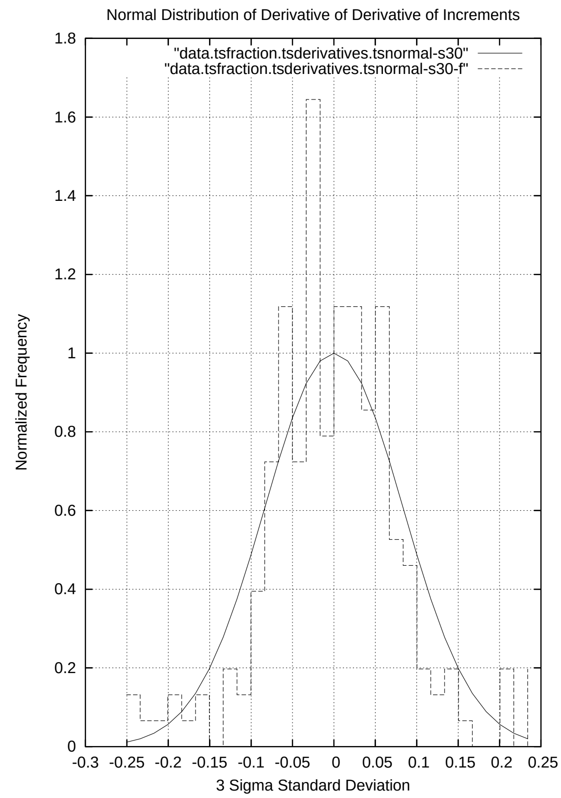

Figure C.76: United States Electronic Component Shipments, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.71.

C.4.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.4.5. Figure C.83 is a graph of the Hurst coefficient data time series data shown in Figure C.70. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.84 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.71. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.83 implies that the variance of the rate of revenue returns, (per month,) in the United States Electronic Component Shipments, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[49], t, $p(t) = erf(1/\sqrt{2t})$ which

[49]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional

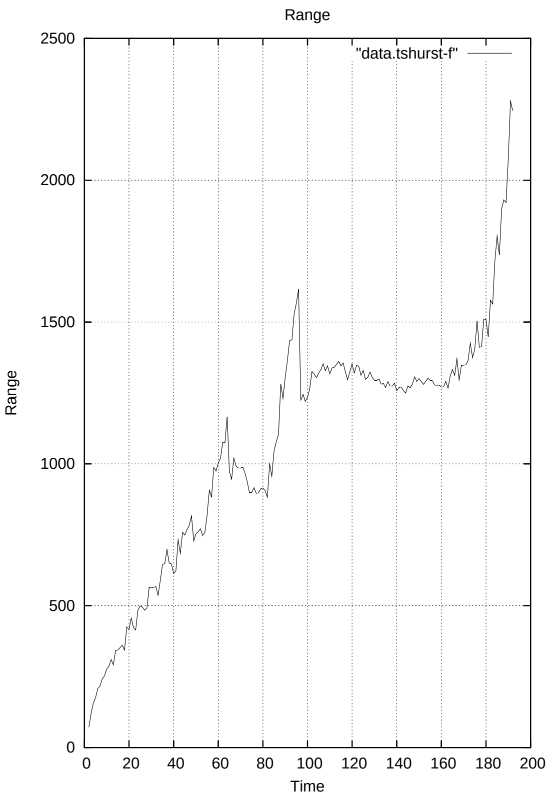

Figure C.77: United States Electronic Component Shipments, range of the time series data shown in Figure C.70.

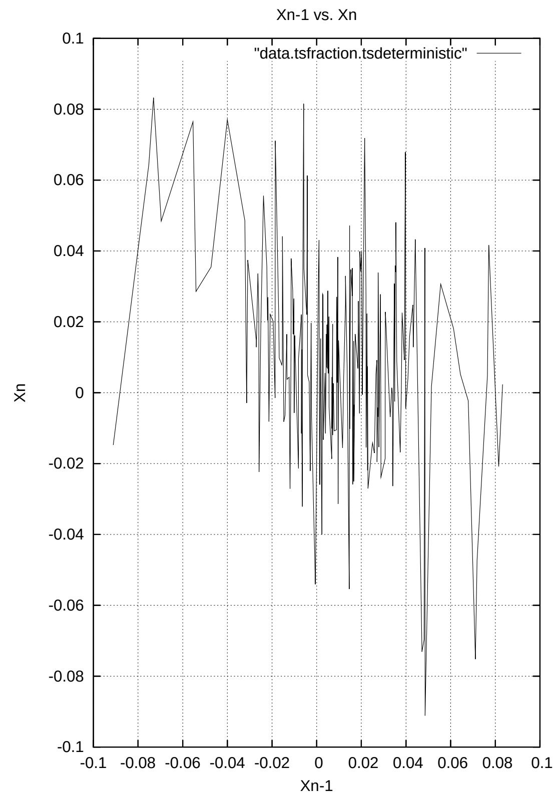

Figure C.78: United States Electronic Component Shipments, deterministic map of the normalized increments of the time series data shown in Figure C.71.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.87, and, C.88 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Electronic Component Shipments for the near term and far term, respectively [Pet91, pp. 83-84][50]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.83, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.755693, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.155}$$

to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[50]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

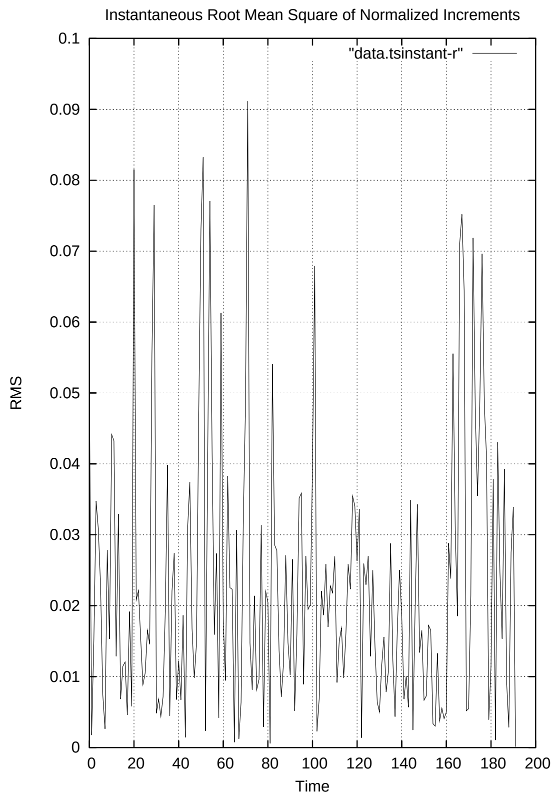

Figure C.79: United States Electronic Component Shipments, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.70.

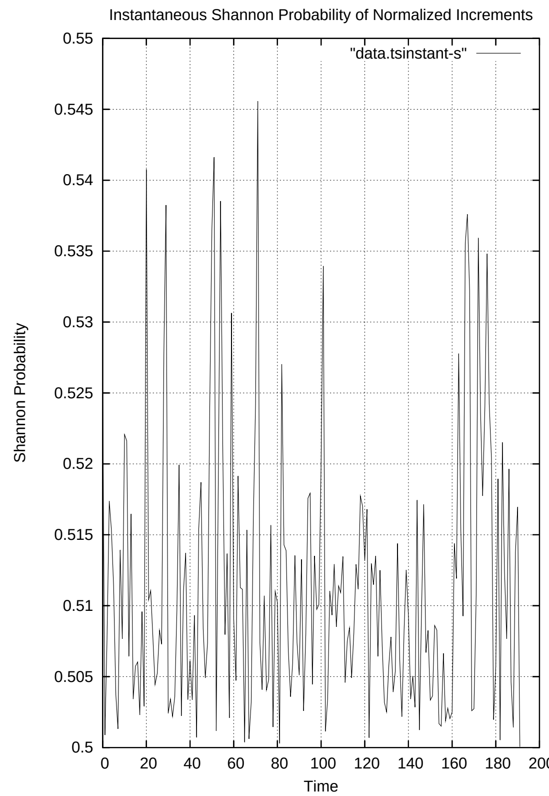

Figure C.80: United States Electronic Component Shipments, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.70.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.755693} \tag{C.156}$$

$$\propto (t_2 - t_1)^{1.511386} \tag{C.157}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[51]. A Hurst coefficient of 0.755693, (for the near future, and 0.621033 for the distant future.) implies

[51] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.4.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Electronic Component Shipments. See

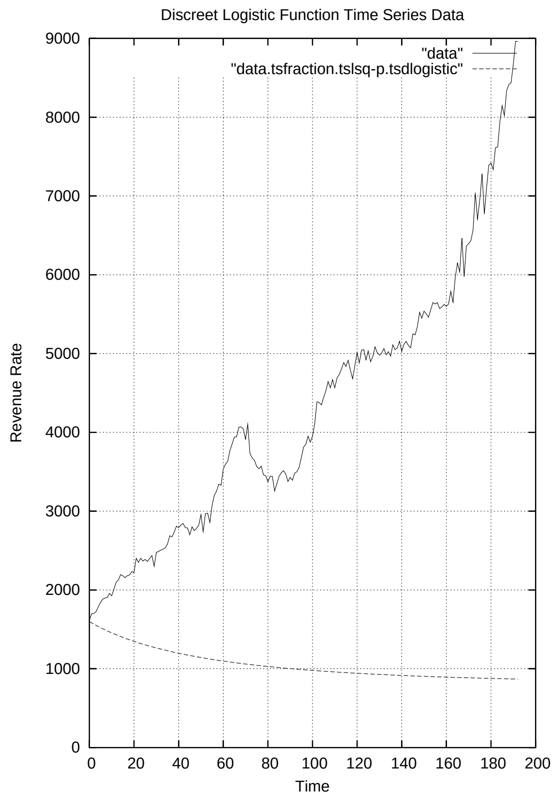

Figure C.81: United States Electronic Component Shipments, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.71 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

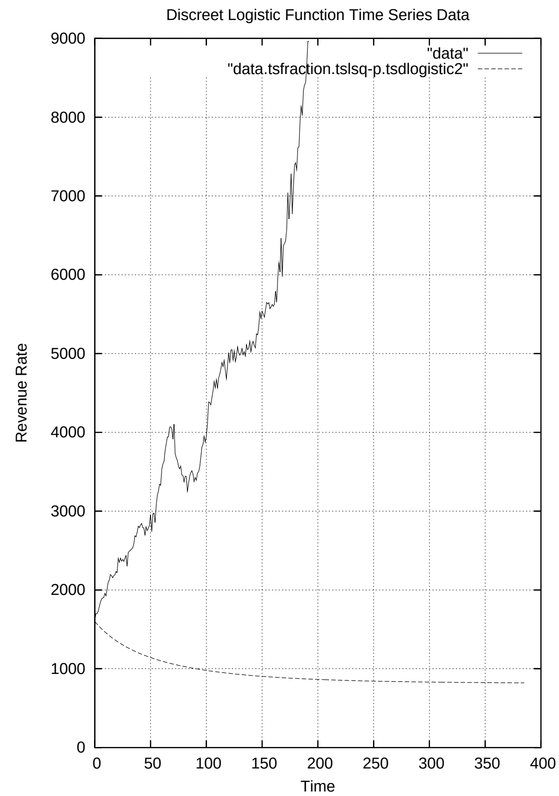

Figure C.82: United States Electronic Component Shipments, logistic function estimates of Figure C.81 with the time scale expanded by a factor of two.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 75.569300% [Pet91, pp. 66] for the near future, and 0.621033 for the distant future. Likewise, there is a 75.569300% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 75.569300% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Electronic Component Shipments are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.158}$$

also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

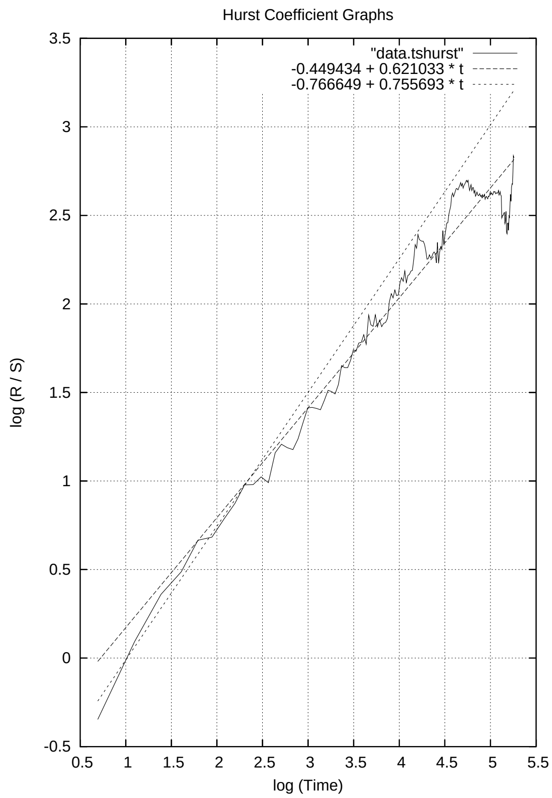

Figure C.83: United States Electronic Component Shipments, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.71. The slope of the graph is the Hurst coefficient.

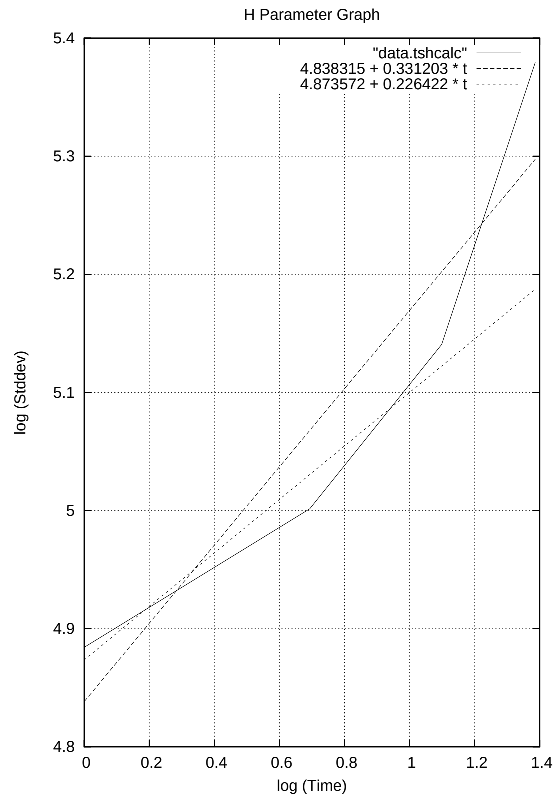

Figure C.84: United States Electronic Component Shipments, H parameter data for the normalized increments of the time series data shown in Figure C.71 The slope of the graph is the H parameter.

$$= 0.755693^n \tag{C.159}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.71, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.009362 \cdot 100$ percent, on the average, with a standard deviation of $0.028297 \cdot 100$ percent, and a root mean square error value of $0.029736 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.160}$$

$$\propto (t_2 - t_1)^{0.755693} \tag{C.161}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.161 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.162}$$

$$\propto \sqrt{(t_2 - t_1)} \tag{C.163}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.164}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[52].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \propto \frac{X\,(rt)}{r^H} \tag{C.165}$$

$$\propto \frac{X\,(rt)}{r^{0.755693}} \tag{C.166}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.84, to provide a least squares approximation to the H parameter for the United States Electronic Component Shipments. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.226422 for the near future, and 0.331203 for the distant future.

Figures C.83 and C.84 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.71. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.71, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.85 and C.86 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Note that the H parameter data is not linear, and the long term predictability is better than the short term predictability, indicating that the least squares approximation is low.

[52] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

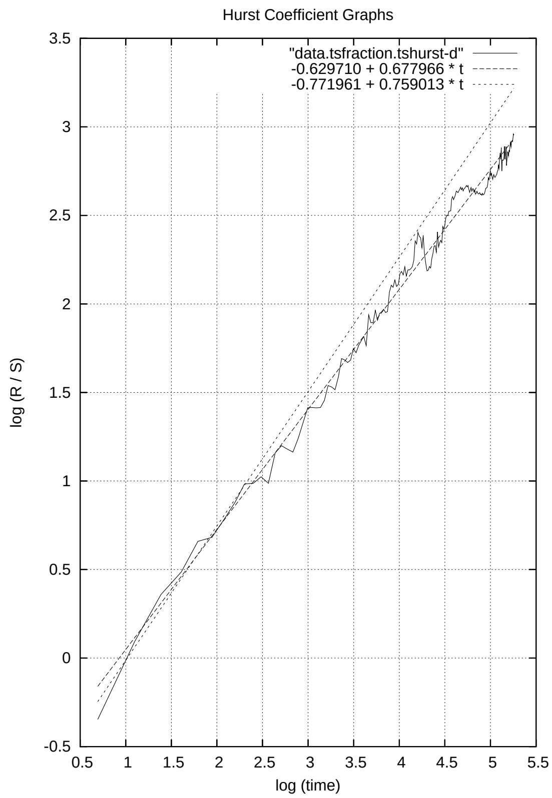

Figure C.85: United States Electronic Component Shipments, traditional Hurst coefficient data for the time series data shown in Figure C.70. The slope of the graph is the Hurst coefficient, and is 0.759013 for the near term, and 0.677966 for the far term.

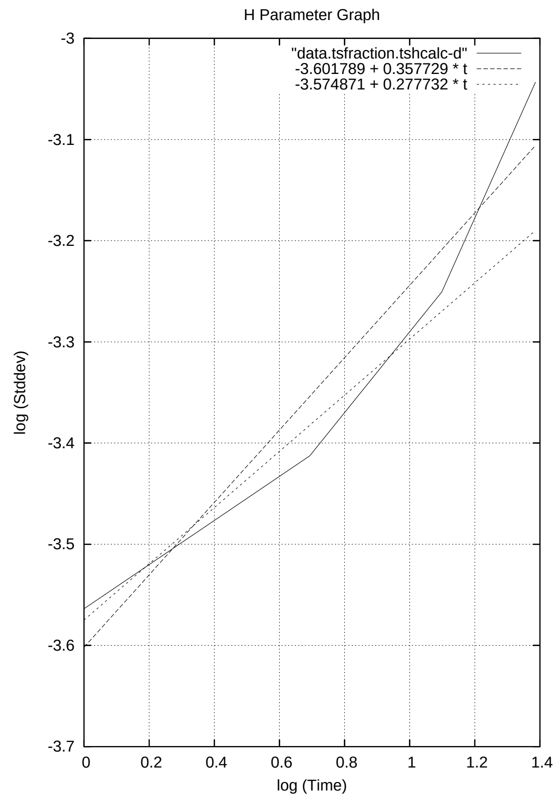

Figure C.86: United States Electronic Component Shipments, traditional H parameter data for the time series data shown in Figure C.70 The slope of the graph is the H parameter, and is 0.277732 for the near term, and 0.357729 for the far term.

C.4.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.4.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.72. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.71. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronic Component Shipments, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Electronic Component Shipments, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.71, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.009362 + 1)}{\ln(2)} = 0.013444 \tag{C.167}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.71, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.010583 + 1)}{\ln(2)} = 0.015188 \tag{C.168}$$

Note that if the mean is not constant in Figure C.71, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.70:

$$bits = 0.010340 \tag{C.169}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.70:

$$bits = 0.012810 \tag{C.170}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.4.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.012810t} \tag{C.171}$$

therefore:

$$C(p) = 0.012810 \tag{C.172}$$

and, *tsshannon* 0.012810 gives:

$$C(0.566532) = 0.012810 \tag{C.173}$$

therefore:

$$\begin{aligned} 2^{C(0.566532)} &= 2^{0.012810} && \text{(C.174)} \\ &= 1.008919 && \text{(C.175)} \\ &= 0.891875\% && \text{(C.176)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.566532) - 1 && \text{(C.177)} \\ &= 0.133064 && \text{(C.178)} \\ &= 13.306400\% && \text{(C.179)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the United States Electronic Component Shipments executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 13.306400% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 86.693600% will be held in “reserve” with a 56.653200% chance of making twice the 13.306400% back, (and a 43.346800% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.891875%, or a doubling of its rate of revenue returns, (per month,) in 78.064012 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 13.306400% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.891875%, per month, on average.

Note that the metrics presented in this section are representative of the United States Electronic Component Shipments as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 13.306400% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Electronic Component Shipments's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.891875% per month.

As another simple example, a company re-invests 13.306400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 13.306400% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.891875% per month.

As an example of "product portfolio" management, suppose a company re-invests 13.306400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 13.306400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 13.306400$ percent for the second product, implying that the company should diversify its product line[53]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 13.306400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Electronic Component Shipments, as a standard bench mark, then the optimal number will be $\frac{1}{0.133064}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.71, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex

[53]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 13.306400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 13.306400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 13.306400 of the rate of revenue returns per month does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 13.306400% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.133064}$ products seems consistent with the industry, also.

C.4.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Electronic Component Shipments, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Electronic Component Shipments time series is 0.009362, and 0.029736respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 10.587747.

If this value seems consistent number of companies in the United States Electronic Component Shipments, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Electronic Component Shipments are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.548379, which would be the value which should be used in Section C.4.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.4.5 is greater than the average Shannon probability for the companies participating in the United States Electronic Component Shipments, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.4.5. The maximum exploitability for the United States Electronic Component Shipments is derived in Section C.4.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Electronic Component Shipments is 0.548379, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.657419 in the United States Electronic Component Shipments. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.180}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Electronic Component Shipments would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.4.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.72. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.71. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronic Component Shipments, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.4.5, is derived from the United States Electronic Component Shipments metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.4.9.

An additional exploitable strategy may be time itself. Equations C.157, C.161, and, C.159, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Electronic Component Shipments, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.87, and, C.88 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Electronic Component Shipments for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Electronic Component Shipments. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[54]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.87, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.159, $0.755693^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.157, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.88, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with

[54] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

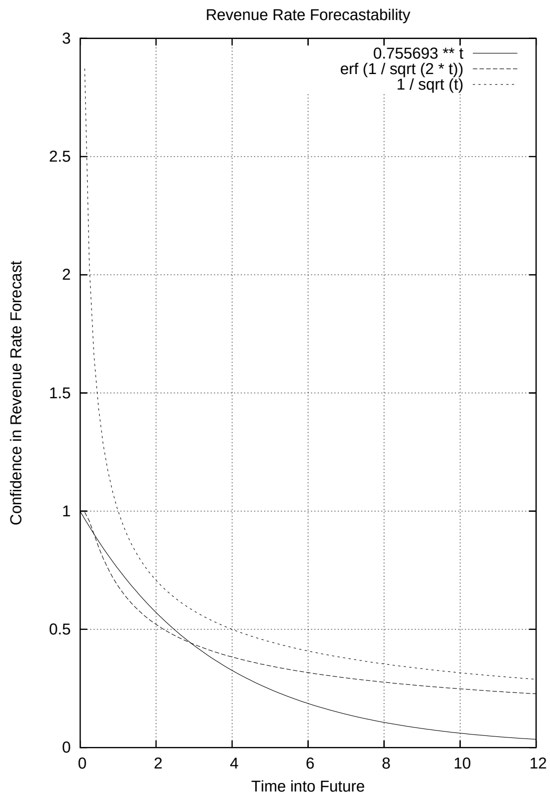

Figure C.87: United States Electronic Component Shipments, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.755693^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

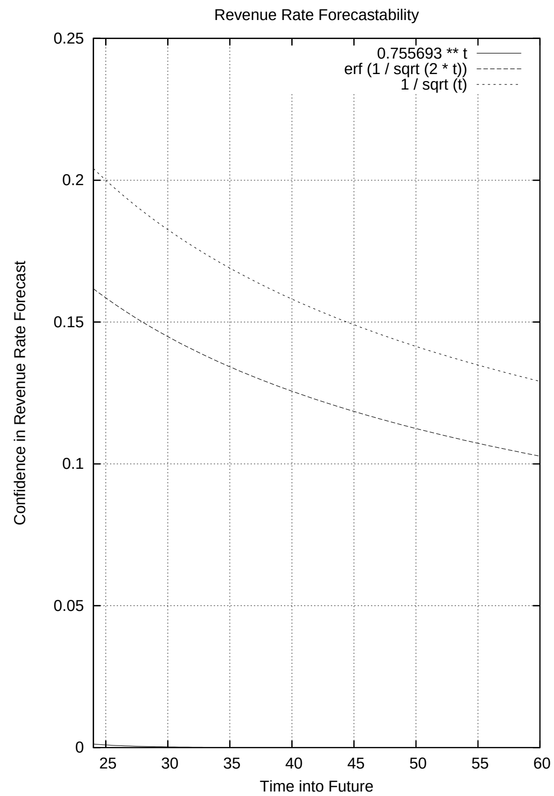

Figure C.88: United States Electronic Component Shipments, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

numbers from the venture community[55]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with $\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.4.4, Equation C.159, and the preceeding section, approximately 3 times the value where $0.755693^n = 0.5$ could be interpreted as an approximation to the "average" product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.755693}$ many months seems consistent with the author's experience in the industry.

[55] For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

C.4.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.4.9. Figure C.89 represents a constructional simulation of the time series data presented in Figure C.70. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.71. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.71 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.90 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.73.

C.4.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.4.3. One of the issues of analysis, as mentioned in Section C.4.7, is to determine the maximum Shannon probability for the time series presented in Figure C.70. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.91 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.70. Figure C.92 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.70. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.566532, as derived in Section C.4.5 to 0.658031. This process, essentially, extracts the random statistical data from the time series presented in Figure C.70, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.70, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Electronic Component Shipments movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.656250, as compared with the predicted value from the program *tsshannonmax* of 0.658031.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.4.1, Figure C.73, it would appear that the United States Electronic Component Shipments's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.316062 of the rate of returns every month is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any month, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.181}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.71 of 0.029736, vs. an "theoretical optimal" value of 0.316062 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard

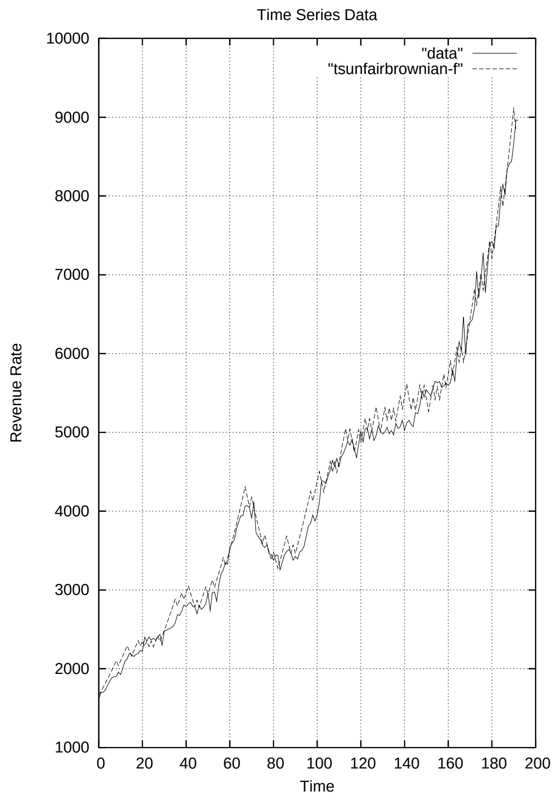

Figure C.89: United States Electronic Component Shipments, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.029736. This data is superimposed on the data presented in Figure C.70.

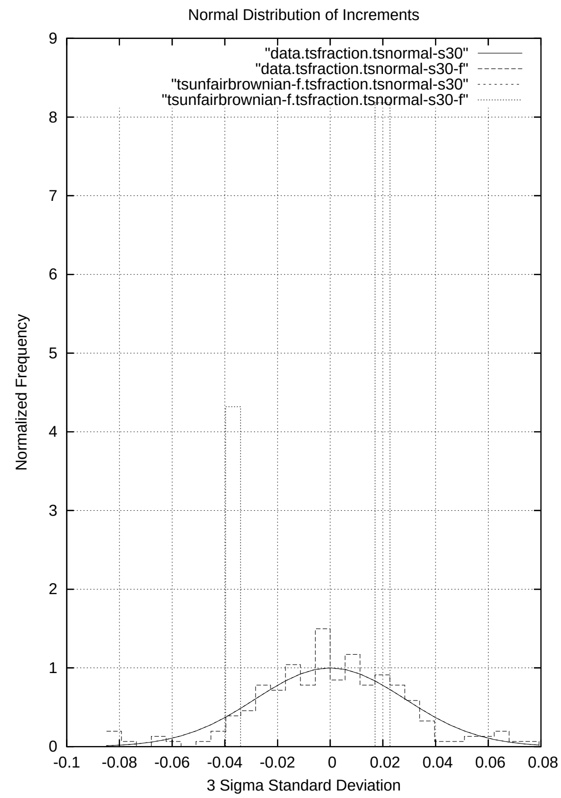

Figure C.90: United States Electronic Component Shipments, normalized histogram of the normalized increments of the time series data shown in Figure C.89, empirical and simulated. The empirical data has a mean of 0.009362, with a standard deviation of 0.028297. By comparison, the simulated data has a mean of 0.009185 with a standard deviation of 0.028356. This data is superimposed on the data presented in Figure C.73. The area under the four curves is identical.

deviation, or about 15.865% per month, which is unacceptably high. However, it is not clear why the United States Electronic Component Shipments is running at a value of 0.029736, which seems very conservative. However, a re-investment strategy of 0.029736 per month does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the United States Electronic Component Shipments is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx 0.5$, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.316062}{2.5}$, compared with an operational value, from Figure C.73 of 0.029736.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the United States Electronic Component Shipments are not running an optimal re-investment

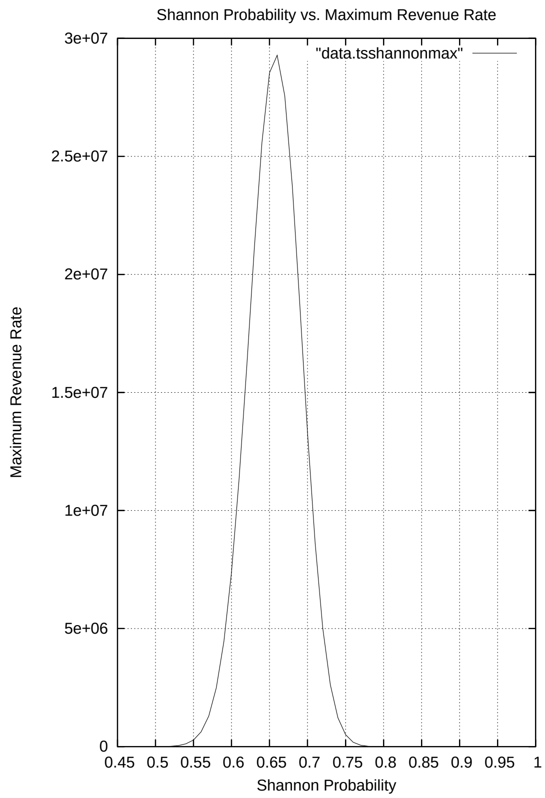

Figure C.91: United States Electronic Component Shipments, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.658031.

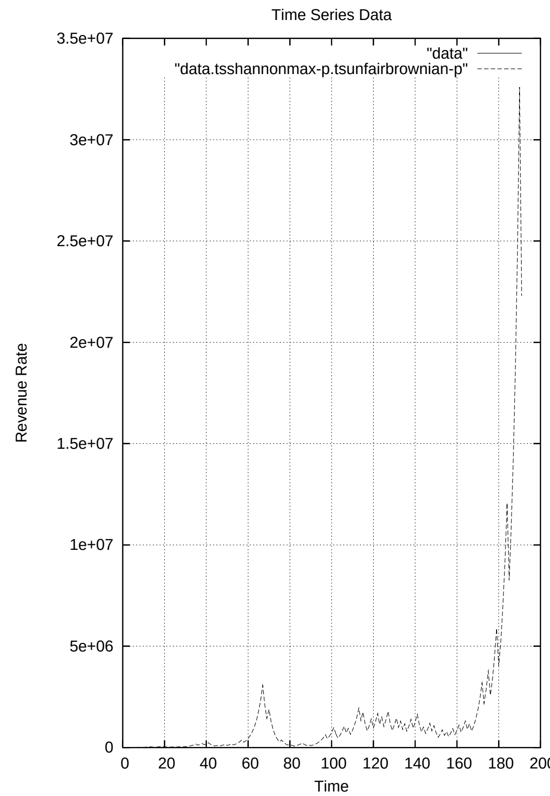

Figure C.92: United States Electronic Component Shipments, maximum rate of revenue returns, per month, at a Shannon probability, of 0.658031, corresponding to a “wager” fraction of 0.316062.

strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn’t. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.70. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.658031 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.566532 is not just marginally lower than the maximum Shannon probability of 0.658031. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.4.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.72. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.71. These values will be used in a fixed increment Brownian fractal analysis of the United States Electronic Component Shipments, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.4.6 and D.4.7. As a subjective evaluation of the "quality" of the analysis of the United States Electronic Component Shipments, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.70 from Figure C.71, and the Shannon probability as calculated by counting the total number of months that the United States Electronic Component Shipments movement was positive, as presented in Section C.4.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.182}$$

$$0.656250 \approx \frac{\frac{0.009362}{0.029736} + 1}{2} \tag{C.183}$$

$$0.656250 \approx 0.657419 \tag{C.184}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.656250 \approx 0.657419 \approx 0.658031 \tag{C.185}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.4.5, should be compared. The four methods used were the mean of Figure C.71, the constant in the least squares approximation to Figure C.71, the least squares exponential approximation to Figure C.70, and the logarithmic returns of Figure C.70, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.013444 \approx 0.015188 \approx 0.010340 \approx 0.012810 \tag{C.186}$$

It is implied in Section C.4.5, Subsection C.4.5 and in Section C.4.8 that, a Brownian motion with fixed increments fractal may "model" the United States Electronic Component Shipments. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.187}$$

$$0.029736\,(2\cdot 0.656250-1) \approx \frac{0.028297\,(2\cdot 0.656250-1)}{2\sqrt{0.656250\,(1-0.656250)}} \tag{C.188}$$

$$0.029736\cdot 0.312500 \approx 0.028297\cdot 0.328976 \tag{C.189}$$

$$0.009292 \approx 0.009309 \tag{C.190}$$

and, equating to the mean:

$$0.009362 \approx 0.009292 \approx 0.009309 \tag{C.191}$$

where, as in Equation C.184 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.70 from Figure C.71, and the Shannon probability as calculated by counting the total number of months that the United States Electronic Component Shipments movement was positive, as presented in Section C.4.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[56], where the absolute value is presented in Figure C.72, and the root mean square value is presented in Figure C.71:

$$0.022871 \approx 0.029736 \tag{C.192}$$

Note, that if the United States Electronic Component Shipments could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.70 from Figure C.71 should be zero. It is 0.019053.

C.5 United States Electronic Component Production

For the analysis, the data was in the directory ../markets/electronic.components.production[57].

The data in this section is presented in tabular form in Section D.5.

C.5.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.5.1. Figure C.93 is a graph of the time series data for the United States Electronic Component Production.

Figure C.94 is a graph of the normalized increments of the time series data presented in Figure C.93. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.95 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.94. The data presented was made by running the Unix utility sed(1) on the normalized increments time series

[56] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[57] Data from the United States Department of Commerce, 1980—1994, by months, as an index, 1987 = 100.

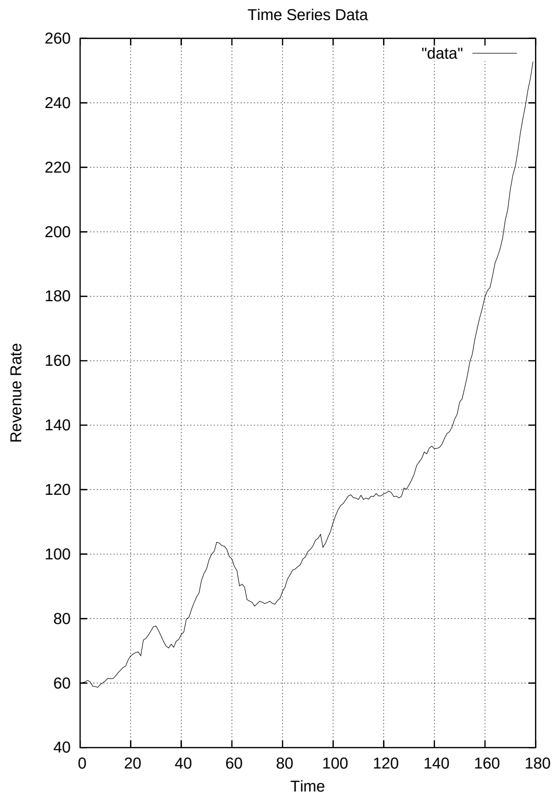

Figure C.93: United States Electronic Component Production, time series data.

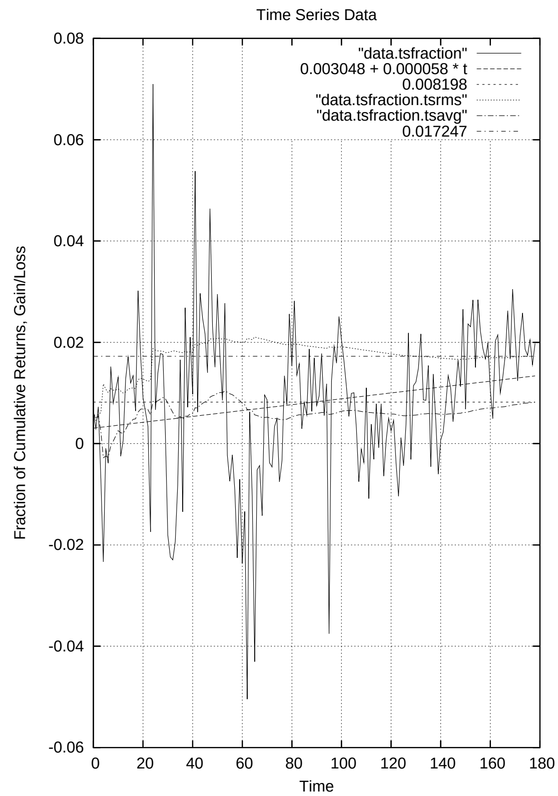

Figure C.94: United States Electronic Component Production, normalized increments of the time series data presented in Figure C.93. The mean is 0.008198 with a standard deviation of 0.015216. The formula for the least squares approximation is $0.003048 + 0.000058t$, and the root mean squared value is 0.017247. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000058, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[58].

[58]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the

Figure C.96 is the normalized histogram of the normalized increments of the time series data shown in Figure C.94. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

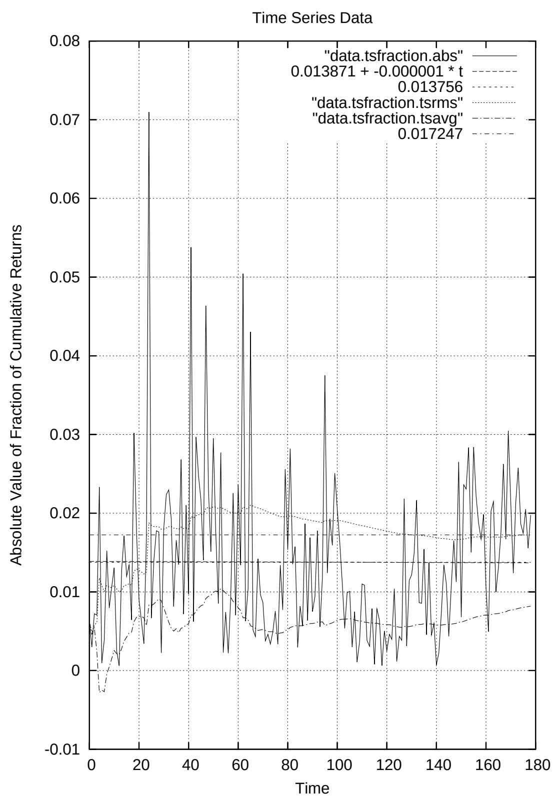

Figure C.95: United States Electronic Component Production, absolute value of the normalized increments of the time series data presented in Figure C.94. The mean is 0.013756 with a standard deviation of 0.010433. The formula for the least squares approximation is $0.013871 + -0.000001t$, and the root mean square value, from Figure C.94, is 0.017247. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.94, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

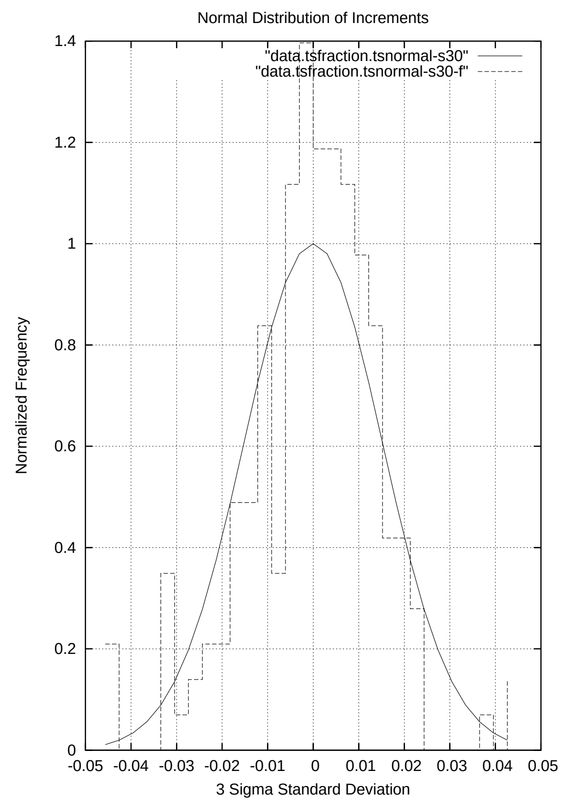

Figure C.96: United States Electronic Component Production, normalized histogram of the normalized increments of the time series data shown in Figure C.94. The data has a mean of 0.008198, with a standard deviation of 0.015216. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 6.808000, with a critical value of 42.557000.

normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.96.

Figure C.97 is the statistical estimate for the data presented in Figure C.94, as derived by the program *tsstatest*, which is briefly described in appendix B.

```
For  a mean  of  0.008153,   with  a confidence   level  of  0.900000
     that  the  error  did  not  exceed  0.000815,   1211  samples  would  be  required.
     (With  180  samples,   the  estimated   error  is  0.002114  = 25.935125   percent.)
For  a standard   deviation   of  0.017247,   with  a confidence   level  of  0.900000
     that  the  error  did  not  exceed  0.001725,   136  samples  would  be  required.
     (With  180  samples,   the  estimated   error  is  0.001495  = 8.669140   percent.)
```

Figure C.97: United States Electronic Component Production, statistical estimates of the normalized increments of the time series shown in Figure C.94. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.94.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.563187$, as derived in Section C.5.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.98 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.94. In principle, if the distribution of the normalized increments presented in Figure C.96 is Gaussian in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.99 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.94. In principle, if the distribution of the normalized increments presented in Figure C.96 is an integrated Gaussian distribution in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43.

Figure C.100 is the range of values of the time series shown in Figure C.93. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.100 would be a square root function[59]. Figure C.101 is the deterministic map of the normalized increments of the time series data shown in Figure C.94. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.96 would seem to indicate that the time series data for the United States Electronic Component Production represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

[59]Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.100 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

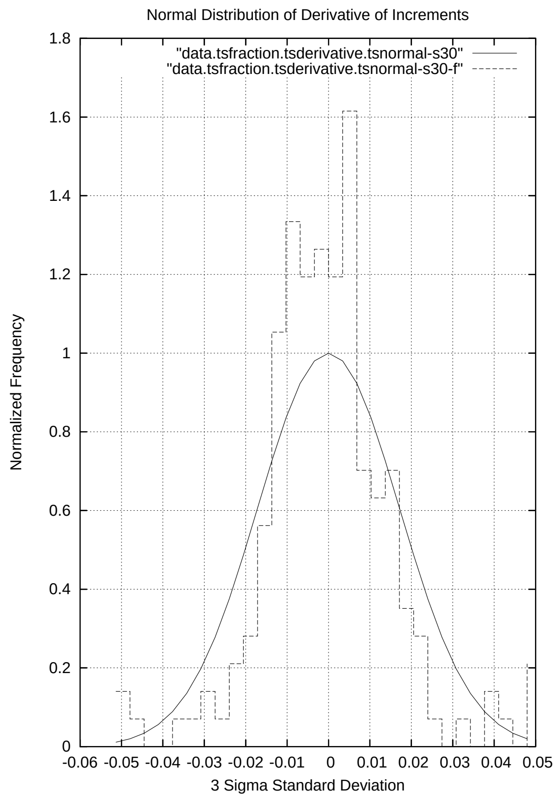

Figure C.98: United States Electronic Component Production, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.94.

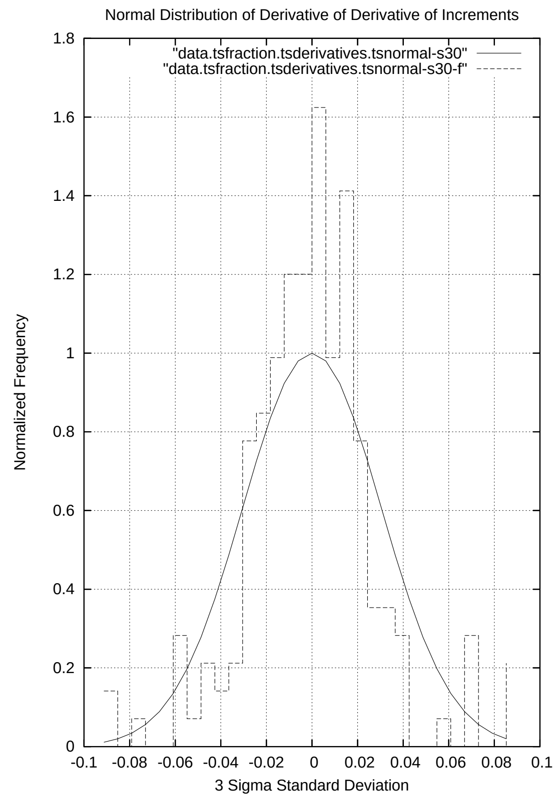

Figure C.99: United States Electronic Component Production, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.94.

C.5.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[60]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.102 is the instantaneous value of the root mean square of the normalized increments for the United States Electronic Component Production, and Figure C.103 is the instantaneous Shannon probability for the normalized increments.

[60]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

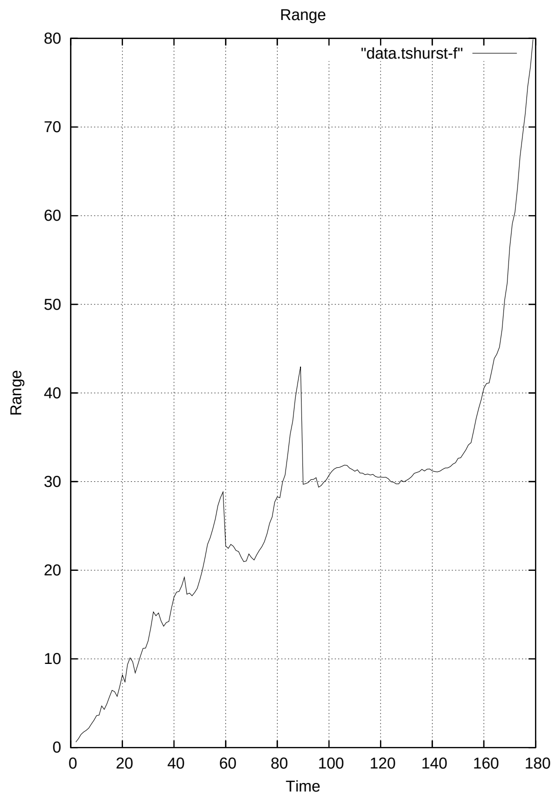

Figure C.100: United States Electronic Component Production, range of the time series data shown in Figure C.93.

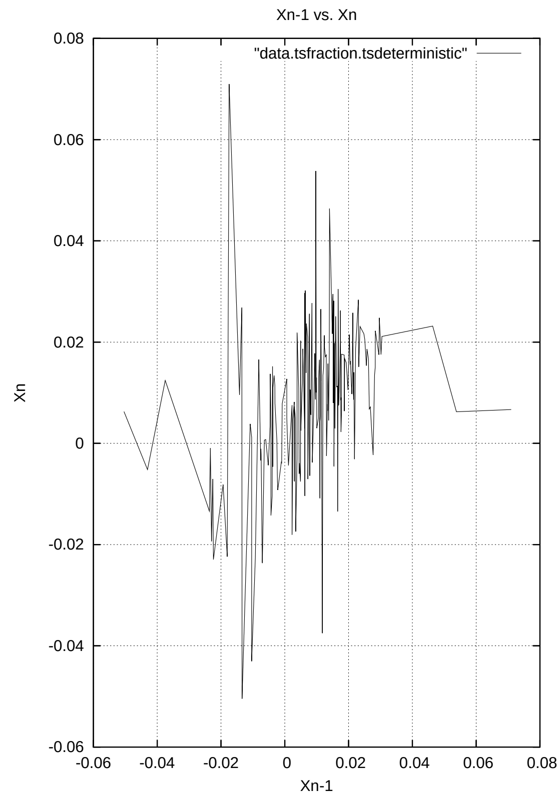

Figure C.101: United States Electronic Component Production, deterministic map of the normalized increments of the time series data shown in Figure C.94.

C.5.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.5.4. Figure C.104 is a graph of the logistic function estimates of the time series data for the United States Electronic Component Production. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[61]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.104 is a graph of the logistic function for the time series data presented in Figure C.93. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.94. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.94. Figure C.105 is the same graph, but with the time scale expanded by a factor of two.

[61]For example, in Figures C.104 and C.105, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.5.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

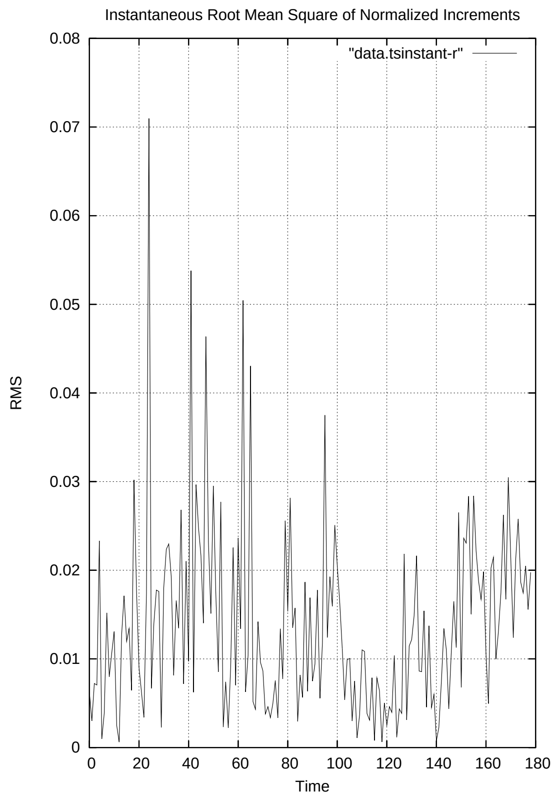

Figure C.102: United States Electronic Component Production, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.93.

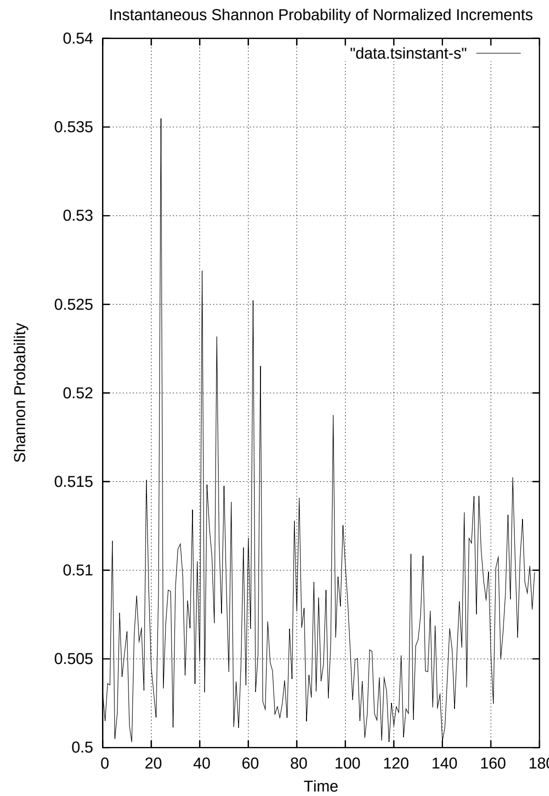

Figure C.103: United States Electronic Component Production, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.93.

C.5.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.5.5. Figure C.106 is a graph of the Hurst coefficient data time series data shown in Figure C.93. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.107 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.94. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.106 implies that the variance of the rate of revenue returns, (per month,) in the United States Electronic Component Production, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[62], t, $p(t) = erf(1/\sqrt{2t})$ which

[62]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning

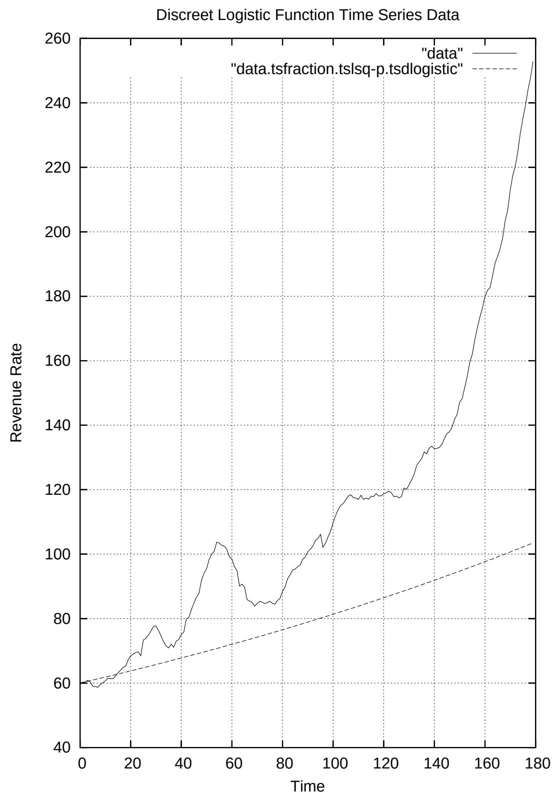

Figure C.104: United States Electronic Component Production, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.94 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

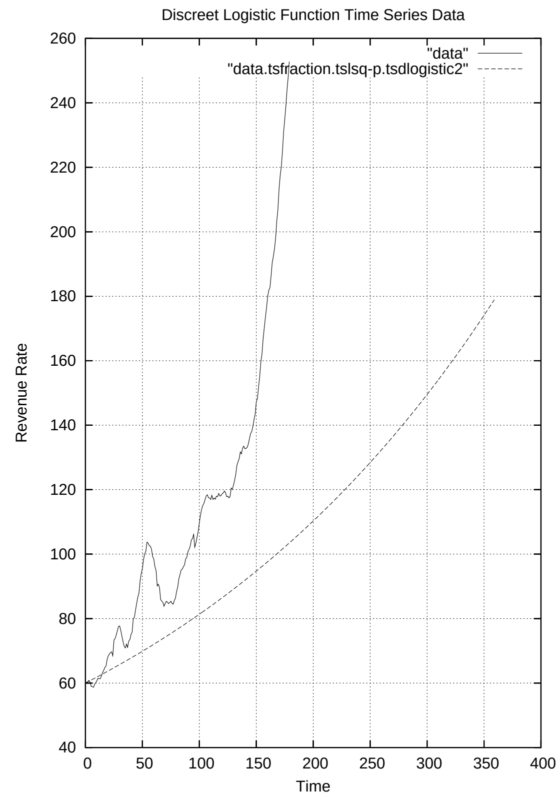

Figure C.105: United States Electronic Component Production, logistic function estimates of Figure C.104 with the time scale expanded by a factor of two.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.110, and, C.111 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Electronic Component Production for the near term and far term, respectively [Pet91, pp. 83-84][63]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.106, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient

that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[63]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

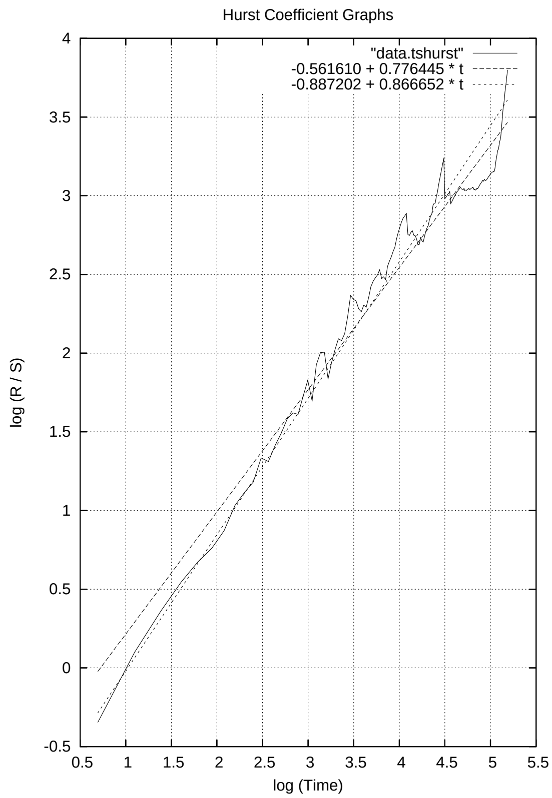

Figure C.106: United States Electronic Component Production, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.94. The slope of the graph is the Hurst coefficient.

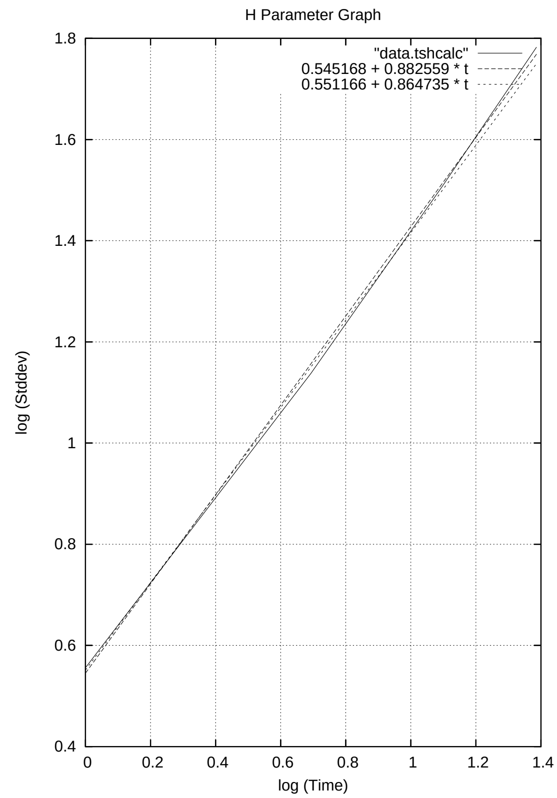

Figure C.107: United States Electronic Component Production, H parameter data for the normalized increments of the time series data shown in Figure C.94 The slope of the graph is the H parameter.

of 0.866652, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.193}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.866652} \tag{C.194}$$

$$\propto (t_2 - t_1)^{1.733304} \tag{C.195}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[64]. A Hurst coefficient of 0.866652, (for the near future, and 0.776445 for the distant future.) implies

[64] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 86.665200% [Pet91, pp. 66] for the near future, and 0.776445 for the distant future. Likewise, there is a 86.665200% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 86.665200% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Electronic Component Production are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.196}$$
$$= 0.866652^n \tag{C.197}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.94, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.008198 \cdot 100$ percent, on the average, with a standard deviation of $0.015216 \cdot 100$ percent, and a root mean square error value of $0.017247 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.198}$$
$$\propto (t_2 - t_1)^{0.866652} \tag{C.199}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.199 reduces to, [Sch91, pp. 129]:

$$R(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.200}$$
$$\propto \sqrt{(t_2 - t_1)} \tag{C.201}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.5.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Electronic Component Production. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.202}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[65].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \quad \propto \quad \frac{X\,(rt)}{r^H} \tag{C.203}$$

$$\propto \quad \frac{X\,(rt)}{r^{0.866652}} \tag{C.204}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.107, to provide a least squares approximation to the H parameter for the United States Electronic Component Production. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.864735 for the near future, and 0.882559 for the distant future.

Figures C.106 and C.107 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.94. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.94, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.108 and C.109 was made using the -d option.

C.5.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.5.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.95. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.94. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronic Component Production, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Electronic Component Production, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.94, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln{(0.008198 + 1)}}{\ln{(2)}} = 0.011779 \tag{C.205}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.94, and Equation 2.17 from Section 2.3.2 in Chapter 2:

[65] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

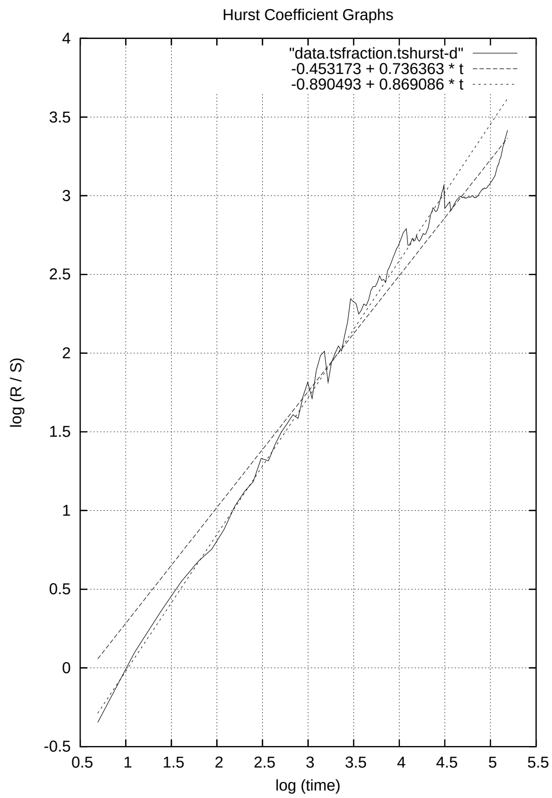

Figure C.108: United States Electronic Component Production, traditional Hurst coefficient data for the time series data shown in Figure C.93. The slope of the graph is the Hurst coefficient, and is 0.869086 for the near term, and 0.736363 for the far term.

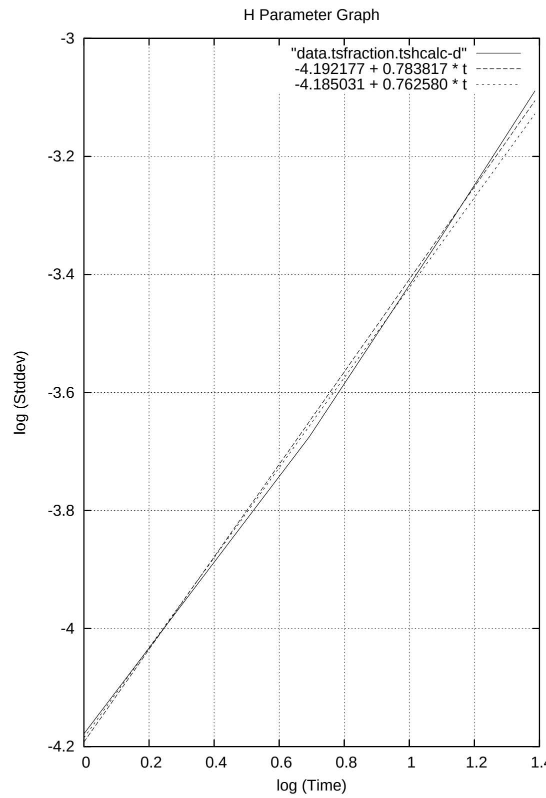

Figure C.109: United States Electronic Component Production, traditional H parameter data for the time series data shown in Figure C.93 The slope of the graph is the H parameter, and is 0.762580 for the near term, and 0.783817 for the far term.

$$bits = \frac{\ln(0.003048 + 1)}{\ln(2)} = 0.004391 \tag{C.206}$$

Note that if the mean is not constant in Figure C.94, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.93:

$$bits = 0.009588 \tag{C.207}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.93:

$$bits = 0.011551 \tag{C.208}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.5.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.011551t} \tag{C.209}$$

therefore:

$$C\left(p\right) = 0.011551 \tag{C.210}$$

and, *tsshannon* 0.011551 gives:

$$C\left(0.563187\right) = 0.011551 \tag{C.211}$$

therefore:

$$2^{C(0.563187)} = 2^{0.011551} \tag{C.212}$$

$$= 1.008039 \tag{C.213}$$

$$= 0.803868\% \tag{C.214}$$

and:

$$2p - 1 = (2 \cdot 0.563187) - 1 \tag{C.215}$$

$$= 0.126374 \tag{C.216}$$

$$= 12.637400\% \tag{C.217}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the United States Electronic Component Production executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 12.637400% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 87.362600% will be held in "reserve" with a 56.318700% chance of making twice the 12.637400% back, (and a 43.681300% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.803868%, or a doubling of its rate of revenue returns, (per month,) in 86.572591 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 12.637400% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.803868%, per month, on average.

Note that the metrics presented in this section are representative of the United States Electronic Component Production as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 12.637400% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Electronic Component Production's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.803868% per month.

As another simple example, a company re-invests 12.637400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 12.637400% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.803868% per month.

As an example of "product portfolio" management, suppose a company re-invests 12.637400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 12.637400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 12.637400$ percent for the second product, implying that the company should diversify its product line[66]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 12.637400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3:1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Electronic Component Production, as a standard bench mark, then the optimal number will be $\frac{1}{0.126374}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.94, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 12.637400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 12.637400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 12.637400 of the rate of revenue returns per month does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 12.637400% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.126374}$ products seems consistent with the industry, also.

[66]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

C.5.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Electronic Component Production, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Electronic Component Production time series is 0.008198, and 0.017247respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 27.560100.

If this value seems consistent number of companies in the United States Electronic Component Production, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Electronic Component Production are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.545271, which would be the value which should be used in Section C.5.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.5.5 is greater than the average Shannon probability for the companies participating in the United States Electronic Component Production, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.5.5. The maximum exploitability for the United States Electronic Component Production is derived in Section C.5.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Electronic Component Production is 0.545271, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.737665 in the United States Electronic Component Production. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.218}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Electronic Component Production would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.5.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.95. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.94. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronic Component Production, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.5.5, is derived from the United States Electronic Component Production metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.5.9.

An additional exploitable strategy may be time itself. Equations C.195, C.199, and, C.197, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Electronic Component Production, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.110, and, C.111 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Electronic Component Production for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Electronic Component Production. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[67]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.110, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.197, $0.866652^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.195, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.111, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[68]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with $\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.5.4, Equation C.197, and the preceeding section, approximately 3 times the value where $0.866652^n = 0.5$ could be interpreted as an approximation to the "average" product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.866652}$ many months seems consistent with the author's experience in the industry.

C.5.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.5.9. Figure C.112 represents a constructional simulation of the time series data presented in Figure C.93. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data.

[67]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[68]For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

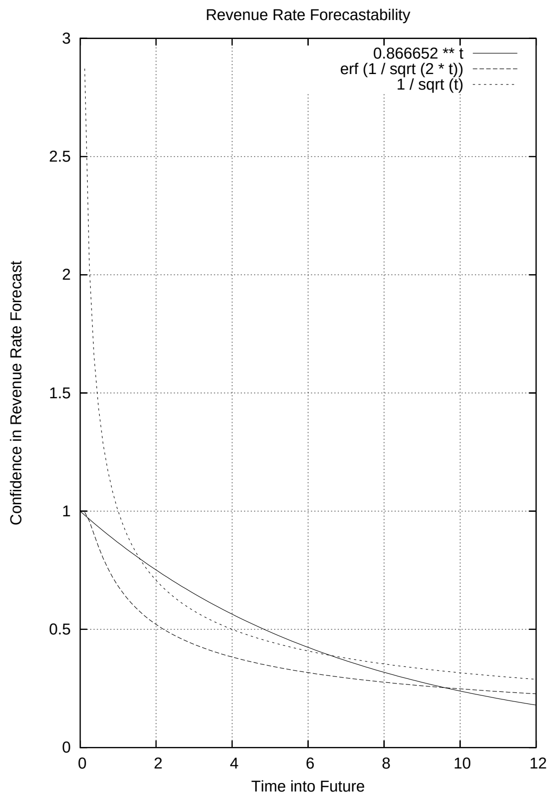

Figure C.110: United States Electronic Component Production, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.866652^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

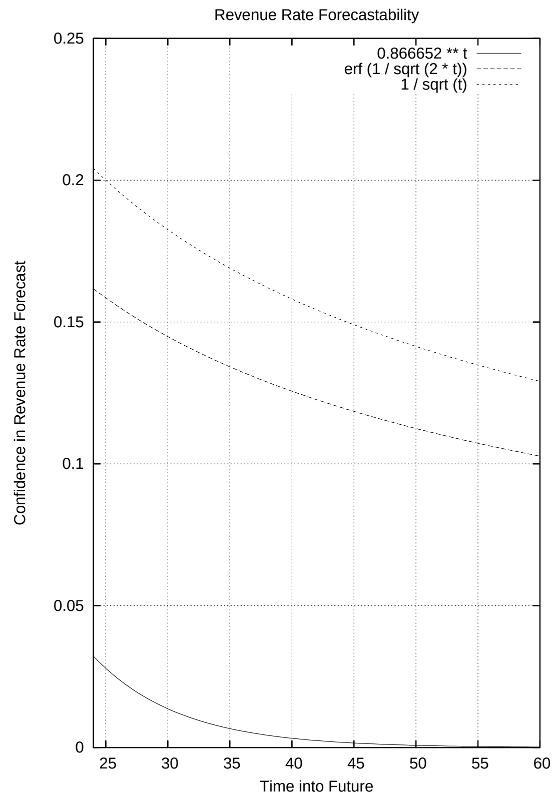

Figure C.111: United States Electronic Component Production, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.94. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.94 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.113 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.96.

C.5.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.5.3. One of the issues of analysis, as mentioned in Section C.5.7, is to determine the maximum Shannon probability for the time series presented in Figure C.93. Potentially,

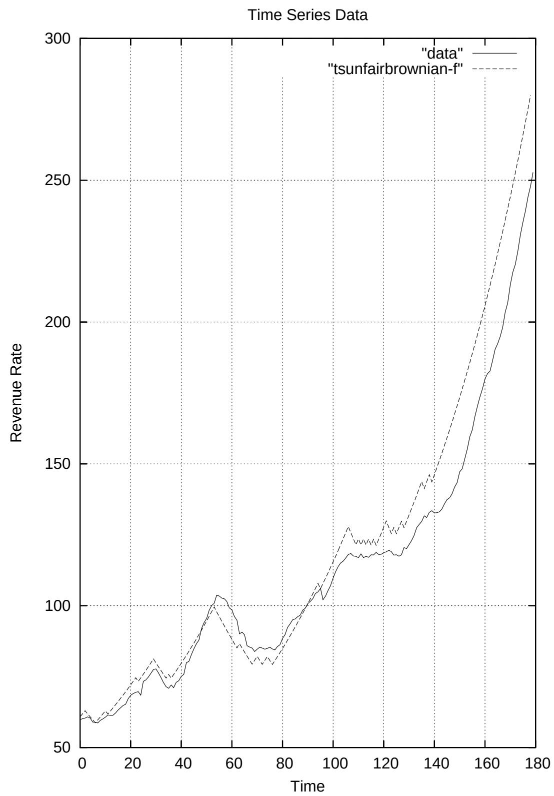

Figure C.112: United States Electronic Component Production, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.017247. This data is superimposed on the data presented in Figure C.93.

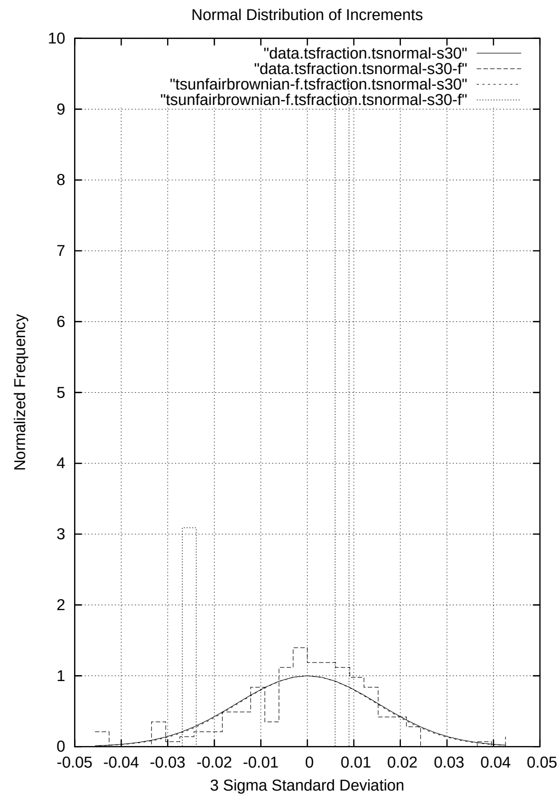

Figure C.113: United States Electronic Component Production, normalized histogram of the normalized increments of the time series data shown in Figure C.112, empirical and simulated. The empirical data has a mean of 0.008198, with a standard deviation of 0.015216. By comparison, the simulated data has a mean of 0.008720 with a standard deviation of 0.014922. This data is superimposed on the data presented in Figure C.96. The area under the four curves is identical.

this could be exploited with an aggressive fiscal strategy. Figure C.114 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.93. Figure C.115 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.93. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.563187, as derived in Section C.5.5 to 0.755556. This process, essentially, extracts the random statistical data from the time series presented in Figure C.93, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

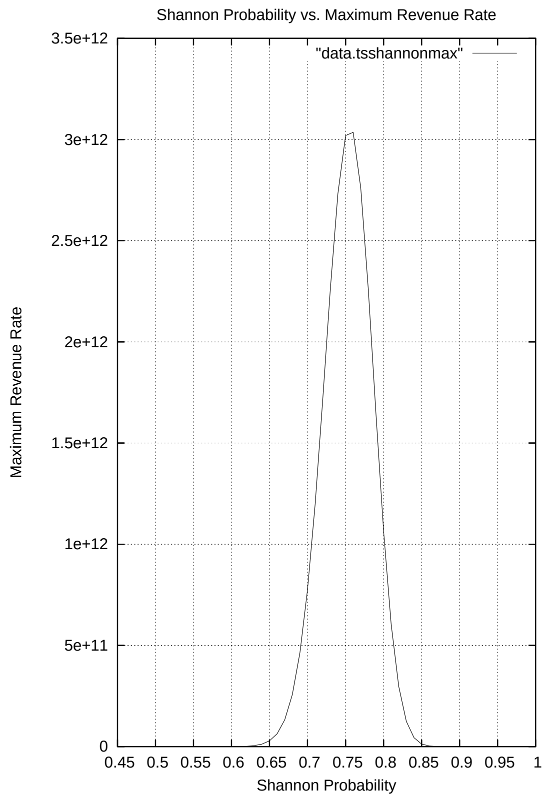

Figure C.114: United States Electronic Component Production, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.755556.

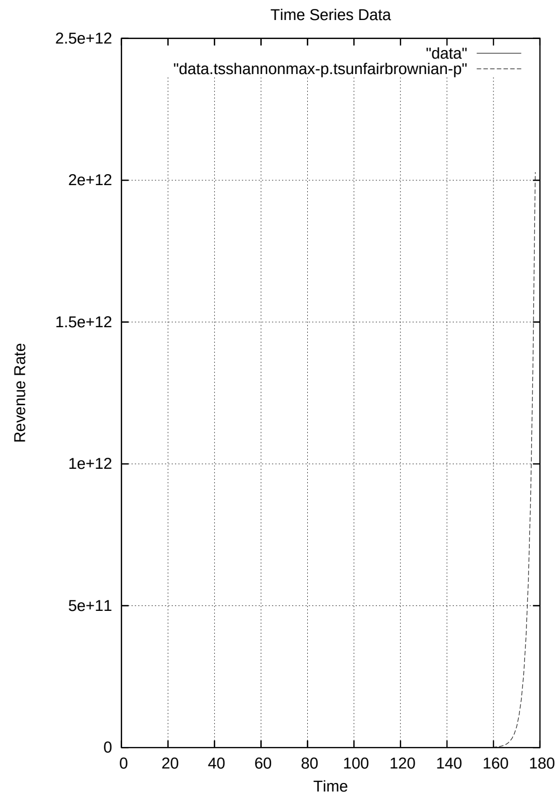

Figure C.115: United States Electronic Component Production, maximum rate of revenue returns, per month, at a Shannon probability, of 0.755556, corresponding to a "wager" fraction of 0.511112.

If it is assumed that the time series data set, presented in Figure C.93, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Electronic Component Production movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.754190, as compared with the predicted value from the program *tsshannonmax* of 0.755556.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.5.1, Figure C.96, it would appear that the United States Electronic Component Production's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.511112 of the rate of returns every month is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any month, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.219}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.94 of 0.017247, vs. an "theoretical optimal" value of 0.511112 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per month, which is unacceptably high. However, it is not clear why the United States Electronic Component Production is running at a value of 0.017247, which seems very conservative. However, a re-investment strategy of 0.017247 per month does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the United States Electronic Component Production is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx 0.5$, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.511112}{2.5}$, compared with an operational value, from Figure C.96 of 0.017247.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the United States Electronic Component Production are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.93. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.755556 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.563187 is not just marginally lower than the maximum Shannon probability of 0.755556. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.5.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.95. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.94. These values will be used in a fixed increment Brownian fractal analysis of the United States Electronic Component Production, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.5.6 and D.5.7. As a subjective evaluation of the "quality" of the analysis of the United States Electronic Component Production, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.93 from Figure C.94, and the Shannon probability as calculated by counting the total number of months that the United States Electronic Component Production movement was positive, as presented in Section C.5.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.220}$$

$$0.754190 \approx \frac{\frac{0.008198}{0.017247} + 1}{2} \tag{C.221}$$

$$0.754190 \approx 0.737665 \tag{C.222}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.754190 \approx 0.737665 \approx 0.755556 \tag{C.223}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.5.5, should be compared. The four methods used were the mean of Figure C.94, the constant in the least squares approximation to Figure C.94, the least squares exponential approximation to Figure C.93, and the logarithmic returns of Figure C.93, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.011779 \approx 0.004391 \approx 0.009588 \approx 0.011551 \tag{C.224}$$

It is implied in Section C.5.5, Subsection C.5.5 and in Section C.5.8 that, a Brownian motion with fixed increments fractal may "model" the United States Electronic Component Production. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P - 1) \approx \frac{\sigma\,(2P - 1)}{2\sqrt{P\,(1 - P)}} \tag{C.225}$$

$$0.017247\,(2 \cdot 0.754190 - 1) \approx \frac{0.015216\,(2 \cdot 0.754190 - 1)}{2\sqrt{0.754190\,(1 - 0.754190)}} \tag{C.226}$$

$$0.017247 \cdot 0.508380 \approx 0.015216 \cdot 0.590362 \tag{C.227}$$

$$0.008768 \approx 0.008983 \tag{C.228}$$

and, equating to the mean:

$$0.008198 \approx 0.008768 \approx 0.008983 \tag{C.229}$$

where, as in Equation C.222 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.93 from Figure C.94, and the Shannon probability as calculated by counting the total number of months that the United States Electronic Component Production movement was positive, as presented in Section C.5.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[69], where the absolute value is presented in Figure C.95, and the root mean square value is presented in Figure C.94:

$$0.013756 \approx 0.017247 \tag{C.230}$$

Note, that if the United States Electronic Component Production could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.93 from Figure C.94 should be zero. It is 0.010433.

C.6 United States Electronics Market

For the analysis, the data was in the directory ../markets/electronics[70].

The data in this section is presented in tabular form in Section D.6.

C.6.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.6.1. Figure C.116 is a graph of the time series data for the United States Electronics Market.

Figure C.117 is a graph of the normalized increments of the time series data presented in Figure C.116. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.118 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.117. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[71].

Figure C.119 is the normalized histogram of the normalized increments of the time series data shown in Figure C.117. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.119.

Figure C.120 is the statistical estimate for the data presented in Figure C.117, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.554410$, as derived in Section C.6.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set

[69]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[70]Data from the United States Department of Commerce, 1980—1994, by months, in millions of dollars, US.

[71]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

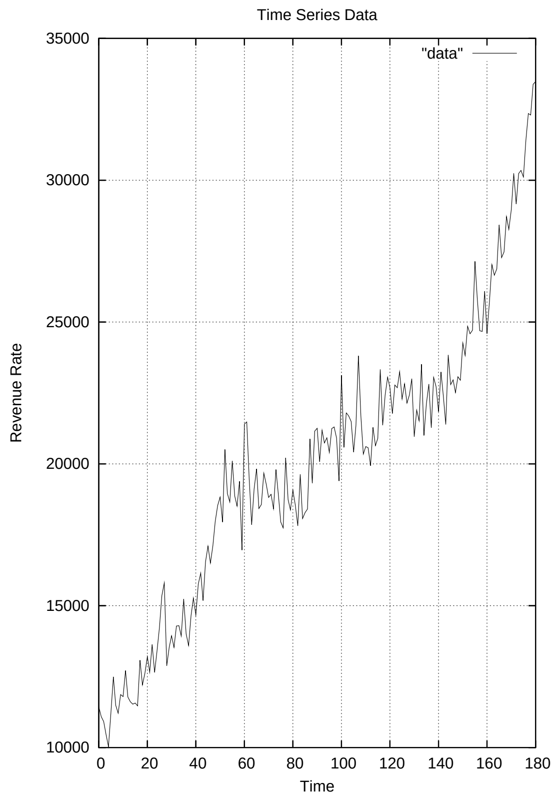

Figure C.116: United States Electronics Market, time series data.

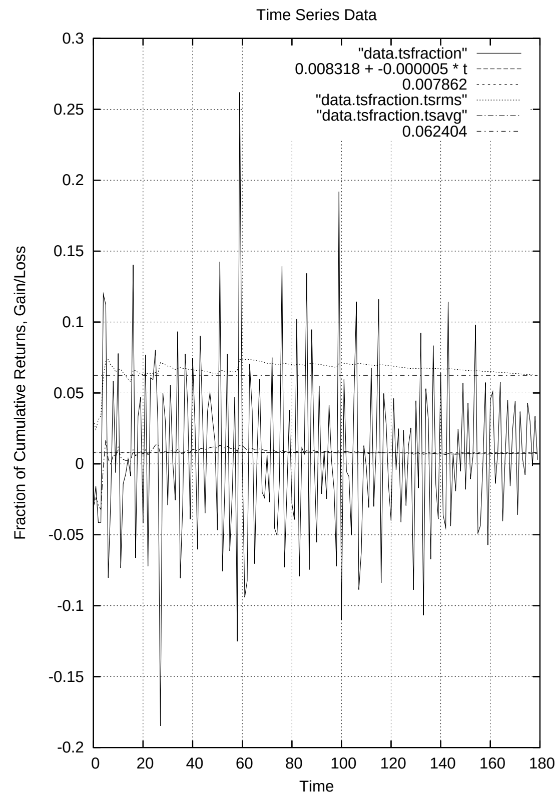

Figure C.117: United States Electronics Market, normalized increments of the time series data presented in Figure C.116. The mean is 0.007862 with a standard deviation of 0.062079. The formula for the least squares approximation is $0.008318 + -0.000005t$, and the root mean squared value is 0.062404. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000005, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.121 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.117. In principle, if the distribution of the normalized increments presented in Figure C.119

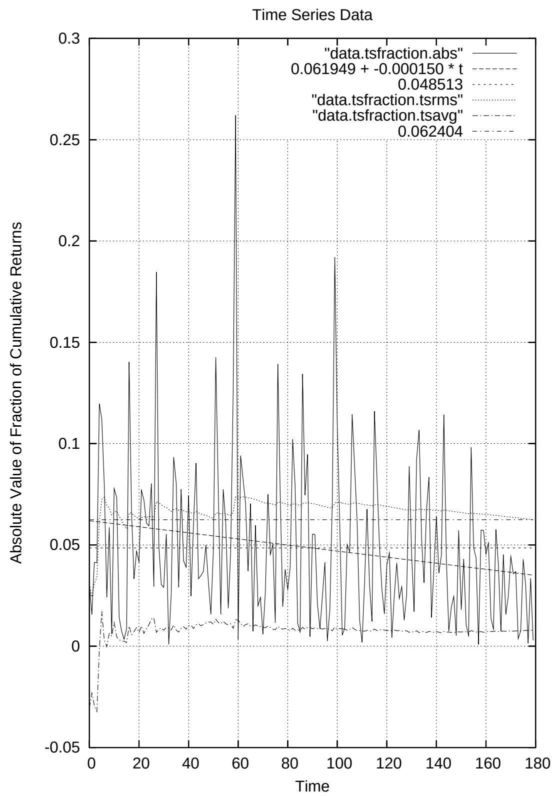

Figure C.118: United States Electronics Market, absolute value of the normalized increments of the time series data presented in Figure C.117. The mean is 0.048513 with a standard deviation of 0.039361. The formula for the least squares approximation is $0.061949 + -0.000150t$, and the root mean square value, from Figure C.117, is 0.062404. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.117, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

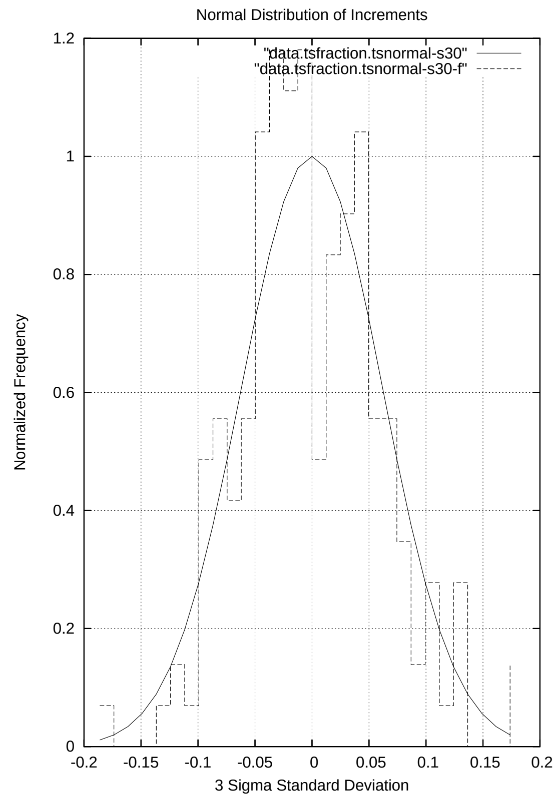

Figure C.119: United States Electronics Market, normalized histogram of the normalized increments of the time series data shown in Figure C.117. The data has a mean of 0.007862, with a standard deviation of 0.062079. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 2.817000, with a critical value of 42.557000.

is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.122 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.117. In principle, if the distribution of the normalized increments presented in Figure C.119 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

```
For  a mean  of 0.007819,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000782,   17235  samples  would  be  required.
     (With  181  samples,   the  estimated   error  is 0.007630   = 97.580043   percent.)
For  a standard   deviation   of 0.062404,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.006240,   136  samples  would  be  required.
     (With  181  samples,   the  estimated   error  is 0.005395   = 8.645159   percent.)
```

Figure C.120: United States Electronics Market, statistical estimates of the normalized increments of the time series shown in Figure C.117. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.117.

Figure C.123 is the range of values of the time series shown in Figure C.116. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.123 would be a square root function[72]. Figure C.124 is the deterministic map of the normalized increments of the time series data shown in Figure C.117. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.119 would seem to indicate that the time series data for the United States Electronics Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.6.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[73]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.125 is the instantaneous value of the root mean square of the normalized increments for the United States Electronics Market, and Figure C.126 is the instantaneous Shannon probability for the normalized increments.

C.6.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.6.4. Figure C.127 is a graph of the logistic function estimates of the time series data for the United States Electronics Market. The reader is cautioned that these

[72]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.123 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[73]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

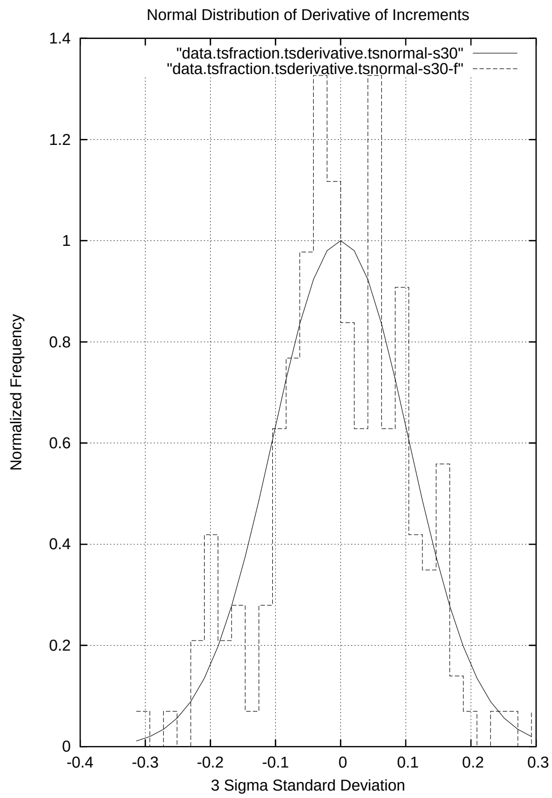

Figure C.121: United States Electronics Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.117.

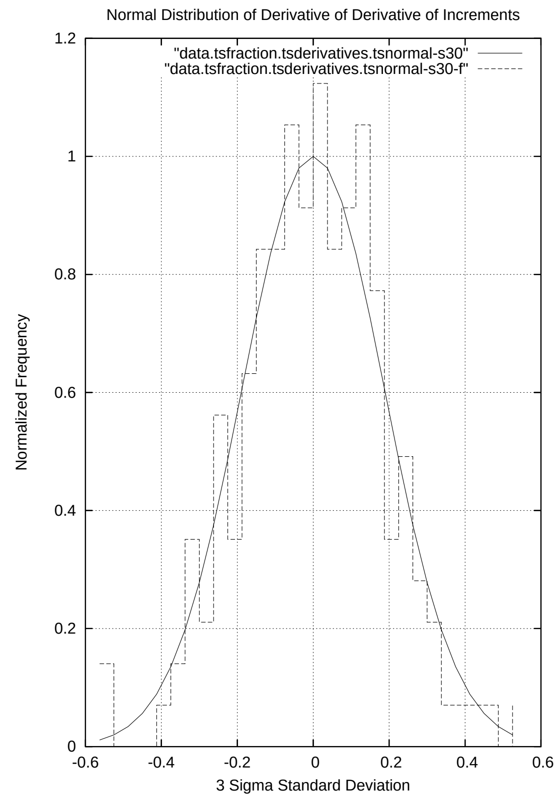

Figure C.122: United States Electronics Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.117.

graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[74]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.127 is a graph of the logistic function for the time series data presented in Figure C.116. The data presented was made by running the program *tsdlogistic,* which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.117. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.117. Figure C.128 is the same graph, but with the time scale expanded by a factor of two.

[74]For example, in Figures C.127 and C.128, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.6.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

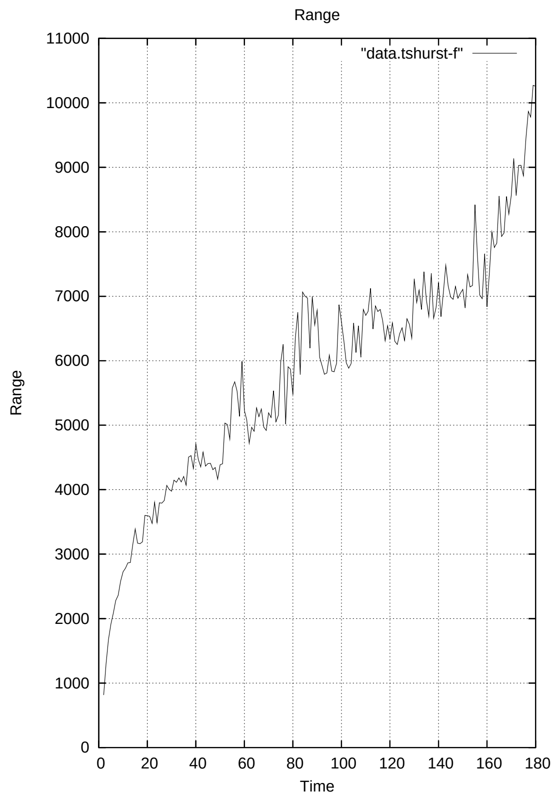

Figure C.123: United States Electronics Market, range of the time series data shown in Figure C.116.

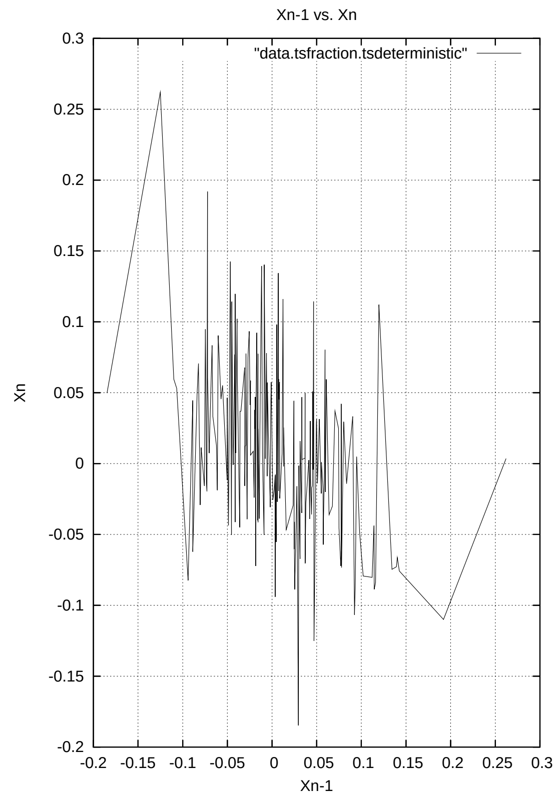

Figure C.124: United States Electronics Market, deterministic map of the normalized increments of the time series data shown in Figure C.117.

C.6.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.6.5. Figure C.129 is a graph of the Hurst coefficient data time series data shown in Figure C.116. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.130 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.117. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.129 implies that the variance of the rate of revenue returns, (per month,) in the United States Electronics Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[75], t, $p(t) = erf(1/\sqrt{2t})$ which is

[75]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp.

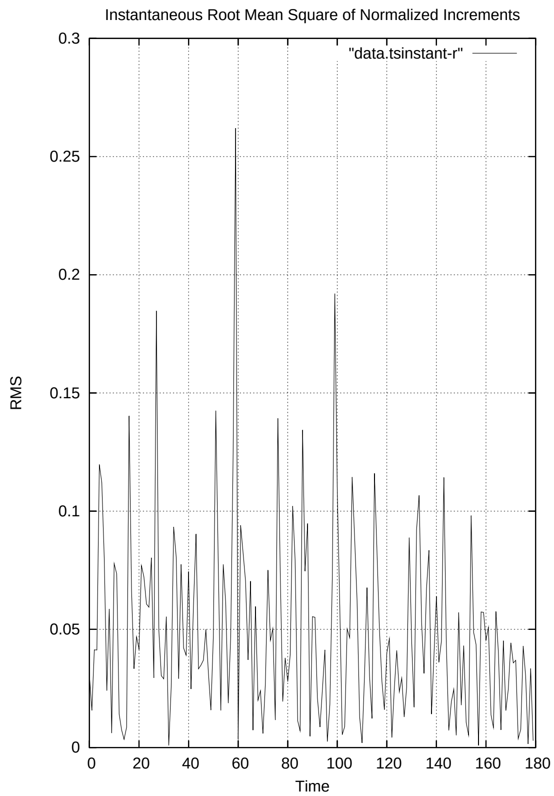

Figure C.125: United States Electronics Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.116.

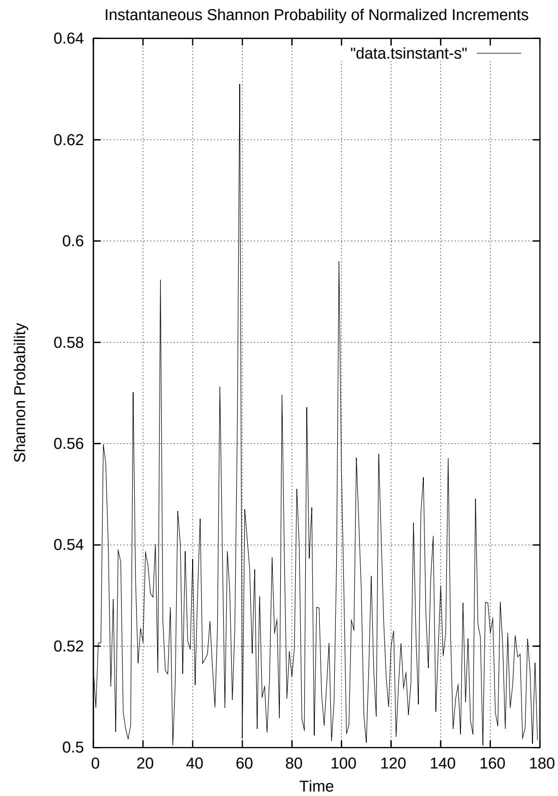

Figure C.126: United States Electronics Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.116.

approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.133, and, C.134 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Electronics Market for the near term and far term, respectively [Pet91, pp. 83-84][76]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.129, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.684410, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.231}$$

153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[76]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

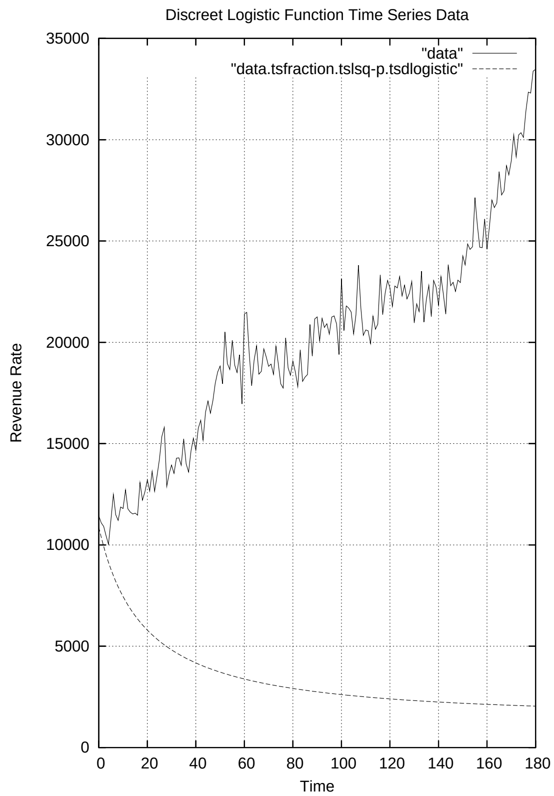

Figure C.127: United States Electronics Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.117 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

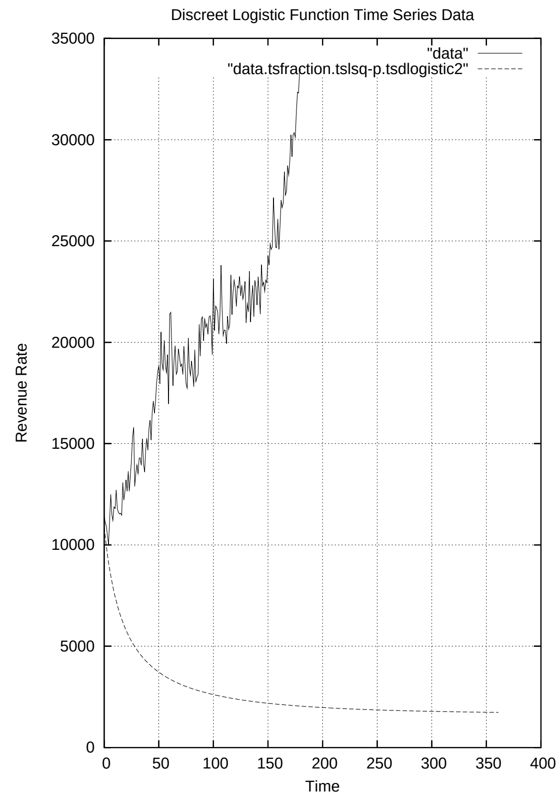

Figure C.128: United States Electronics Market, logistic function estimates of Figure C.127 with the time scale expanded by a factor of two.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.684410} \quad (C.232)$$

$$\propto (t_2 - t_1)^{1.368820} \quad (C.233)$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[77]. A Hurst coefficient of 0.684410, (for the near future, and 0.399911 for the distant future.) implies

[77] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.6.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Electronics Market. See also [Pet91,

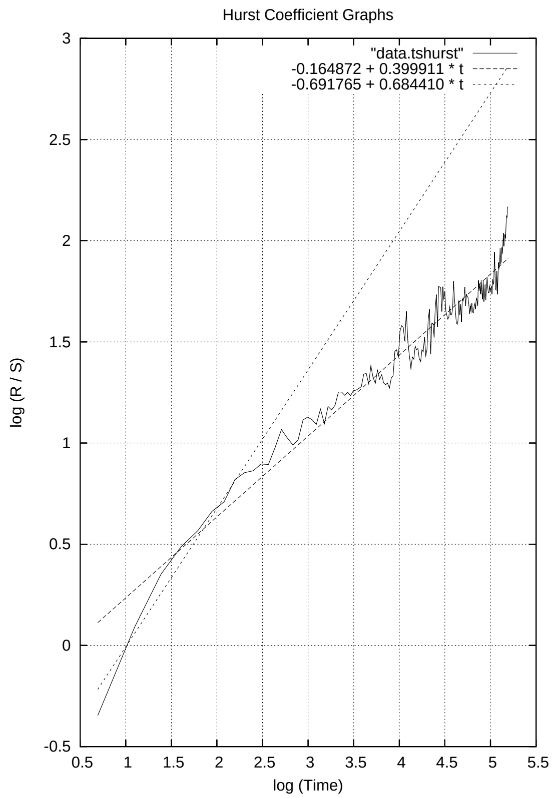

Figure C.129: United States Electronics Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.117. The slope of the graph is the Hurst coefficient.

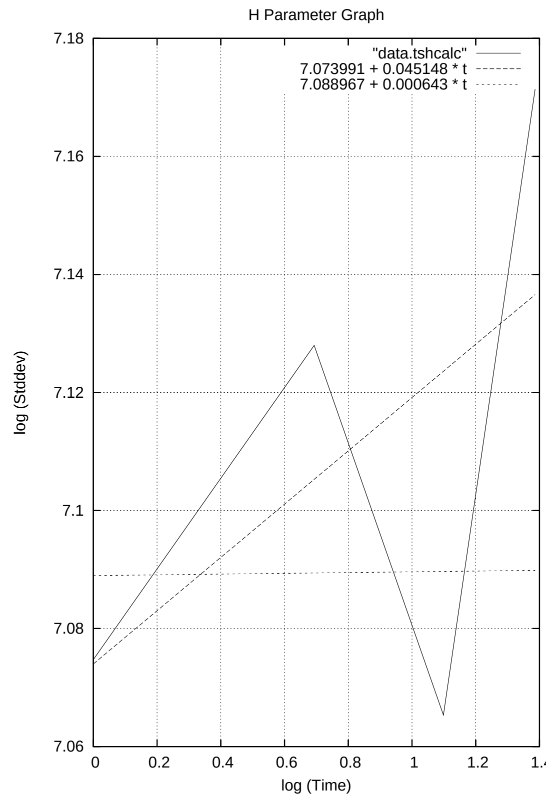

Figure C.130: United States Electronics Market, H parameter data for the normalized increments of the time series data shown in Figure C.117 The slope of the graph is the H parameter.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 68.441000% [Pet91, pp. 66] for the near future, and 0.399911 for the distant future. Likewise, there is a 68.441000% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 68.441000% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Electronics Market are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.234}$$
$$= 0.684410^n \tag{C.235}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.117, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would

pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.007862 \cdot 100$ percent, on the average, with a standard deviation of $0.062079 \cdot 100$ percent, and a root mean square error value of $0.062404 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^H \tag{C.236}$$

$$\propto \left(t_2 - t_1\right)^{0.684410} \tag{C.237}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.237 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.238}$$

$$\propto \sqrt{\left(t_2 - t_1\right)} \tag{C.239}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.240}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[78].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \propto \frac{X\left(rt\right)}{r^H} \tag{C.241}$$

$$\propto \frac{X\left(rt\right)}{r^{0.684410}} \tag{C.242}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.130, to provide a least squares approximation to the H parameter for the United States Electronics Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.000643 for the near future, and 0.045148 for the distant future.

Figures C.129 and C.130 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.117. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.117, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.131 and C.132 was made using the -d option.

[78] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

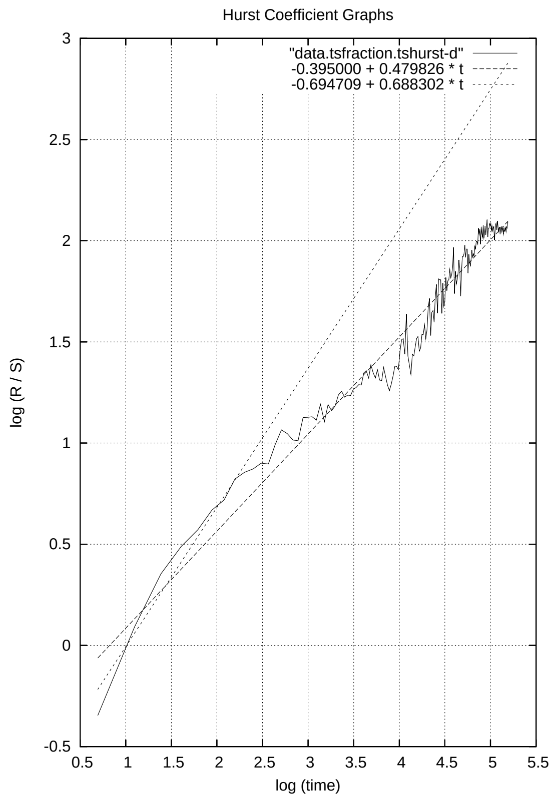

Figure C.131: United States Electronics Market, traditional Hurst coefficient data for the time series data shown in Figure C.116. The slope of the graph is the Hurst coefficient, and is 0.688302 for the near term, and 0.479826 for the far term.

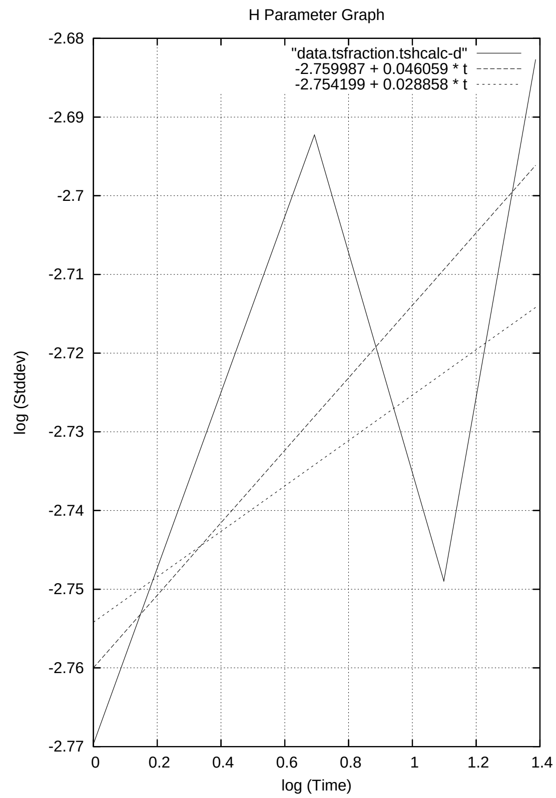

Figure C.132: United States Electronics Market, traditional H parameter data for the time series data shown in Figure C.116 The slope of the graph is the H parameter, and is 0.028858 for the near term, and 0.046059 for the far term.

Observations on the Hurst Coefficient Analysis

Note that the H parameter data is not linear, and both the short term and long term predictability are better than the mid term predictability. This is also indicated by a Hurst coefficient of 0.399911, which is less than 0.5, and would tend to indicate that there is a predisposition to antipersistence, or ergodic, market behavior. What this means is that the system is mean reverting, and that a month where rate of revenue returns increased, will have a predisposition to be followed by a month where the rate of revenue returns decrease, and vice versa. See [Pet91, pp. 64], [Fed88, pp. 170], [PJS92, pp. 496] [Sch91, pp. 130], [Ç93, pp. 172].

C.6.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.6.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.118. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.117. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronics Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Electronics Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.117, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.007862 + 1)}{\ln(2)} = 0.011298 \tag{C.243}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.117, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.008318 + 1)}{\ln(2)} = 0.011951 \tag{C.244}$$

Note that if the mean is not constant in Figure C.117, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.116:

$$bits = 0.007056 \tag{C.245}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.116:

$$bits = 0.008559 \tag{C.246}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.6.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.008559t} \tag{C.247}$$

therefore:

$$C(p) = 0.008559 \tag{C.248}$$

and, *tsshannon* 0.008559 gives:

$$C(0.554410) = 0.008559 \tag{C.249}$$

therefore:

$$2^{C(0.554410)} = 2^{0.008559} \tag{C.250}$$
$$= 1.005950 \tag{C.251}$$
$$= 0.595028\% \tag{C.252}$$

and:

$$
\begin{aligned}
2p - 1 &= (2 \cdot 0.554410) - 1 && \text{(C.253)} \\
&= 0.108820 && \text{(C.254)} \\
&= 10.882000\% && \text{(C.255)}
\end{aligned}
$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the United States Electronics Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 10.882000% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 89.118000% will be held in "reserve" with a 55.441000% chance of making twice the 10.882000% back, (and a 44.559000% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.595028%, or a doubling of its rate of revenue returns, (per month,) in 116.836079 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 10.882000% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.595028%, per month, on average.

Note that the metrics presented in this section are representative of the United States Electronics Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 10.882000% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Electronics Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.595028% per month.

As another simple example, a company re-invests 10.882000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 10.882000% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.595028% per month.

As an example of "product portfolio" management, suppose a company re-invests 10.882000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 10.882000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 10.882000$ percent for the second product, implying that the company should diversify its product line[79]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated

[79]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 10.882000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Electronics Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.108820}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.117, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 10.882000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 10.882000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 10.882000 of the rate of revenue returns per month does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 10.882000% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.108820}$ products seems consistent with the industry, also.

C.6.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Electronics Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Electronics Market time series is 0.007862, and 0.062404respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 2.018869.

If this value seems consistent number of companies in the United States Electronics Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Electronics Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.544334, which would be the value which should be used in Section C.6.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.6.5 is greater than the average Shannon probability for the companies participating in the United States Electronics Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.6.5. The maximum exploitability for the United States Electronics Market is derived in Section C.6.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Electronics Market is 0.544334, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.562993 in the United States Electronics Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.256}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Electronics Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.6.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.118. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.117. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Electronics Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.6.5, is derived from the United States Electronics Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.6.9.

An additional exploitable strategy may be time itself. Equations C.233, C.237, and, C.235, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Electronics Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.133, and, C.134 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Electronics Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Electronics Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[80]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.133, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.235, $0.684410^n = 0.5$ months of operations. Since the optimal amount of

[80] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

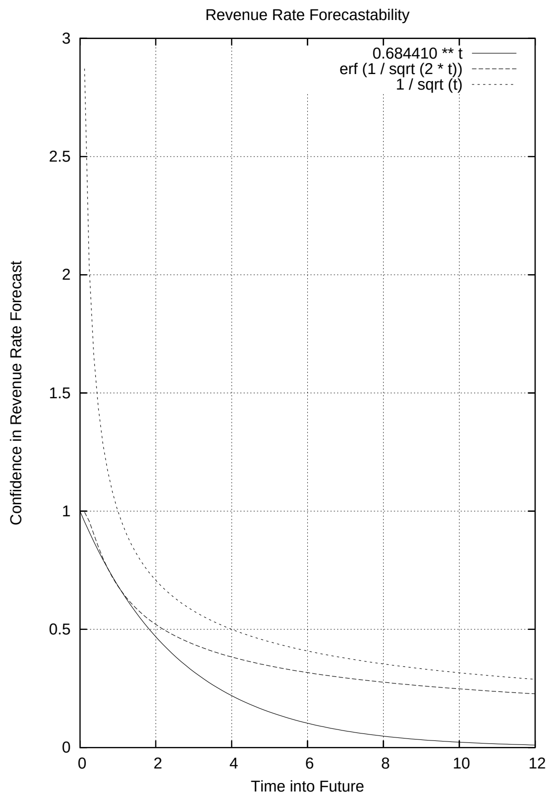

Figure C.133: United States Electronics Market, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.684410^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

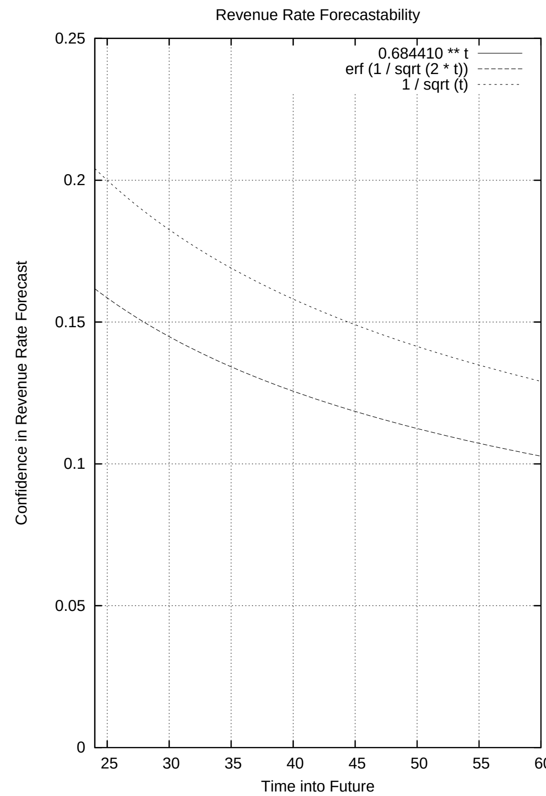

Figure C.134: United States Electronics Market, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

inventory and, from Equation C.233, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.134, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[81]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with

[81] For example, see “IEEE Engineering Management Review,” Volume 23 Number 3, Fall 1995, pp. 83

$\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.6.4, Equation C.235, and the preceeding section, approximately 3 times the value where $0.684410^n = 0.5$ could be interpreted as an approximation to the “average” product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.684410}$ many months seems consistent with the author’s experience in the industry.

C.6.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.6.9. Figure C.135 represents a constructional simulation of the time series data presented in Figure C.116. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.117. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.117 represent a relatively complex process, that may not be “modeled” with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.136 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.119.

C.6.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.6.3. One of the issues of analysis, as mentioned in Section C.6.7, is to determine the maximum Shannon probability for the time series presented in Figure C.116. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.137 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.116. Figure C.138 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.116. This represents a “what if” the investment strategy was changed from a Shannon probability of 0.554410, as derived in Section C.6.5 to 0.524862. This process, essentially, extracts the random statistical data from the time series presented in Figure C.116, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be “modeled” with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.116, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Electronics Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.522222, as compared with the predicted value from the program *tsshannonmax* of 0.524862.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.6.1, Figure C.119, it would appear that the United States Electronics Market’s normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to “wager” a fraction of 0.049724 of the rate of returns every month is overly

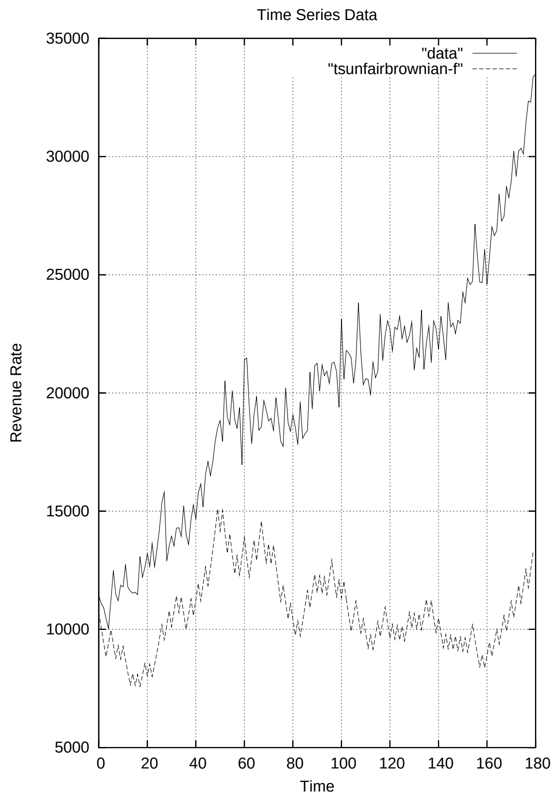

Figure C.135: United States Electronics Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.062404. This data is superimposed on the data presented in Figure C.116.

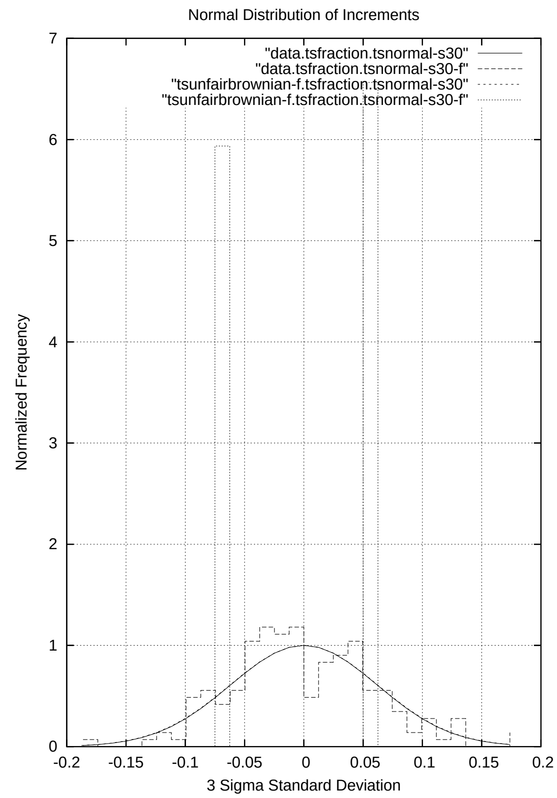

Figure C.136: United States Electronics Market, normalized histogram of the normalized increments of the time series data shown in Figure C.135, empirical and simulated. The empirical data has a mean of 0.007862, with a standard deviation of 0.062079. By comparison, the simulated data has a mean of 0.003138 with a standard deviation of 0.062500. This data is superimposed on the data presented in Figure C.119. The area under the four curves is identical.

aggressive, since in the classical Brownian scenario, the maximum loss, in any month, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.257}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.117 of 0.062404, vs. an "theoretical optimal" value of 0.049724 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per month, which is unacceptably high. However, it is not clear why the United States Electronics

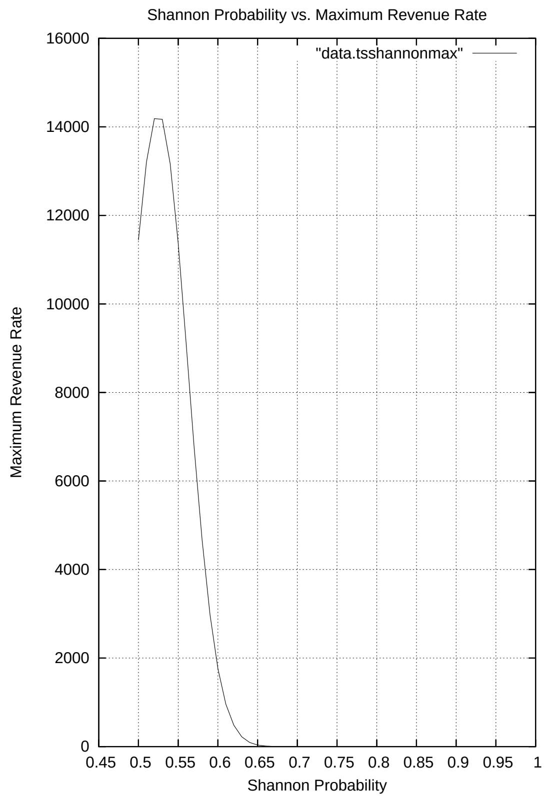

Figure C.137: United States Electronics Market, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.524862.

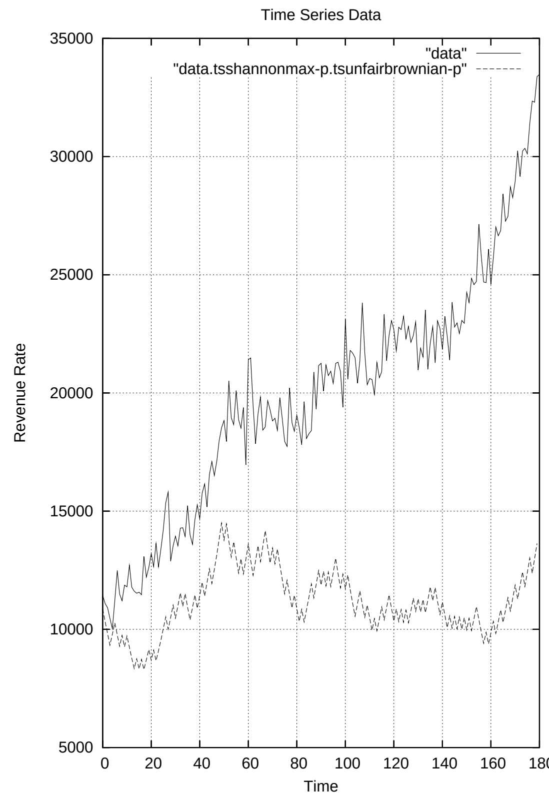

Figure C.138: United States Electronics Market, maximum rate of revenue returns, per month, at a Shannon probability, of 0.524862, corresponding to a "wager" fraction of 0.049724.

Market is running at a value of 0.062404, which seems very conservative. However, a re-investment strategy of 0.062404 per month does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the United States Electronics Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx 0.5$, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.049724}{2.5}$, compared with an operational value, from Figure C.119 of 0.062404.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the United States Electronics Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.116. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.524862 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.554410 is not just marginally lower than the maximum Shannon probability of 0.524862. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.6.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.118. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.117. These values will be used in a fixed increment Brownian fractal analysis of the United States Electronics Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.6.6 and D.6.7. As a subjective evaluation of the "quality" of the analysis of the United States Electronics Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.116 from Figure C.117, and the Shannon probability as calculated by counting the total number of months that the United States Electronics Market movement was positive, as presented in Section C.6.9:

$$P \approx \frac{\frac{avg}{rms}+1}{2} \tag{C.258}$$

$$0.522222 \approx \frac{\frac{0.007862}{0.062404}+1}{2} \tag{C.259}$$

$$0.522222 \approx 0.562993 \tag{C.260}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.522222 \approx 0.562993 \approx 0.524862 \tag{C.261}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.6.5, should be compared. The four methods used were the mean of Figure C.117, the constant in the least squares approximation to Figure C.117, the least squares exponential approximation to Figure C.116, and the logarithmic returns of Figure C.116, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.011298 \approx 0.011951 \approx 0.007056 \approx 0.008559 \tag{C.262}$$

It is implied in Section C.6.5, Subsection C.6.5 and in Section C.6.8 that, a Brownian motion with fixed increments fractal may "model" the United States Electronics Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$\begin{aligned} rms\,(2P-1) &\approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} && \text{(C.263)} \\ 0.062404\,(2 \cdot 0.522222 - 1) &\approx \frac{0.062079\,(2 \cdot 0.522222 - 1)}{2\sqrt{0.522222\,(1 - 0.522222)}} && \text{(C.264)} \\ 0.062404 \cdot 0.044444 &\approx 0.062079 \cdot 0.044488 && \text{(C.265)} \\ 0.002774 &\approx 0.002762 && \text{(C.266)} \end{aligned}$$

and, equating to the mean:

$$0.007862 \approx 0.002774 \approx 0.002762 \tag{C.267}$$

where, as in Equation C.260 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.116 from Figure C.117, and the Shannon probability as calculated by counting the total number of months that the United States Electronics Market movement was positive, as presented in Section C.6.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[82], where the absolute value is presented in Figure C.118, and the root mean square value is presented in Figure C.117:

$$0.048513 \approx 0.062404 \tag{C.268}$$

Note, that if the United States Electronics Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.116 from Figure C.117 should be zero. It is 0.039361.

Observations on the Qualitative Verification of Fixed Increment Approximation Analysis

In the equation:

$$0.007862 \approx 0.002774 \approx 0.002762 \tag{C.269}$$

Note that the mean is unusually large, in relation to values for $rms(2P-1)$ and $\frac{\sigma(2P-1)}{2\sqrt{P(1-P)}}$, respectively-in principle, they should all be equal. Also note that the standard deviation of the increments, 0.062079, is much larger than the mean. These issues, coupled with a Hurst coefficient of 0.399911, probably prohibit "modeling" the United States Electronics Market with the methodologies presented in this manuscript. Note, however, the poor accuracy performance of the methodology was estimated by the equation, above, so was anticipated. As an aside, it is not

[82] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

clear what market mechanisms create the numbers in the equation above, or a Hurst coefficient that is indicative of an antipersistent rate of revenue returns. Investigation of the metric methodologies for this market place, perhaps, may be interesting and provide some insight.

C.7 United States Office Computer Market

For the analysis, the data was in the directory ../markets/computer.office[83].

The data in this section is presented in tabular form in Section D.7.

C.7.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.7.1. Figure C.139 is a graph of the time series data for the United States Office Computer Market.

Figure C.140 is a graph of the normalized increments of the time series data presented in Figure C.139. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.141 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.140. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[84].

Figure C.142 is the normalized histogram of the normalized increments of the time series data shown in Figure C.140. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.142.

Figure C.143 is the statistical estimate for the data presented in Figure C.140, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.589554$, as derived in Section C.7.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.144 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.140. In principle, if the distribution of the normalized increments presented in Figure C.142 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.145 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.140. In principle, if the distribution of the normalized increments presented in Figure C.142 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

[83]Data from the United States Department of Commerce, 1982—1994, by months, as an index, 1987 = 100.

[84]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

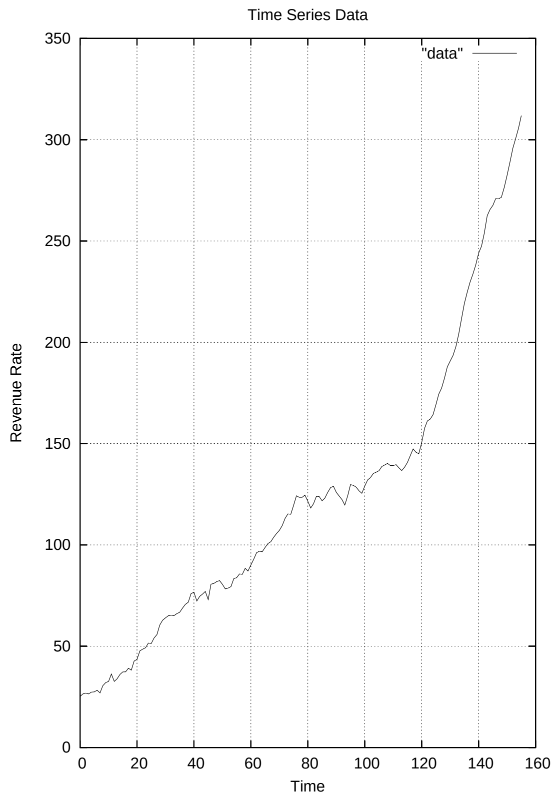

Figure C.139: United States Office Computer Market, time series data.

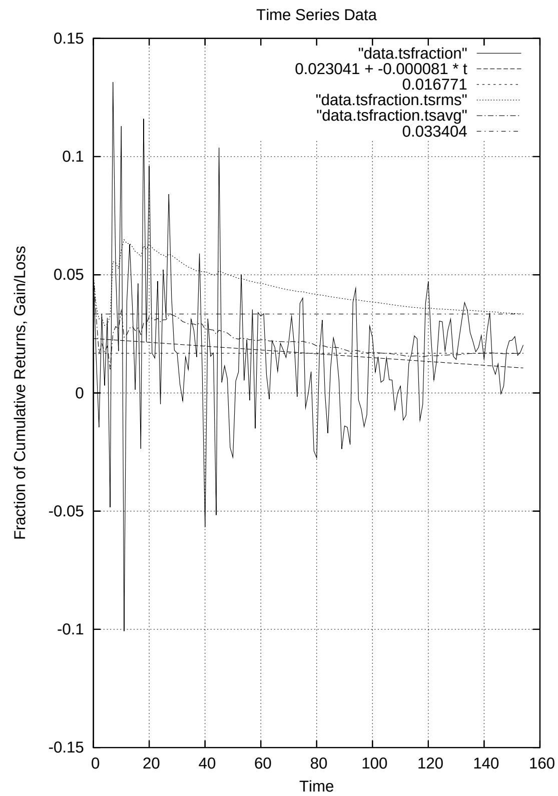

Figure C.140: United States Office Computer Market, normalized increments of the time series data presented in Figure C.139. The mean is 0.016771 with a standard deviation of 0.028983. The formula for the least squares approximation is $0.023041 + -0.000081t$, and the root mean squared value is 0.033404. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000081, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.146 is the range of values of the time series shown in Figure C.139. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.146 would be a square root function[85]. Figure C.147 is the deterministic map of the normalized increments of the time

[85]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.146 are a computational artifact—caused by not using the -m

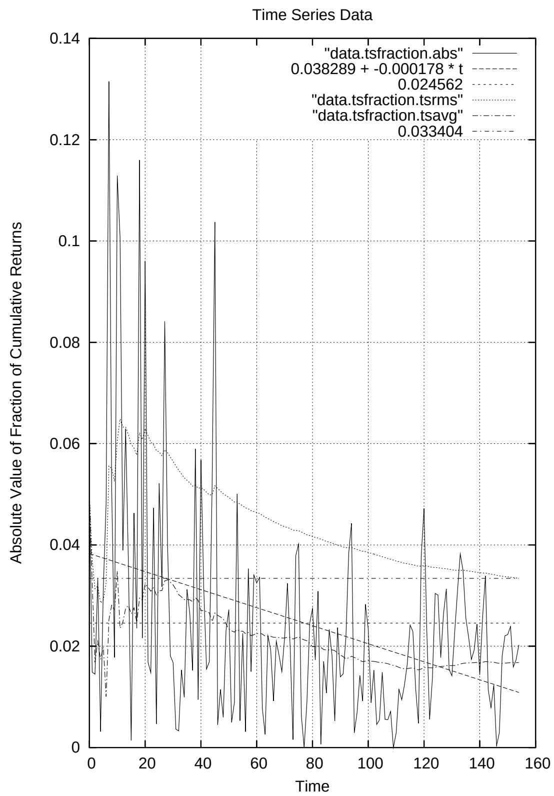

Figure C.141: United States Office Computer Market, absolute value of the normalized increments of the time series data presented in Figure C.140. The mean is 0.024562 with a standard deviation of 0.022713. The formula for the least squares approximation is $0.038289 + -0.000178t$, and the root mean square value, from Figure C.140, is 0.033404. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.140, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

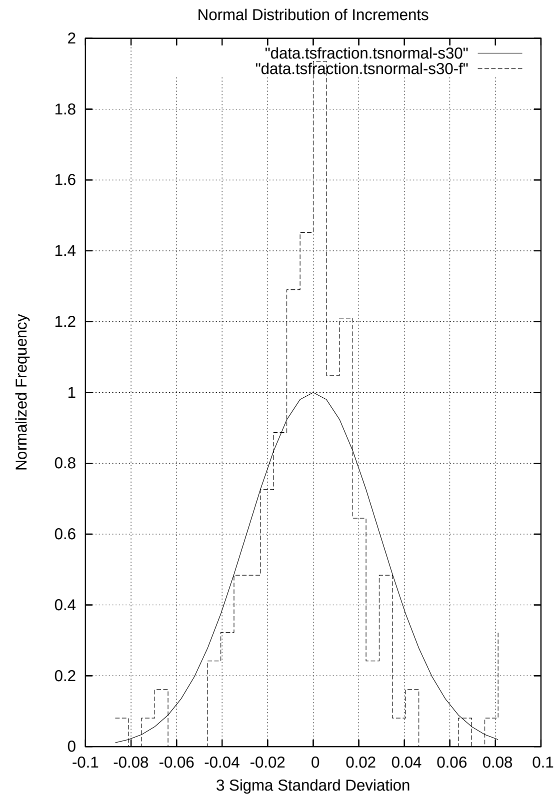

Figure C.142: United States Office Computer Market, normalized histogram of the normalized increments of the time series data shown in Figure C.140. The data has a mean of 0.016771, with a standard deviation of 0.028983. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 8.722000, with a critical value of 42.557000.

series data shown in Figure C.140. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

option to the program *tshurst*, which is computationally inefficient.

```
For a mean of 0.016663, with a confidence level of 0.900000
    that the error did not exceed 0.001666, 1088 samples would be required.
    (With 156 samples, the estimated error is 0.004399 = 26.400221 percent.)
For a standard deviation of 0.033404, with a confidence level of 0.900000
    that the error did not exceed 0.003340, 136 samples would be required.
    (With 156 samples, the estimated error is 0.003111 = 9.312150 percent.)
```

Figure C.143: United States Office Computer Market, statistical estimates of the normalized increments of the time series shown in Figure C.140. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.140.

Observations on the Time Series Increments Analysis

Figure C.142 would seem to indicate that the time series data for the United States Office Computer Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.7.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[86]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.148 is the instantaneous value of the root mean square of the normalized increments for the United States Office Computer Market, and Figure C.149 is the instantaneous Shannon probability for the normalized increments.

C.7.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.7.4. Figure C.150 is a graph of the logistic function estimates of the time series data for the United States Office Computer Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[87]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.150 is a graph of the logistic function for the time series data presented in Figure C.139. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters

[86]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[87]For example, in Figures C.150 and C.151, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.7.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

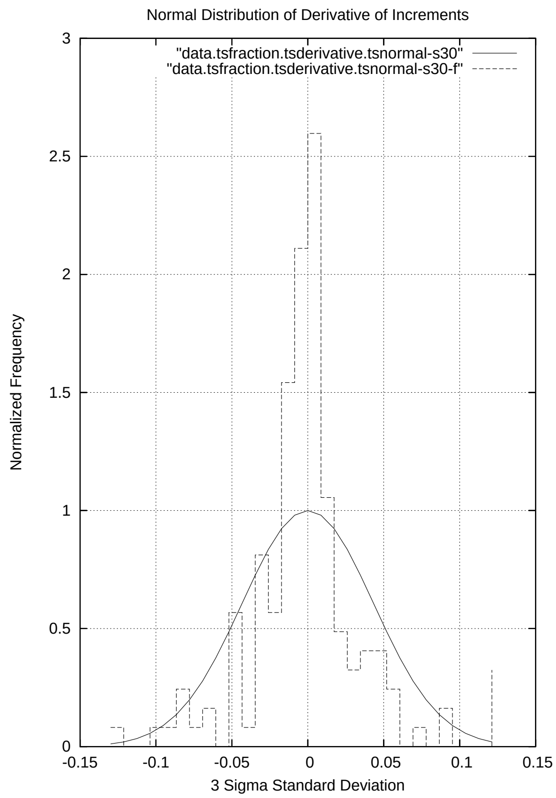

Figure C.144: United States Office Computer Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.140.

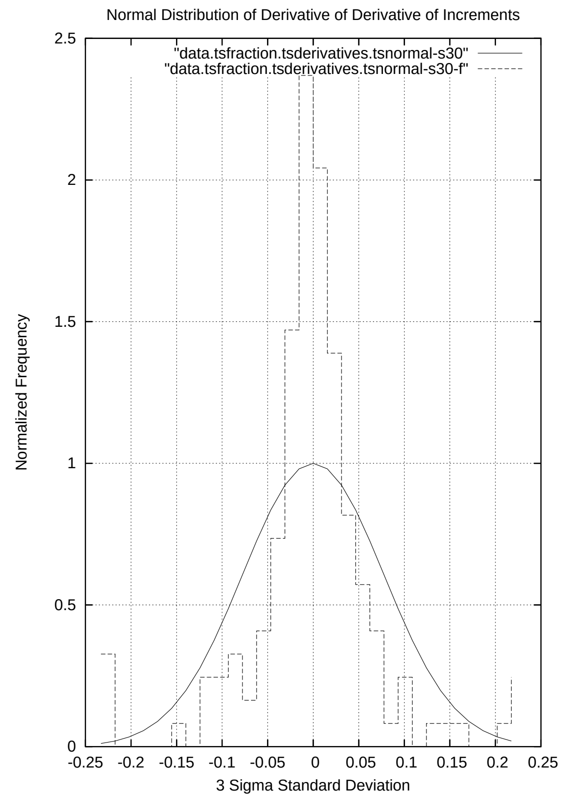

Figure C.145: United States Office Computer Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.140.

extracted from the time series data as suggested in Figure C.140. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.140. Figure C.151 is the same graph, but with the time scale expanded by a factor of two.

C.7.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.7.5. Figure C.152 is a graph of the Hurst coefficient data time series data shown in Figure C.139. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.153 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.140. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.152 implies that the variance of the rate of revenue returns, (per month,) in the United States Office Computer Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a

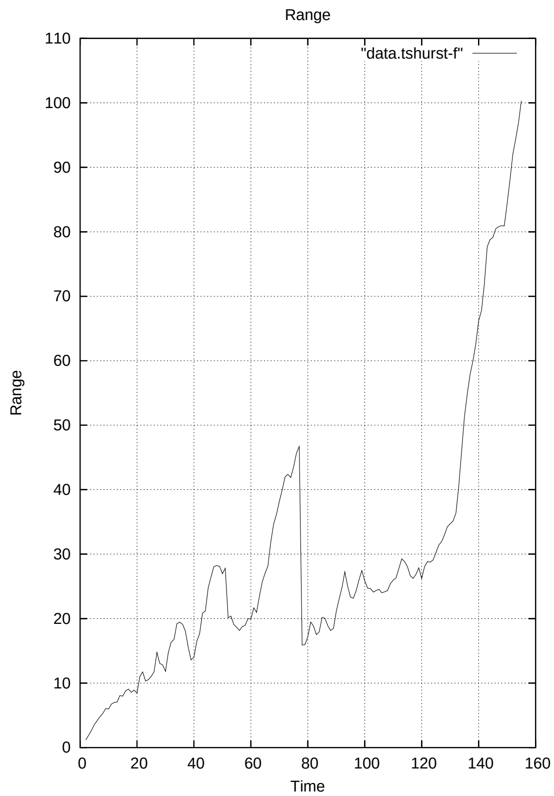

Figure C.146: United States Office Computer Market, range of the time series data shown in Figure C.139.

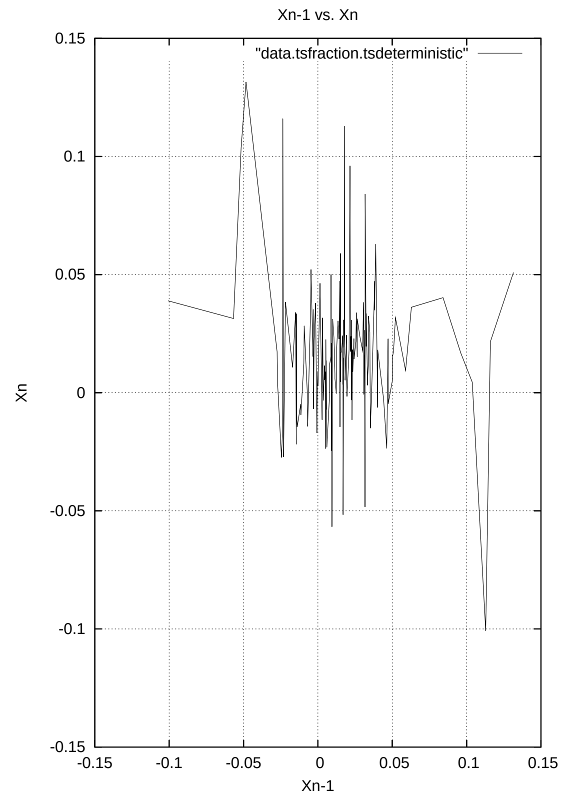

Figure C.147: United States Office Computer Market, deterministic map of the normalized increments of the time series data shown in Figure C.140.

period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[88], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.156, and, C.157 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Office Computer Market for the near term and far term, respectively [Pet91, pp. 83-84][89]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.152, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient

[88]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[89]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

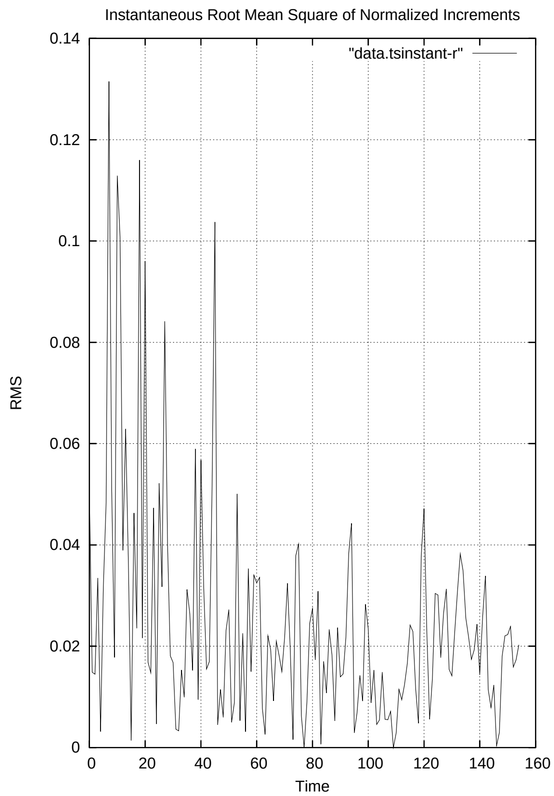

Figure C.148: United States Office Computer Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.139.

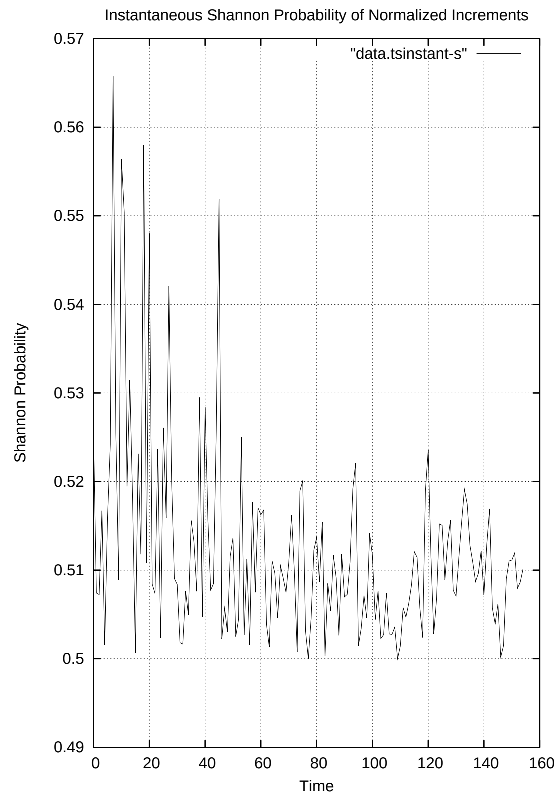

Figure C.149: United States Office Computer Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.139.

of 0.888451, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \quad \text{(C.270)}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.888451} \quad \text{(C.271)}$$

$$\propto (t_2 - t_1)^{1.776902} \quad \text{(C.272)}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[90]. A Hurst coefficient of 0.888451, (for the near future, and 0.723276 for the distant future.) implies

[90]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient

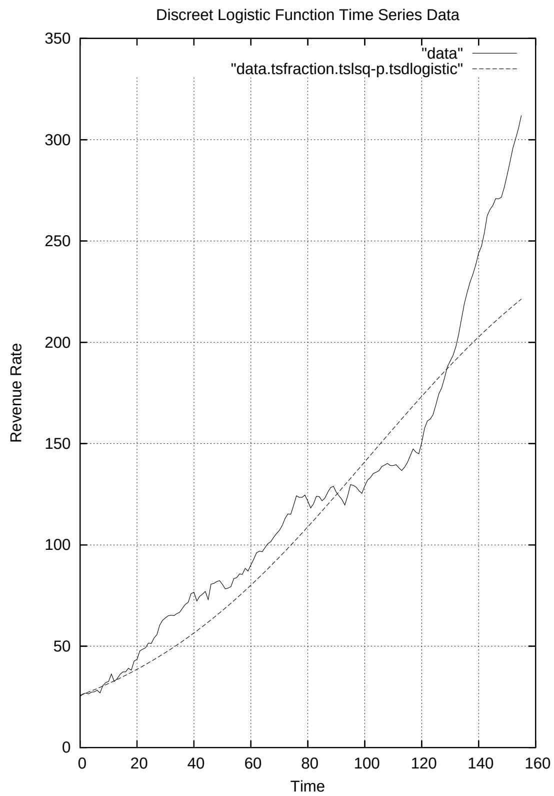

Figure C.150: United States Office Computer Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.140 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

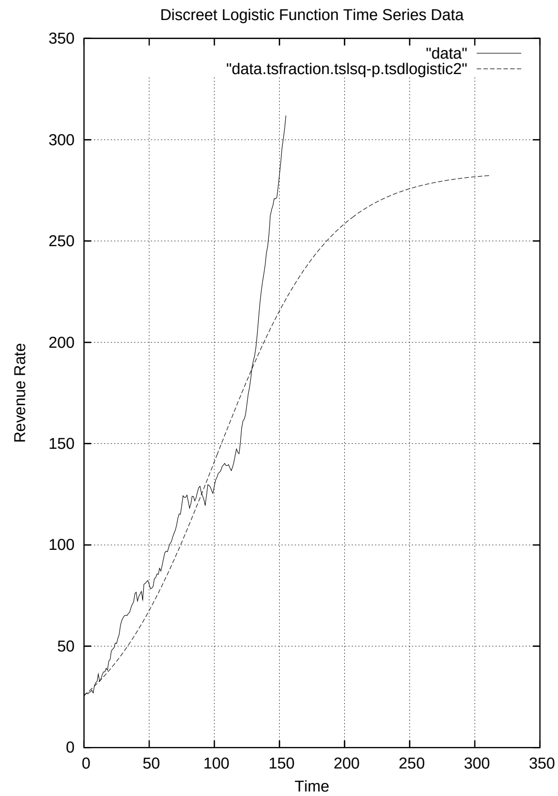

Figure C.151: United States Office Computer Market, logistic function estimates of Figure C.150 with the time scale expanded by a factor of two.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 88.845100% [Pet91, pp. 66] for the near future, and 0.723276 for the distant future. Likewise, there is a 88.845100% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 88.845100% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Office Computer Market are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.7.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Office Computer Market. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

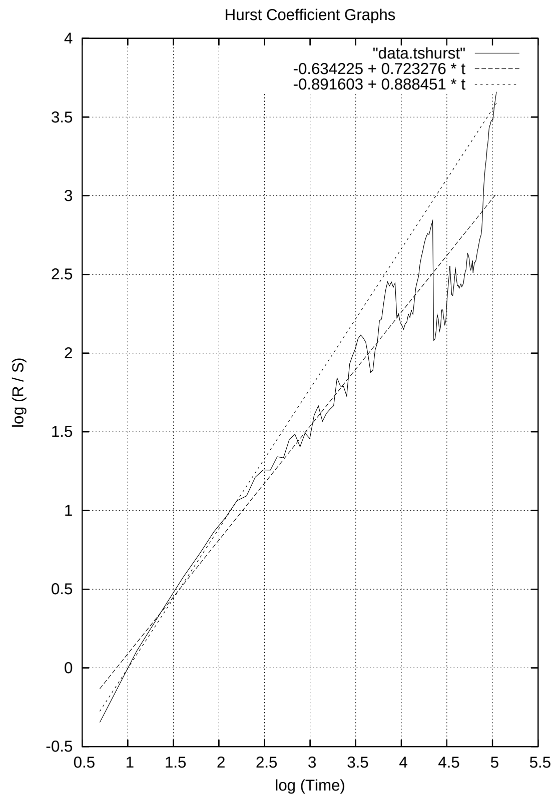

Figure C.152: United States Office Computer Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.140. The slope of the graph is the Hurst coefficient.

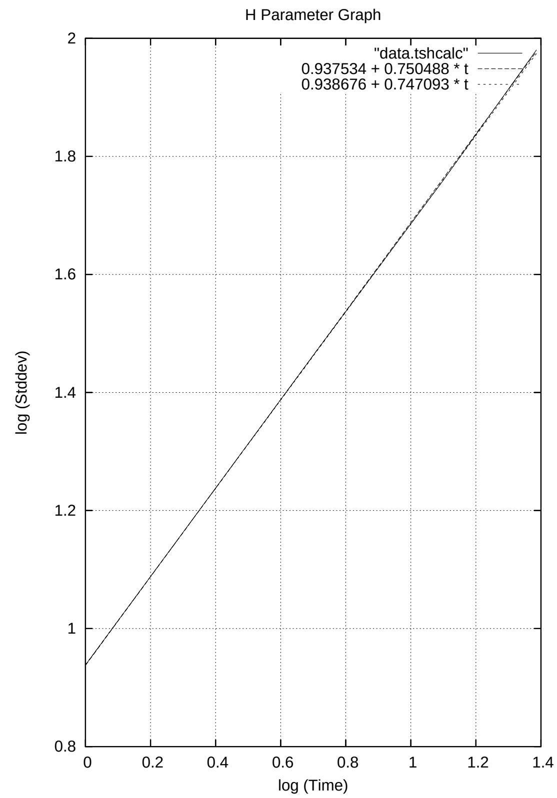

Figure C.153: United States Office Computer Market, H parameter data for the normalized increments of the time series data shown in Figure C.140 The slope of the graph is the H parameter.

$$p_a(n) = H^n \tag{C.273}$$
$$= 0.888451^n \tag{C.274}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.140, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.016771 \cdot 100$ percent, on the average, with a standard deviation of $0.028983 \cdot 100$ percent, and a root mean square error value of $0.033404 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.275}$$

$$\propto \quad (t_2 - t_1)^{0.888451} \tag{C.276}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.276 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.277}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.278}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.279}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[91].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \quad \propto \quad \frac{X\,(rt)}{r^H} \tag{C.280}$$

$$\propto \quad \frac{X\,(rt)}{r^{0.888451}} \tag{C.281}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.153, to provide a least squares approximation to the H parameter for the United States Office Computer Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.747093 for the near future, and 0.750488 for the distant future.

Figures C.152 and C.153 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.140. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.140, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.154 and C.155 was made using the -d option.

C.7.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.7.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.141. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.140. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Office Computer Market, and may, or may not, provide adequate accuracy for projections.

[91]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

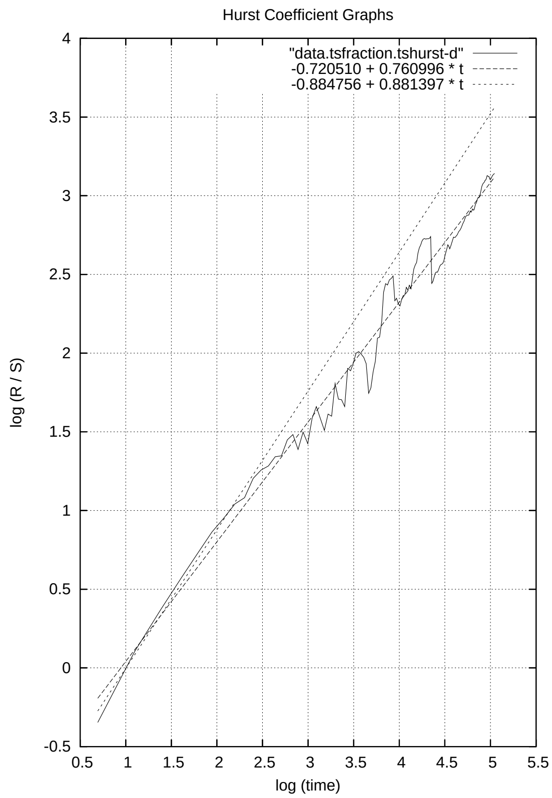

Figure C.154: United States Office Computer Market, traditional Hurst coefficient data for the time series data shown in Figure C.139. The slope of the graph is the Hurst coefficient, and is 0.881397 for the near term, and 0.760996 for the far term.

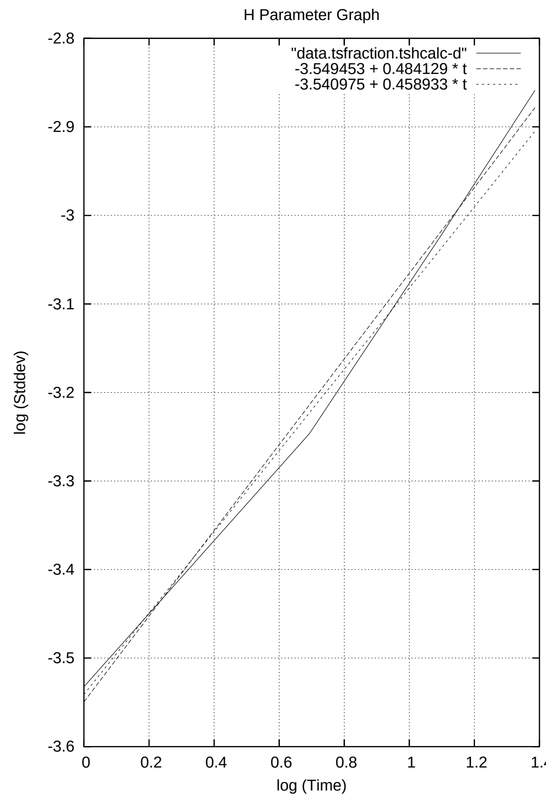

Figure C.155: United States Office Computer Market, traditional H parameter data for the time series data shown in Figure C.139 The slope of the graph is the H parameter, and is 0.458933 for the near term, and 0.484129 for the far term.

For an organization operating in the United States Office Computer Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.140, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.016771 + 1)}{\ln(2)} = 0.023995 \tag{C.282}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.140, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.023041 + 1)}{\ln(2)} = 0.032864 \tag{C.283}$$

Note that if the mean is not constant in Figure C.140, this method will not provide accurate results. And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.139:

$$bits = 0.019653 \tag{C.284}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.139:

$$bits = 0.023266 \tag{C.285}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.7.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.023266t} \tag{C.286}$$

therefore:

$$C(p) = 0.023266 \tag{C.287}$$

and, *tsshannon* 0.023266 gives:

$$C(0.589554) = 0.023266 \tag{C.288}$$

therefore:

$$\begin{aligned} 2^{C(0.589554)} &= 2^{0.023266} && \text{(C.289)} \\ &= 1.016258 && \text{(C.290)} \\ &= 1.625750\% && \text{(C.291)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.589554) - 1 && \text{(C.292)} \\ &= 0.179108 && \text{(C.293)} \\ &= 17.910800\% && \text{(C.294)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the United States Office Computer Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 17.910800% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 82.089200% will be held in “reserve” with a 58.955400% chance of making twice the 17.910800% back, (and a 41.044600% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 1.625750%, or a doubling of its rate of revenue returns, (per month,) in 42.981174 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 17.910800% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.625750%, per month, on average.

Note that the metrics presented in this section are representative of the United States Office Computer Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 17.910800% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Office Computer Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.625750% per month.

As another simple example, a company re-invests 17.910800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 17.910800% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.625750% per month.

As an example of "product portfolio" management, suppose a company re-invests 17.910800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 17.910800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 17.910800$ percent for the second product, implying that the company should diversify its product line[92]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 17.910800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Office Computer Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.179108}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.140, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex

[92]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 17.910800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 17.910800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 17.910800 of the rate of revenue returns per month does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 17.910800% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.179108}$ products seems consistent with the industry, also.

C.7.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Office Computer Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Office Computer Market time series is 0.016771, and 0.033404respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 15.030105.

If this value seems consistent number of companies in the United States Office Computer Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Office Computer Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.564751, which would be the value which should be used in Section C.7.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.7.5 is greater than the average Shannon probability for the companies participating in the United States Office Computer Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.7.5. The maximum exploitability for the United States Office Computer Market is derived in Section C.7.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Office Computer Market is 0.564751, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.751033 in the United States Office Computer Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.295}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Office Computer Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.7.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.141. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.140. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Office Computer Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.7.5, is derived from the United States Office Computer Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.7.9.

An additional exploitable strategy may be time itself. Equations C.272, C.276, and, C.274, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Office Computer Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.156, and, C.157 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Office Computer Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Office Computer Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[93]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.156, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.274, $0.888451^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.272, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.157, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[94]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with

[93] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[94] For example, see "IEEE Engineering Management Review," Volume 23 Number 3, Fall 1995, pp. 83

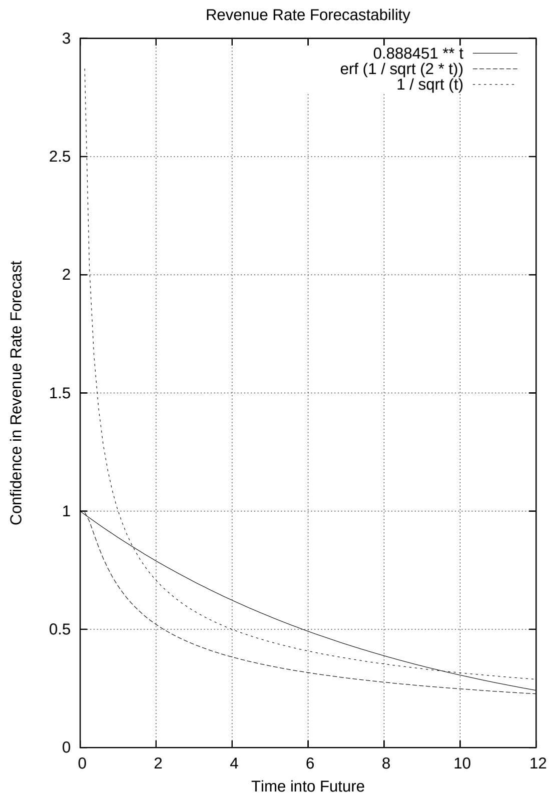

Figure C.156: United States Office Computer Market, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.888451^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

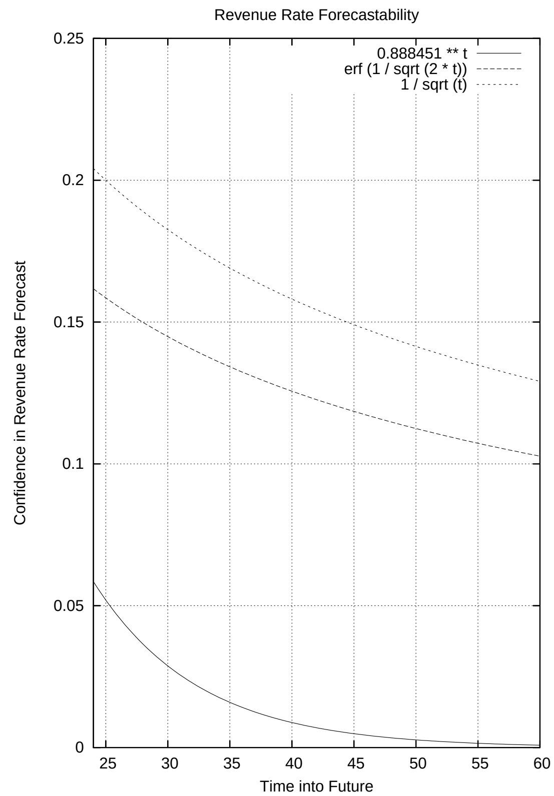

Figure C.157: United States Office Computer Market, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

$\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.7.4, Equation C.274, and the preceeding section, approximately 3 times the value where $0.888451^n = 0.5$ could be interpreted as an approximation to the "average" product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.888451}$ many months seems consistent with the author's experience in the industry.

C.7.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.7.9. Figure C.158 represents a constructional simulation of the time series data presented in Figure C.139. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data.

The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.140. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.140 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.159 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.142.

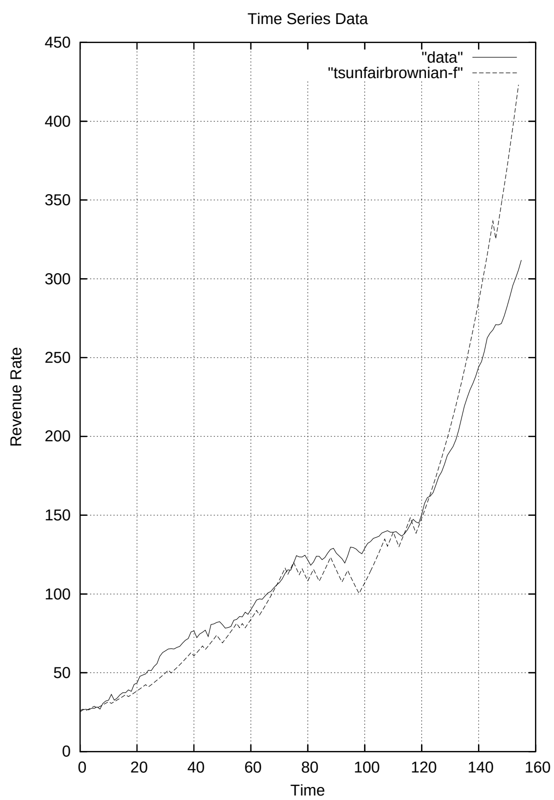

Figure C.158: United States Office Computer Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.033404. This data is superimposed on the data presented in Figure C.139.

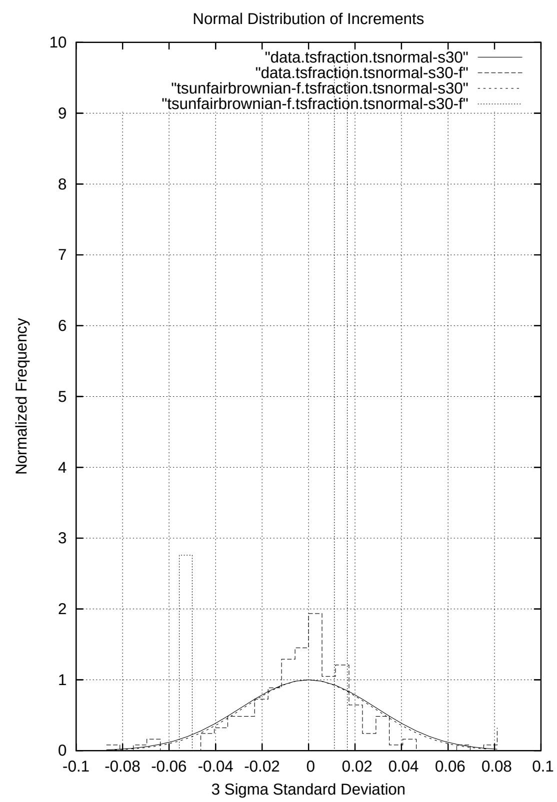

Figure C.159: United States Office Computer Market, normalized histogram of the normalized increments of the time series data shown in Figure C.158, empirical and simulated. The empirical data has a mean of 0.016771, with a standard deviation of 0.028983. By comparison, the simulated data has a mean of 0.018654 with a standard deviation of 0.027800. This data is superimposed on the data presented in Figure C.142. The area under the four curves is identical.

C.7.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.7.3. One of the issues of analysis, as mentioned in Section C.7.7, is to determine the maximum Shannon probability for the time series presented in Figure C.139. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.160 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.139. Figure C.161 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.139. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.589554, as derived in Section C.7.5 to 0.782051. This process, essentially, extracts the random statistical data from the time series presented in Figure C.139, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

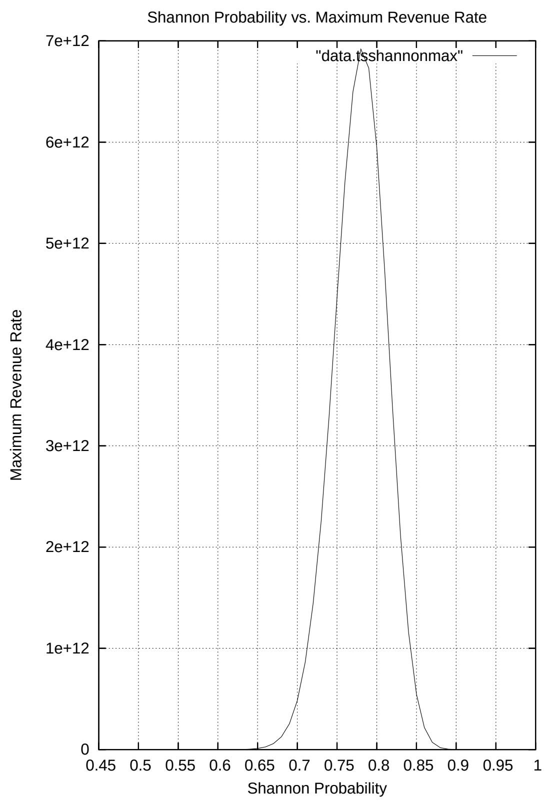

Figure C.160: United States Office Computer Market, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.782051.

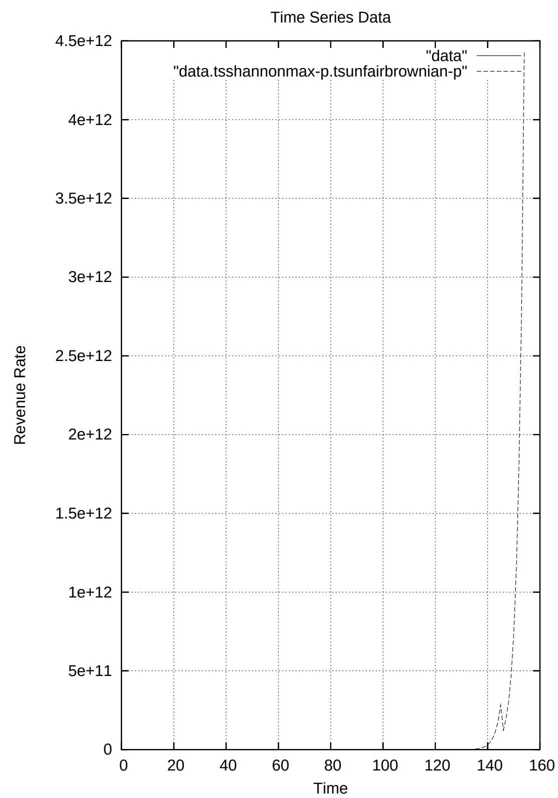

Figure C.161: United States Office Computer Market, maximum rate of revenue returns, per month, at a Shannon probability, of 0.782051, corresponding to a "wager" fraction of 0.564102.

If it is assumed that the time series data set, presented in Figure C.139, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Office Computer Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.780645, as compared with the predicted value from the program *tsshannonmax* of 0.782051.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.7.1, Figure C.142, it would appear that the United States Office Computer Market's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.564102 of the rate of returns every month is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any month, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.296}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.140 of 0.033404, vs. an "theoretical optimal" value of 0.564102 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per month, which is unacceptably high. However, it is not clear why the United States Office Computer Market is running at a value of 0.033404, which seems very conservative. However, a re-investment strategy of 0.033404 per month does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the United States Office Computer Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx$ 0.5, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.564102}{2.5}$, compared with an operational value, from Figure C.142 of 0.033404.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the United States Office Computer Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.139. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.782051 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.589554 is not just marginally lower than the maximum Shannon probability of 0.782051. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.7.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.141. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.140. These values will be used in a fixed increment Brownian fractal analysis of the United States Office Computer Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.7.6 and D.7.7. As a subjective evaluation of the "quality" of the analysis of the United States Office Computer Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.139 from Figure C.140, and the Shannon probability as calculated by counting the total number of months that the United States Office Computer Market movement was positive, as presented in Section C.7.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.297}$$

$$0.780645 \approx \frac{\frac{0.016771}{0.033404} + 1}{2} \tag{C.298}$$

$$0.780645 \approx 0.751033 \tag{C.299}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.780645 \approx 0.751033 \approx 0.782051 \tag{C.300}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.7.5, should be compared. The four methods used were the mean of Figure C.140, the constant in the least squares approximation to Figure C.140, the least squares exponential approximation to Figure C.139, and the logarithmic returns of Figure C.139, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.023995 \approx 0.032864 \approx 0.019653 \approx 0.023266 \tag{C.301}$$

It is implied in Section C.7.5, Subsection C.7.5 and in Section C.7.8 that, a Brownian motion with fixed increments fractal may "model" the United States Office Computer Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P - 1) \approx \frac{\sigma\,(2P - 1)}{2\sqrt{P\,(1 - P)}} \tag{C.302}$$

$$0.033404\,(2 \cdot 0.780645 - 1) \approx \frac{0.028983\,(2 \cdot 0.780645 - 1)}{2\sqrt{0.780645\,(1 - 0.780645)}} \quad \text{(C.303)}$$

$$0.033404 \cdot 0.561290 \approx 0.028983 \cdot 0.678199 \quad \text{(C.304)}$$

$$0.018749 \approx 0.019656 \quad \text{(C.305)}$$

and, equating to the mean:

$$0.016771 \approx 0.018749 \approx 0.019656 \quad \text{(C.306)}$$

where, as in Equation C.299 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.139 from Figure C.140, and the Shannon probability as calculated by counting the total number of months that the United States Office Computer Market movement was positive, as presented in Section C.7.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[95], where the absolute value is presented in Figure C.141, and the root mean square value is presented in Figure C.140:

$$0.024562 \approx 0.033404 \quad \text{(C.307)}$$

Note, that if the United States Office Computer Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.139 from Figure C.140 should be zero. It is 0.022713.

C.8 United States Information Systems Market

For the analysis, the data was in the directory ../markets/information.systems[96].

The data in this section is presented in tabular form in Section D.8.

C.8.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.8.1. Figure C.162 is a graph of the time series data for the United States Information Systems Market.

Figure C.163 is a graph of the normalized increments of the time series data presented in Figure C.162. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.164 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.163. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[97].

[95] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[96] Data from the United States Department of Commerce, 1979—1994, by months, in millions of dollars, US.

[97] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

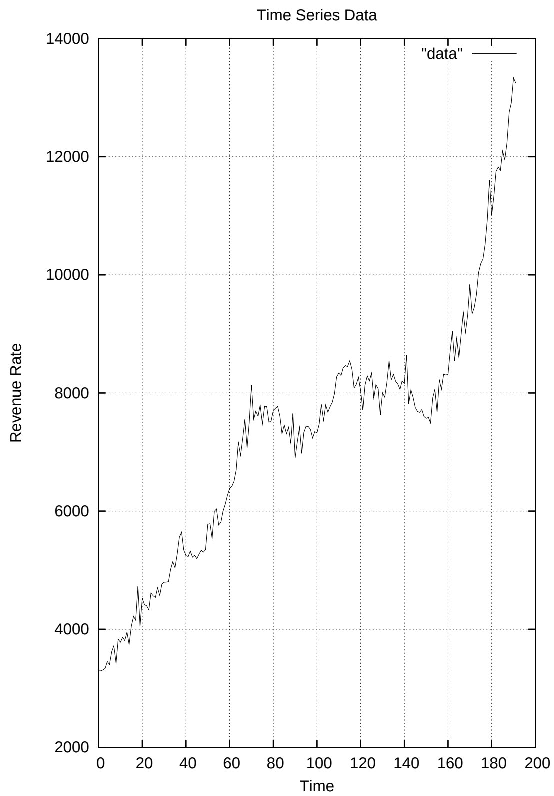

Figure C.162: United States Information Systems Market, time series data.

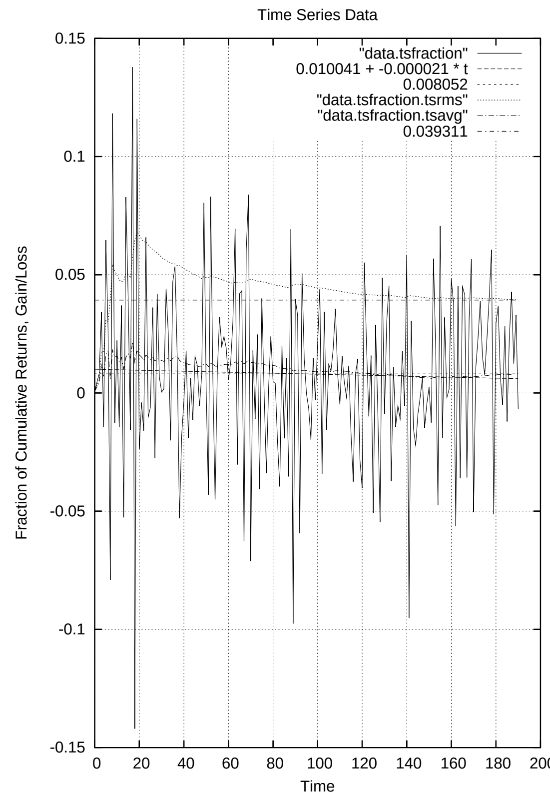

Figure C.163: United States Information Systems Market, normalized increments of the time series data presented in Figure C.162. The mean is 0.008052 with a standard deviation of 0.038579. The formula for the least squares approximation is $0.010041 + -0.000021t$, and the root mean squared value is 0.039311. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000021, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.165 is the normalized histogram of the normalized increments of the time series data shown in Figure C.163. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data

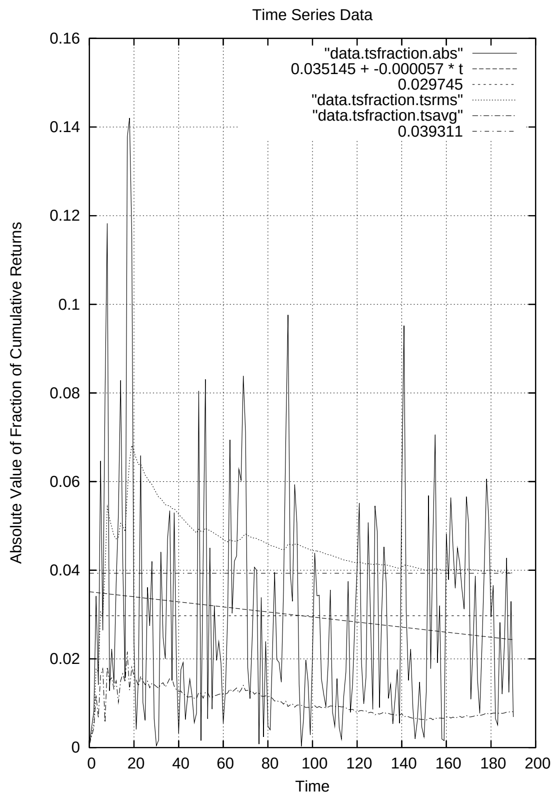

Figure C.164: United States Information Systems Market, absolute value of the normalized increments of the time series data presented in Figure C.163. The mean is 0.029745 with a standard deviation of 0.025769. The formula for the least squares approximation is $0.035145 + -0.000057t$, and the root mean square value, from Figure C.163, is 0.039311. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.163, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

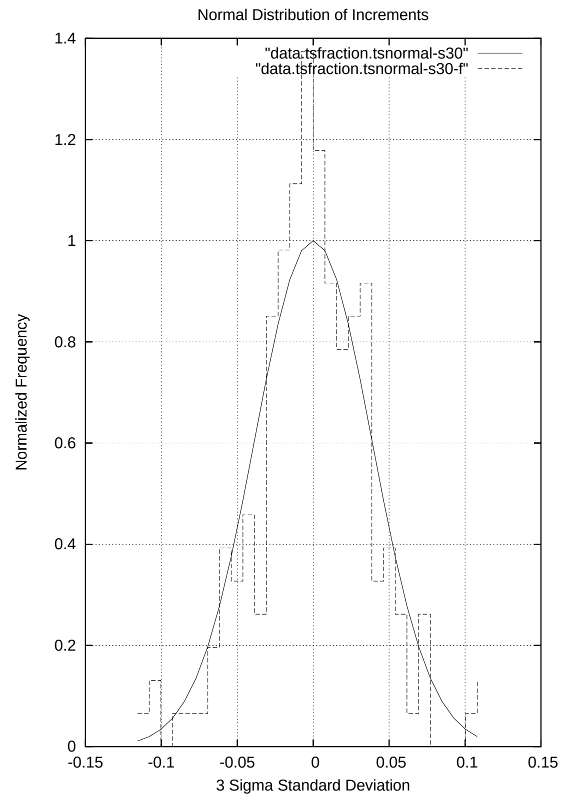

Figure C.165: United States Information Systems Market, normalized histogram of the normalized increments of the time series data shown in Figure C.163. The data has a mean of 0.008052, with a standard deviation of 0.038579. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 2.862000, with a critical value of 42.557000.

presented in Figure C.165.

Figure C.166 is the statistical estimate for the data presented in Figure C.163, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.560125$, as derived in Section C.8.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set

```
For a mean of 0.008010, with a confidence level of 0.900000
    that the error did not exceed 0.000801, 6518 samples would be required.
    (With 192 samples, the estimated error is 0.004667 = 58.261979 percent.)
For a standard deviation of 0.039311, with a confidence level of 0.900000
    that the error did not exceed 0.003931, 136 samples would be required.
    (With 192 samples, the estimated error is 0.003300 = 8.393859 percent.)
```

Figure C.166: United States Information Systems Market, statistical estimates of the normalized increments of the time series shown in Figure C.163. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.163.

sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.167 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.163. In principle, if the distribution of the normalized increments presented in Figure C.165 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.168 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.163. In principle, if the distribution of the normalized increments presented in Figure C.165 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.169 is the range of values of the time series shown in Figure C.162. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.169 would be a square root function[98]. Figure C.170 is the deterministic map of the normalized increments of the time series data shown in Figure C.163. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.165 would seem to indicate that the time series data for the United States Information Systems Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.8.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

[98]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.169 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

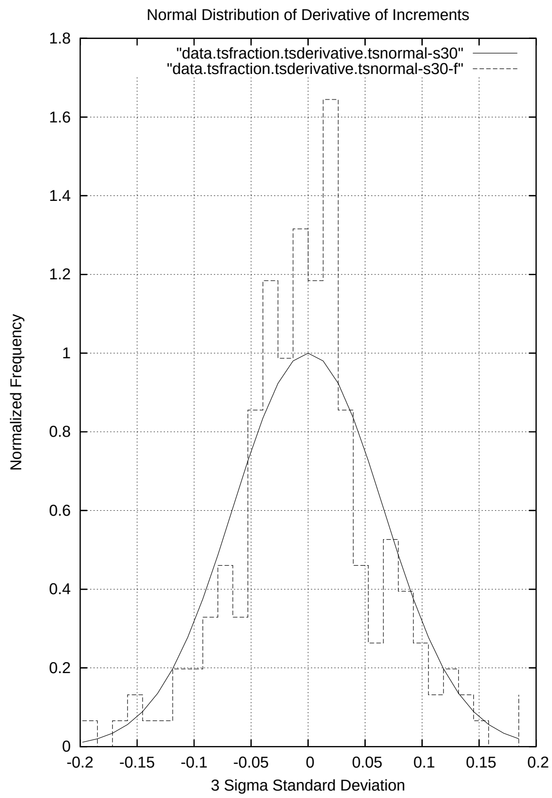

Figure C.167: United States Information Systems Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.163.

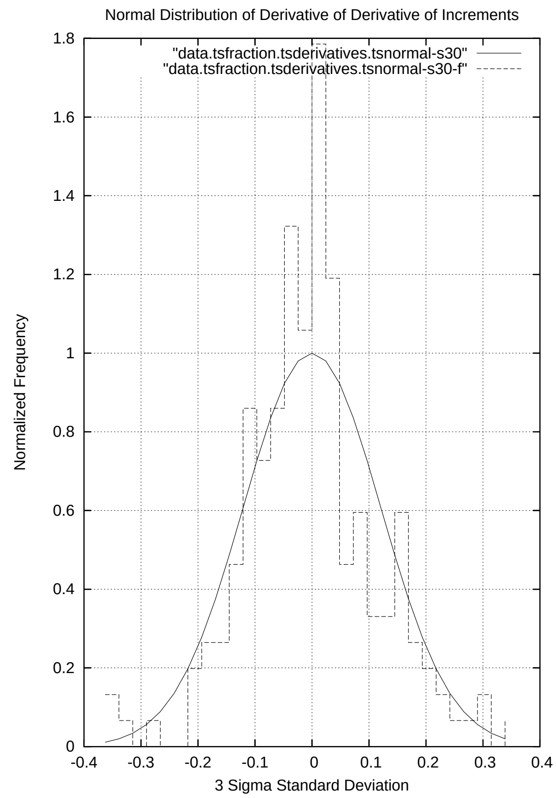

Figure C.168: United States Information Systems Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.163.

mean square of the instantaneous fraction of change[99]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.171 is the instantaneous value of the root mean square of the normalized increments for the United States Information Systems Market, and Figure C.172 is the instantaneous Shannon probability for the normalized increments.

C.8.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.8.4. Figure C.173 is a graph of the logistic function estimates of the time series data for the United States Information Systems Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is

[99]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

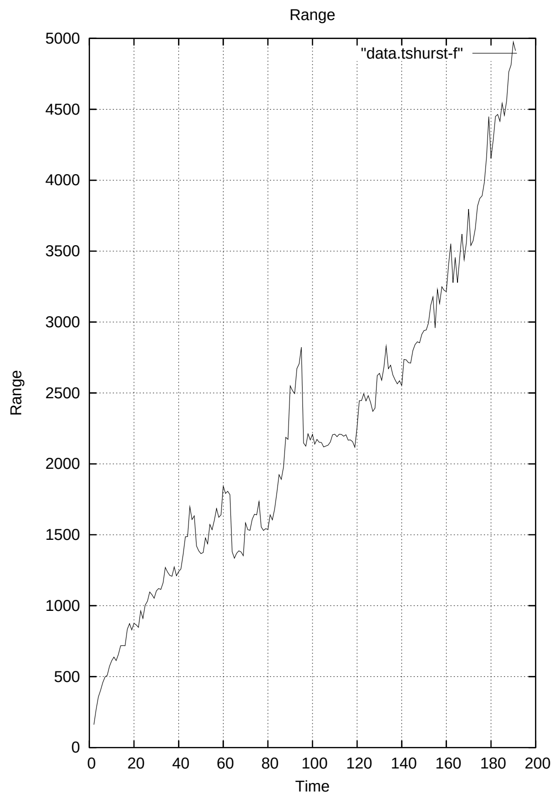

Figure C.169: United States Information Systems Market, range of the time series data shown in Figure C.162.

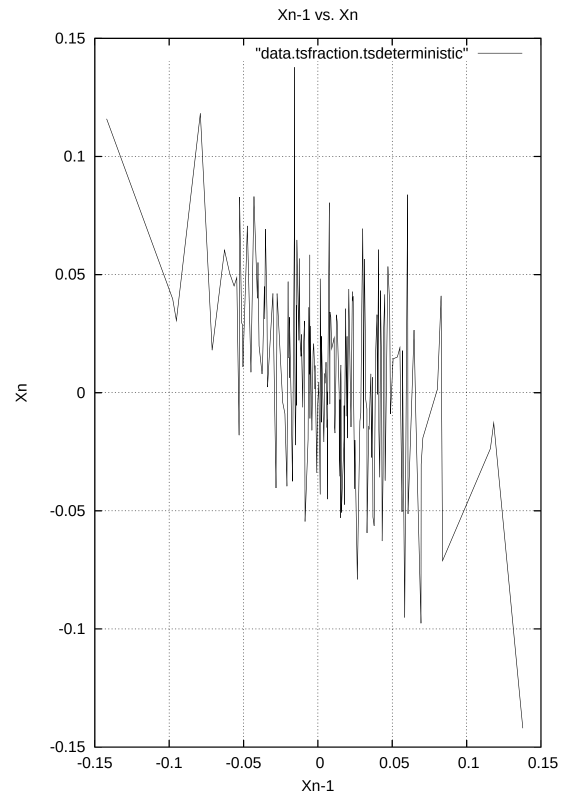

Figure C.170: United States Information Systems Market, deterministic map of the normalized increments of the time series data shown in Figure C.163.

required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[100]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.173 is a graph of the logistic function for the time series data presented in Figure C.162. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.163. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.163. Figure C.174 is the same graph, but with the time scale expanded by a factor of two.

C.8.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.8.5. Figure C.175 is a graph of the Hurst coefficient data time series data shown in Figure C.162. The slope of the graph is the Hurst coefficient. The data for this figure

[100]For example, in Figures C.173 and C.174, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.8.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

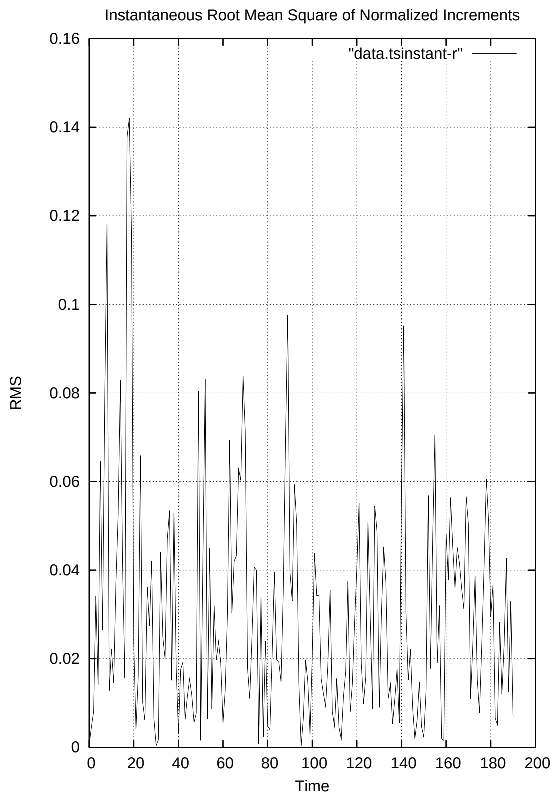

Figure C.171: United States Information Systems Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.162.

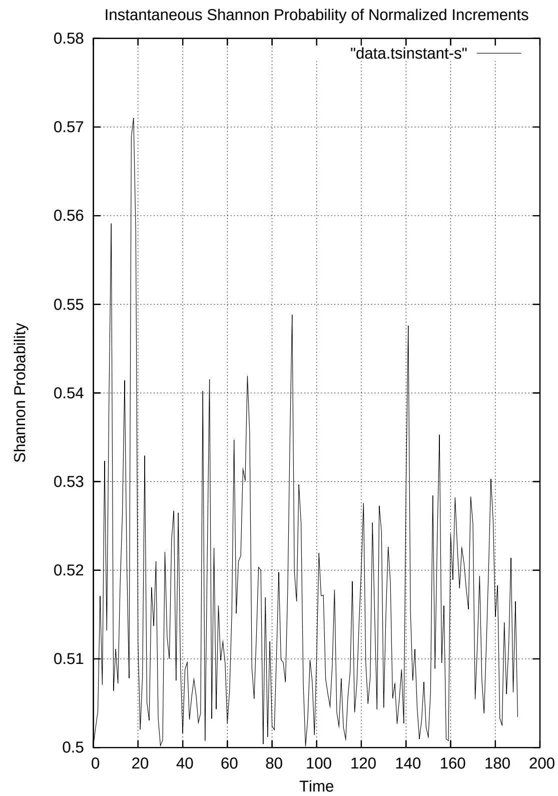

Figure C.172: United States Information Systems Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.162.

was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.176 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.163. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.175 implies that the variance of the rate of revenue returns, (per month,) in the United States Information Systems Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[101], t, $p(t) = erf(1/\sqrt{2t})$ which is

[101] It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

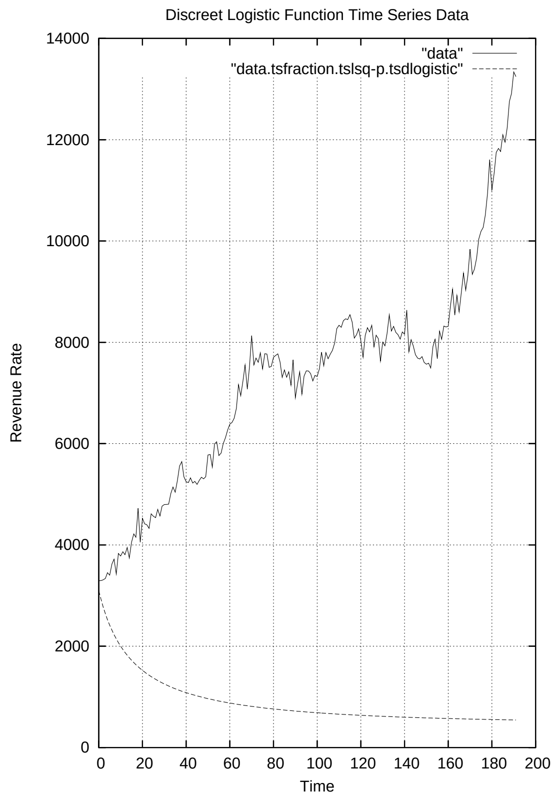

Figure C.173: United States Information Systems Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.163 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

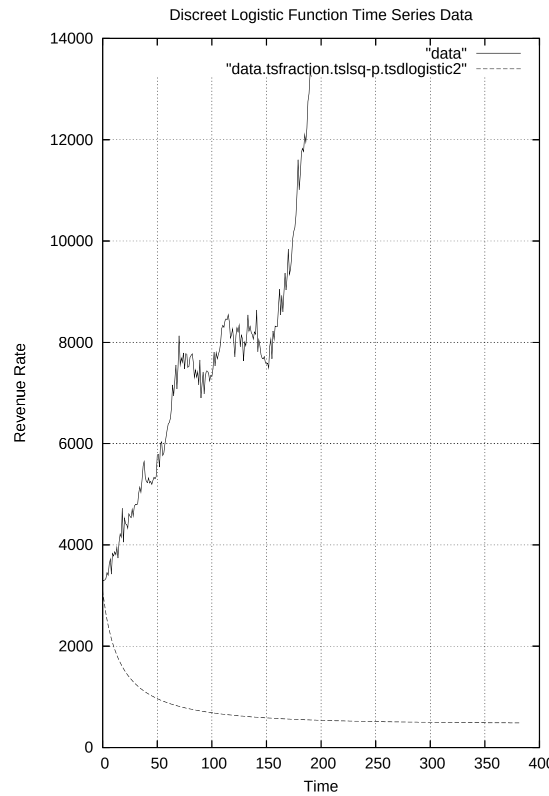

Figure C.174: United States Information Systems Market, logistic function estimates of Figure C.173 with the time scale expanded by a factor of two.

approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.179, and, C.180 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Information Systems Market for the near term and far term, respectively [Pet91, pp. 83-84][102]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.175, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.710108, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.308}$$

[102]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

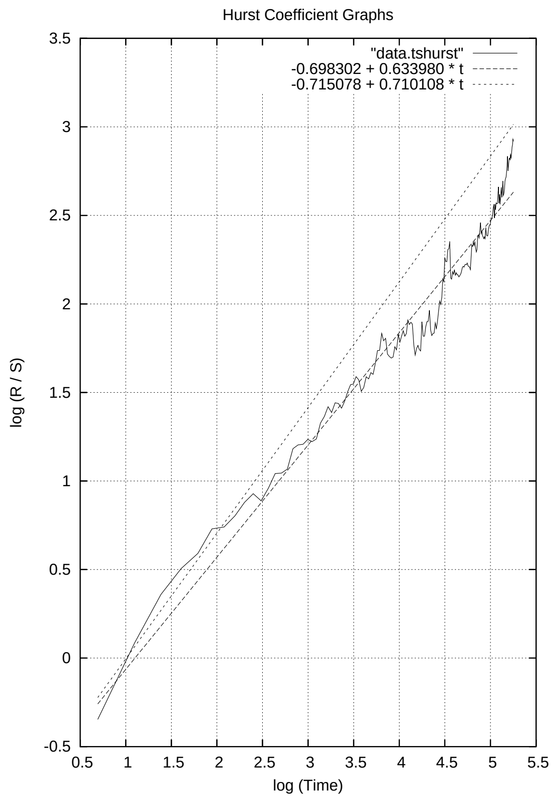

Figure C.175: United States Information Systems Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.163. The slope of the graph is the Hurst coefficient.

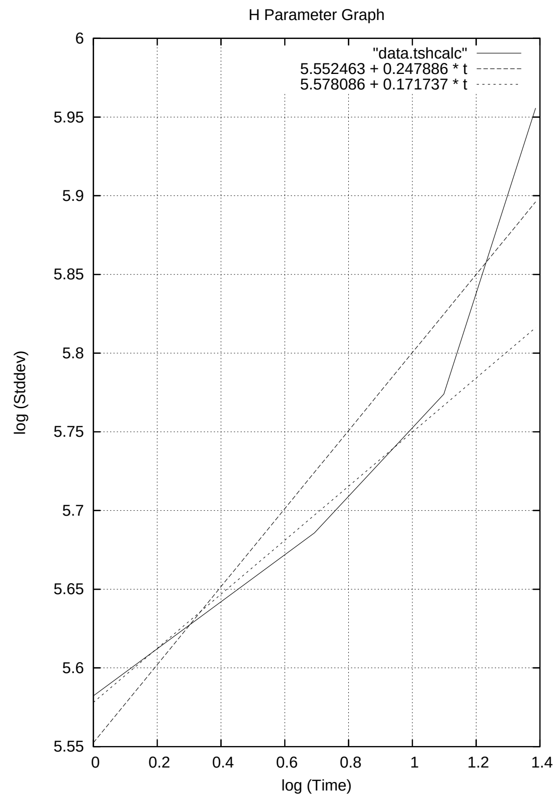

Figure C.176: United States Information Systems Market, H parameter data for the normalized increments of the time series data shown in Figure C.163 The slope of the graph is the H parameter.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.710108} \quad (C.309)$$

$$\propto (t_2 - t_1)^{1.420216} \quad (C.310)$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[103]. A Hurst coefficient of 0.710108, (for the near future, and 0.633980 for the distant future.) implies

[103] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.8.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Information Systems Market. See

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 71.010800% [Pet91, pp. 66] for the near future, and 0.633980 for the distant future. Likewise, there is a 71.010800% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 71.010800% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Information Systems Market are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.311}$$

$$= 0.710108^n \tag{C.312}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.163, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.008052 \cdot 100$ percent, on the average, with a standard deviation of $0.038579 \cdot 100$ percent, and a root mean square error value of $0.039311 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.313}$$

$$\propto (t_2 - t_1)^{0.710108} \tag{C.314}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.314 reduces to, [Sch91, pp. 129]:

$$R(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.315}$$

$$\propto \sqrt{(t_2 - t_1)} \tag{C.316}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.317}$$

also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[104].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.318}$$

$$\propto \frac{X(rt)}{r^{0.710108}} \tag{C.319}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.176, to provide a least squares approximation to the H parameter for the United States Information Systems Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.171737 for the near future, and 0.247886 for the distant future.

Figures C.175 and C.176 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.163. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.163, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.177 and C.178 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Note that the H parameter data is not linear, and the long term predictability is better than the short term predictability, indicating that the least squares approximation is low.

C.8.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.8.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.164. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.163. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Information Systems Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Information Systems Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.163, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.008052 + 1)}{\ln(2)} = 0.011570 \tag{C.320}$$

[104] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

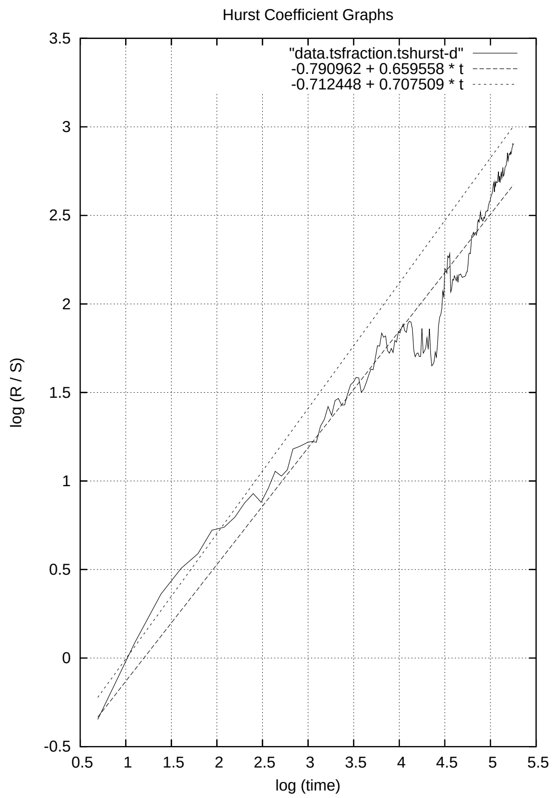

Figure C.177: United States Information Systems Market, traditional Hurst coefficient data for the time series data shown in Figure C.162. The slope of the graph is the Hurst coefficient, and is 0.707509 for the near term, and 0.659558 for the far term.

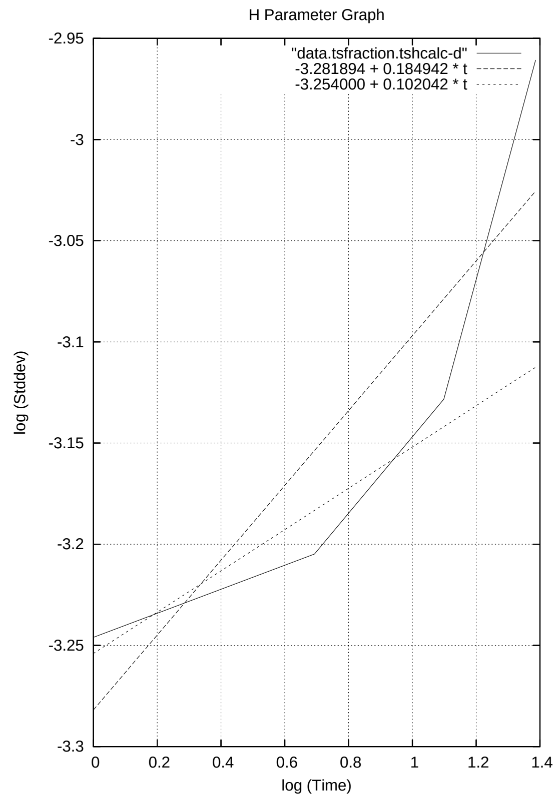

Figure C.178: United States Information Systems Market, traditional H parameter data for the time series data shown in Figure C.162 The slope of the graph is the H parameter, and is 0.102042 for the near term, and 0.184942 for the far term.

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.163, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.010041 + 1)}{\ln(2)} = 0.014414 \tag{C.321}$$

Note that if the mean is not constant in Figure C.163, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.162:

$$bits = 0.007623 \tag{C.322}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.162:

$$bits = 0.010456 \tag{C.323}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.8.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.010456t} \tag{C.324}$$

therefore:

$$C(p) = 0.010456 \tag{C.325}$$

and, *tsshannon* 0.010456 gives:

$$C(0.560125) = 0.010456 \tag{C.326}$$

therefore:

$$\begin{aligned} 2^{C(0.560125)} &= 2^{0.010456} && \text{(C.327)} \\ &= 1.007274 && \text{(C.328)} \\ &= 0.727387\% && \text{(C.329)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.560125) - 1 && \text{(C.330)} \\ &= 0.120250 && \text{(C.331)} \\ &= 12.025000\% && \text{(C.332)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the United States Information Systems Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 12.025000% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 87.975000% will be held in "reserve" with a 56.012500% chance of making twice the 12.025000% back, (and a 43.987500% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.727387%, or a doubling of its rate of revenue returns, (per month,) in 95.638868 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 12.025000% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.727387%, per month, on average.

Note that the metrics presented in this section are representative of the United States Information Systems Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 12.025000% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Information Systems Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.727387% per month.

As another simple example, a company re-invests 12.025000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 12.025000% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.727387% per month.

As an example of "product portfolio" management, suppose a company re-invests 12.025000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 12.025000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 12.025000$ percent for the second product, implying that the company should diversify its product line[105]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 12.025000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Information Systems Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.120250}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.163, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 12.025000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 12.025000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

Observations on the Fixed Increment Approximation for Fiscal Strategy

A re-investment of 12.025000 of the rate of revenue returns per month does not seem inconsistent with the industry averages, since it includes investments in research and development, additional manufacturing infrastructure, advertising, etc. Additionally, a product mix of 12.025000% "proprietary" and the remainder "industry standard" products seems consistent with the industry analyst "20/80" rule. The value of one standard deviation, 84.13%, of the revenue return rate being generated by $\frac{1}{0.120250}$ products seems consistent with the industry, also.

[105] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^{H} = P_1^{H} + P_2^{H} + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

C.8.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Information Systems Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Information Systems Market time series is 0.008052, and 0.039311respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 5.210454.

If this value seems consistent number of companies in the United States Information Systems Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Information Systems Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.544866, which would be the value which should be used in Section C.8.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.8.5 is greater than the average Shannon probability for the companies participating in the United States Information Systems Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.8.5. The maximum exploitability for the United States Information Systems Market is derived in Section C.8.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Information Systems Market is 0.544866, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.602414 in the United States Information Systems Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.333}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Information Systems Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.8.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.164. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.163. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Information Systems Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.8.5, is derived from the United States Information Systems Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.8.9.

An additional exploitable strategy may be time itself. Equations C.310, C.314, and, C.312, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Information Systems Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.179, and, C.180 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Information Systems Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the “forecastability” of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in “timeliness” of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the “average” product or service life cycle in the United States Information Systems Market. When the “forecastability” of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the “forecastability” of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of “market windows.[106]” The conceptual approximation will probably predict a “conservative” or “pessimistic” value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.179, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.312, $0.710108^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.310, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

Observations on the Fixed Increment Approximation for Operational Strategy

As an interesting interpretation of Figure C.180, and evaluating the approximation $\frac{1}{\sqrt{t}}$ at 60 months gives a probability that the market will still have the same agenda of about 0.12909945, or about 1 in 8. This is commensurate with numbers from the venture community[107]. Of course new venture backed companies fail for many reasons, but market appropriateness to product portfolio 60 months in the future may be a major contributor. Additionally, the success rate of development projects of 8 month duration, which have a market success rate of about 1 in 3, seems consistent with $\frac{1}{\sqrt{3}} = 0.353553391$. Naturally, projects fail in the market for many reasons, but market appropriateness, in a dynamic market environment may be a major contributor to failure.

As mentioned in Section C.8.4, Equation C.312, and the preceeding section, approximately 3 times the value where $0.710108^n = 0.5$ could be interpreted as an approximation to the “average” product life cycle. This seems consistent with the 6 to 12 month life cycles quoted by many industry analyst. In addition, maintaining inventory levels that do not exceed the anticipated requirements of $\frac{\ln 0.5}{\ln 0.710108}$ many months seems consistent with the author’s experience in the industry.

C.8.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.8.9. Figure C.181 represents a constructional simulation of the time series data presented in Figure C.162. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data.

[106]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

[107]For example, see “IEEE Engineering Management Review,” Volume 23 Number 3, Fall 1995, pp. 83

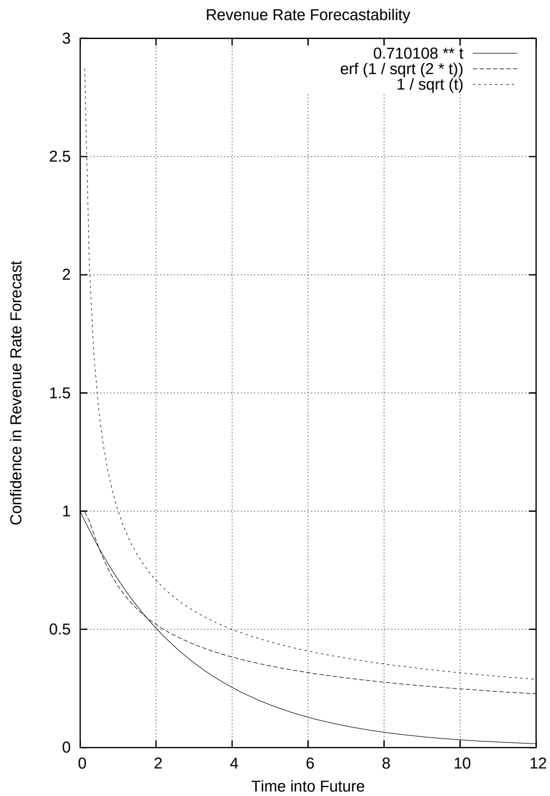

Figure C.179: United States Information Systems Market, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.710108^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

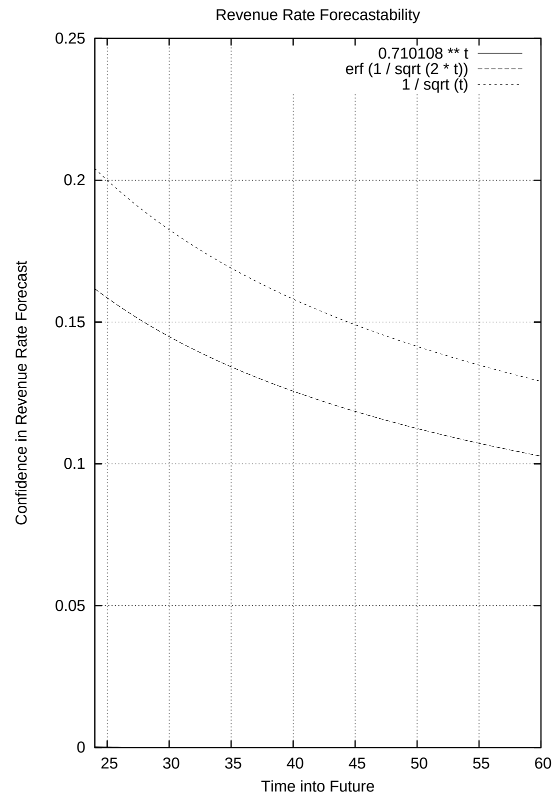

Figure C.180: United States Information Systems Market, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.163. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.163 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.182 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.165.

C.8.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.8.3. One of the issues of analysis, as mentioned in Section C.8.7, is to determine the maximum Shannon probability for the time series presented in Figure C.162. Potentially,

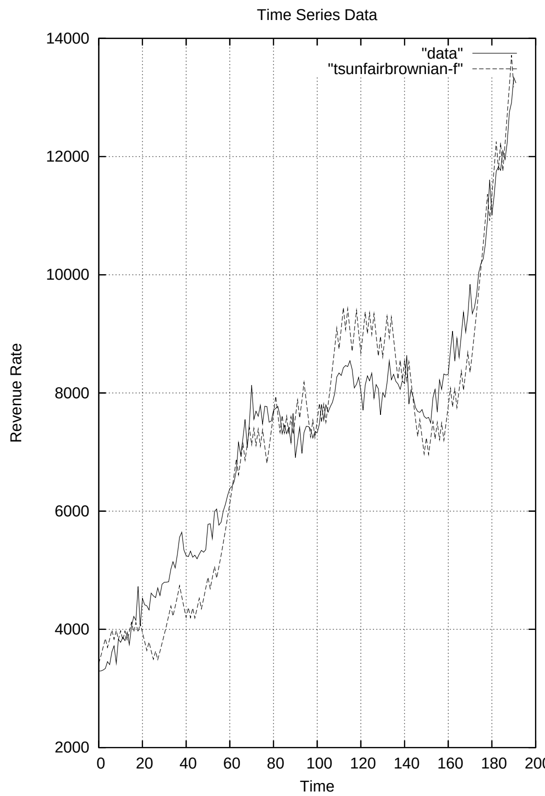

Figure C.181: United States Information Systems Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.039311. This data is superimposed on the data presented in Figure C.162.

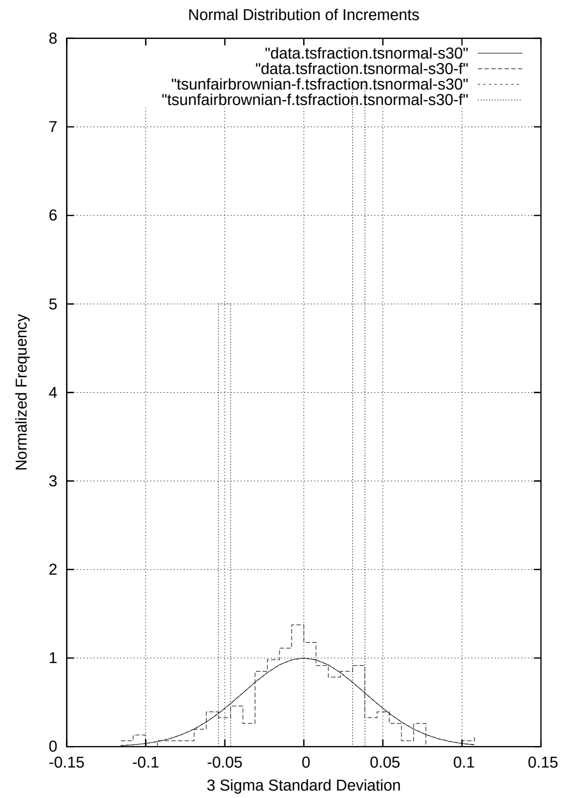

Figure C.182: United States Information Systems Market, normalized histogram of the normalized increments of the time series data shown in Figure C.181, empirical and simulated. The empirical data has a mean of 0.008052, with a standard deviation of 0.038579. By comparison, the simulated data has a mean of 0.007862 with a standard deviation of 0.038619. This data is superimposed on the data presented in Figure C.165. The area under the four curves is identical.

this could be exploited with an aggressive fiscal strategy. Figure C.183 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.162. Figure C.184 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.162. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.560125, as derived in Section C.8.5 to 0.604167. This process, essentially, extracts the random statistical data from the time series presented in Figure C.162, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

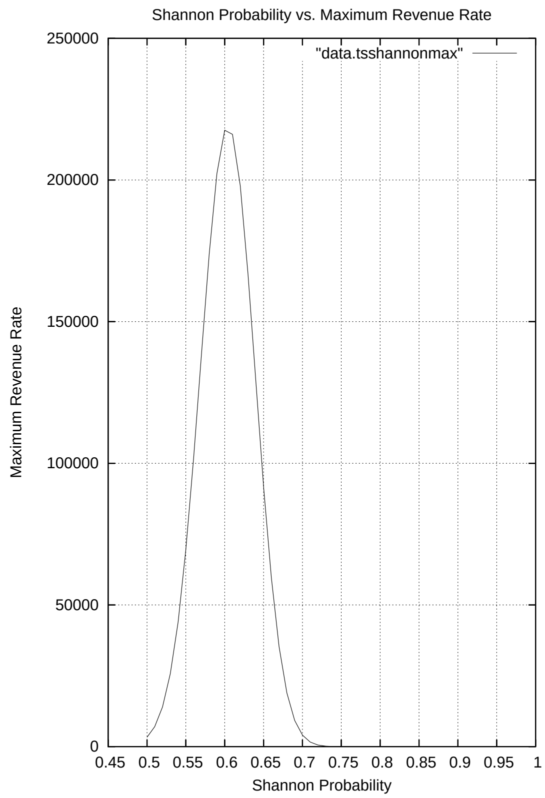

Figure C.183: United States Information Systems Market, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.604167.

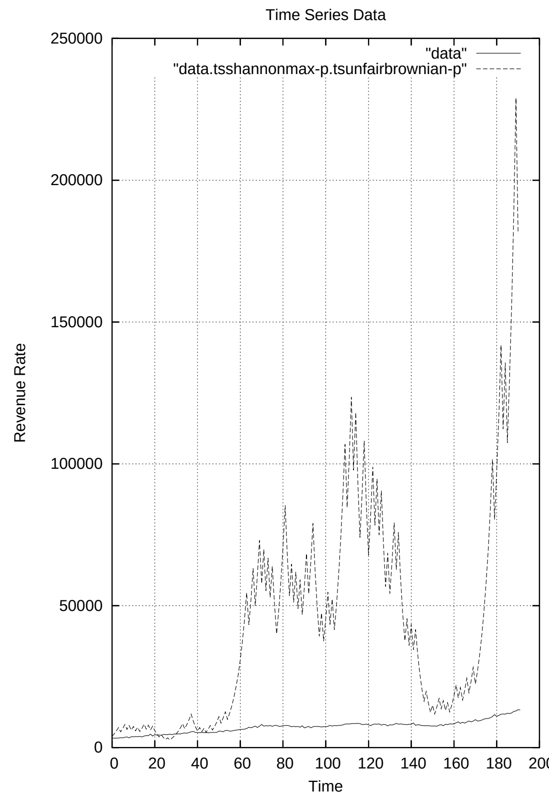

Figure C.184: United States Information Systems Market, maximum rate of revenue returns, per month, at a Shannon probability, of 0.604167, corresponding to a "wager" fraction of 0.208334.

If it is assumed that the time series data set, presented in Figure C.162, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Information Systems Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.602094, as compared with the predicted value from the program *tsshannonmax* of 0.604167.

Observations on the Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

Note that these simulations are base on a very, perhaps overly, simplified model. For example, from Section C.8.1, Figure C.165, it would appear that the United States Information Systems Market's normalized increments are characterized by fractional Brownian motion—but the simulations used classical Brownian motion as the model. One consequence of this is that a re-investment strategy that is to "wager" a fraction of 0.208334 of the rate of returns every month is overly aggressive, since in the classical Brownian scenario, the maximum loss, in any month, was no more that what was "wagered." However, in the fractional Brownian scenario, much more can be lost. From Equation 2.60,

$$\frac{avg}{rms^2} = \frac{f_{opt}}{rms} = K \tag{C.334}$$

where, under the optimum classical Brownian scenario, K is unity, or $avg = rms^2$. Notice that, since $f = rms$, whether the scenario is optimal or not, that the operational "wager" fraction, from Figure C.163 of 0.039311, vs. an "theoretical optimal" value of 0.208334 seems overly conservative. Additionally, notice that, at least in principle, the chance of failure in the fractional Brownian scenario, which is more accurate, would correspond to 1 standard deviation, or about 15.865% per month, which is unacceptably high. However, it is not clear why the United States Information Systems Market is running at a value of 0.039311, which seems very conservative. However, a re-investment strategy of 0.039311 per month does not seem inconsistent with a failure rate, on the Fortune 500 list, which it is inferred that the United States Information Systems Market is similar to, of about 50% in ten years, which corresponds to $(1 - p_f)^{120} \approx 0.5$, or p_f, the probability of failure, is 0.005759576, which is, approximately, 2.5 standard deviations, meaning that to be consistent with the large companies in the Fortune 500, the re-investment rate should be, approximately, $\frac{0.208334}{2.5}$, compared with an operational value, from Figure C.165 of 0.039311.

An interesting, and intriguing, interpretation and discussion of the maximum Shannon probability, is an explanation as to why the companies in the United States Information Systems Market are not running an optimal re-investment strategy. This seems enigmatic, since those companies that run, on a long term average, below the optimally maximal value would seem to be eclipsed by those that didn't. And those that run above the optimally maximal value would be over extended, and become financially destitute during market down turns, which is inevitable in a fractal time series as presented in Figure C.162. It would seem that the natural selection process of the competitive environment would allow only those companies that run near the optimally maximal value to survive, in the long run. One possible explanation, foremost, is that the analytical methodology presented herein is inappropriate. Another explanation is that the gross margins are less than the fraction 0.604167 of the rate of revenue returns, and thus could not accommodate such an aggressive re-investment strategy. If this is the case, then it presents an intriguing issue. If, in a capitalistic market, the natural outcome of the competitive situation, according to game-theoretic analysis, is that there will be many competitors, each making minimal gross margins, then how do the companies grow their markets? Naturally, those that run the most efficient will have lower costs, making larger percentage of rate of revenue returns re-investment possible. Yet another interpretation is that the number of competitors would grow at an exponential rate, but all of them would make minimal returns. However, an operational Shannon probability of 0.560125 is not just marginally lower than the maximum Shannon probability of 0.604167. There is a significant disparity which is difficult to explain. It would seem that the game-theoretic eventual outcome of a competitive market place would be a solution that hinders growth, wealth and jobs creation, etc., which does not seem consistent with capitalistic theory. On the other hand, is there an optimum number of competitors in a market place, where the gross margins can be higher, permitting wealth and job creation, and also a competitive situation? If this analysis is correct, and that should be subject to scrutiny, then it would appear that this is the case. But this brings up another issue—that of taxation, and other contributions to the social welfare function. If there is an optimum number of competitors in the market place, that maximizes wealth and job creation, then, perhaps by lemma, there is also an optimal value of taxation rate, and other contributions to the social welfare function, that will permit maximal industrial growth, and thus maximal growth in the tax base. But this would seem to be inconsistent with the work of Kenneth Arrow and the so called Impossibility Theorem, which states that such optimizations can not be determined because the ordering of priorities are intransitive. All very perplexing, since the simulation of the maximum Shannon probability in the next section seems to indicate that such an aggressive re-investment strategy is, indeed, feasible.

Yet another possibility for the industry not running at maximum Shannon probability is the high cost of expansion of operations. Some of these industries require very sophisticated manufacturing processes, which have high barrier costs.

Additionally, as mentioned in both [BdL95, pp. 29], and [Art88, pp. 8], optimal efficiency may not be attainable in increasing-return economic scenarios.

C.8.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.164. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.163. These values will be used in a fixed increment Brownian fractal analysis of the United States Information Systems Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.8.6 and D.8.7. As a subjective evaluation of the "quality" of the analysis of the United States Information Systems Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.162 from Figure C.163, and the Shannon probability as calculated by counting the total number of months that the United States Information Systems Market movement was positive, as presented in Section C.8.9:

$$P \approx \frac{\frac{avg}{rms}+1}{2} \tag{C.335}$$

$$0.602094 \approx \frac{\frac{0.008052}{0.039311}+1}{2} \tag{C.336}$$

$$0.602094 \approx 0.602414 \tag{C.337}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.602094 \approx 0.602414 \approx 0.604167 \tag{C.338}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.8.5, should be compared. The four methods used were the mean of Figure C.163, the constant in the least squares approximation to Figure C.163, the least squares exponential approximation to Figure C.162, and the logarithmic returns of Figure C.162, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.011570 \approx 0.014414 \approx 0.007623 \approx 0.010456 \tag{C.339}$$

It is implied in Section C.8.5, Subsection C.8.5 and in Section C.8.8 that, a Brownian motion with fixed increments fractal may "model" the United States Information Systems Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.340}$$

$$0.039311\,(2\cdot 0.602094-1) \approx \frac{0.038579\,(2\cdot 0.602094-1)}{2\sqrt{0.602094\,(1-0.602094)}} \tag{C.341}$$

$$0.039311\cdot 0.204188 \approx 0.038579\cdot 0.208583 \tag{C.342}$$

$$0.008027 \approx 0.008047 \tag{C.343}$$

and, equating to the mean:

$$0.008052 \approx 0.008027 \approx 0.008047 \tag{C.344}$$

where, as in Equation C.337 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.162 from Figure C.163, and the Shannon probability as calculated by counting the total number of months that the United States Information Systems Market movement was positive, as presented in Section C.8.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[108], where the absolute value is presented in Figure C.164, and the root mean square value is presented in Figure C.163:

$$0.029745 \approx 0.039311 \tag{C.345}$$

Note, that if the United States Information Systems Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.162 from Figure C.163 should be zero. It is 0.025769.

C.9 Dow Jones Average

For the analysis, the data was in the directory ../markets/dj[109].

The data in this section is presented in tabular form in Section D.9. Note that in this analysis, the rate of revenue returns means the increase or decrease in the value, or price, of the stocks in the Dow Jones Average, and not stock yield, or dividends. This is included for comparative purposes.

C.9.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.9.1. Figure C.185 is a graph of the time series data for the Dow Jones Average.

Figure C.186 is a graph of the normalized increments of the time series data presented in Figure C.185. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.187 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.186. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[110].

Figure C.188 is the normalized histogram of the normalized increments of the time series data shown in Figure C.186. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.188.

Figure C.189 is the statistical estimate for the data presented in Figure C.186, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.563735$, as derived in Section C.9.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set

[108] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[109] Data from Dow Jones News Information Retrieval Service, 1981—1994, by months, as an index.

[110] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

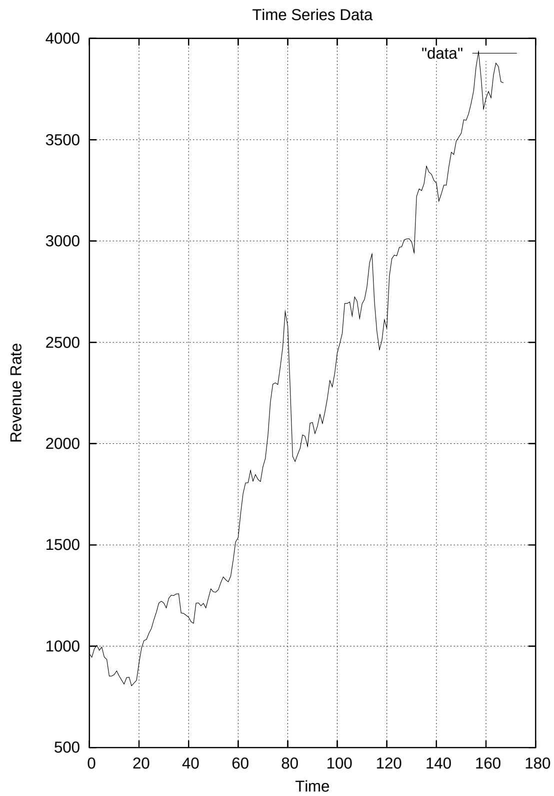

Figure C.185: Dow Jones Average, time series data.

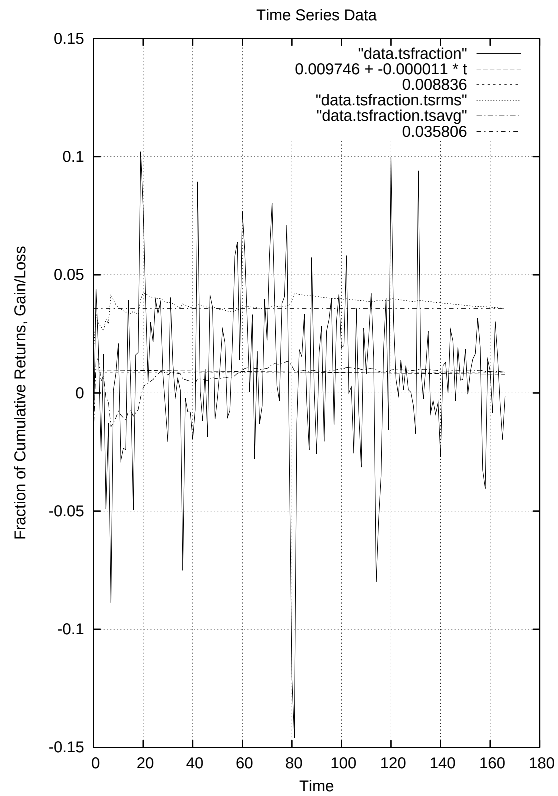

Figure C.186: Dow Jones Average, normalized increments of the time series data presented in Figure C.185. The mean is 0.008836 with a standard deviation of 0.034803. The formula for the least squares approximation is $0.009746 + -0.000011t$, and the root mean squared value is 0.035806. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000011, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.190 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.186. In principle, if the distribution of the normalized increments presented in Figure C.188

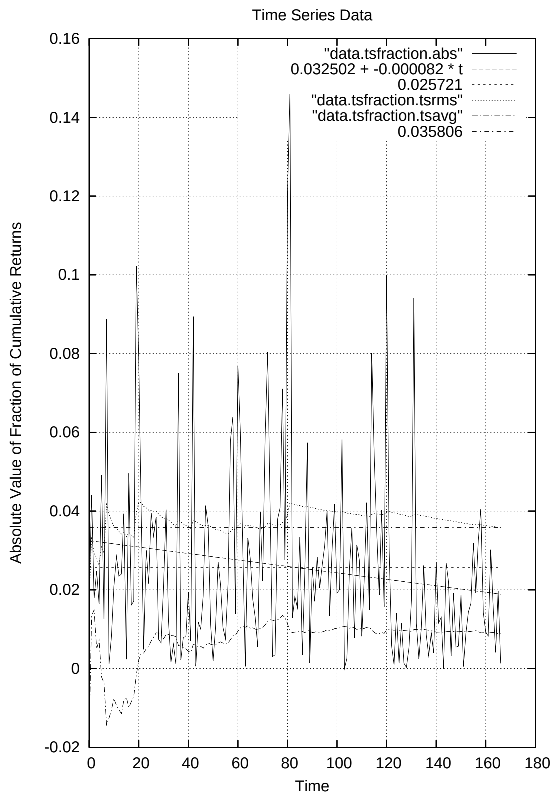

Figure C.187: Dow Jones Average, absolute value of the normalized increments of the time series data presented in Figure C.186. The mean is 0.025721 with a standard deviation of 0.024985. The formula for the least squares approximation is $0.032502 + -0.000082t$, and the root mean square value, from Figure C.186, is 0.035806. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.186, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

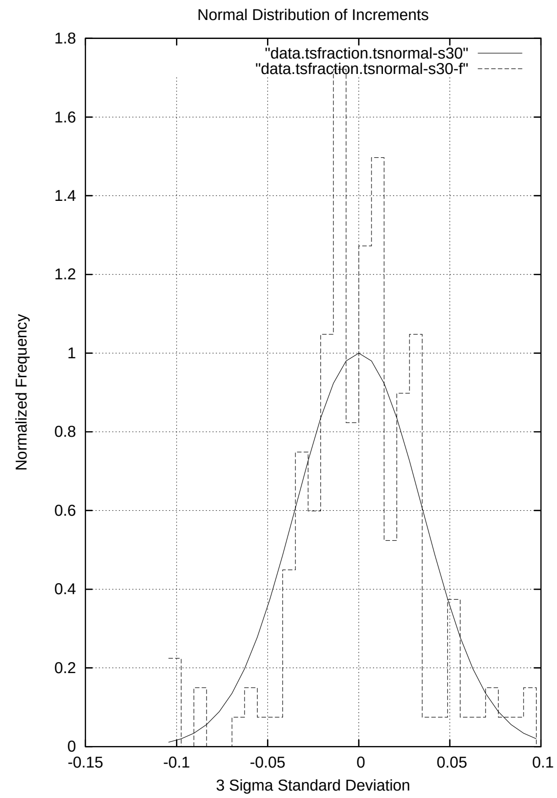

Figure C.188: Dow Jones Average, normalized histogram of the normalized increments of the time series data shown in Figure C.186. The data has a mean of 0.008836, with a standard deviation of 0.034803. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 8.043000, with a critical value of 42.557000.

is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.191 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.186. In principle, if the distribution of the normalized increments presented in Figure C.188 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

```
For  a mean  of 0.008784,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000878,   4496  samples  would  be  required.
     (With  168  samples,   the  estimated   error  is 0.004544   = 51.731212   percent.)
For  a standard   deviation   of 0.035806,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.003581,   136  samples  would  be  required.
     (With  168  samples,   the  estimated   error  is 0.003213   = 8.973412   percent.)
```

Figure C.189: Dow Jones Average, statistical estimates of the normalized increments of the time series shown in Figure C.186. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.186.

Figure C.192 is the range of values of the time series shown in Figure C.185. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.192 would be a square root function[111]. Figure C.193 is the deterministic map of the normalized increments of the time series data shown in Figure C.186. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.188 would seem to indicate that the time series data for the Dow Jones Average represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.9.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[112]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.194 is the instantaneous value of the root mean square of the normalized increments for the Dow Jones Average, and Figure C.195 is the instantaneous Shannon probability for the normalized increments.

C.9.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.9.4. Figure C.196 is a graph of the logistic function estimates of the time series data for the Dow Jones Average. The reader is cautioned that these graphs are constructed

[111] Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.192 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[112] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is $\approx$ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

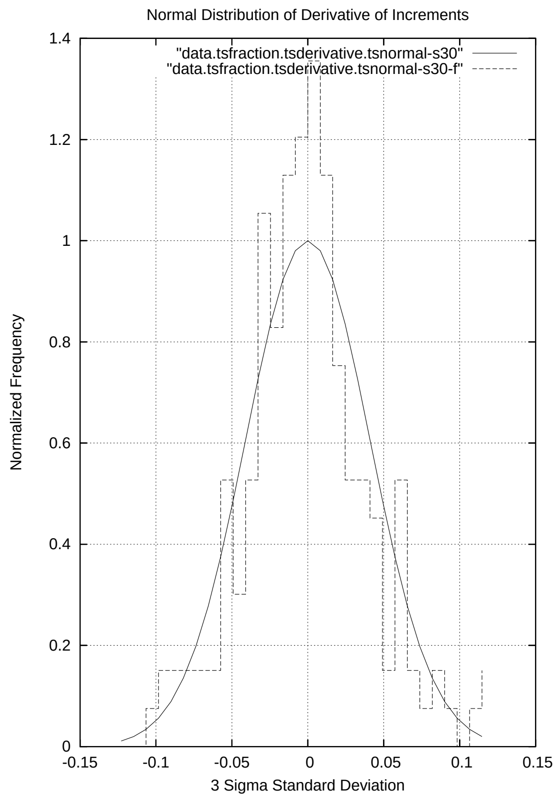

Figure C.190: Dow Jones Average, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.186.

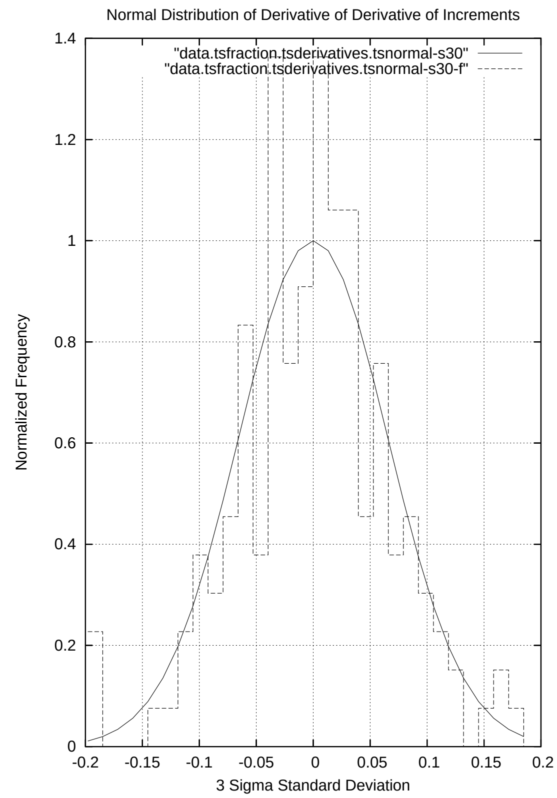

Figure C.191: Dow Jones Average, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.186.

using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[113]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.196 is a graph of the logistic function for the time series data presented in Figure C.185. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.186. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.186. Figure C.197 is the same graph, but with the time scale expanded by a factor of two.

[113]For example, in Figures C.196 and C.197, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.9.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

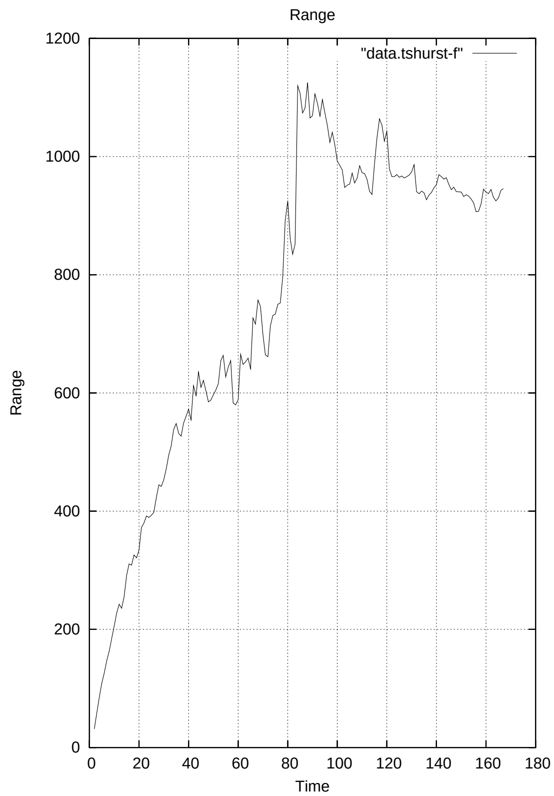

Figure C.192: Dow Jones Average, range of the time series data shown in Figure C.185.

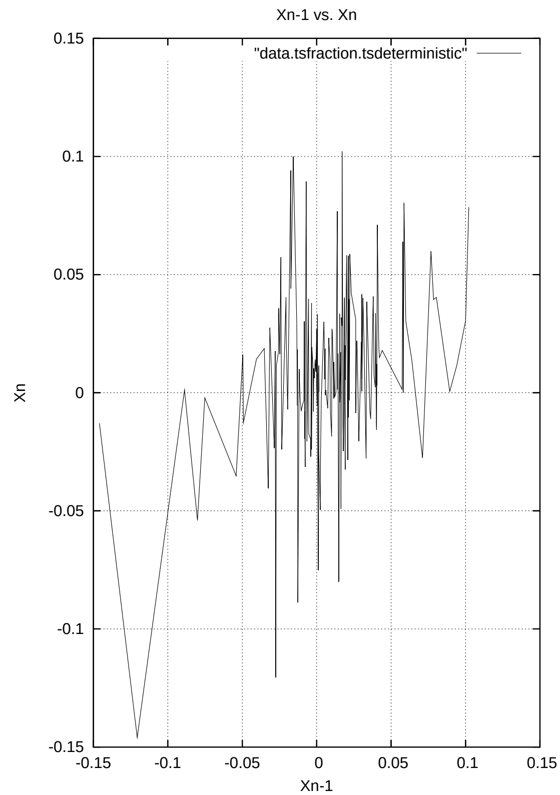

Figure C.193: Dow Jones Average, deterministic map of the normalized increments of the time series data shown in Figure C.186.

C.9.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.9.5. Figure C.198 is a graph of the Hurst coefficient data time series data shown in Figure C.185. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.199 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.186. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.198 implies that the variance of the rate of revenue returns, (per month,) in the Dow Jones Average, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[114], t, $p(t) = erf(1/\sqrt{2t})$ which

[114]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp.

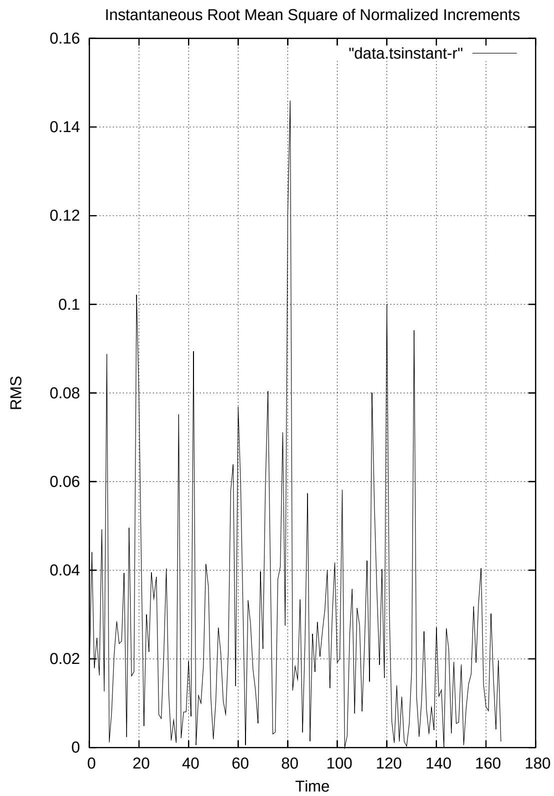

Figure C.194: Dow Jones Average, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.185.

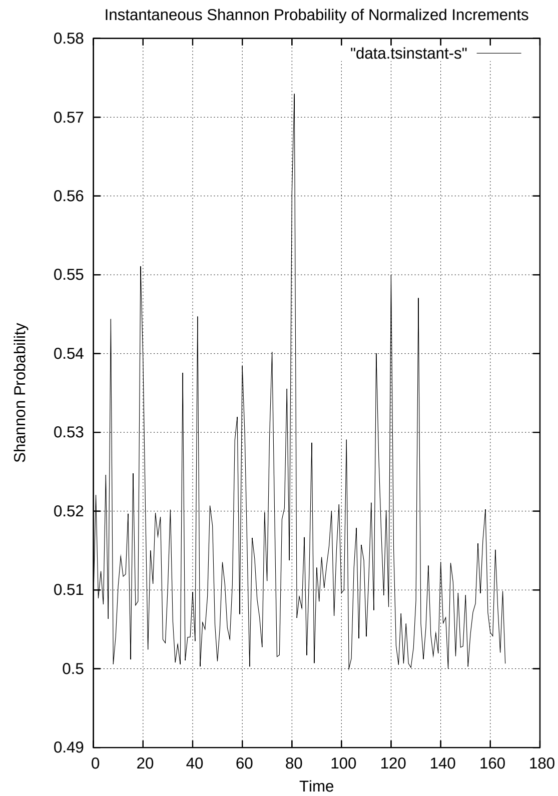

Figure C.195: Dow Jones Average, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.185.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.202, and, C.203 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Dow Jones Average for the near term and far term, respectively [Pet91, pp. 83-84][115]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.198, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.891560, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.346}$$

153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[115] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

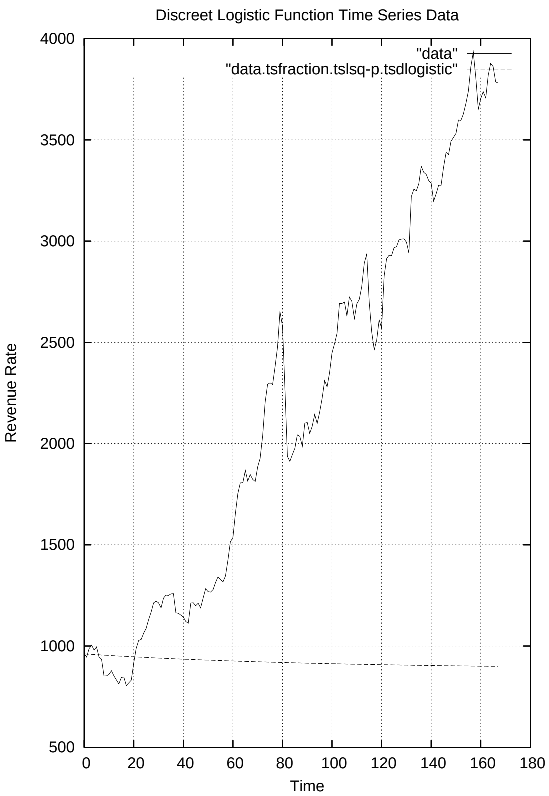

Figure C.196: Dow Jones Average, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.186 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

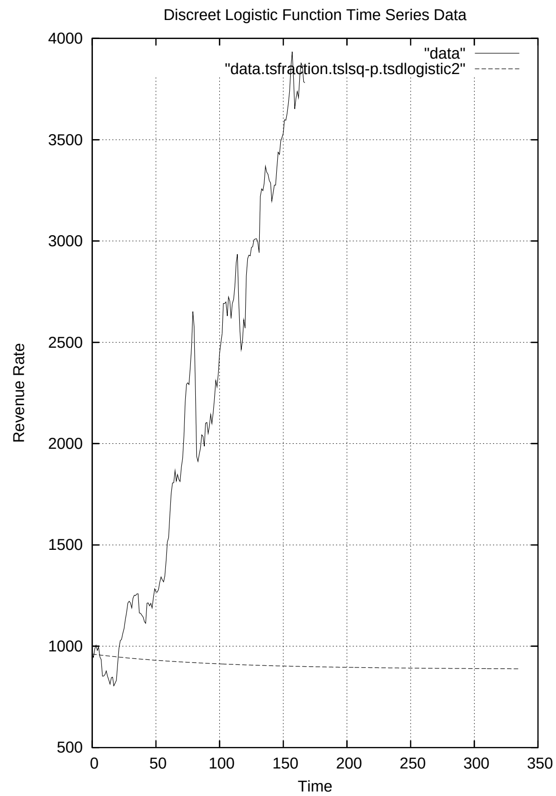

Figure C.197: Dow Jones Average, logistic function estimates of Figure C.196 with the time scale expanded by a factor of two.

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot 0.891560} \tag{C.347}$$

$$\propto \quad (t_2 - t_1)^{1.783120} \tag{C.348}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[116]. A Hurst coefficient of 0.891560, (for the near future, and 0.566146 for the distant future.) implies

[116] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.9.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Dow Jones Average. See also [Pet91, pp. 67, 83-84]

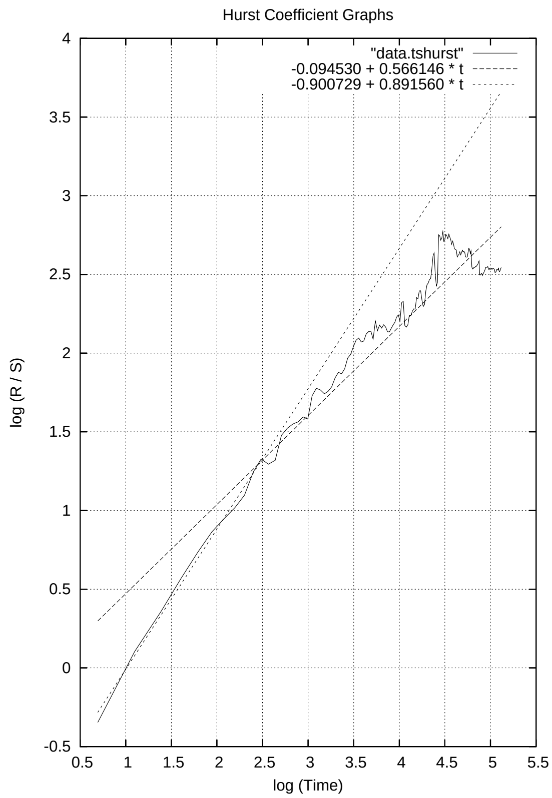

Figure C.198: Dow Jones Average, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.186. The slope of the graph is the Hurst coefficient.

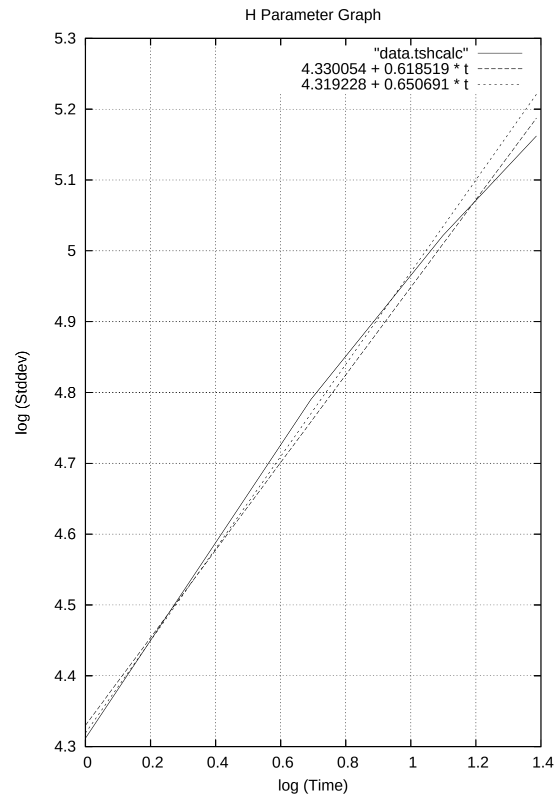

Figure C.199: Dow Jones Average, H parameter data for the normalized increments of the time series data shown in Figure C.186 The slope of the graph is the H parameter.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 89.156000% [Pet91, pp. 66] for the near future, and 0.566146 for the distant future. Likewise, there is a 89.156000% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 89.156000% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the Dow Jones Average are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.349}$$

$$= 0.891560^n \tag{C.350}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.186, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would

and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.008836 \cdot 100$ percent, on the average, with a standard deviation of $0.034803 \cdot 100$ percent, and a root mean square error value of $0.035806 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^H \tag{C.351}$$
$$\propto\; (t_2 - t_1)^{0.891560} \tag{C.352}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.352 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^{\frac{1}{2}} \tag{C.353}$$
$$\propto\; \sqrt{(t_2 - t_1)} \tag{C.354}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.355}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[117].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \;\propto\; \frac{X\,(rt)}{r^H} \tag{C.356}$$
$$\propto\; \frac{X\,(rt)}{r^{0.891560}} \tag{C.357}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.199, to provide a least squares approximation to the H parameter for the Dow Jones Average. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.650691 for the near future, and 0.618519 for the distant future.

Figures C.198 and C.199 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.186. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.186, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.200 and C.201 was made using the -d option.

[117] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

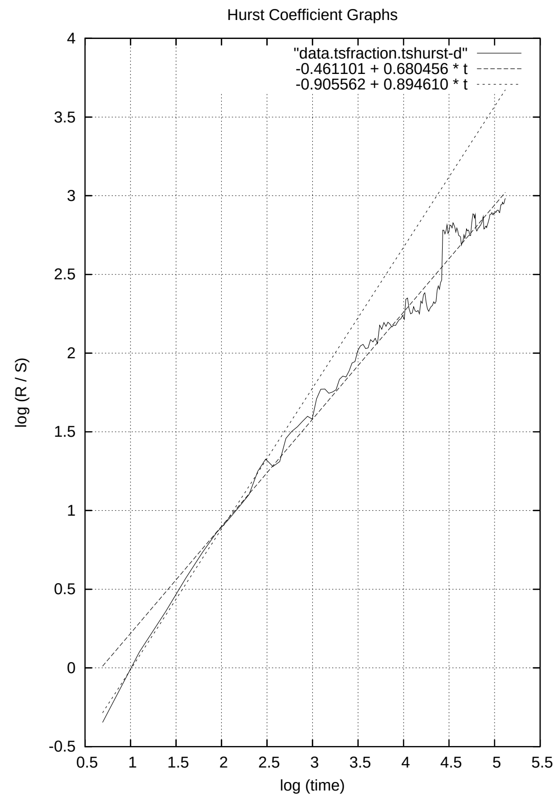

Figure C.200: Dow Jones Average, traditional Hurst coefficient data for the time series data shown in Figure C.185. The slope of the graph is the Hurst coefficient, and is 0.894610 for the near term, and 0.680456 for the far term.

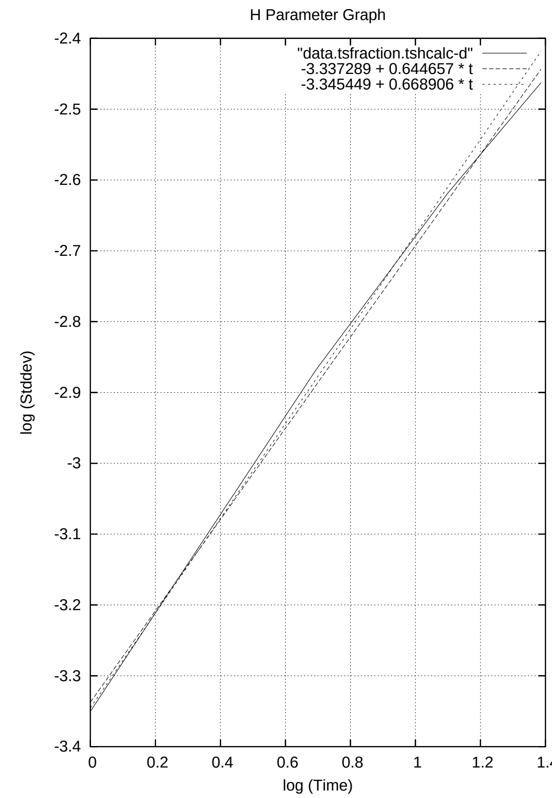

Figure C.201: Dow Jones Average, traditional H parameter data for the time series data shown in Figure C.185 The slope of the graph is the H parameter, and is 0.668906 for the near term, and 0.644657 for the far term.

C.9.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.9.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.187. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.186. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Dow Jones Average, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Dow Jones Average, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in

Chapter B, and is presented in Figure C.186, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.008836 + 1)}{\ln(2)} = 0.012692 \tag{C.358}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.186, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.009746 + 1)}{\ln(2)} = 0.013992 \tag{C.359}$$

Note that if the mean is not constant in Figure C.186, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.185:

$$bits = 0.014084 \tag{C.360}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.185:

$$bits = 0.011753 \tag{C.361}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.9.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.011753t} \tag{C.362}$$

therefore:

$$C(p) = 0.011753 \tag{C.363}$$

and, *tsshannon* 0.011753 gives:

$$C(0.563735) = 0.011753 \tag{C.364}$$

therefore:

$$\begin{aligned} 2^{C(0.563735)} &= 2^{0.011753} && \text{(C.365)} \\ &= 1.008180 && \text{(C.366)} \\ &= 0.817983\% && \text{(C.367)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.563735) - 1 && \text{(C.368)} \\ &= 0.127470 && \text{(C.369)} \\ &= 12.747000\% && \text{(C.370)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the Dow Jones Average executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 12.747000% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 87.253000% will be held in “reserve” with a 56.373500% chance of making twice the 12.747000% back, (and a 43.626500% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.817983%, or a doubling of its rate of revenue returns, (per month,) in 85.084659 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 12.747000% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.817983%, per month, on average.

Note that the metrics presented in this section are representative of the Dow Jones Average as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 12.747000% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the Dow Jones Average's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.817983% per month.

As another simple example, a company re-invests 12.747000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 12.747000% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.817983% per month.

As an example of "product portfolio" management, suppose a company re-invests 12.747000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 12.747000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 12.747000$ percent for the second product, implying that the company should diversify its product line[118]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 12.747000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Dow Jones Average, as a standard bench mark, then the optimal number will be $\frac{1}{0.127470}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.186, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider

[118]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

the issue of product mix. In this interpretation, 12.747000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 12.747000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.9.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Dow Jones Average, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Dow Jones Average time series is 0.008836, and 0.035806respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 6.891981.

If this value seems consistent number of companies in the Dow Jones Average, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Dow Jones Average are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.547000, which would be the value which should be used in Section C.9.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.9.5 is greater than the average Shannon probability for the companies participating in the Dow Jones Average, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.9.5. The maximum exploitability for the Dow Jones Average is derived in Section C.9.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Dow Jones Average is 0.547000, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.623387 in the Dow Jones Average. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.371}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Dow Jones Average would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.9.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.187. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.186. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Dow Jones Average, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.9.5, is derived from the Dow Jones Average metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.9.9.

An additional exploitable strategy may be time itself. Equations C.348, C.352, and, C.350, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Dow Jones Average, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.202, and, C.203 compare methods of approximation of the "forecastability" of rate of revenue returns in the Dow Jones Average for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Dow Jones Average. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[119]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.202, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.350, $0.891560^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.348, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.9.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.9.9. Figure C.204 represents a constructional simulation of the time series data presented in Figure C.185. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.186. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.186 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.205 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.188.

C.9.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.9.3. One of the issues of analysis, as mentioned in Section C.9.7, is to determine the maximum Shannon probability for the time series presented in Figure C.185. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.206 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.185. Figure C.207 was constructed using *tsunfairbrownian* program,

[119]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

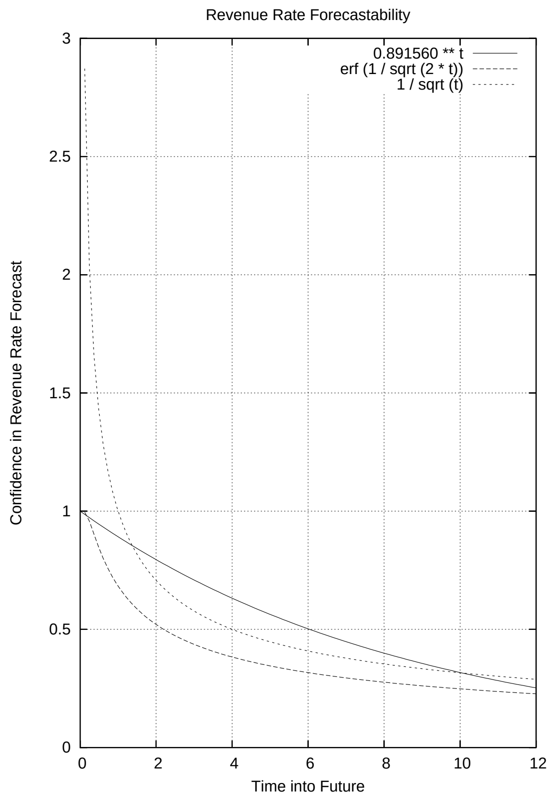

Figure C.202: Dow Jones Average, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.891560^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

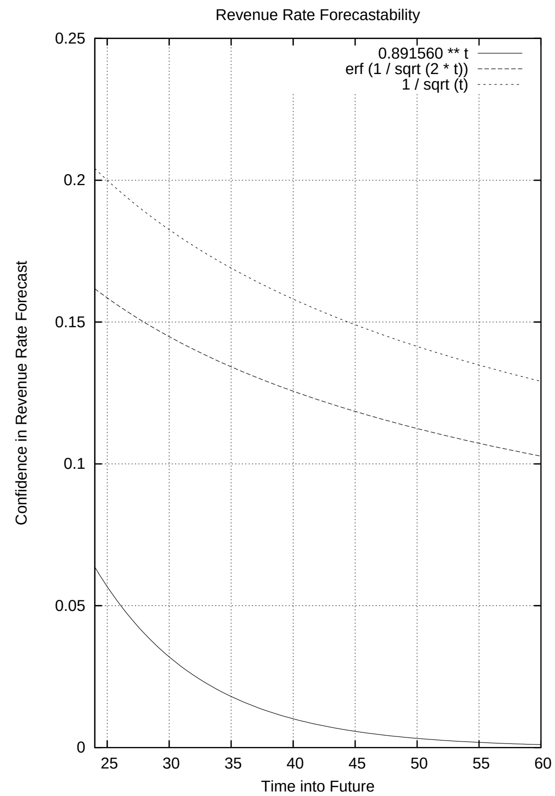

Figure C.203: Dow Jones Average, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.185. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.563735, as derived in Section C.9.5 to 0.636905. This process, essentially, extracts the random statistical data from the time series presented in Figure C.185, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.185, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the Dow Jones Average movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.634731, as compared with the predicted value from the program *tsshannonmax* of 0.636905.

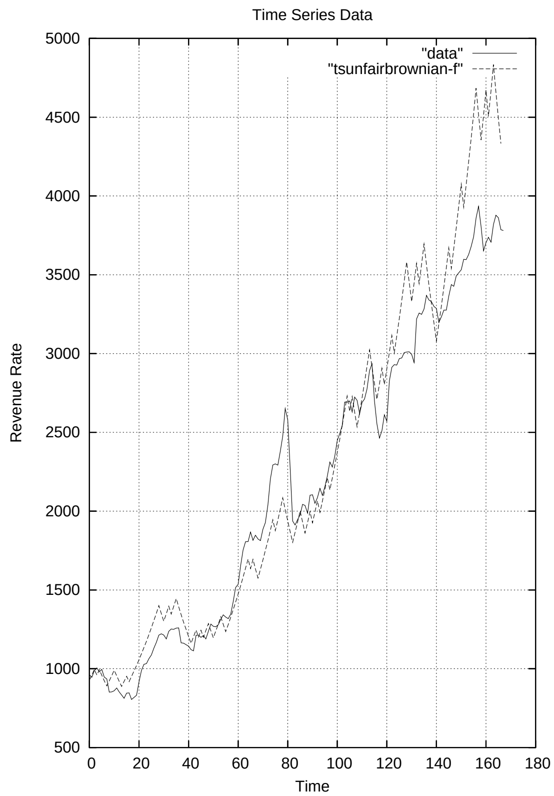

Figure C.204: Dow Jones Average, Time series data, empirical and simulated, using the program *tsunfair-brownian* with f = 0.035806. This data is superimposed on the data presented in Figure C.185.

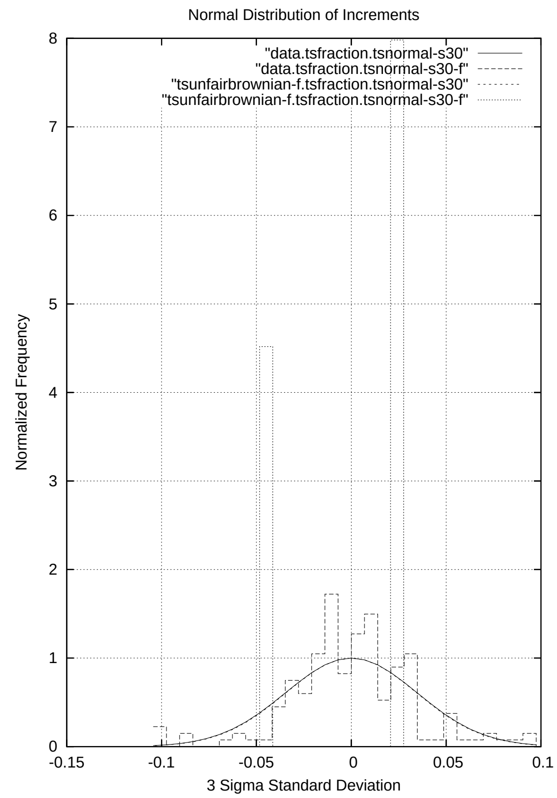

Figure C.205: Dow Jones Average, normalized histogram of the normalized increments of the time series data shown in Figure C.204, empirical and simulated. The empirical data has a mean of 0.008836, with a standard deviation of 0.034803. By comparison, the simulated data has a mean of 0.009922 with a standard deviation of 0.034508. This data is superimposed on the data presented in Figure C.188. The area under the four curves is identical.

C.9.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.187. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.186. These values will be used in a fixed increment Brownian fractal analysis of the Dow Jones Average, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.9.6 and D.9.7. As a subjective evaluation of the "quality" of the analysis of the Dow Jones Average, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.185 from Figure C.186, and the Shannon probability as calculated by counting the total number of months that the Dow Jones Average movement was positive, as presented in Section C.9.9:

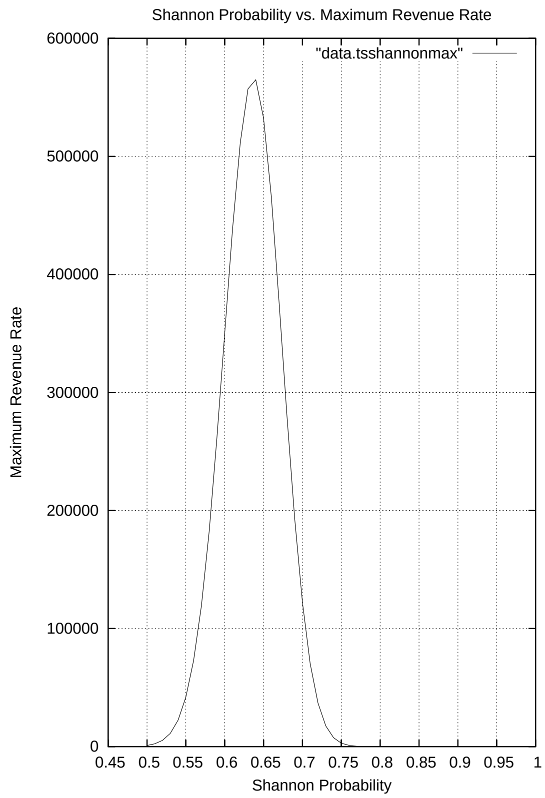

Figure C.206: Dow Jones Average, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.636905.

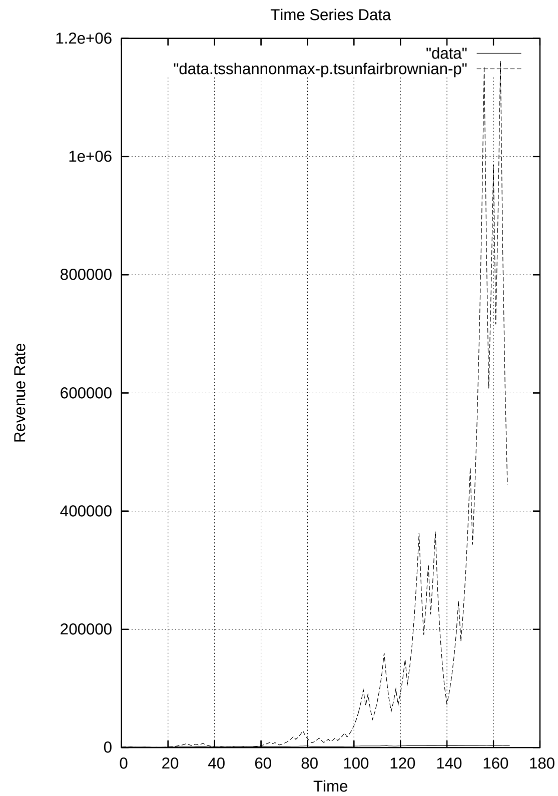

Figure C.207: Dow Jones Average, maximum rate of revenue returns, per month, at a Shannon probability, of 0.636905, corresponding to a “wager” fraction of 0.273810.

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.372}$$

$$0.634731 \approx \frac{\frac{0.008836}{0.035806} + 1}{2} \tag{C.373}$$

$$0.634731 \approx 0.623387 \tag{C.374}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.634731 \approx 0.623387 \approx 0.636905 \tag{C.375}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.9.5, should be compared. The four methods used were the mean of Figure C.186, the constant in the least squares approximation to

Figure C.186, the least squares exponential approximation to Figure C.185, and the logarithmic returns of Figure C.185, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.012692 \approx 0.013992 \approx 0.014084 \approx 0.011753 \tag{C.376}$$

It is implied in Section C.9.5, Subsection C.9.5 and in Section C.9.8 that, a Brownian motion with fixed increments fractal may "model" the Dow Jones Average. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.377}$$

$$0.035806\,(2 \cdot 0.634731 - 1) \approx \frac{0.034803\,(2 \cdot 0.634731 - 1)}{2\sqrt{0.634731\,(1-0.634731)}} \tag{C.378}$$

$$0.035806 \cdot 0.269461 \approx 0.034803 \cdot 0.279811 \tag{C.379}$$

$$0.009648 \approx 0.009738 \tag{C.380}$$

and, equating to the mean:

$$0.008836 \approx 0.009648 \approx 0.009738 \tag{C.381}$$

where, as in Equation C.374 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.185 from Figure C.186, and the Shannon probability as calculated by counting the total number of months that the Dow Jones Average movement was positive, as presented in Section C.9.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[120], where the absolute value is presented in Figure C.187, and the root mean square value is presented in Figure C.186:

$$0.025721 \approx 0.035806 \tag{C.382}$$

Note, that if the Dow Jones Average could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.185 from Figure C.186 should be zero. It is 0.024985.

C.10 Cirrus Logic Stock

For the analysis, the data was in the directory ../markets/crus[121].

The data in this section is presented in tabular form in Section D.10. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Cirrus Logic Stock. This is included for "theoretical" comparative purposes, and has no meaning, unless it is considered as a "future."

[120] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[121] Cirrus Logic stock price, November 15, 1994, through April 8, 1996, inclusive. The data is by days.

C.10.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.10.1. Figure C.208 is a graph of the time series data for the Cirrus Logic Stock.

Figure C.209 is a graph of the normalized increments of the time series data presented in Figure C.208. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.210 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.209. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[122].

Figure C.211 is the normalized histogram of the normalized increments of the time series data shown in Figure C.209. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.211.

Figure C.212 is the statistical estimate for the data presented in Figure C.209, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.521329$, as derived in Section C.10.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.213 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.209. In principle, if the distribution of the normalized increments presented in Figure C.211 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.214 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.209. In principle, if the distribution of the normalized increments presented in Figure C.211 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.215 is the range of values of the time series shown in Figure C.208. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.215 would be a square root function[123]. Figure C.216 is the deterministic map of the normalized increments of the time series data shown in Figure C.209. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.211 would seem to indicate that the time series data for the Cirrus Logic Stock represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

[122] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[123] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.215 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

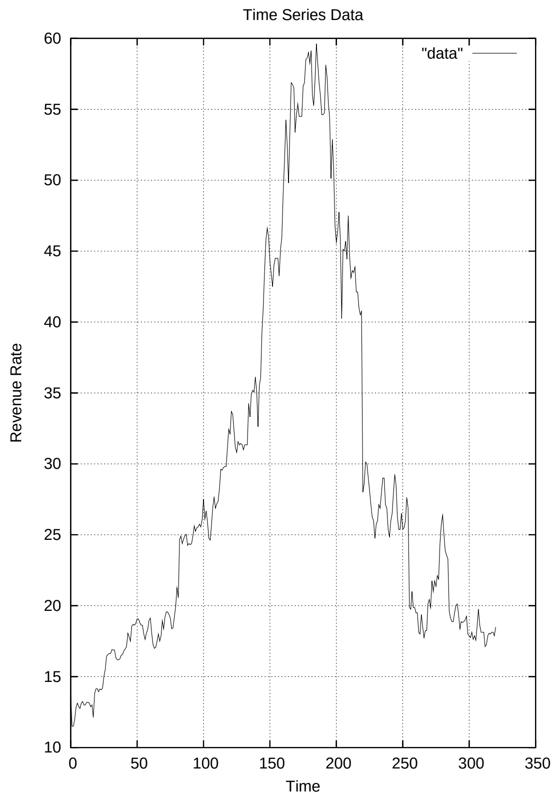

Figure C.208: Cirrus Logic Stock, time series data.

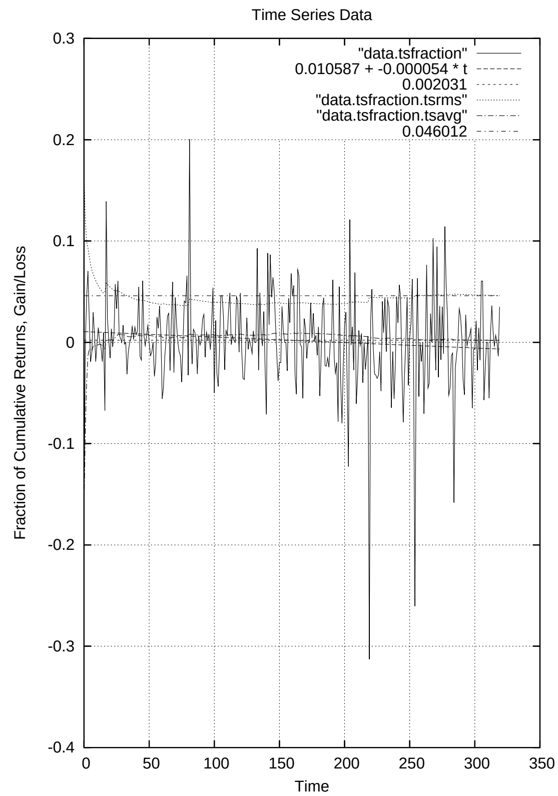

Figure C.209: Cirrus Logic Stock, normalized increments of the time series data presented in Figure C.208. The mean is 0.002031 with a standard deviation of 0.046039. The formula for the least squares approximation is $0.010587 + -0.000054t$, and the root mean squared value is 0.046012. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000054, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

C.10.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1

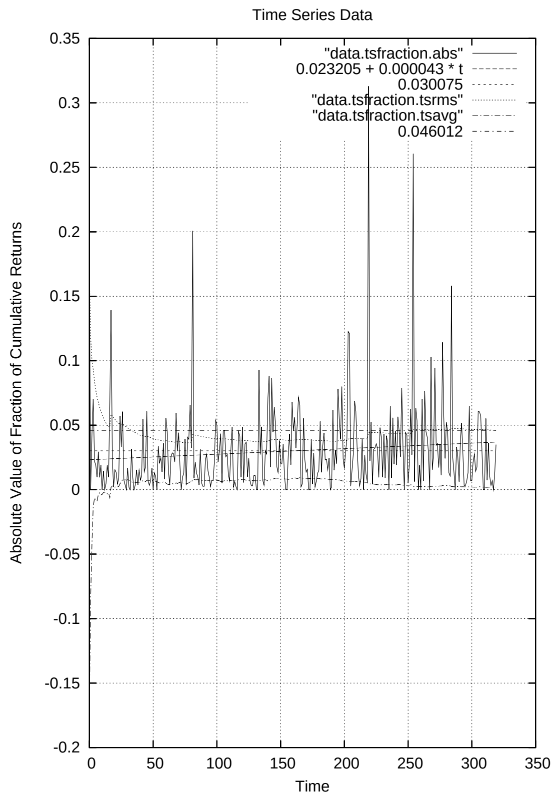

Figure C.210: Cirrus Logic Stock, absolute value of the normalized increments of the time series data presented in Figure C.209. The mean is 0.030075 with a standard deviation of 0.034876. The formula for the least squares approximation is $0.023205 + 0.000043t$, and the root mean square value, from Figure C.209, is 0.046012. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments presented in Figure C.209, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

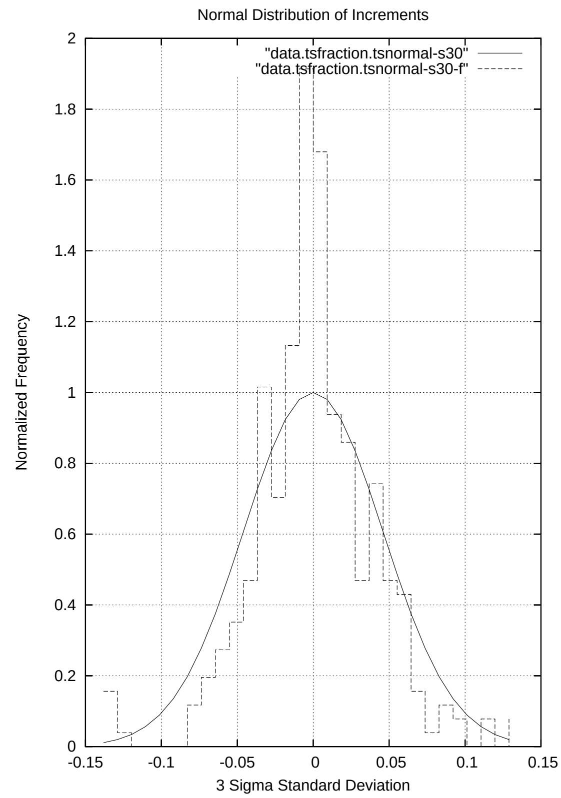

Figure C.211: Cirrus Logic Stock, normalized histogram of the normalized increments of the time series data shown in Figure C.209. The data has a mean of 0.002031, with a standard deviation of 0.046039. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 4.817000, with a critical value of 42.557000.

for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[124]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the

[124] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

```
For a mean of 0.002025, with a confidence level of 0.900000
    that the error did not exceed 0.000202, 139735 samples would be required.
    (With 321 samples, the estimated error is 0.004224 = 208.640688 percent.)
For a standard deviation of 0.046012, with a confidence level of 0.900000
    that the error did not exceed 0.004601, 136 samples would be required.
    (With 321 samples, the estimated error is 0.002987 = 6.491719 percent.)
```

Figure C.212: Cirrus Logic Stock, statistical estimates of the normalized increments of the time series shown in Figure C.209. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.209.

Shannon probability of the instantaneous fraction of change.

Figure C.217 is the instantaneous value of the root mean square of the normalized increments for the Cirrus Logic Stock, and Figure C.218 is the instantaneous Shannon probability for the normalized increments.

C.10.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.10.4. Figure C.219 is a graph of the logistic function estimates of the time series data for the Cirrus Logic Stock. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[125]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.219 is a graph of the logistic function for the time series data presented in Figure C.208. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.209. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.209. Figure C.220 is the same graph, but with the time scale expanded by a factor of two.

C.10.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.10.5. Figure C.221 is a graph of the Hurst coefficient data time series data shown in Figure C.208. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.222 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.209. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.221 implies that the variance of the rate of revenue returns, (per day,) in the Cirrus Logic Stock, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating

[125]For example, in Figures C.219 and C.220, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.10.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

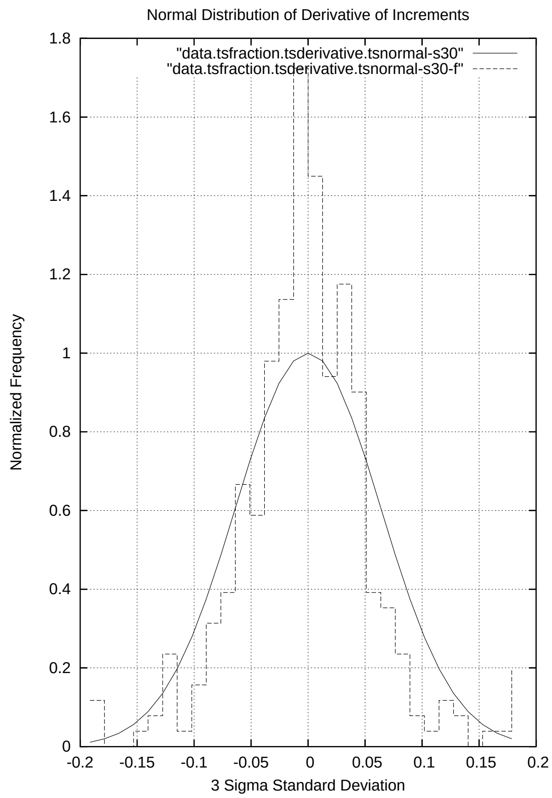

Figure C.213: Cirrus Logic Stock, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.209.

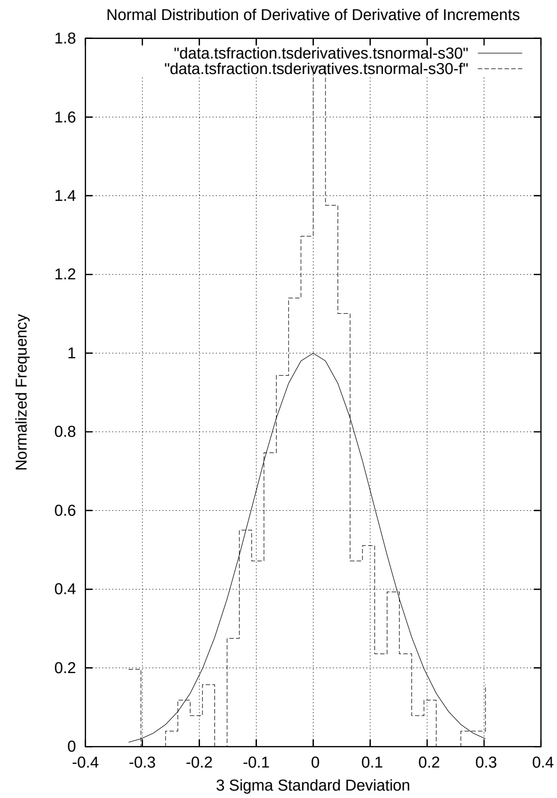

Figure C.214: Cirrus Logic Stock, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.209.

sometime in the future goes down with increasing time[126], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.225, and, C.226 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Cirrus Logic Stock for the near term and far term, respectively [Pet91, pp. 83-84][127]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per day.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.221, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.887589, so that:

[126]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[127]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

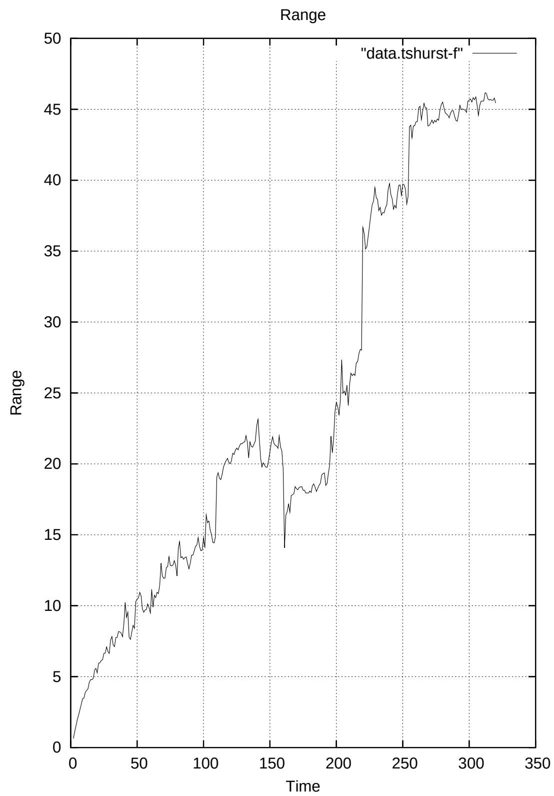

Figure C.215: Cirrus Logic Stock, range of the time series data shown in Figure C.208.

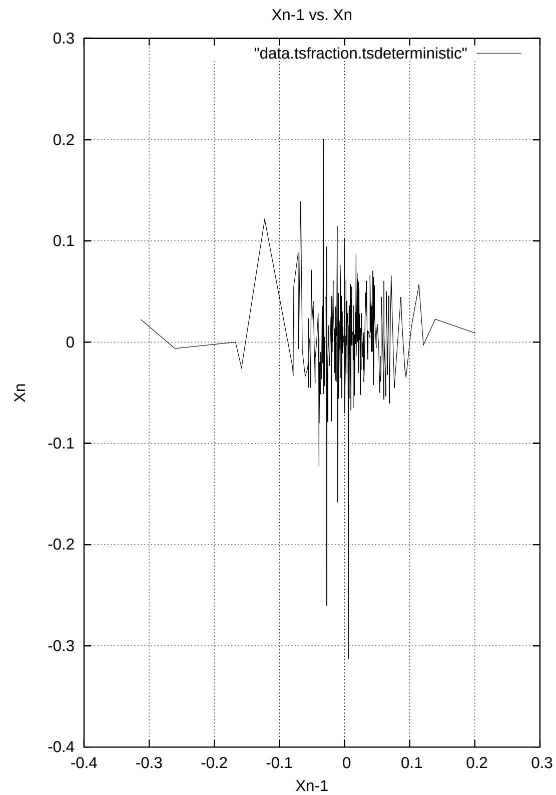

Figure C.216: Cirrus Logic Stock, deterministic map of the normalized increments of the time series data shown in Figure C.209.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \quad \text{(C.383)}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.887589} \quad \text{(C.384)}$$

$$\propto (t_2 - t_1)^{1.775178} \quad \text{(C.385)}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per day,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[128]. A Hurst coefficient of 0.887589, (for the near future, and 0.706220 for the distant future.) implies that the likelihood of the rate of revenue returns, (per day,) for any two consecutive days being the same is 88.758900% [Pet91,

[128]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term"

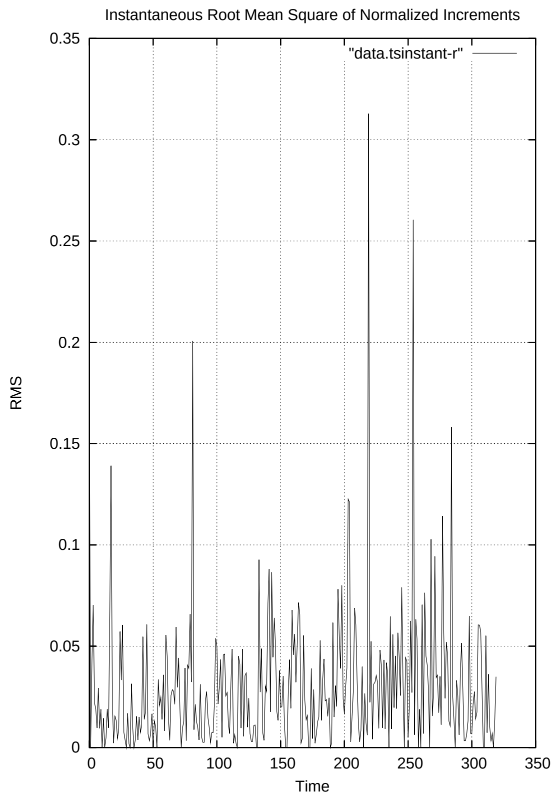

Figure C.217: Cirrus Logic Stock, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.208.

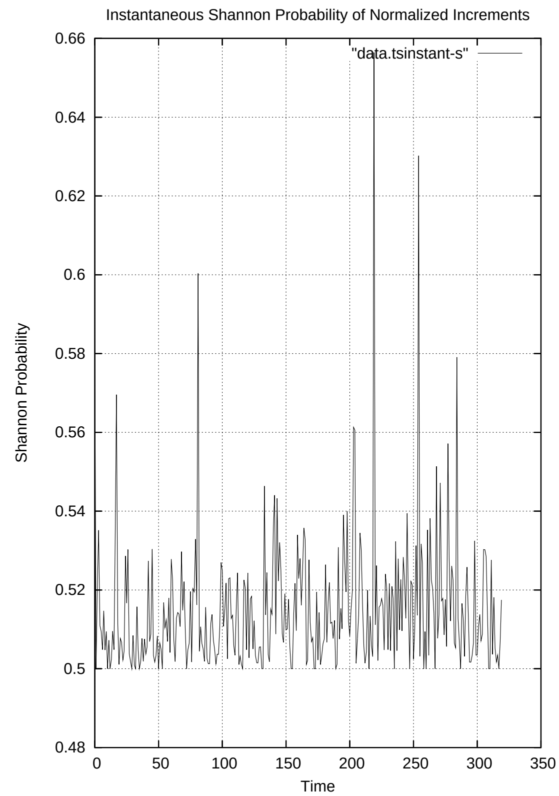

Figure C.218: Cirrus Logic Stock, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.208.

pp. 66] for the near future, and 0.706220 for the distant future. Likewise, there is a 88.758900% chance of the rate of revenue returns, (per day,) movements being the same in consecutive time periods—ie., if, in a given day, the rate of revenue returns, (per day,) is increasing, there is a 88.758900% that the rate of revenue returns, (per day,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per day,) for the Cirrus Logic Stock are over time, since the probability of having n many consecutive days of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many days of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.386}$$

$$= 0.887589^n \tag{C.387}$$

and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.10.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Cirrus Logic Stock. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

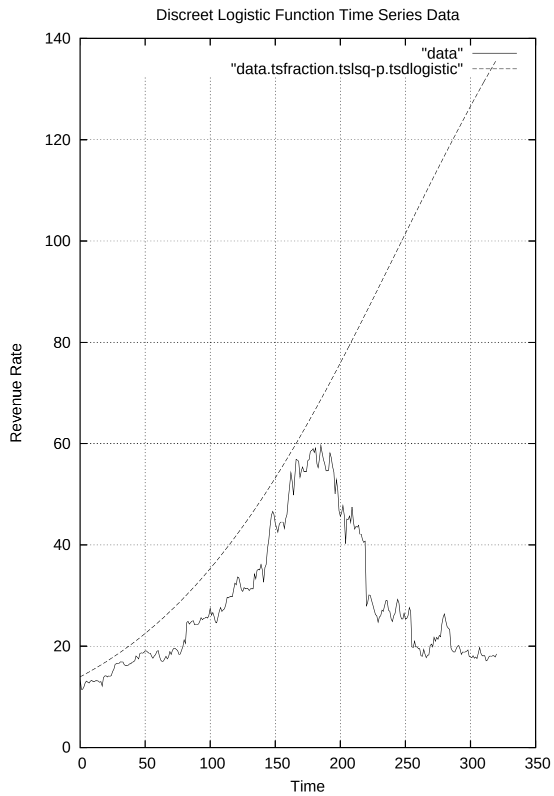

Figure C.219: Cirrus Logic Stock, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.209 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

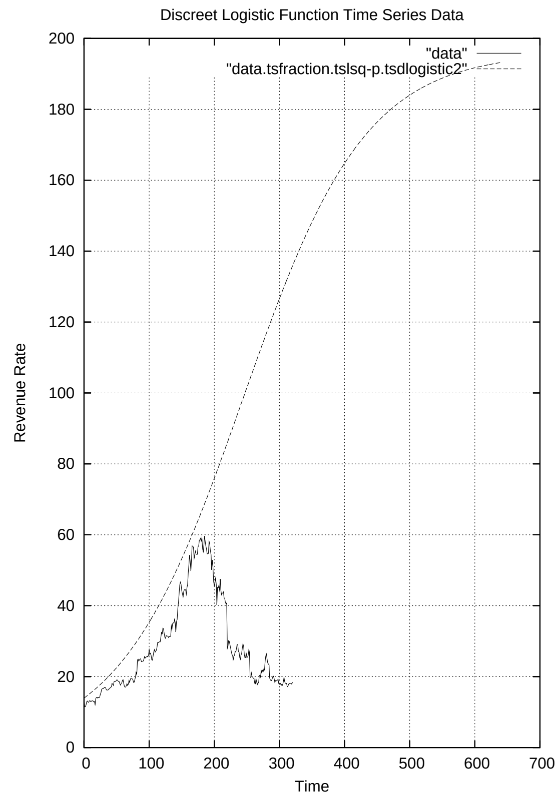

Figure C.220: Cirrus Logic Stock, logistic function estimates of Figure C.219 with the time scale expanded by a factor of two.

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.209, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next day's rate of revenue returns would be the same as the current day's revenue rate. Interestingly, it is $0.002031 \cdot 100$ percent, on the average, with a standard deviation of $0.046039 \cdot 100$ percent, and a root mean square error value of $0.046012 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per day,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^H \tag{C.388}$$

$$\propto \quad (t_2 - t_1)^{0.887589} \tag{C.389}$$

where R is the range of values in the increments of the rate of revenue returns, (per day.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of

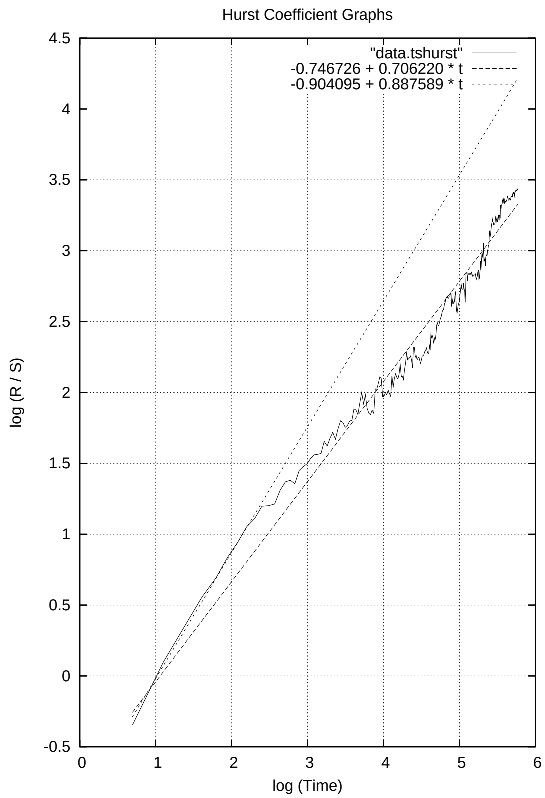

Figure C.221: Cirrus Logic Stock, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.209. The slope of the graph is the Hurst coefficient.

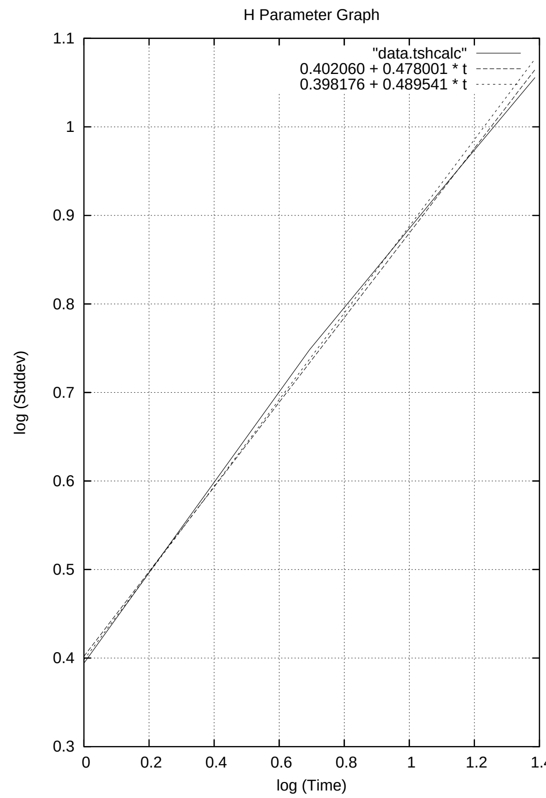

Figure C.222: Cirrus Logic Stock, H parameter data for the normalized increments of the time series data shown in Figure C.209 The slope of the graph is the H parameter.

revenue returns, (per day,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per day) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.389 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^{\frac{1}{2}} \tag{C.390}$$

$$\propto\; \sqrt{(t_2 - t_1)} \tag{C.391}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per day,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.392}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per day,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[129].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.393}$$

$$\propto \frac{X(rt)}{r^{0.887589}} \tag{C.394}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.222, to provide a least squares approximation to the H parameter for the Cirrus Logic Stock. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.489541 for the near future, and 0.478001 for the distant future.

Figures C.221 and C.222 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.209. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.209, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.223 and C.224 was made using the -d option.

C.10.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.10.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.210. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.209. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Cirrus Logic Stock, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Cirrus Logic Stock, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.209, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.002031 + 1)}{\ln(2)} = 0.002927 \tag{C.395}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.209, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.010587 + 1)}{\ln(2)} = 0.015194 \tag{C.396}$$

Note that if the mean is not constant in Figure C.209, this method will not provide accurate results.

[129]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

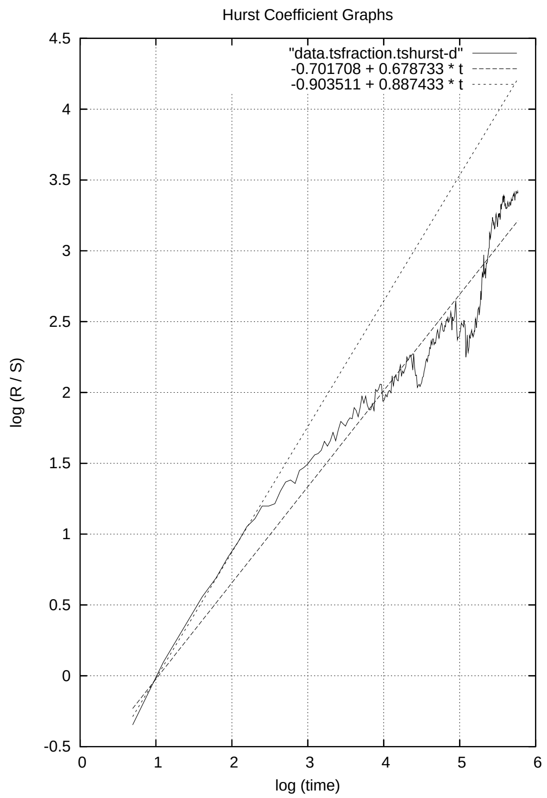

Figure C.223: Cirrus Logic Stock, traditional Hurst coefficient data for the time series data shown in Figure C.208. The slope of the graph is the Hurst coefficient, and is 0.887433 for the near term, and 0.678733 for the far term.

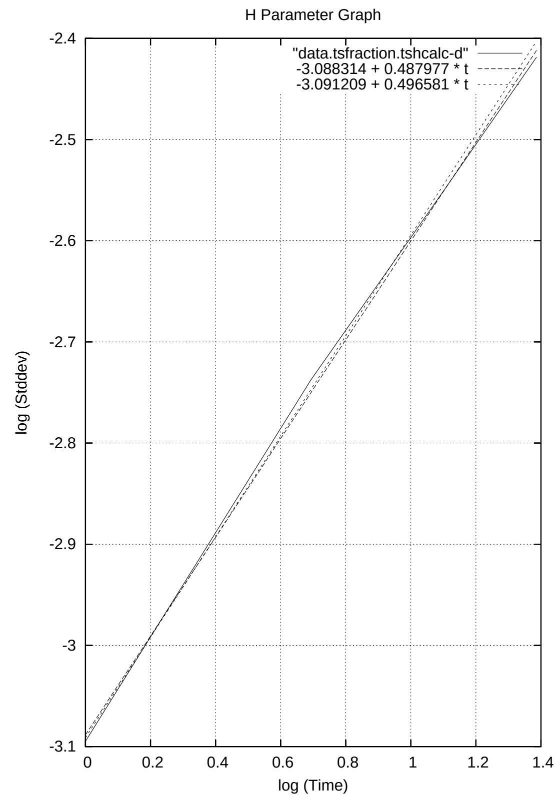

Figure C.224: Cirrus Logic Stock, traditional H parameter data for the time series data shown in Figure C.208 The slope of the graph is the H parameter, and is 0.496581 for the near term, and 0.487977 for the far term.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.208:

$$bits = 0.001768 \tag{C.397}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.208:

$$bits = 0.001313 \tag{C.398}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.10.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.001313t} \tag{C.399}$$

therefore:

$$C(p) = 0.001313 \tag{C.400}$$

and, *tsshannon* 0.001313 gives:

$$C(0.521329) = 0.001313 \tag{C.401}$$

therefore:

$$2^{C(0.521329)} = 2^{0.001313} \tag{C.402}$$
$$= 1.000911 \tag{C.403}$$
$$= 0.091052\% \tag{C.404}$$

and:

$$2p - 1 = (2 \cdot 0.521329) - 1 \tag{C.405}$$
$$= 0.042658 \tag{C.406}$$
$$= 4.265800\% \tag{C.407}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Cirrus Logic Stock executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every day, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 4.265800% of its rate of revenue returns, (per day.) As a conceptual model, the remaining 95.734200% will be held in "reserve" with a 52.132900% chance of making twice the 4.265800% back, (and a 47.867100% chance of making 0.0,) in one day, on the average, for an average growth in its rate of revenue returns, (per day,) of 0.091052%, or a doubling of its rate of revenue returns, (per day,) in 761.614623 days.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 4.265800% per day of the rate of revenue returns, (per day,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.091052%, per day, on average.

Note that the metrics presented in this section are representative of the Cirrus Logic Stock as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 4.265800% of its rate of revenue returns, (per day,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some days, depending on the Cirrus Logic Stock's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other days, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.091052% per day.

As another simple example, a company re-invests 4.265800% of its rate of revenue returns, (per day,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 4.265800% per day investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.091052% per day.

As an example of "product portfolio" management, suppose a company re-invests 4.265800% of its rate of revenue returns, (per day,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 4.265800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 4.265800$ percent for the second product, implying that the company should diversify its product line[130]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 4.265800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Cirrus Logic Stock, as a standard bench mark, then the optimal number will be $\frac{1}{0.042658}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.209, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 4.265800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 4.265800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.10.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Cirrus Logic Stock, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Cirrus Logic Stock time series is 0.002031, and 0.046012respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.959329.

If this value seems consistent number of companies in the Cirrus Logic Stock, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Cirrus Logic Stock are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.522533, which would be the value which should be used in Section C.10.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.10.5 is greater than the average Shannon probability for the companies participating in the Cirrus Logic Stock, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.10.5. The maximum exploitability for the Cirrus Logic Stock is derived in Section C.10.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in

[130] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Cirrus Logic Stock is 0.522533, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.522070 in the Cirrus Logic Stock. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.408}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Cirrus Logic Stock would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.10.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.210. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.209. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Cirrus Logic Stock, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.10.5, is derived from the Cirrus Logic Stock metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.10.9.

An additional exploitable strategy may be time itself. Equations C.385, C.389, and, C.387, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Cirrus Logic Stock, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.225, and, C.226 compare methods of approximation of the "forecastability" of rate of revenue returns in the Cirrus Logic Stock for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Cirrus Logic Stock. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[131]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.225, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over

[131] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

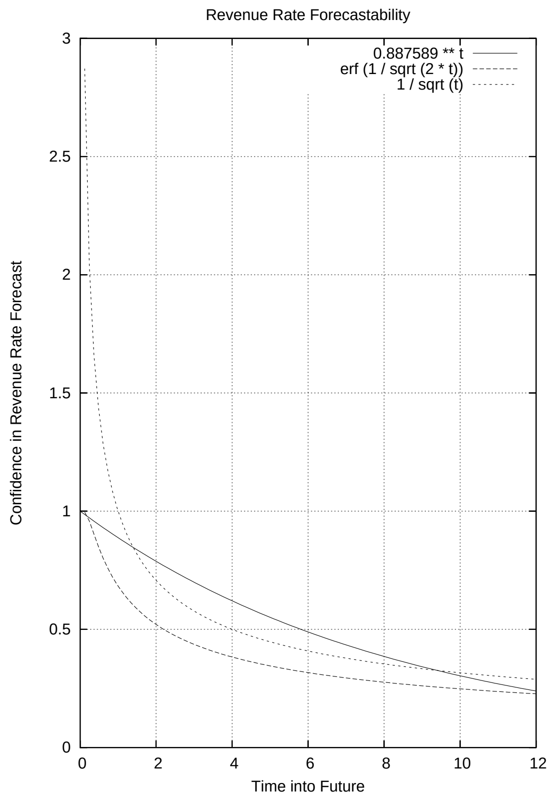

Figure C.225: Cirrus Logic Stock, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.887589^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

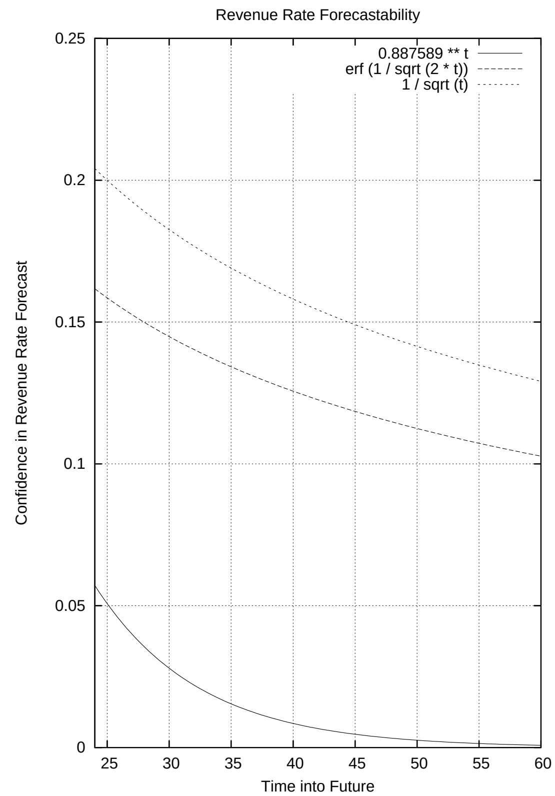

Figure C.226: Cirrus Logic Stock, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.387, $0.887589^n = 0.5$ days of operations. Since the optimal amount of inventory and, from Equation C.385, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.10.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.10.9. Figure C.227 represents a constructional simulation of the time series data presented in Figure C.208. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented

in Figure C.209. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.209 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.228 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.211.

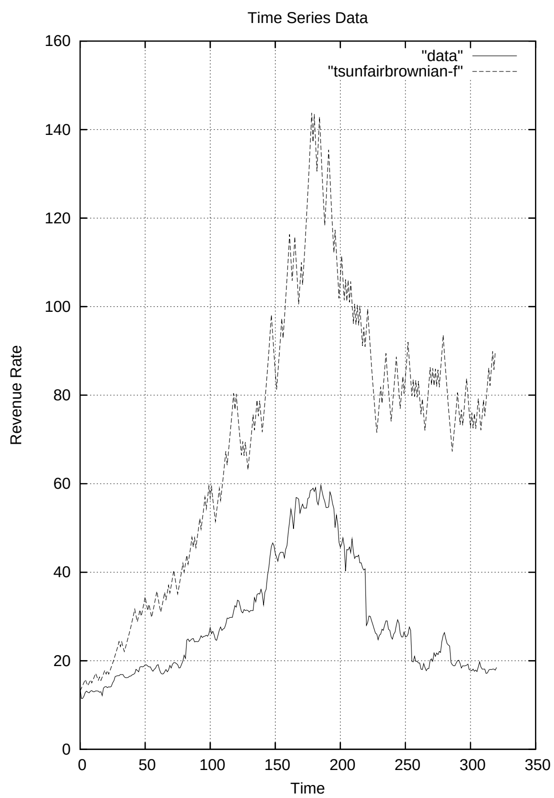

Figure C.227: Cirrus Logic Stock, Time series data, empirical and simulated, using the program *tsunfair-brownian* with f = 0.046012. This data is superimposed on the data presented in Figure C.208.

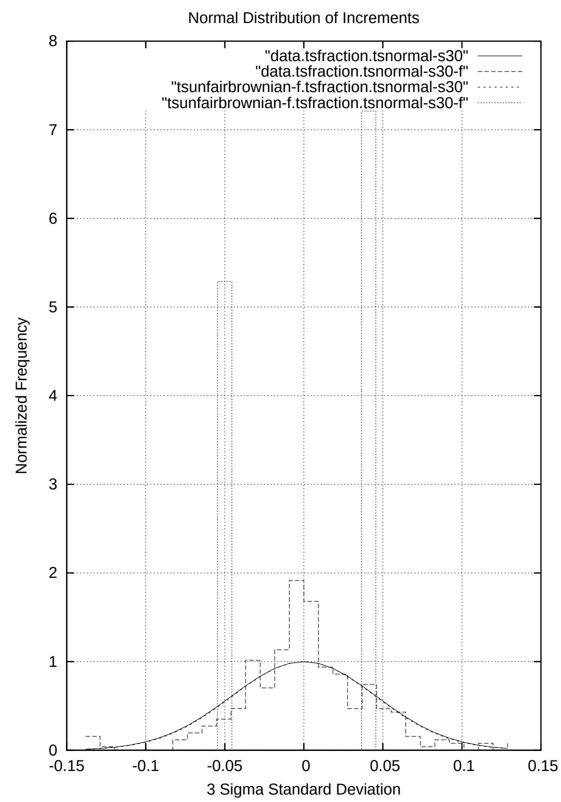

Figure C.228: Cirrus Logic Stock, normalized histogram of the normalized increments of the time series data shown in Figure C.227, empirical and simulated. The empirical data has a mean of 0.002031, with a standard deviation of 0.046039. By comparison, the simulated data has a mean of 0.007068 with a standard deviation of 0.045537. This data is superimposed on the data presented in Figure C.211. The area under the four curves is identical.

C.10.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.10.3. One of the issues of analysis, as mentioned in Section C.10.7, is to determine the maximum Shannon probability for the time series presented in Figure C.208. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.229 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.208. Figure C.230 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.208. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.521329, as derived in Section C.10.5 to 0.576324. This process, essentially, extracts the random statistical data from the time series presented in Figure C.208, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.208, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of days that the Cirrus Logic Stock movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.575000, as compared with the predicted value from the program *tsshannonmax* of 0.576324.

C.10.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.210. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.209. These values will be used in a fixed increment Brownian fractal analysis of the Cirrus Logic Stock, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.10.6 and D.10.7. As a subjective evaluation of the "quality" of the analysis of the Cirrus Logic Stock, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.208 from Figure C.209, and the Shannon probability as calculated by counting the total number of days that the Cirrus Logic Stock movement was positive, as presented in Section C.10.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.409}$$

$$0.575000 \approx \frac{\frac{0.002031}{0.046012} + 1}{2} \tag{C.410}$$

$$0.575000 \approx 0.522070 \tag{C.411}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.575000 \approx 0.522070 \approx 0.576324 \tag{C.412}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.10.5, should be compared. The four methods used were the mean of Figure C.209, the constant in the least squares approximation to Figure C.209, the least squares exponential approximation to Figure C.208, and the logarithmic returns of Figure C.208,

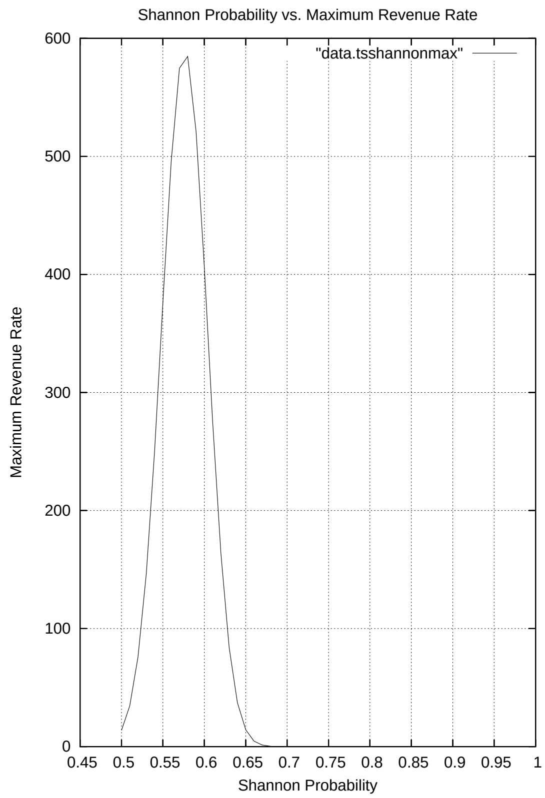

Figure C.229: Cirrus Logic Stock, maximum rate of revenue returns, per day, vs. Shannon probability. The maximum rate of revenue returns, per day, occurs at a Shannon probability of 0.576324.

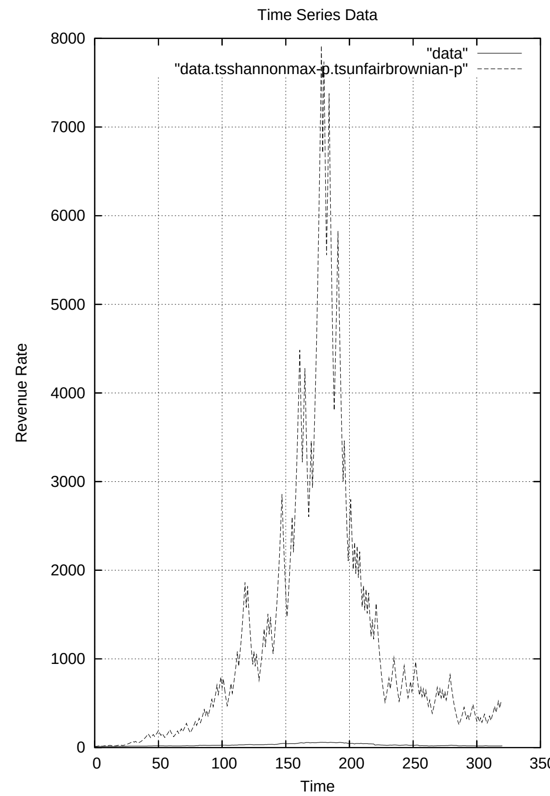

Figure C.230: Cirrus Logic Stock, maximum rate of revenue returns, per day, at a Shannon probability, of 0.576324, corresponding to a "wager" fraction of 0.152648.

derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.002927 \approx 0.015194 \approx 0.001768 \approx 0.001313 \tag{C.413}$$

It is implied in Section C.10.5, Subsection C.10.5 and in Section C.10.8 that, a Brownian motion with fixed increments fractal may "model" the Cirrus Logic Stock. Using Equation 2.104 from Chapter 2, Section 2.5:

$$\begin{aligned}
rms\,(2P-1) &\approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} && \text{(C.414)}\\
0.046012\,(2\cdot 0.575000-1) &\approx \frac{0.046039\,(2\cdot 0.575000-1)}{2\sqrt{0.575000\,(1-0.575000)}} && \text{(C.415)}\\
0.046012\cdot 0.150000 &\approx 0.046039\cdot 0.151717 && \text{(C.416)}\\
0.006902 &\approx 0.006985 && \text{(C.417)}
\end{aligned}$$

and, equating to the mean:

$$0.002031 \approx 0.006902 \approx 0.006985 \qquad \text{(C.418)}$$

where, as in Equation C.411 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.208 from Figure C.209, and the Shannon probability as calculated by counting the total number of days that the Cirrus Logic Stock movement was positive, as presented in Section C.10.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[132], where the absolute value is presented in Figure C.210, and the root mean square value is presented in Figure C.209:

$$0.030075 \approx 0.046012 \qquad \text{(C.419)}$$

Note, that if the Cirrus Logic Stock could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.208 from Figure C.209 should be zero. It is 0.034876.

C.11 United States Gross Domestic Product

For the analysis, the data was in the directory ../markets/us.gdp[133].

The data in this section is presented in tabular form in Section D.11. Note that in this analysis, the rate of revenue returns means the increase or decrease in the United States Gross Domestic Product. This is included for comparative purposes.

C.11.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.11.1. Figure C.231 is a graph of the time series data for the United States Gross Domestic Product.

Figure C.232 is a graph of the normalized increments of the time series data presented in Figure C.231. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.233 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.232. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[134].

Figure C.234 is the normalized histogram of the normalized increments of the time series data shown in Figure C.232. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.234.

[132] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[133] Data from the United States Department of Commerce, 1979—1994, by months, in billions of 1987 dollars, US.

[134] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

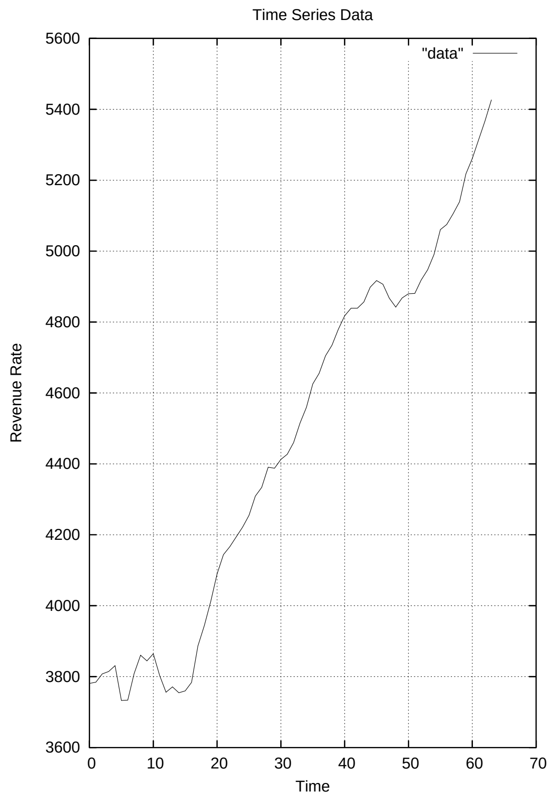

Figure C.231: United States Gross Domestic Product, time series data.

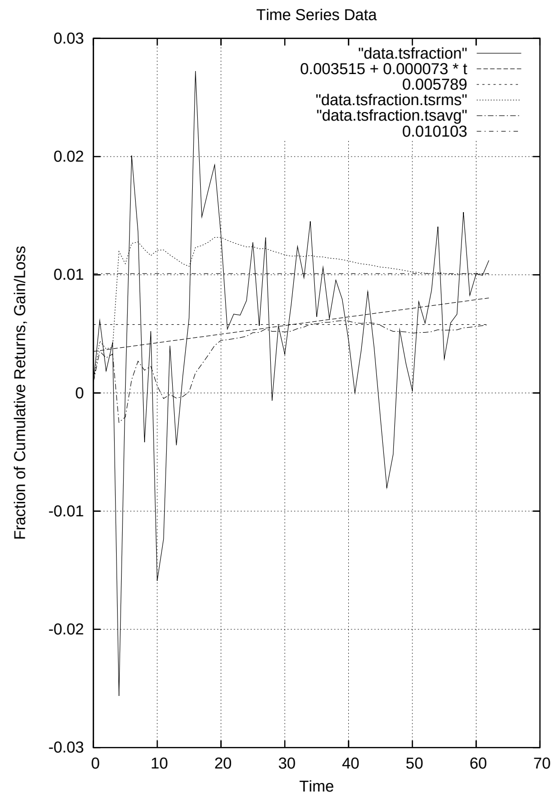

Figure C.232: United States Gross Domestic Product, normalized increments of the time series data presented in Figure C.231. The mean is 0.005789 with a standard deviation of 0.008347. The formula for the least squares approximation is $0.003515 + 0.000073t$, and the root mean squared value is 0.010103. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000073, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.235 is the statistical estimate for the data presented in Figure C.232, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.553093$, as derived in Section C.11.5. See Chapter 2,

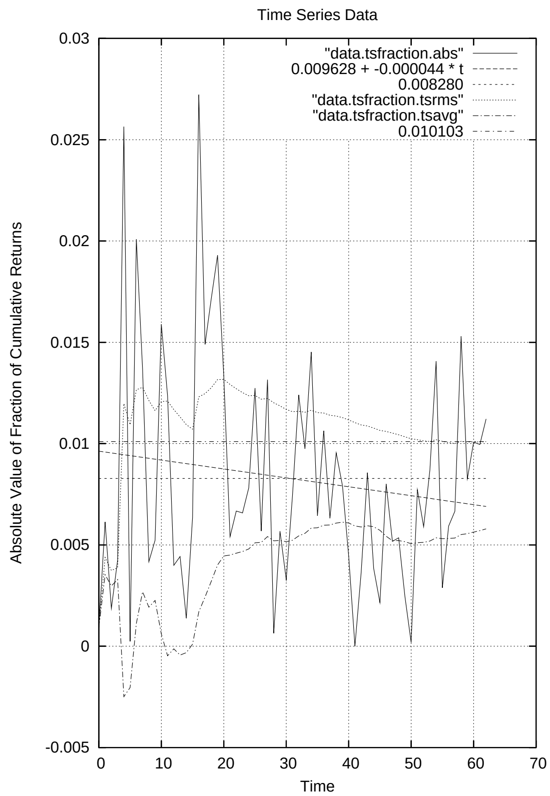

Figure C.233: United States Gross Domestic Product, absolute value of the normalized increments of the time series data presented in Figure C.232. The mean is 0.008280 with a standard deviation of 0.005836. The formula for the least squares approximation is $0.009628 + -0.000044t$, and the root mean square value, from Figure C.232, is 0.010103. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.232, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

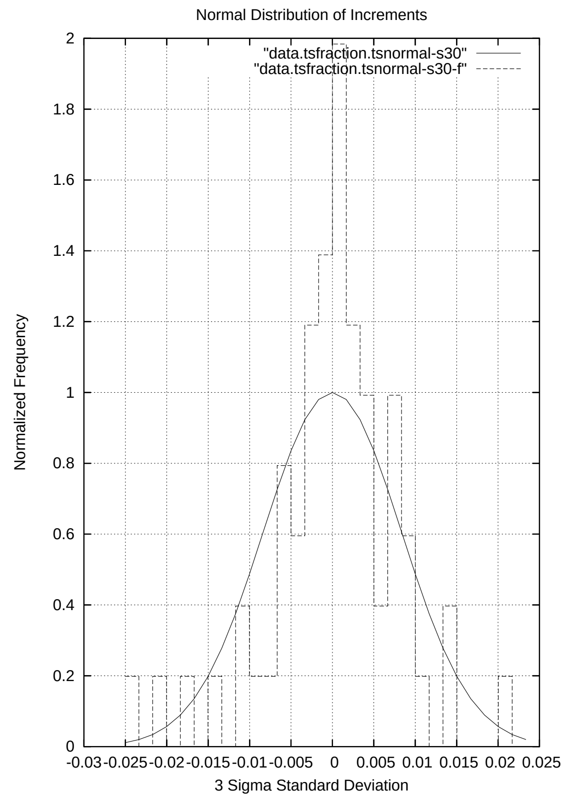

Figure C.234: United States Gross Domestic Product, normalized histogram of the normalized increments of the time series data shown in Figure C.232. The data has a mean of 0.005789, with a standard deviation of 0.008347. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 8.124000, with a critical value of 42.557000.

Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.236 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.232. In principle, if the distribution of the normalized increments presented in Figure C.234 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

```
For a mean of 0.005698, with a confidence level of 0.900000
    that the error did not exceed 0.000570, 851 samples would be required.
    (With 64 samples, the estimated error is 0.002077 = 36.454172 percent.)
For a standard deviation of 0.010103, with a confidence level of 0.900000
    that the error did not exceed 0.001010, 136 samples would be required.
    (With 64 samples, the estimated error is 0.001469 = 14.538589 percent.)
```

Figure C.235: United States Gross Domestic Product, statistical estimates of the normalized increments of the time series shown in Figure C.232. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.232.

The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.237 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.232. In principle, if the distribution of the normalized increments presented in Figure C.234 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.238 is the range of values of the time series shown in Figure C.231. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.238 would be a square root function[135]. Figure C.239 is the deterministic map of the normalized increments of the time series data shown in Figure C.232. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.234 would seem to indicate that the time series data for the United States Gross Domestic Product represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.11.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[136]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.240 is the instantaneous value of the root mean square of the normalized increments for the United States Gross Domestic Product, and Figure C.241 is the instantaneous Shannon probability for the normalized increments.

[135]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.238 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[136]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

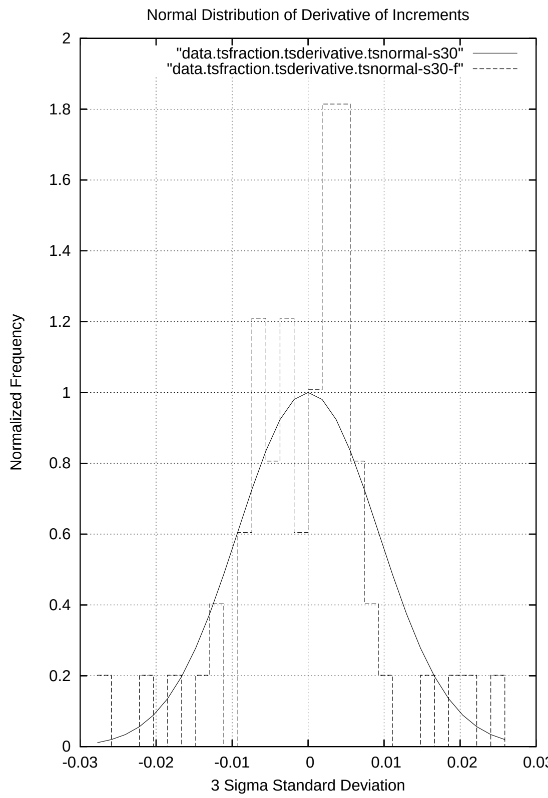

Figure C.236: United States Gross Domestic Product, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.232.

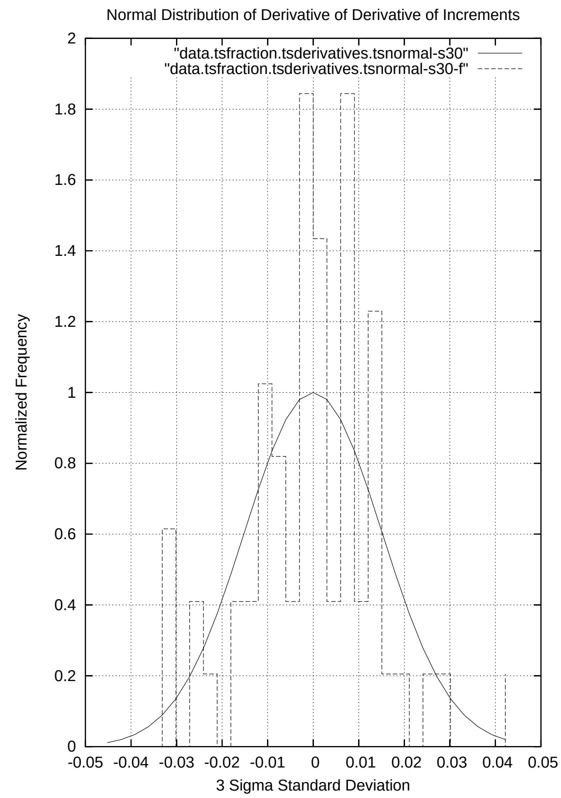

Figure C.237: United States Gross Domestic Product, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.232.

C.11.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.11.4. Figure C.242 is a graph of the logistic function estimates of the time series data for the United States Gross Domestic Product. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[137]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.242 is a graph of the logistic function for the time series data presented in Figure C.231. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.232. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.232. Figure C.243 is the same graph, but with the time scale expanded by a factor of two.

[137]For example, in Figures C.242 and C.243, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.11.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function

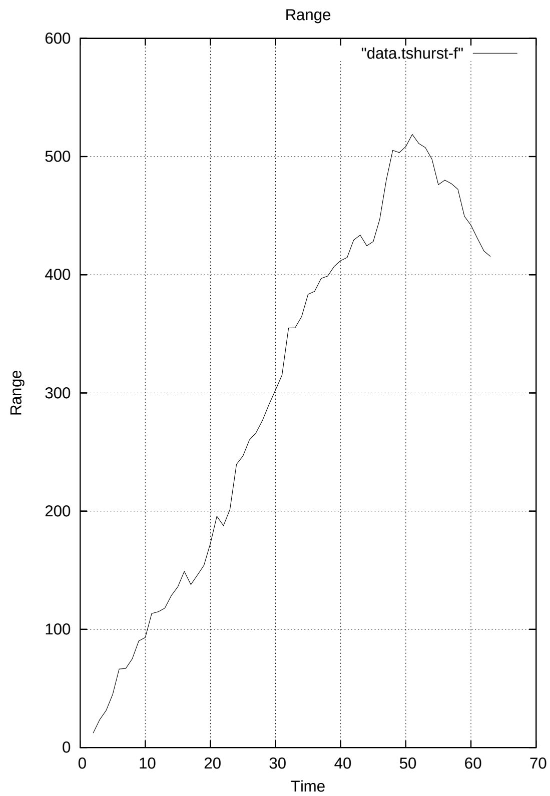

Figure C.238: United States Gross Domestic Product, range of the time series data shown in Figure C.231.

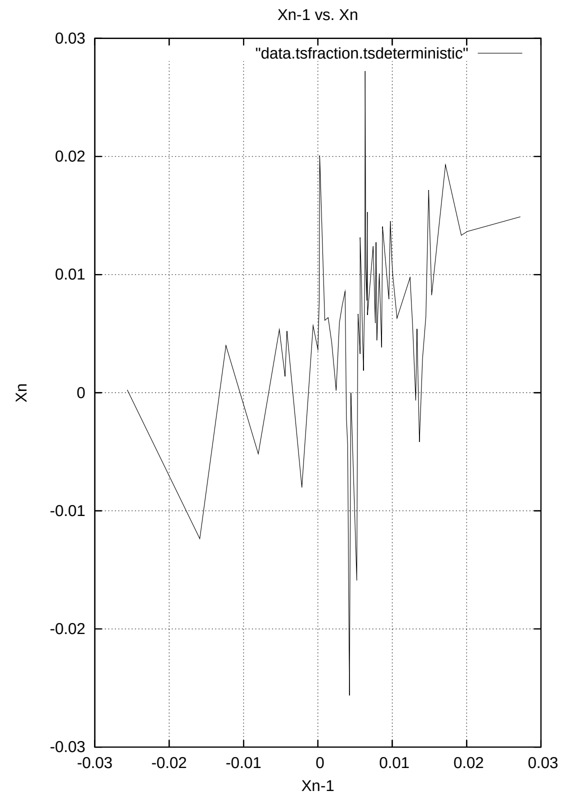

Figure C.239: United States Gross Domestic Product, deterministic map of the normalized increments of the time series data shown in Figure C.232.

C.11.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.11.5. Figure C.244 is a graph of the Hurst coefficient data time series data shown in Figure C.231. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.245 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.232. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.244 implies that the variance of the rate of revenue returns, (per month,) in the United States Gross Domestic Product, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[138], t, $p(t) = erf(1/\sqrt{2t})$ which

of time

[138]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional

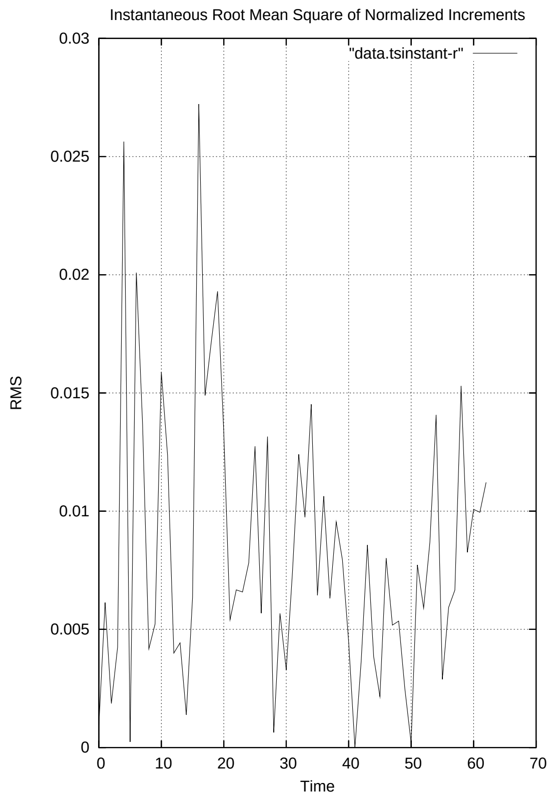

Figure C.240: United States Gross Domestic Product, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.231.

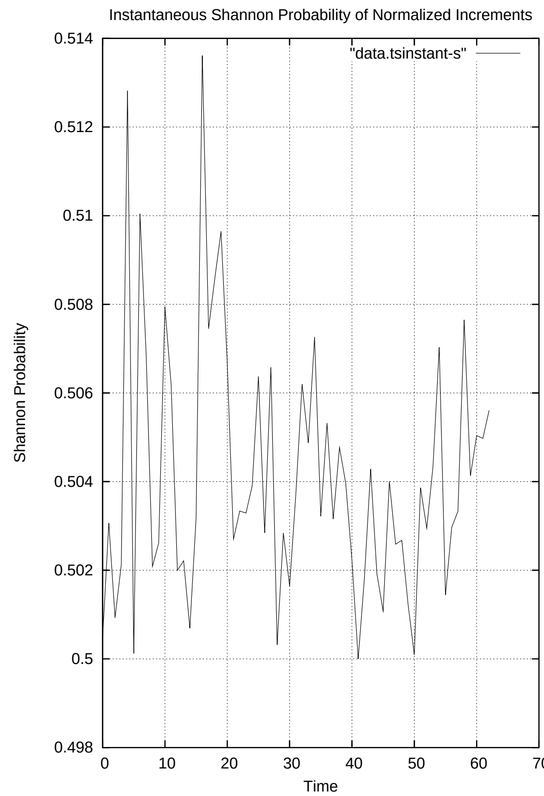

Figure C.241: United States Gross Domestic Product, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.231.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.248, and, C.249 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Gross Domestic Product for the near term and far term, respectively [Pet91, pp. 83-84][139]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.244, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.935237, so that:

to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[139]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

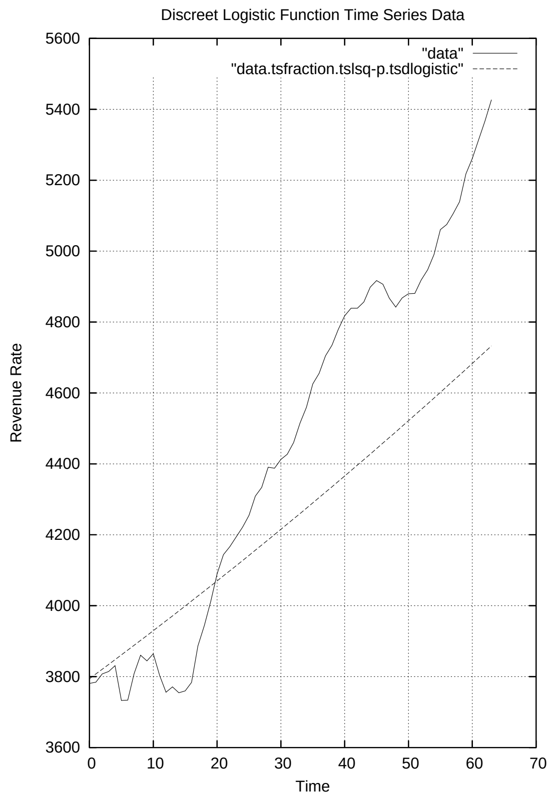

Figure C.242: United States Gross Domestic Product, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.232 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

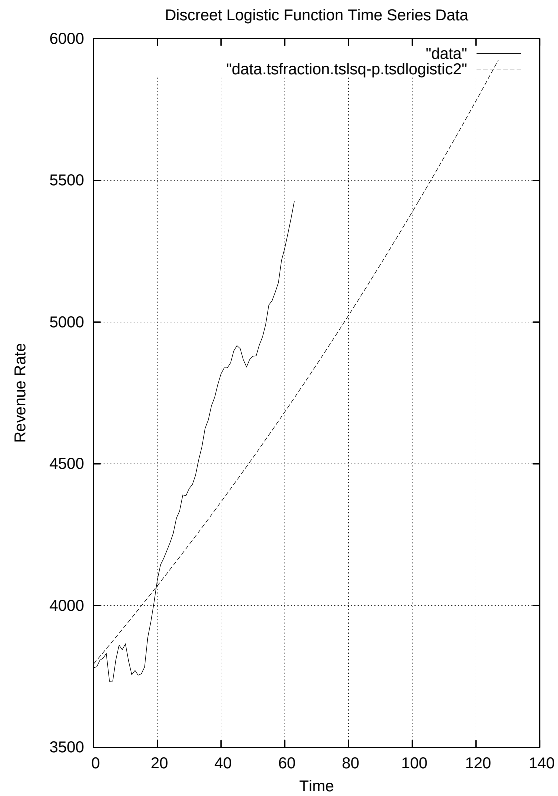

Figure C.243: United States Gross Domestic Product, logistic function estimates of Figure C.242 with the time scale expanded by a factor of two.

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.420}$$
$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot 0.935237} \tag{C.421}$$
$$\propto \quad (t_2 - t_1)^{1.870474} \tag{C.422}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[140]. A Hurst coefficient of 0.935237, (for the near future, and 0.858488 for the distant future.) implies

[140] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the

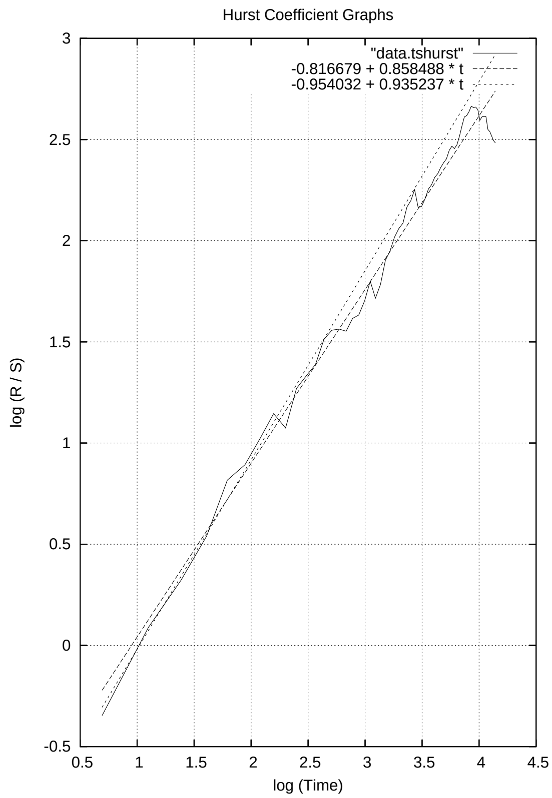

Figure C.244: United States Gross Domestic Product, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.232. The slope of the graph is the Hurst coefficient.

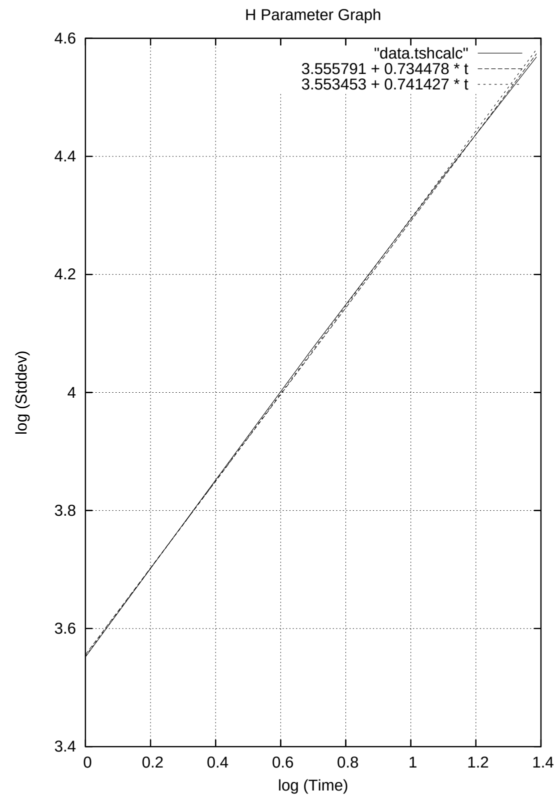

Figure C.245: United States Gross Domestic Product, H parameter data for the normalized increments of the time series data shown in Figure C.232 The slope of the graph is the H parameter.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 93.523700% [Pet91, pp. 66] for the near future, and 0.858488 for the distant future. Likewise, there is a 93.523700% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 93.523700% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Gross Domestic Product are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) \quad = \quad H^n \tag{C.423}$$

"long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.11.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Gross Domestic Product. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

$$= \quad 0.935237^n \tag{C.424}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.232, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.005789 \cdot 100$ percent, on the average, with a standard deviation of $0.008347 \cdot 100$ percent, and a root mean square error value of $0.010103 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^H \tag{C.425}$$

$$\propto \quad \left(t_2 - t_1\right)^{0.935237} \tag{C.426}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.426 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.427}$$

$$\propto \quad \sqrt{\left(t_2 - t_1\right)} \tag{C.428}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.429}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[141].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \quad \propto \quad \frac{X\left(rt\right)}{r^H} \tag{C.430}$$

$$\propto \quad \frac{X\left(rt\right)}{r^{0.935237}} \tag{C.431}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.245, to provide a least squares approximation to the H parameter for the United States Gross Domestic Product. The superimposed least squares

[141] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.741427 for the near future, and 0.734478 for the distant future.

Figures C.244 and C.245 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.232. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.232, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.246 and C.247 was made using the -d option.

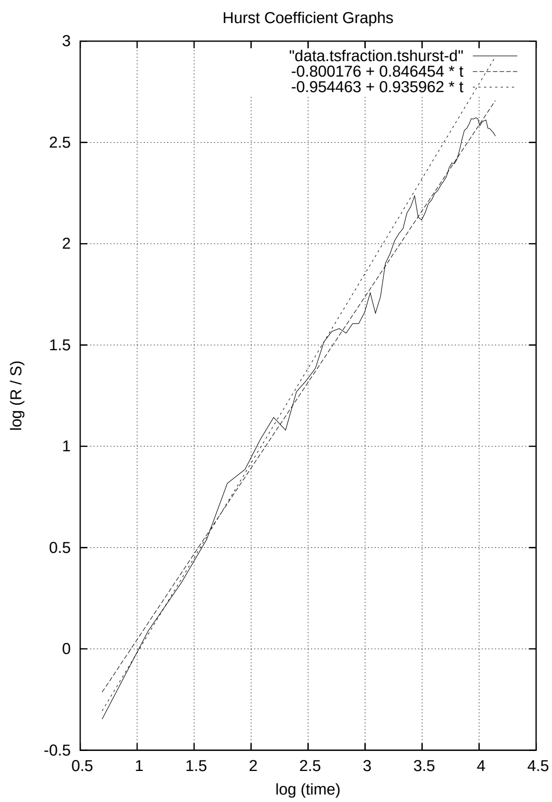

Figure C.246: United States Gross Domestic Product, traditional Hurst coefficient data for the time series data shown in Figure C.231. The slope of the graph is the Hurst coefficient, and is 0.935962 for the near term, and 0.846454 for the far term.

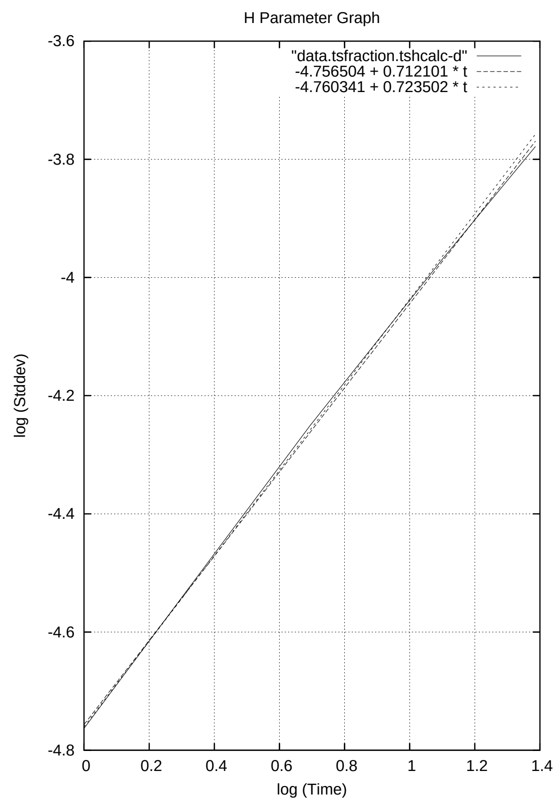

Figure C.247: United States Gross Domestic Product, traditional H parameter data for the time series data shown in Figure C.231 The slope of the graph is the H parameter, and is 0.723502 for the near term, and 0.712101 for the far term.

C.11.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.11.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.233. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.232. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Gross Domestic Product, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Gross Domestic Product, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.232, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln\left(0.005789 + 1\right)}{\ln\left(2\right)} = 0.008328 \tag{C.432}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.232, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln\left(0.003515 + 1\right)}{\ln\left(2\right)} = 0.005062 \tag{C.433}$$

Note that if the mean is not constant in Figure C.232, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.231:

$$bits = 0.008994 \tag{C.434}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.231:

$$bits = 0.008149 \tag{C.435}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.11.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.008149t} \tag{C.436}$$

therefore:

$$C\left(p\right) = 0.008149 \tag{C.437}$$

and, *tsshannon* 0.008149 gives:

$$C\left(0.553093\right) = 0.008149 \tag{C.438}$$

therefore:

$$\begin{aligned} 2^{C(0.553093)} &= 2^{0.008149} && \text{(C.439)} \\ &= 1.005664 && \text{(C.440)} \\ &= 0.566444\% && \text{(C.441)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= \left(2 \cdot 0.553093\right) - 1 && \text{(C.442)} \\ &= 0.106186 && \text{(C.443)} \\ &= 10.618600\% && \text{(C.444)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the United States Gross Domestic Product executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 10.618600% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 89.381400% will be held in "reserve" with a 55.309300% chance of making twice the 10.618600% back, (and a 44.690700% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.566444%, or a doubling of its rate of revenue returns, (per month,) in 122.714443 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 10.618600% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.566444%, per month, on average.

Note that the metrics presented in this section are representative of the United States Gross Domestic Product as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 10.618600% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Gross Domestic Product's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.566444% per month.

As another simple example, a company re-invests 10.618600% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 10.618600% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.566444% per month.

As an example of "product portfolio" management, suppose a company re-invests 10.618600% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 10.618600$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 10.618600$ percent for the second product, implying that the company should diversify its product line[142]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 10.618600%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Gross Domestic Product, as a standard bench mark, then the optimal number will be $\frac{1}{0.106186}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example

[142]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.232, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 10.618600% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 10.618600% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.11.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Gross Domestic Product, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Gross Domestic Product time series is 0.005789, and 0.010103respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 56.715641.

If this value seems consistent number of companies in the United States Gross Domestic Product, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Gross Domestic Product are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.538043, which would be the value which should be used in Section C.11.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.11.5 is greater than the average Shannon probability for the companies participating in the United States Gross Domestic Product, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.11.5. The maximum exploitability for the United States Gross Domestic Product is derived in Section C.11.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Gross Domestic Product is 0.538043, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.786499 in the United States Gross Domestic Product. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.445}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Gross Domestic Product would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.11.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the “average” of the normalized increments presented in Figure C.233. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.232. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Gross Domestic Product, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.11.5, is derived from the United States Gross Domestic Product metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.11.9.

An additional exploitable strategy may be time itself. Equations C.422, C.426, and, C.424, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Gross Domestic Product, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.248, and, C.249 compare methods of approximation of the “forecastability” of rate of revenue returns in the United States Gross Domestic Product for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the “forecastability” of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in “timeliness” of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the “average” product or service life cycle in the United States Gross Domestic Product. When the “forecastability” of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the “forecastability” of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of “market windows.[143]” The conceptual approximation will probably predict a “conservative” or “pessimistic” value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.248, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.424, $0.935237^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.422, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.11.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.11.9. Figure C.250 represents a constructional simulation of the time series data presented in Figure C.231. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.232. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in

[143] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

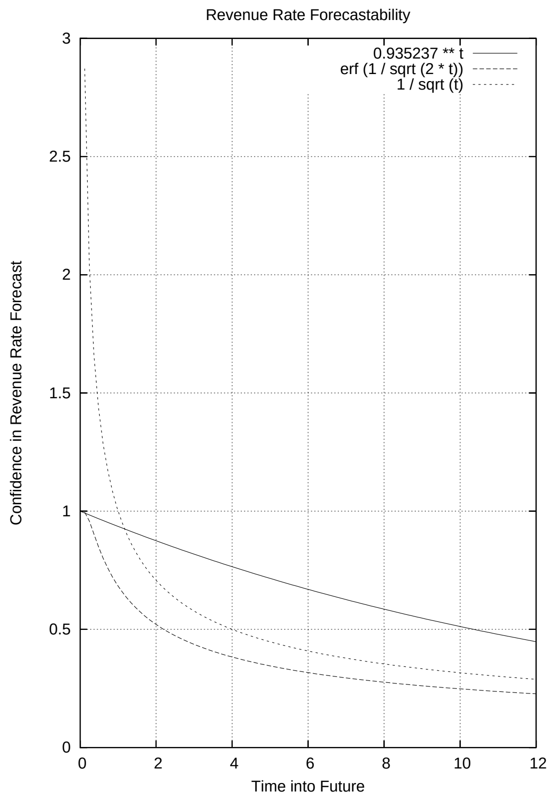

Figure C.248: United States Gross Domestic Product, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.935237^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

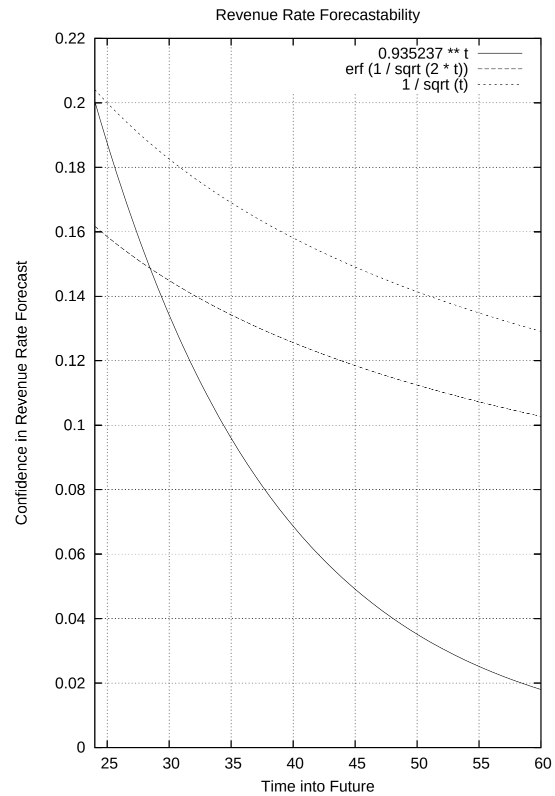

Figure C.249: United States Gross Domestic Product, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

all probability, the normalized increments presented in Figure C.232 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.251 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.234.

C.11.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.11.3. One of the issues of analysis, as mentioned in Section C.11.7, is to determine the maximum Shannon probability for the time series presented in Figure C.231. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.252 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum

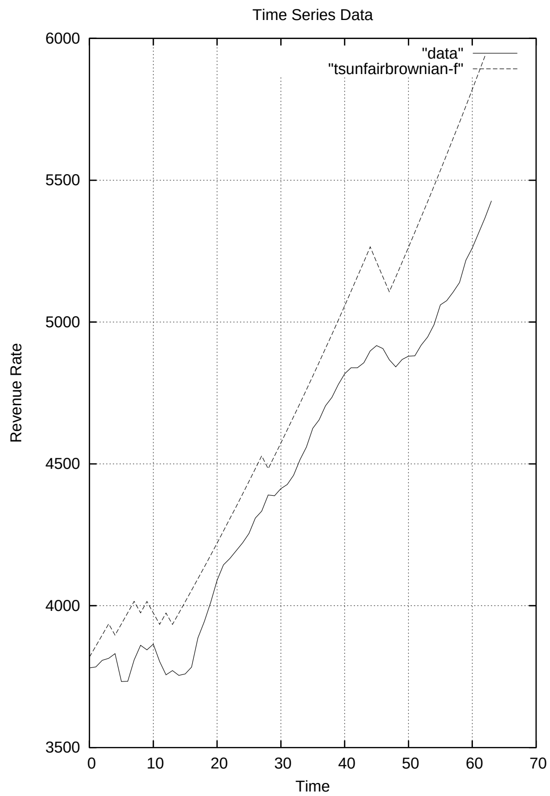

Figure C.250: United States Gross Domestic Product, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.010103. This data is superimposed on the data presented in Figure C.231.

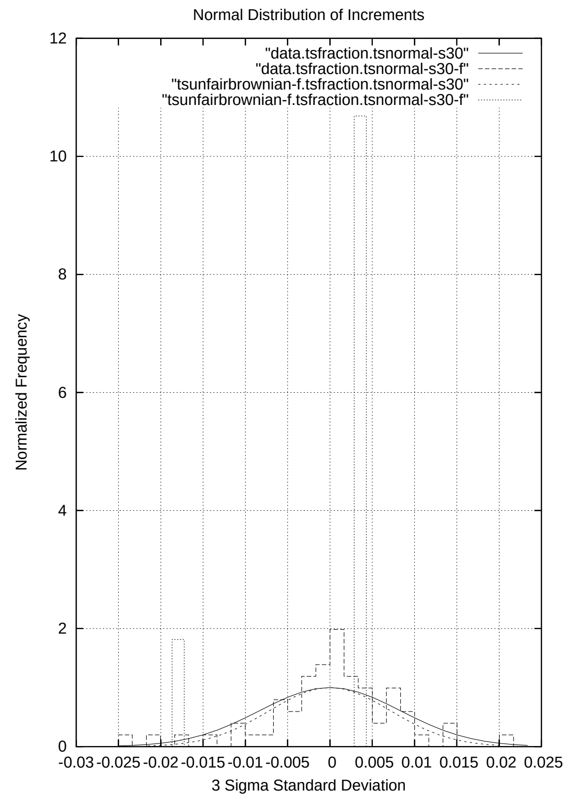

Figure C.251: United States Gross Domestic Product, normalized histogram of the normalized increments of the time series data shown in Figure C.250, empirical and simulated. The empirical data has a mean of 0.005789, with a standard deviation of 0.008347. By comparison, the simulated data has a mean of 0.007170 with a standard deviation of 0.007176. This data is superimposed on the data presented in Figure C.234. The area under the four curves is identical.

Shannon probability for the time series data presented in Figure C.231. Figure C.253 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.231. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.553093, as derived in Section C.11.5 to 0.859375. This process, essentially, extracts the random statistical data from the time series presented in Figure C.231, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.231, constitutes classical Brownian motion,

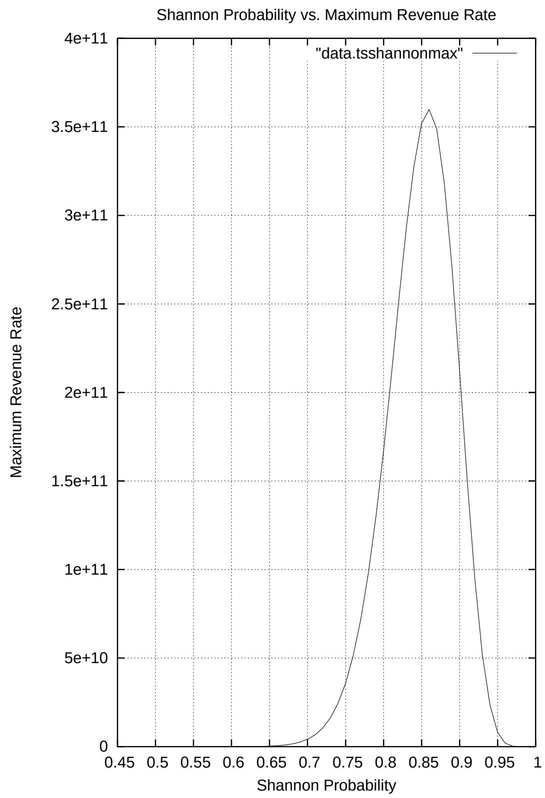

Figure C.252: United States Gross Domestic Product, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.859375.

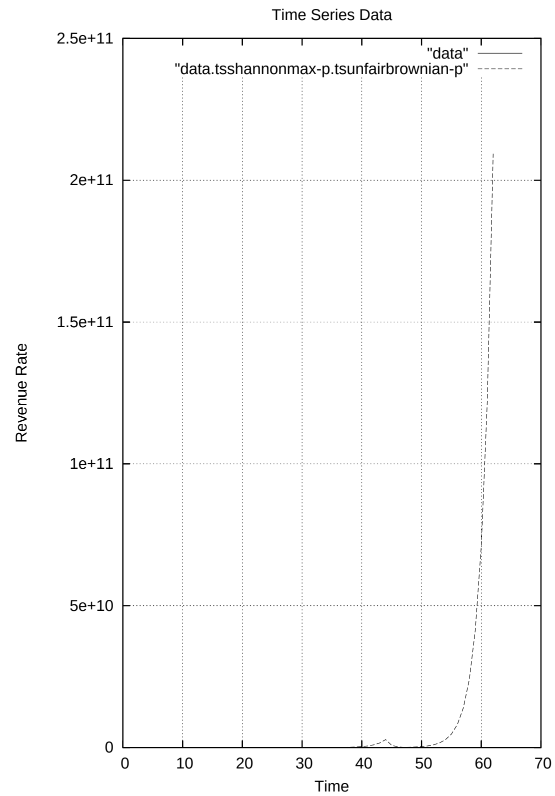

Figure C.253: United States Gross Domestic Product, maximum rate of revenue returns, per month, at a Shannon probability, of 0.859375, corresponding to a "wager" fraction of 0.718750.

then the Shannon probability can be calculated by counting the total number of months that the United States Gross Domestic Product movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.857143, as compared with the predicted value from the program *tsshannonmax* of 0.859375.

C.11.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.233. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.232. These values will be used in a fixed increment Brownian fractal analysis of the United States Gross Domestic Product, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.11.6 and D.11.7. As a subjective evaluation of the "quality" of the analysis of the United States Gross Domestic Product, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.231 from Figure C.232, and the Shannon probability as calculated by counting the total number of months that the United

States Gross Domestic Product movement was positive, as presented in Section C.11.9:

$$
\begin{aligned}
P &\approx \frac{\frac{avg}{rms}+1}{2} && \text{(C.446)}\\
0.857143 &\approx \frac{\frac{0.005789}{0.010103}+1}{2} && \text{(C.447)}\\
0.857143 &\approx 0.786499 && \text{(C.448)}
\end{aligned}
$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.857143 \approx 0.786499 \approx 0.859375 \tag{C.449}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.11.5, should be compared. The four methods used were the mean of Figure C.232, the constant in the least squares approximation to Figure C.232, the least squares exponential approximation to Figure C.231, and the logarithmic returns of Figure C.231, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.008328 \approx 0.005062 \approx 0.008994 \approx 0.008149 \tag{C.450}$$

It is implied in Section C.11.5, Subsection C.11.5 and in Section C.11.8 that, a Brownian motion with fixed increments fractal may "model" the United States Gross Domestic Product. Using Equation 2.104 from Chapter 2, Section 2.5:

$$
\begin{aligned}
rms\,(2P-1) &\approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} && \text{(C.451)}\\
0.010103\,(2\cdot 0.857143-1) &\approx \frac{0.008347\,(2\cdot 0.857143-1)}{2\sqrt{0.857143\,(1-0.857143)}} && \text{(C.452)}\\
0.010103\cdot 0.714286 &\approx 0.008347\cdot 1.020621 && \text{(C.453)}\\
0.007216 &\approx 0.008519 && \text{(C.454)}
\end{aligned}
$$

and, equating to the mean:

$$0.005789 \approx 0.007216 \approx 0.008519 \tag{C.455}$$

where, as in Equation C.448 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.231 from Figure C.232, and the Shannon probability as calculated by counting the total number of months that the United States Gross Domestic Product movement was positive, as presented in Section C.11.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[144], where the absolute value is presented in Figure C.233, and the root mean square value is presented in Figure C.232:

$$0.008280 \approx 0.010103 \tag{C.456}$$

[144] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

Note, that if the United States Gross Domestic Product could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.231 from Figure C.232 should be zero. It is 0.005836.

C.12 United States Employment Figures

For the analysis, the data was in the directory ../markets/us.employment[145].

The data in this section is presented in tabular form in Section D.12. Note that in this analysis, the rate of revenue returns means the increase or decrease in the United States Employment Figures. This is included for comparative purposes. Presumably, the United States Employment Figures represents something of value, or they could be used as a "futures" derivative, and thus, it would be considered that there is a rate of revenue returns.

C.12.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.12.1. Figure C.254 is a graph of the time series data for the United States Employment Figures.

Figure C.255 is a graph of the normalized increments of the time series data presented in Figure C.254. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.256 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.255. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[146].

Figure C.257 is the normalized histogram of the normalized increments of the time series data shown in Figure C.255. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.257.

Figure C.258 is the statistical estimate for the data presented in Figure C.255, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.525655$, as derived in Section C.12.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.259 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.255. In principle, if the distribution of the normalized increments presented in Figure C.257 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.260 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.255. In principle, if the distribution of the normalized increments presented in Figure C.257 is an integrated

[145] Data from the United States Bureau of Labor and Statistics, 1980—1994, by months, in thousands of persons.

[146] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

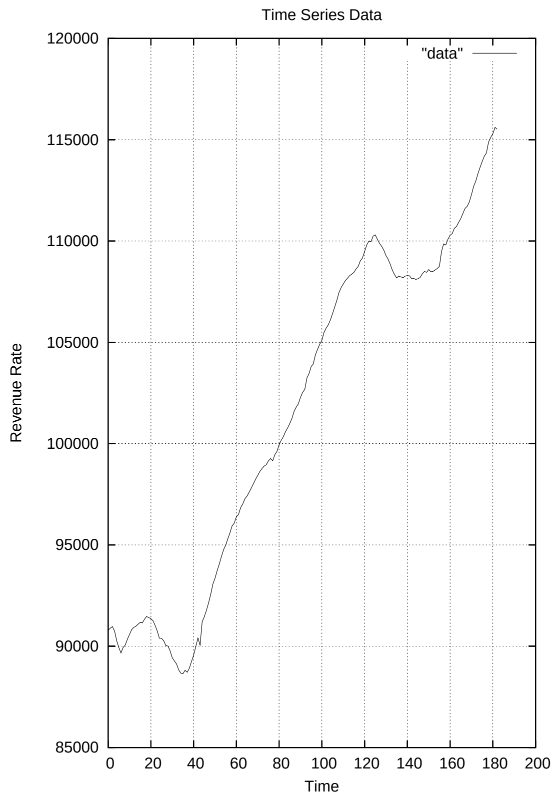

Figure C.254: United States Employment Figures, time series data.

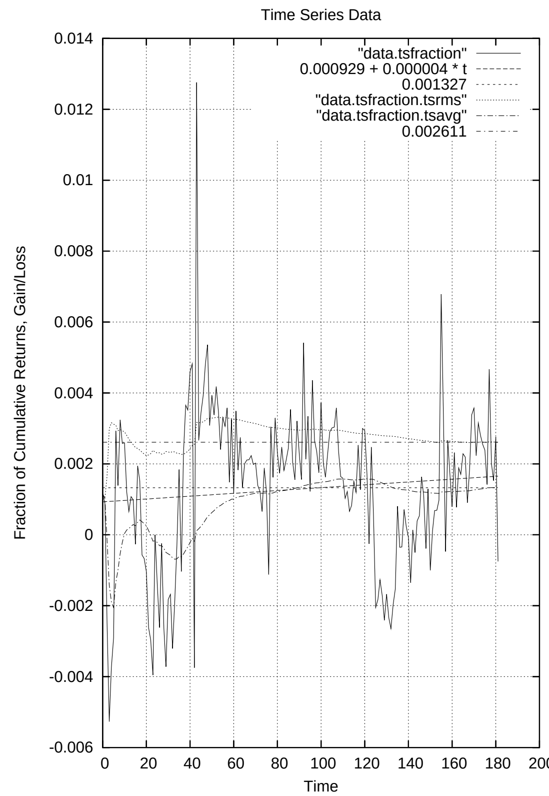

Figure C.255: United States Employment Figures, normalized increments of the time series data presented in Figure C.254. The mean is 0.001327 with a standard deviation of 0.002254. The formula for the least squares approximation is $0.000929 + 0.000004t$, and the root mean squared value is 0.002611. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000004, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Gaussian distribution in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43.

Figure C.261 is the range of values of the time series shown in Figure C.254. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.261

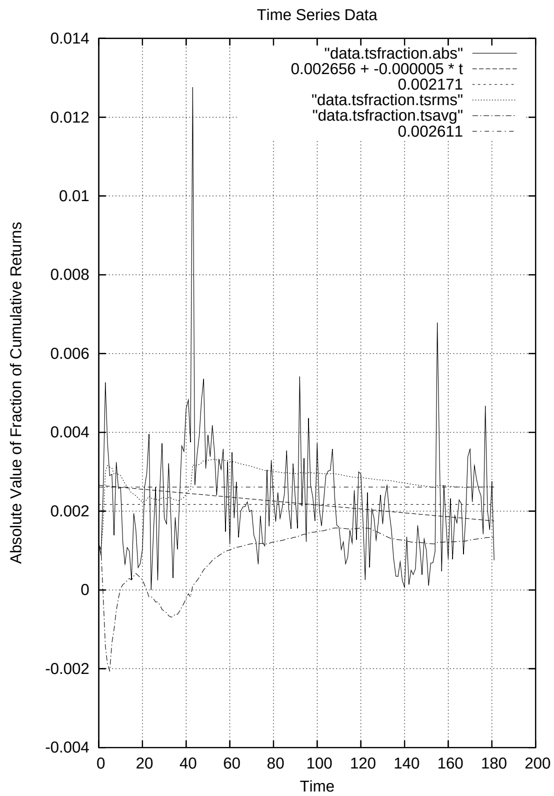

Figure C.256: United States Employment Figures, absolute value of the normalized increments of the time series data presented in Figure C.255. The mean is 0.002171 with a standard deviation of 0.001454. The formula for the least squares approximation is $0.002656 + -0.000005t$, and the root mean square value, from Figure C.255, is 0.002611. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.255, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

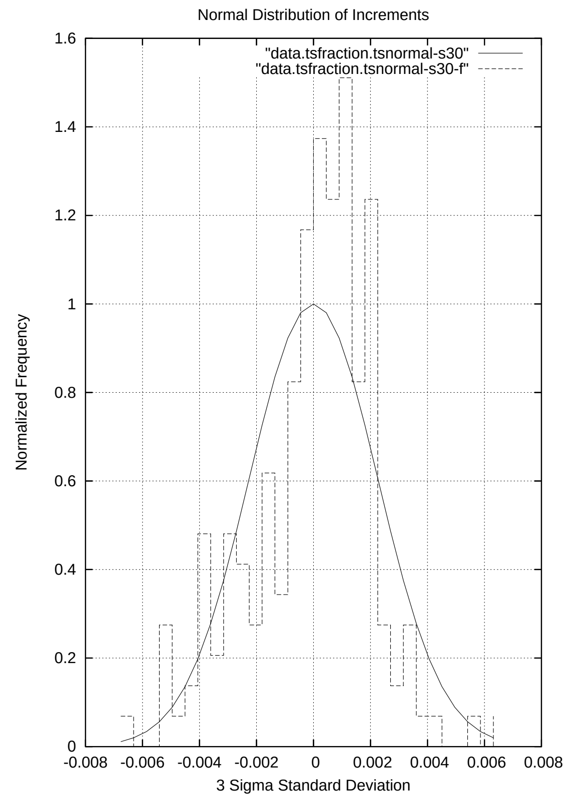

Figure C.257: United States Employment Figures, normalized histogram of the normalized increments of the time series data shown in Figure C.255. The data has a mean of 0.001327, with a standard deviation of 0.002254. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 4.227000, with a critical value of 42.557000.

would be a square root function[147]. Figure C.262 is the deterministic map of the normalized increments of the time series data shown in Figure C.255. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

[147]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.261 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

```
For  a  mean  of  0.001320,   with  a  confidence   level  of  0.900000
     that  the  error  did  not  exceed  0.000132,   1059  samples  would  be  required.
     (With  183  samples,  the  estimated  error  is  0.000317  = 24.046114  percent.)
For  a  standard  deviation  of  0.002611,  with  a  confidence  level  of  0.900000
     that  the  error  did  not  exceed  0.000261,  136  samples  would  be  required.
     (With  183  samples,  the  estimated  error  is  0.000224  = 8.597788  percent.)
```

Figure C.258: United States Employment Figures, statistical estimates of the normalized increments of the time series shown in Figure C.255. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.255.

Observations on the Time Series Increments Analysis

Figure C.257 would seem to indicate that the time series data for the United States Employment Figures represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.12.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[148]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.263 is the instantaneous value of the root mean square of the normalized increments for the United States Employment Figures, and Figure C.264 is the instantaneous Shannon probability for the normalized increments.

C.12.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.12.4. Figure C.265 is a graph of the logistic function estimates of the time series data for the United States Employment Figures. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[149]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.265 is a graph of the logistic function for the time series data presented in Figure C.254. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters

[148] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[149] For example, in Figures C.265 and C.266, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.12.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

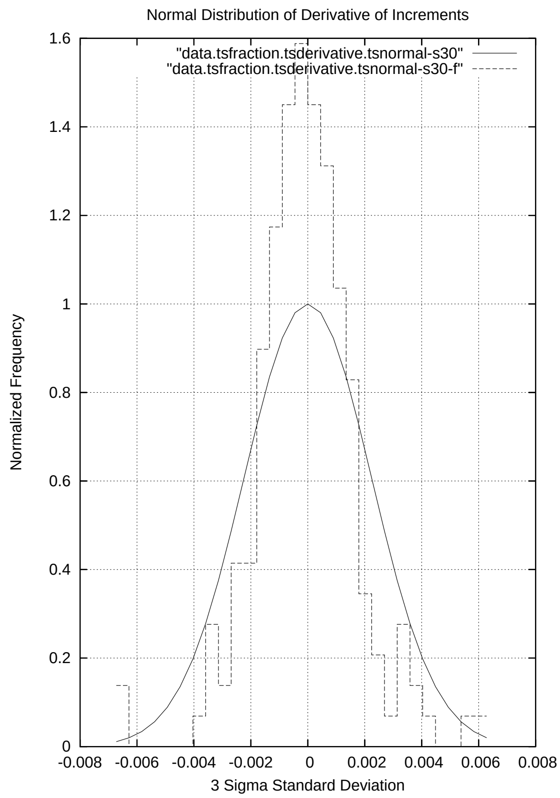

Figure C.259: United States Employment Figures, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.255.

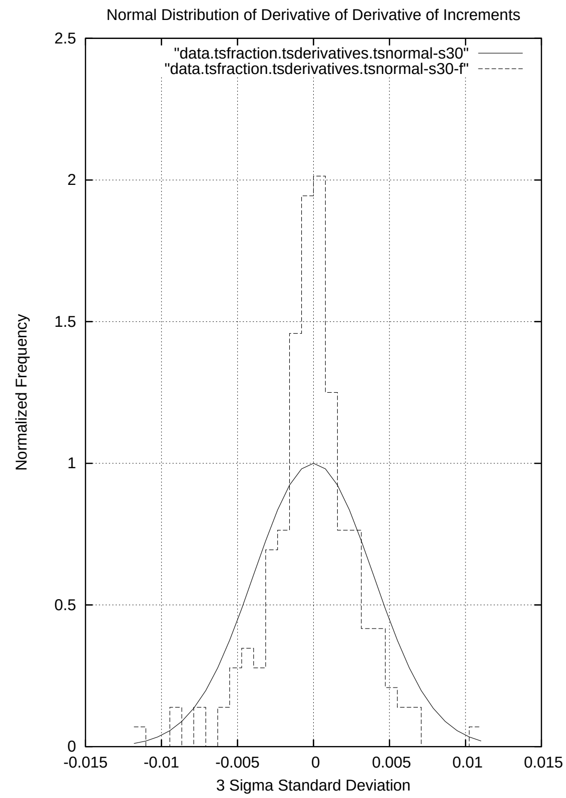

Figure C.260: United States Employment Figures, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.255.

extracted from the time series data as suggested in Figure C.255. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.255. Figure C.266 is the same graph, but with the time scale expanded by a factor of two.

C.12.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.12.5. Figure C.267 is a graph of the Hurst coefficient data time series data shown in Figure C.254. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.268 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.255. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.267 implies that the variance of the rate of revenue returns, (per month,) in the United States Employment Figures, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a

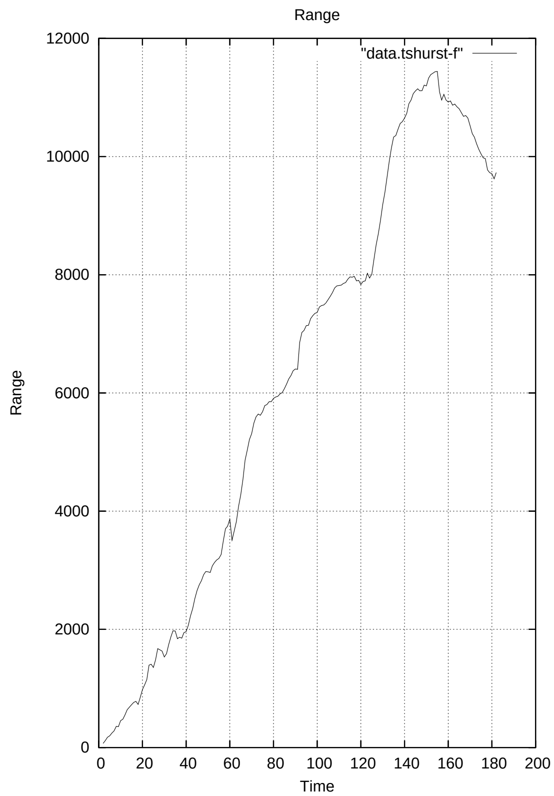

Figure C.261: United States Employment Figures, range of the time series data shown in Figure C.254.

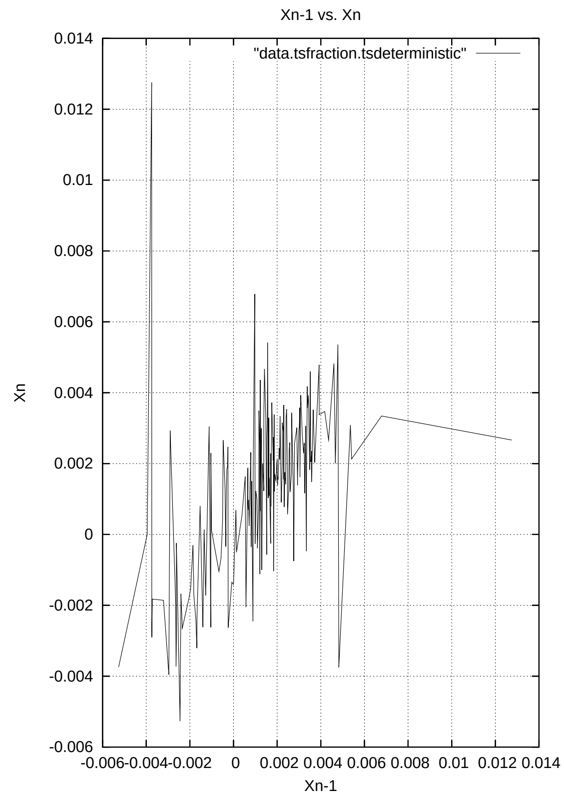

Figure C.262: United States Employment Figures, deterministic map of the normalized increments of the time series data shown in Figure C.255.

period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[150], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.271, and, C.272 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Employment Figures for the near term and far term, respectively [Pet91, pp. 83-84][151]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.267, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient

[150]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[151]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

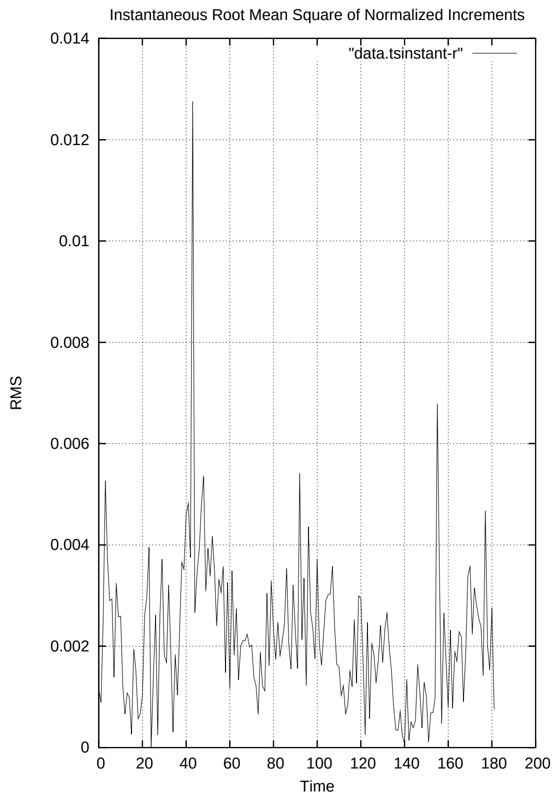

Figure C.263: United States Employment Figures, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.254.

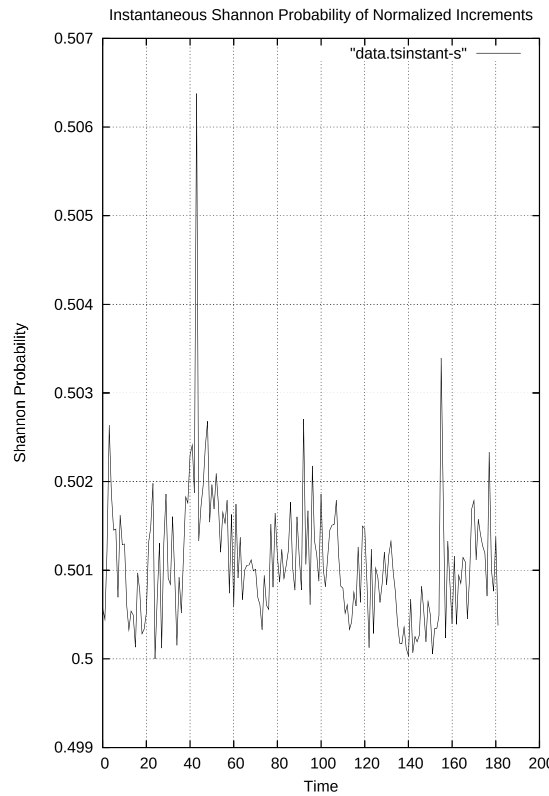

Figure C.264: United States Employment Figures, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.254.

of 0.879967, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \quad \text{(C.457)}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.879967} \quad \text{(C.458)}$$

$$\propto (t_2 - t_1)^{1.759934} \quad \text{(C.459)}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[152]. A Hurst coefficient of 0.879967, (for the near future, and 0.986346 for the distant future.) implies

[152] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient

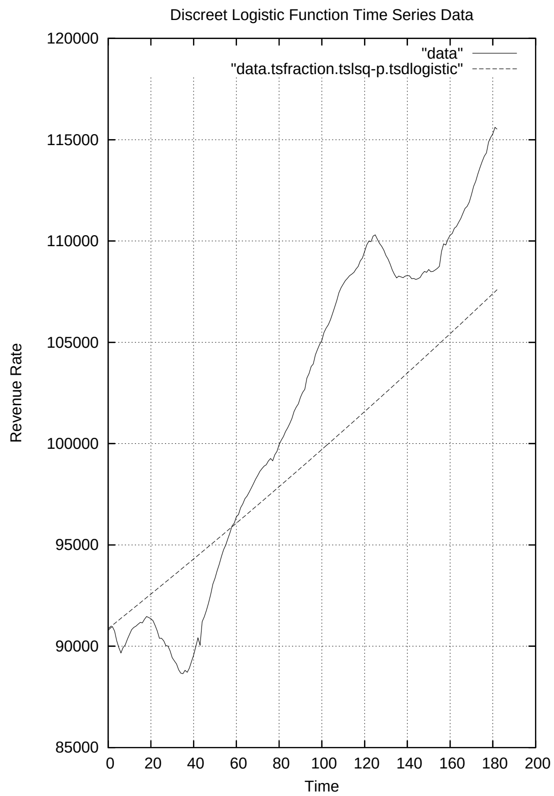

Figure C.265: United States Employment Figures, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.255 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

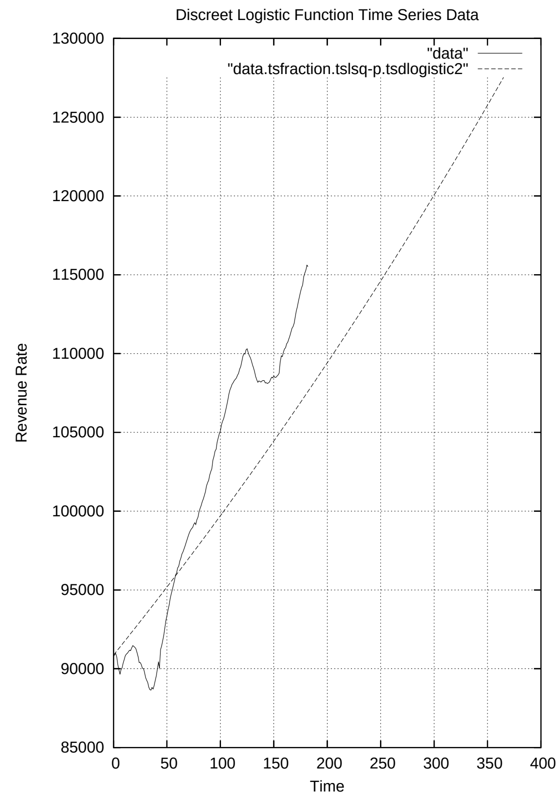

Figure C.266: United States Employment Figures, logistic function estimates of Figure C.265 with the time scale expanded by a factor of two.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 87.996700% [Pet91, pp. 66] for the near future, and 0.986346 for the distant future. Likewise, there is a 87.996700% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 87.996700% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Employment Figures are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.12.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Employment Figures. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

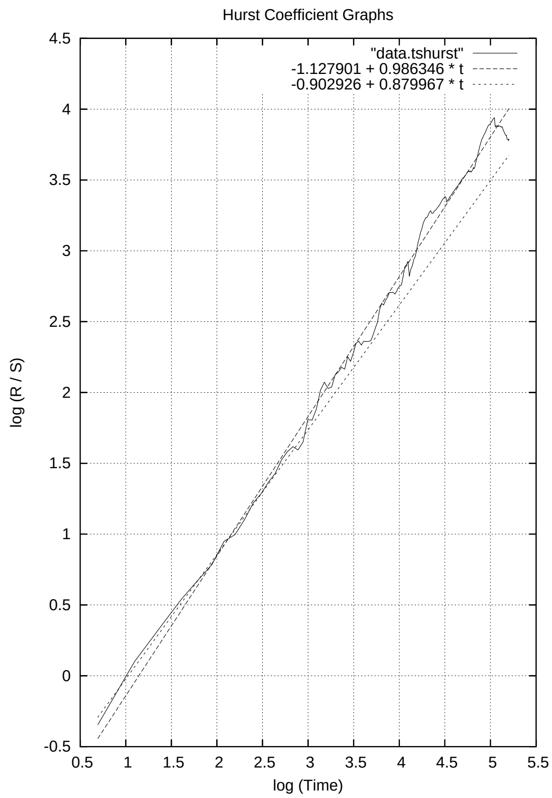

Figure C.267: United States Employment Figures, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.255. The slope of the graph is the Hurst coefficient.

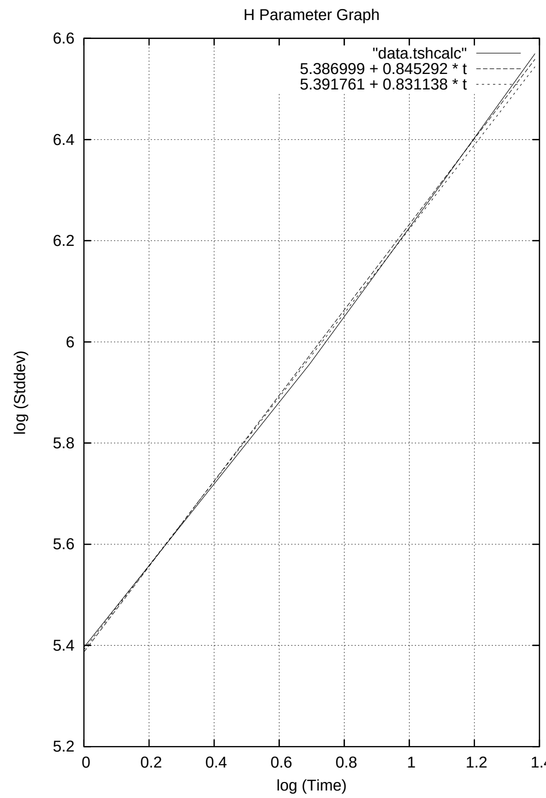

Figure C.268: United States Employment Figures, H parameter data for the normalized increments of the time series data shown in Figure C.255 The slope of the graph is the H parameter.

$$p_a(n) = H^n \tag{C.460}$$
$$= 0.879967^n \tag{C.461}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.255, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.001327 \cdot 100$ percent, on the average, with a standard deviation of $0.002254 \cdot 100$ percent, and a root mean square error value of $0.002611 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.462}$$

$$\propto \quad (t_2 - t_1)^{0.879967} \tag{C.463}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.463 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.464}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.465}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.466}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[153].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\,(t) \quad \propto \quad \frac{X\,(rt)}{r^H} \tag{C.467}$$

$$\propto \quad \frac{X\,(rt)}{r^{0.879967}} \tag{C.468}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.268, to provide a least squares approximation to the H parameter for the United States Employment Figures. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.831138 for the near future, and 0.845292 for the distant future.

Figures C.267 and C.268 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.255. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.255, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.269 and C.270 was made using the -d option.

C.12.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.12.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.256. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.255. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Employment Figures, and may, or may not, provide adequate accuracy for projections.

[153] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

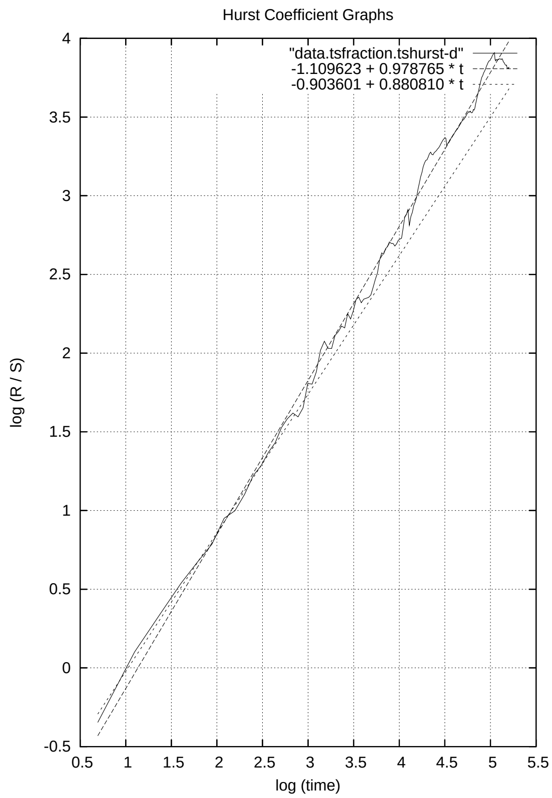

Figure C.269: United States Employment Figures, traditional Hurst coefficient data for the time series data shown in Figure C.254. The slope of the graph is the Hurst coefficient, and is 0.880810 for the near term, and 0.978765 for the far term.

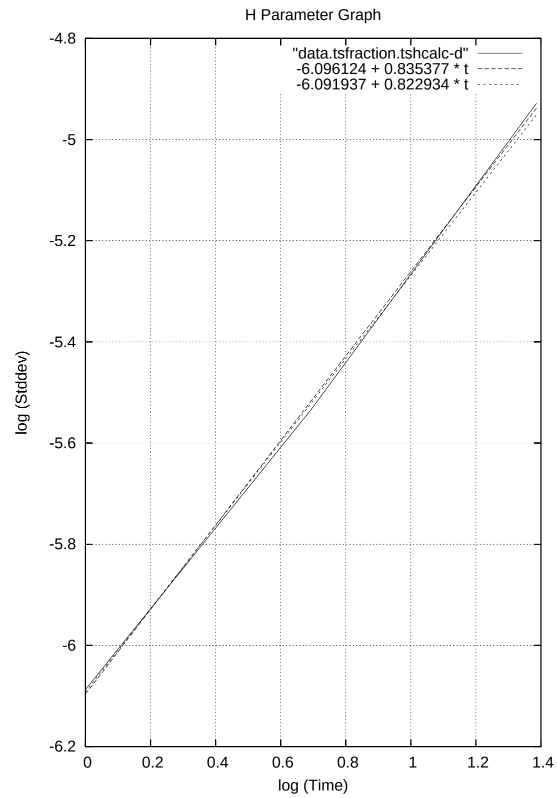

Figure C.270: United States Employment Figures, traditional H parameter data for the time series data shown in Figure C.254 The slope of the graph is the H parameter, and is 0.822934 for the near term, and 0.835377 for the far term.

For an organization operating in the United States Employment Figures, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.255, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.001327 + 1)}{\ln(2)} = 0.001913 \tag{C.469}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.255, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.000929 + 1)}{\ln(2)} = 0.001340 \tag{C.470}$$

Note that if the mean is not constant in Figure C.255, this method will not provide accurate results. And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.254:

$$bits = 0.002205 \tag{C.471}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.254:

$$bits = 0.001900 \tag{C.472}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.12.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.001900t} \tag{C.473}$$

therefore:

$$C(p) = 0.001900 \tag{C.474}$$

and, *tsshannon* 0.001900 gives:

$$C(0.525655) = 0.001900 \tag{C.475}$$

therefore:

$$\begin{aligned} 2^{C(0.525655)} &= 2^{0.001900} && \text{(C.476)} \\ &= 1.001318 && \text{(C.477)} \\ &= 0.131785\% && \text{(C.478)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.525655) - 1 && \text{(C.479)} \\ &= 0.051310 && \text{(C.480)} \\ &= 5.131000\% && \text{(C.481)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the United States Employment Figures executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 5.131000% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 94.869000% will be held in “reserve” with a 52.565500% chance of making twice the 5.131000% back, (and a 47.434500% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.131785%, or a doubling of its rate of revenue returns, (per month,) in 526.315789 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 5.131000% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.131785%, per month, on average.

Note that the metrics presented in this section are representative of the United States Employment Figures as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 5.131000% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Employment Figures's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.131785% per month.

As another simple example, a company re-invests 5.131000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 5.131000% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.131785% per month.

As an example of "product portfolio" management, suppose a company re-invests 5.131000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 5.131000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 5.131000$ percent for the second product, implying that the company should diversify its product line[154]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 5.131000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Employment Figures, as a standard bench mark, then the optimal number will be $\frac{1}{0.051310}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.255, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex

[154]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 5.131000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 5.131000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.12.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Employment Figures, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Employment Figures time series is 0.001327, and 0.002611respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 194.651242.

If this value seems consistent number of companies in the United States Employment Figures, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Employment Figures are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.518214, which would be the value which should be used in Section C.12.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.12.5 is greater than the average Shannon probability for the companies participating in the United States Employment Figures, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.12.5. The maximum exploitability for the United States Employment Figures is derived in Section C.12.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Employment Figures is 0.518214, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.754117 in the United States Employment Figures. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.482}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Employment Figures would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.12.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.256. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.255. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Employment Figures, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.12.5, is derived from the United States Employment Figures metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.12.9.

An additional exploitable strategy may be time itself. Equations C.459, C.463, and, C.461, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Employment Figures, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.271, and, C.272 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Employment Figures for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Employment Figures. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[155]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.271, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.461, $0.879967^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.459, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.12.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.12.9. Figure C.273 represents a constructional simulation of the time series data presented in Figure C.254. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.255. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.255 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.274 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.257.

[155]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

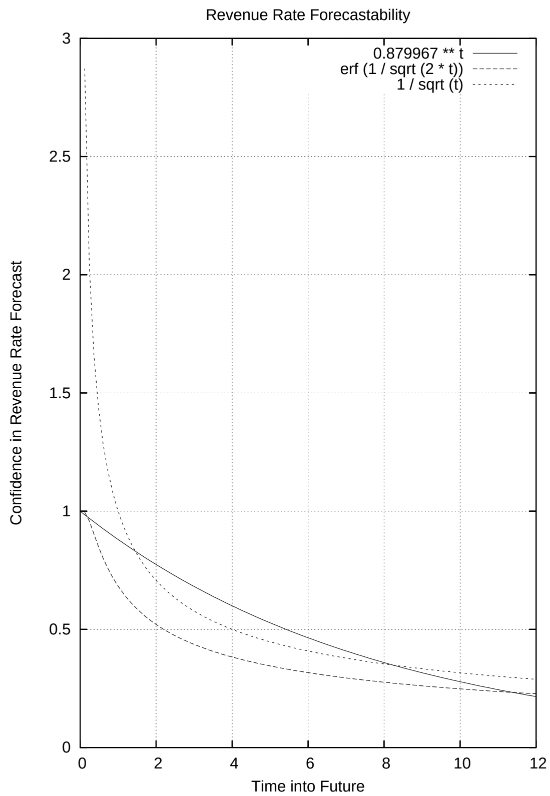

Figure C.271: United States Employment Figures, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.879967^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

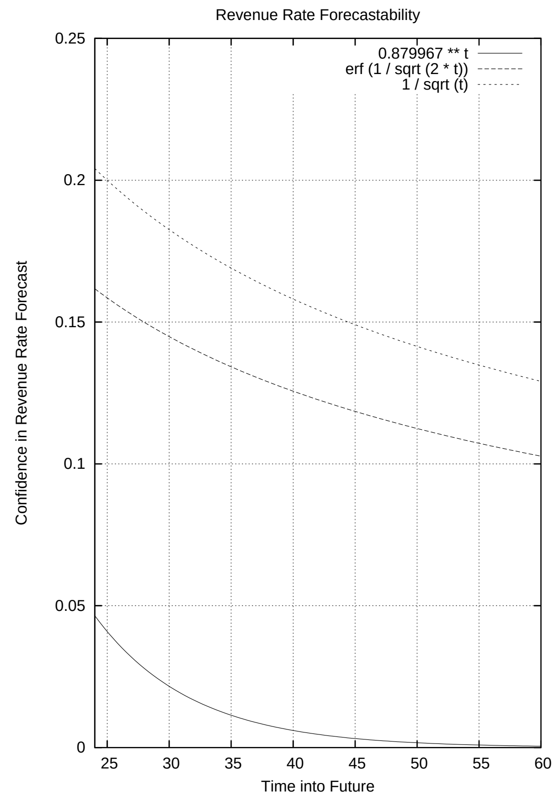

Figure C.272: United States Employment Figures, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

C.12.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.12.3. One of the issues of analysis, as mentioned in Section C.12.7, is to determine the maximum Shannon probability for the time series presented in Figure C.254. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.275 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.254. Figure C.276 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.254. This represents a “what if” the investment strategy was changed from a Shannon probability of 0.525655, as derived in Section C.12.5 to 0.759563. This process, essentially, extracts the random statistical data from the time series presented in Figure C.254, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The “quality” of such a reconstruction should be subject

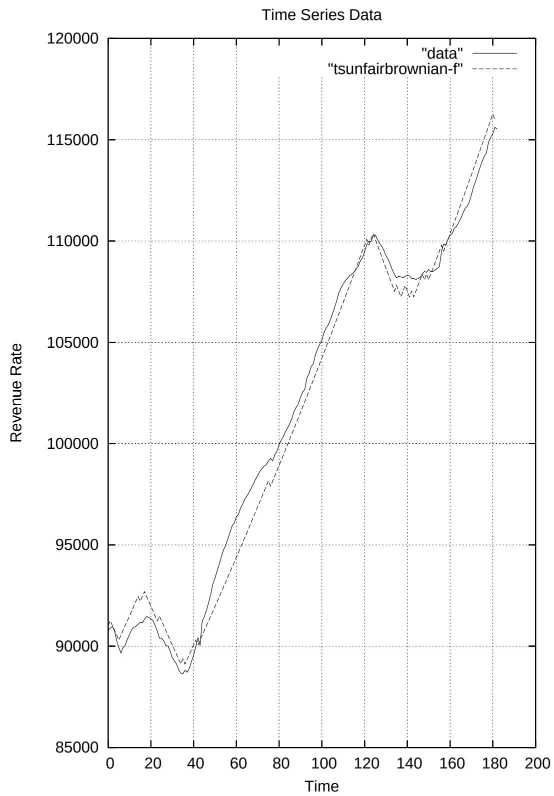

Figure C.273: United States Employment Figures, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.002611. This data is superimposed on the data presented in Figure C.254.

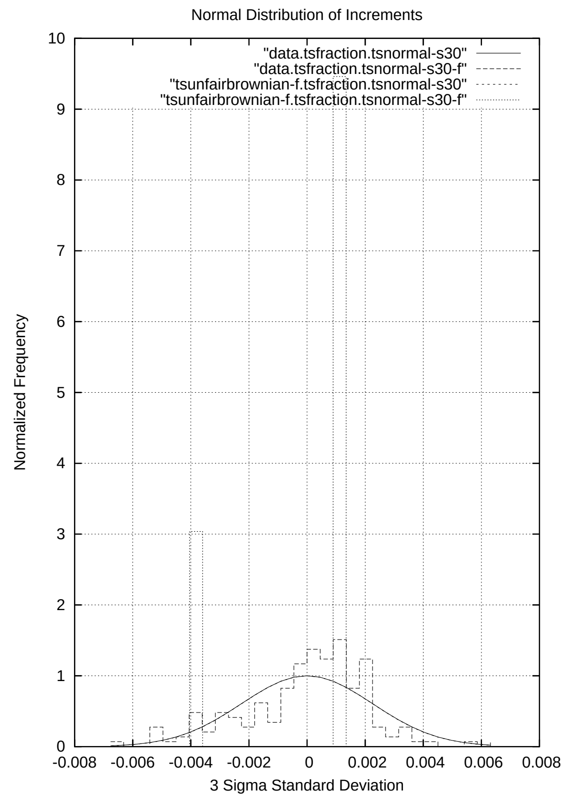

Figure C.274: United States Employment Figures, normalized histogram of the normalized increments of the time series data shown in Figure C.273, empirical and simulated. The empirical data has a mean of 0.001327, with a standard deviation of 0.002254. By comparison, the simulated data has a mean of 0.001342 with a standard deviation of 0.002246. This data is superimposed on the data presented in Figure C.257. The area under the four curves is identical.

to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be “modeled” with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.254, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Employment Figures movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.758242, as compared with the predicted value from the program *tsshannonmax* of 0.759563.

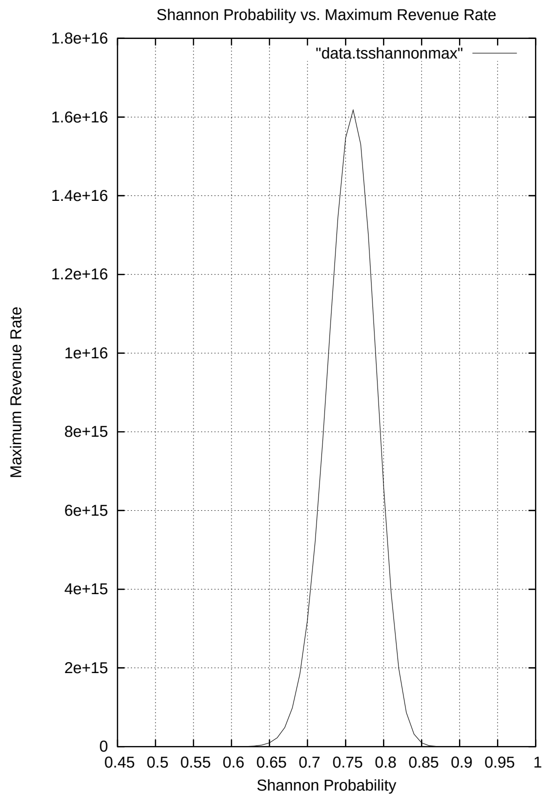

Figure C.275: United States Employment Figures, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.759563.

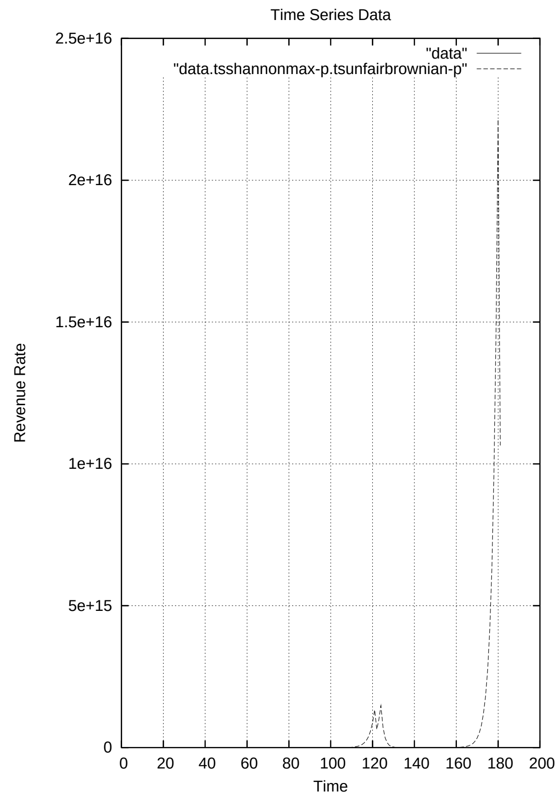

Figure C.276: United States Employment Figures, maximum rate of revenue returns, per month, at a Shannon probability, of 0.759563, corresponding to a “wager” fraction of 0.519126.

C.12.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the “average” of the normalized increments presented in Figure C.256. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.255. These values will be used in a fixed increment Brownian fractal analysis of the United States Employment Figures, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.12.6 and D.12.7. As a subjective evaluation of the “quality” of the analysis of the United States Employment Figures, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.254 from Figure C.255, and the Shannon probability as calculated by counting the total number of months that the United States Employment Figures movement was positive, as presented in Section C.12.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \qquad \text{(C.483)}$$

$$0.758242 \approx \frac{\frac{0.001327}{0.002611}+1}{2} \tag{C.484}$$

$$0.758242 \approx 0.754117 \tag{C.485}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.758242 \approx 0.754117 \approx 0.759563 \tag{C.486}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.12.5, should be compared. The four methods used were the mean of Figure C.255, the constant in the least squares approximation to Figure C.255, the least squares exponential approximation to Figure C.254, and the logarithmic returns of Figure C.254, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.001913 \approx 0.001340 \approx 0.002205 \approx 0.001900 \tag{C.487}$$

It is implied in Section C.12.5, Subsection C.12.5 and in Section C.12.8 that, a Brownian motion with fixed increments fractal may "model" the United States Employment Figures. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.488}$$

$$0.002611\,(2\cdot 0.758242-1) \approx \frac{0.002254\,(2\cdot 0.758242-1)}{2\sqrt{0.758242\,(1-0.758242)}} \tag{C.489}$$

$$0.002611\cdot 0.516484 \approx 0.002254\cdot 0.603159 \tag{C.490}$$

$$0.001349 \approx 0.001360 \tag{C.491}$$

and, equating to the mean:

$$0.001327 \approx 0.001349 \approx 0.001360 \tag{C.492}$$

where, as in Equation C.485 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.254 from Figure C.255, and the Shannon probability as calculated by counting the total number of months that the United States Employment Figures movement was positive, as presented in Section C.12.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[156], where the absolute value is presented in Figure C.256, and the root mean square value is presented in Figure C.255:

$$0.002171 \approx 0.002611 \tag{C.493}$$

Note, that if the United States Employment Figures could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.254 from Figure C.255 should be zero. It is 0.001454.

[156]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

C.13 United States Leading Economic Indicators

For the analysis, the data was in the directory ../markets/us.indicators[157].

The data in this section is presented in tabular form in Section D.13. Note that in this analysis, the rate of revenue returns means the increase or decrease in the United States Leading Economic Indicators. This is included for comparative purposes. Presumably, the United States Leading Economic Indicators represent something of value, or they could be used as a "futures" derivative, and thus, it would be considered that there is a rate of revenue returns.

C.13.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.13.1. Figure C.277 is a graph of the time series data for the United States Leading Economic Indicators.

Figure C.278 is a graph of the normalized increments of the time series data presented in Figure C.277. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.279 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.278. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[158].

Figure C.280 is the normalized histogram of the normalized increments of the time series data shown in Figure C.278. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.280.

Figure C.281 is the statistical estimate for the data presented in Figure C.278, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.518919$, as derived in Section C.13.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.282 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.278. In principle, if the distribution of the normalized increments presented in Figure C.280 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.283 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.278. In principle, if the distribution of the normalized increments presented in Figure C.280 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.284 is the range of values of the time series shown in Figure C.277. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.284

[157] Data from the United States Department of Commerce, 1980—1994, by months, as an index of 1987 = 100.

[158] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

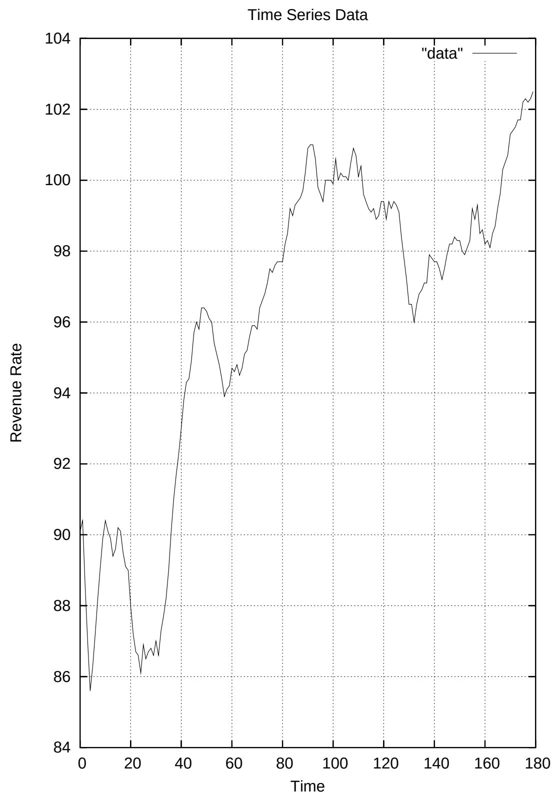

Figure C.277: United States Leading Economic Indicators, time series data.

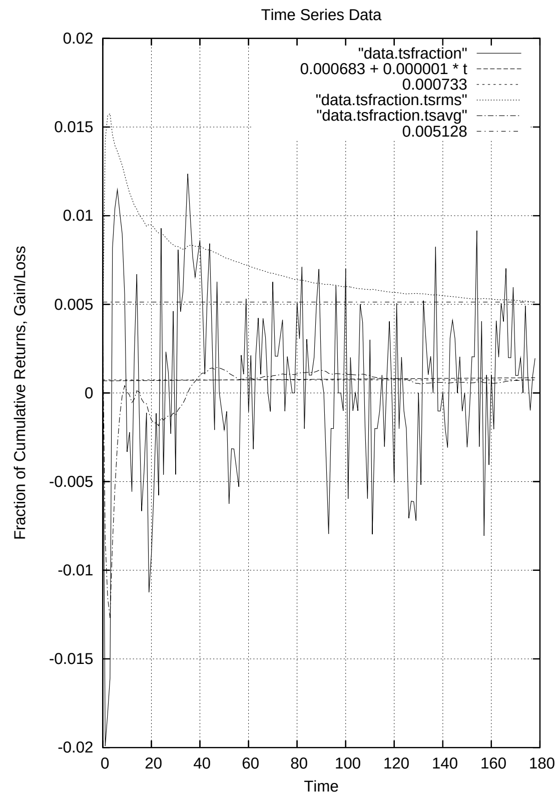

Figure C.278: United States Leading Economic Indicators, normalized increments of the time series data presented in Figure C.277. The mean is 0.000733 with a standard deviation of 0.005089. The formula for the least squares approximation is $0.000683 + 0.000001t$, and the root mean squared value is 0.005128. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000001, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

would be a square root function[159]. Figure C.285 is the deterministic map of the normalized increments of the time series data shown in Figure C.278. The deterministic map is useful for determining if a time series was created by a

[159]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.284 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

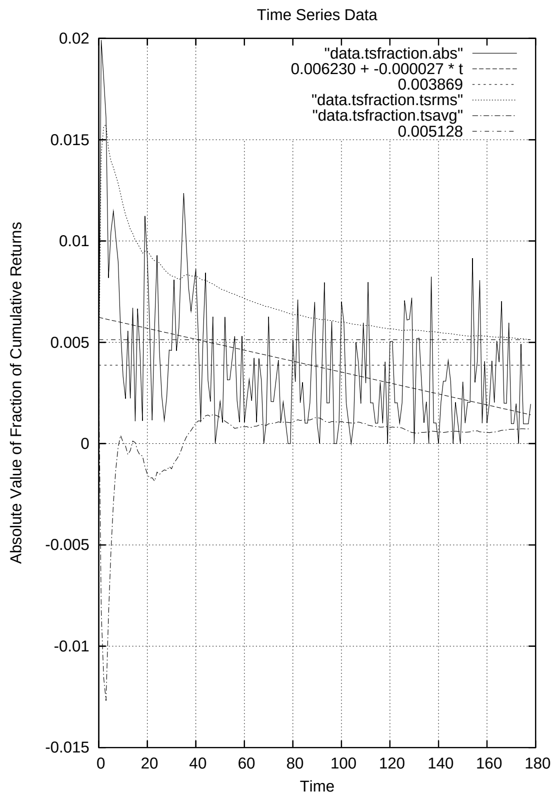

Figure C.279: United States Leading Economic Indicators, absolute value of the normalized increments of the time series data presented in Figure C.278. The mean is 0.003869 with a standard deviation of 0.003375. The formula for the least squares approximation is $0.006230 + -0.000027t$, and the root mean square value, from Figure C.278, is 0.005128. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments presented in Figure C.278, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

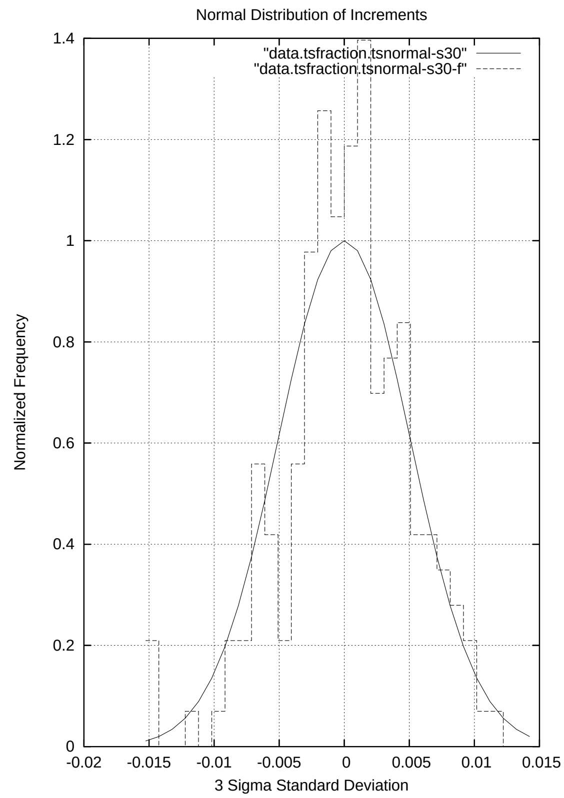

Figure C.280: United States Leading Economic Indicators, normalized histogram of the normalized increments of the time series data shown in Figure C.278. The data has a mean of 0.000733, with a standard deviation of 0.005089. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 4.790000, with a critical value of 42.557000.

deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

```
For a mean of 0.000729, with a confidence level of 0.900000
    that the error did not exceed 0.000073, 13371 samples would be required.
    (With 180 samples, the estimated error is 0.000629 = 86.184948 percent.)
For a standard deviation of 0.005128, with a confidence level of 0.900000
    that the error did not exceed 0.000513, 136 samples would be required.
    (With 180 samples, the estimated error is 0.000445 = 8.669140 percent.)
```

Figure C.281: United States Leading Economic Indicators, statistical estimates of the normalized increments of the time series shown in Figure C.278. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.278.

Observations on the Time Series Increments Analysis

Figure C.280 would seem to indicate that the time series data for the United States Leading Economic Indicators represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.13.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[160]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.286 is the instantaneous value of the root mean square of the normalized increments for the United States Leading Economic Indicators, and Figure C.287 is the instantaneous Shannon probability for the normalized increments.

C.13.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.13.4. Figure C.288 is a graph of the logistic function estimates of the time series data for the United States Leading Economic Indicators. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[161]. The methodology should be regarded as "fragile." It is included for completeness.

[160] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[161] For example, in Figures C.288 and C.289, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.13.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

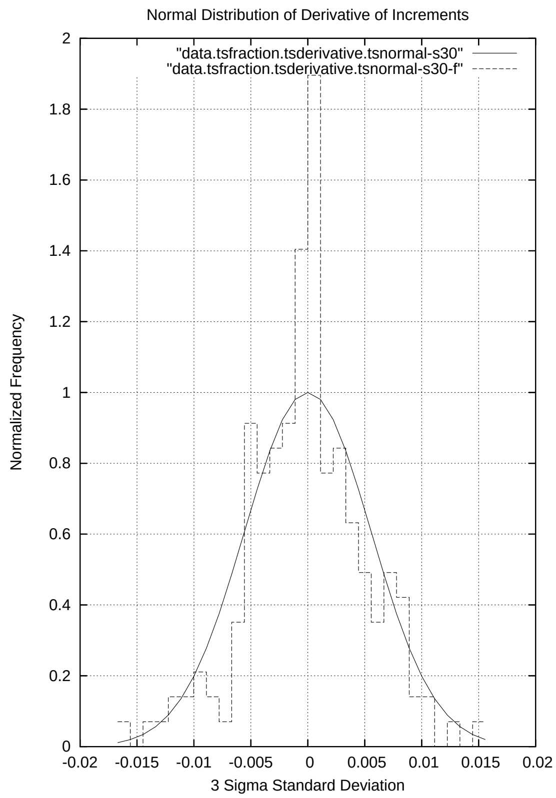

Figure C.282: United States Leading Economic Indicators, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.278.

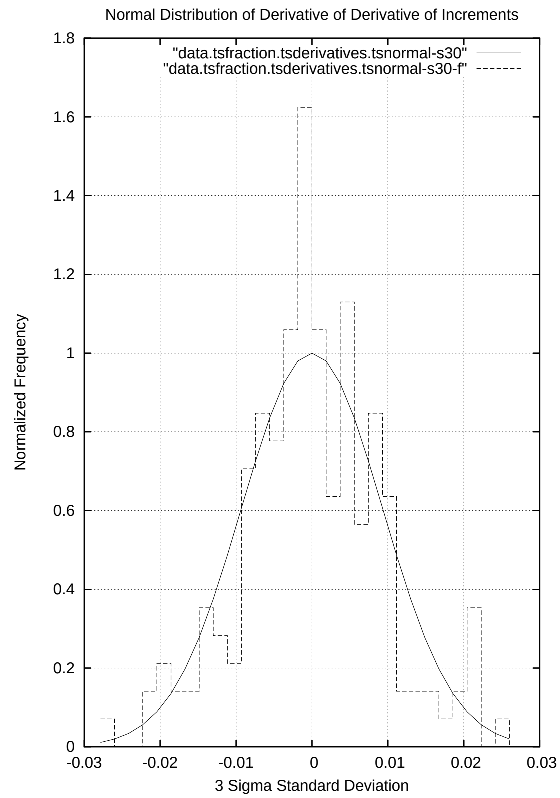

Figure C.283: United States Leading Economic Indicators, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.278.

Figure C.288 is a graph of the logistic function for the time series data presented in Figure C.277. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.278. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.278. Figure C.289 is the same graph, but with the time scale expanded by a factor of two.

C.13.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.13.5. Figure C.290 is a graph of the Hurst coefficient data time series data shown in Figure C.277. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.291 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.278. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.290 implies that the variance of the rate of revenue returns, (per month,) in the United States Leading Economic Indicators, $V(t_2 - t_1)$, over a period of time is proportional to the

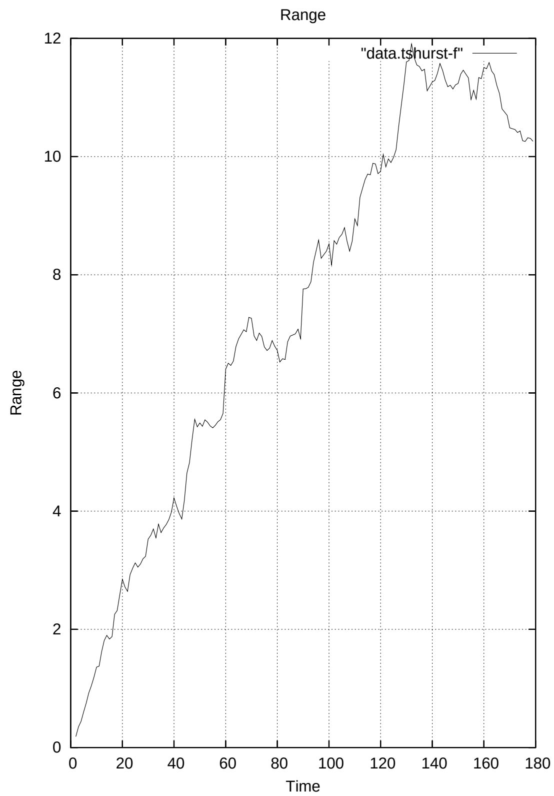

Figure C.284: United States Leading Economic Indicators, range of the time series data shown in Figure C.277.

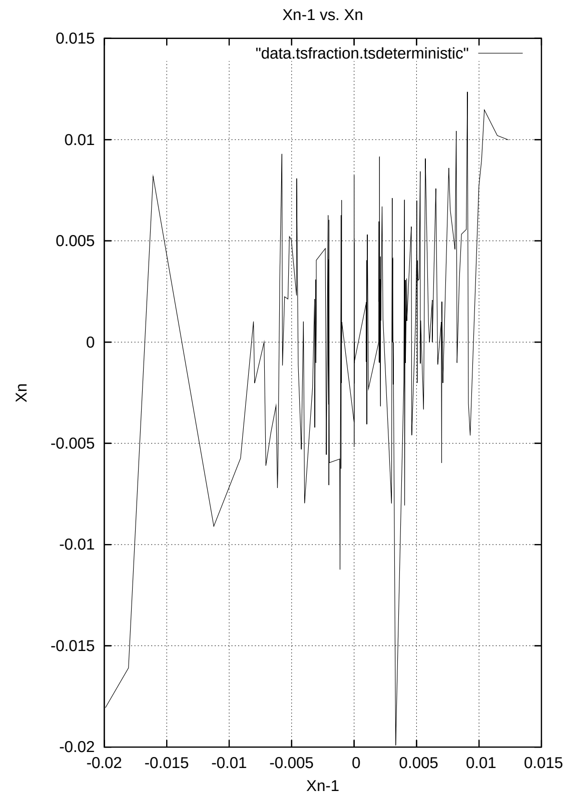

Figure C.285: United States Leading Economic Indicators, deterministic map of the normalized increments of the time series data shown in Figure C.278.

period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[162], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.294, and, C.295 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Leading Economic Indicators for the near term and far term, respectively [Pet91, pp. 83-84][163]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data,

[162]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[163]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

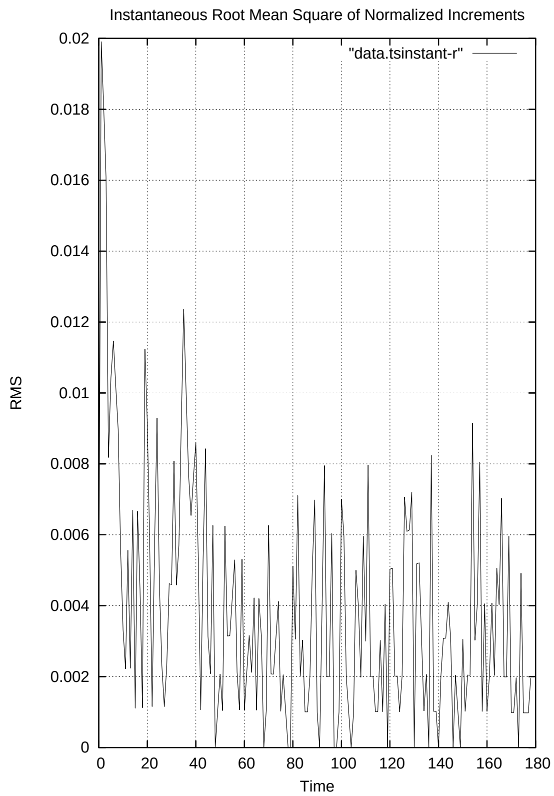

Figure C.286: United States Leading Economic Indicators, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.277.

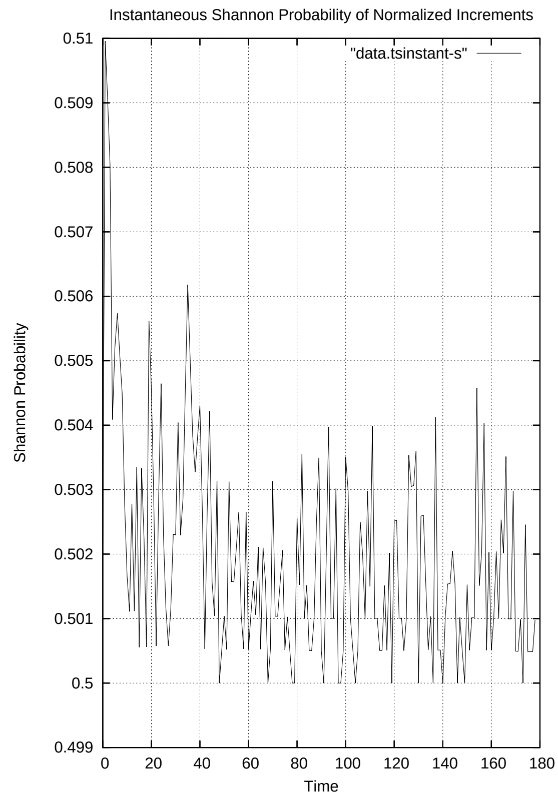

Figure C.287: United States Leading Economic Indicators, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.277.

presented in Figure C.290, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.931126, so that:

$$V\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^{2 \cdot H} \tag{C.494}$$
$$V\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^{2 \cdot 0.931126} \tag{C.495}$$
$$\propto\; (t_2 - t_1)^{1.862252} \tag{C.496}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on

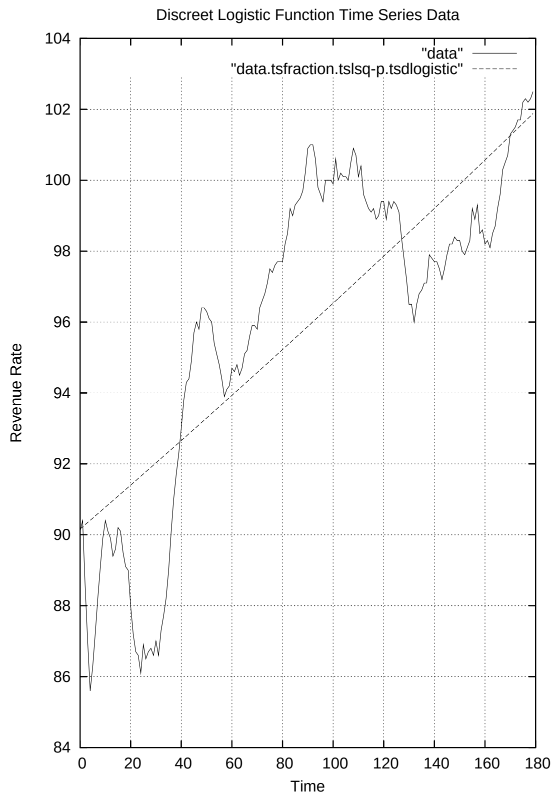

Figure C.288: United States Leading Economic Indicators, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.278 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

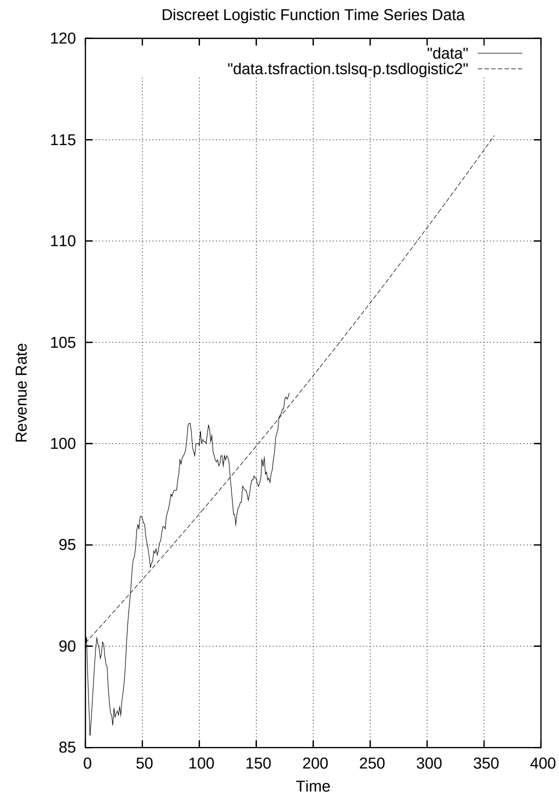

Figure C.289: United States Leading Economic Indicators, logistic function estimates of Figure C.288 with the time scale expanded by a factor of two.

the past[164]. A Hurst coefficient of 0.931126, (for the near future, and 0.714236 for the distant future.) implies that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 93.112600% [Pet91, pp. 66] for the near future, and 0.714236 for the distant future. Likewise, there is a 93.112600% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 93.112600% that the rate of revenue

[164] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.13.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Leading Economic Indicators. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

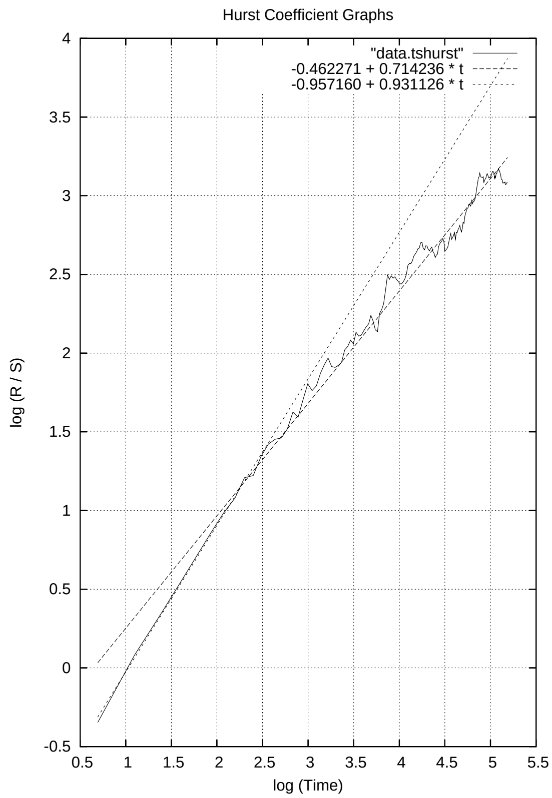

Figure C.290: United States Leading Economic Indicators, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.278. The slope of the graph is the Hurst coefficient.

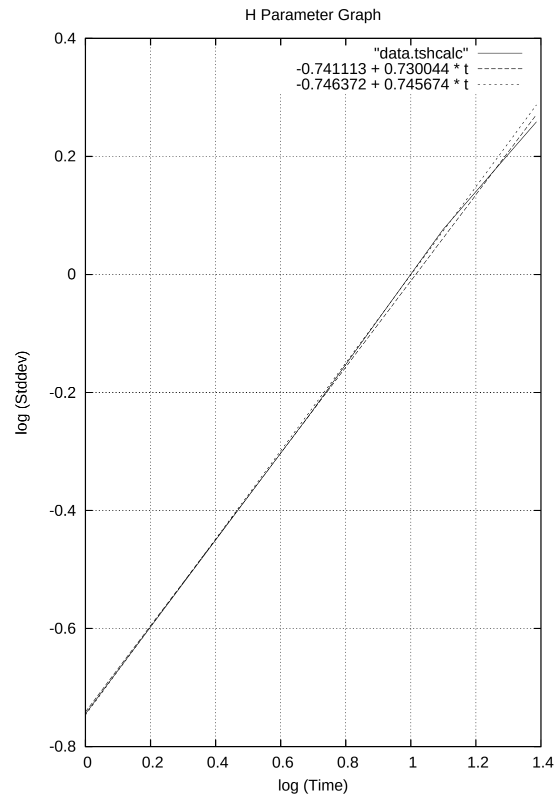

Figure C.291: United States Leading Economic Indicators, H parameter data for the normalized increments of the time series data shown in Figure C.278 The slope of the graph is the H parameter.

returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Leading Economic Indicators are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.497}$$

$$= 0.931126^n \tag{C.498}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.278, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.000733 \cdot 100$ percent, on the average, with a standard deviation of $0.005089 \cdot 100$ percent, and a root mean square error value of $0.005128 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v (t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.499}$$
$$\propto (t_2 - t_1)^{0.931126} \tag{C.500}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.500 reduces to, [Sch91, pp. 129]:

$$R (t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.501}$$
$$\propto \sqrt{(t_2 - t_1)} \tag{C.502}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R (t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.503}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[165].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X (t) \propto \frac{X (rt)}{r^H} \tag{C.504}$$
$$\propto \frac{X (rt)}{r^{0.931126}} \tag{C.505}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.291, to provide a least squares approximation to the H parameter for the United States Leading Economic Indicators. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.745674 for the near future, and 0.730044 for the distant future.

Figures C.290 and C.291 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.278. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.278, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.292 and C.293 was made using the -d option.

[165]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

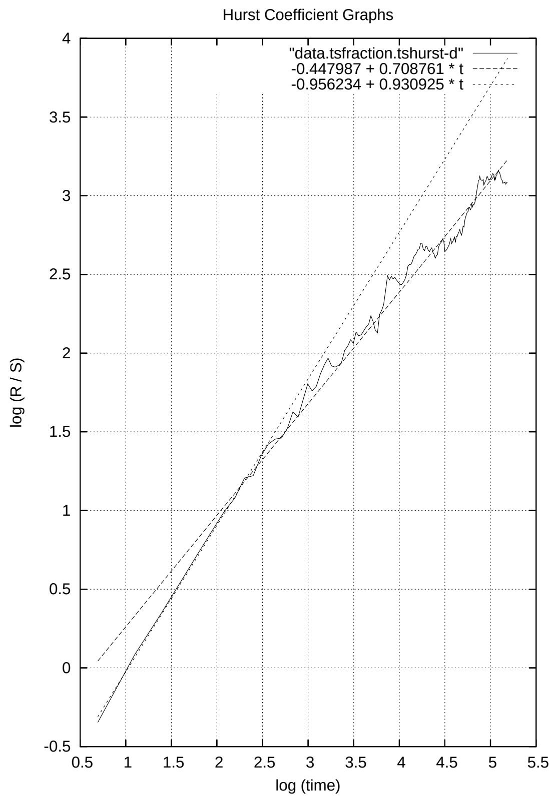

Figure C.292: United States Leading Economic Indicators, traditional Hurst coefficient data for the time series data shown in Figure C.277. The slope of the graph is the Hurst coefficient, and is 0.930925 for the near term, and 0.708761 for the far term.

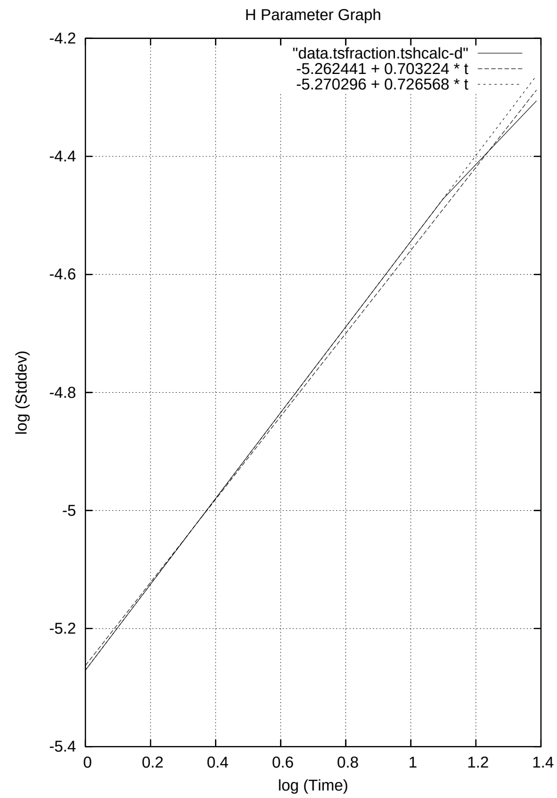

Figure C.293: United States Leading Economic Indicators, traditional H parameter data for the time series data shown in Figure C.277 The slope of the graph is the H parameter, and is 0.726568 for the near term, and 0.703224 for the far term.

C.13.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.13.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.279. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.278. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Leading Economic Indicators, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Leading Economic Indicators, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.278, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.000733 + 1)}{\ln(2)} = 0.001057 \tag{C.506}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.278, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.000683 + 1)}{\ln(2)} = 0.000985 \tag{C.507}$$

Note that if the mean is not constant in Figure C.278, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.277:

$$bits = 0.001105 \tag{C.508}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.277:

$$bits = 0.001033 \tag{C.509}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.13.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.001033t} \tag{C.510}$$

therefore:

$$C(p) = 0.001033 \tag{C.511}$$

and, *tsshannon* 0.001033 gives:

$$C(0.518919) = 0.001033 \tag{C.512}$$

therefore:

$$\begin{aligned} 2^{C(0.518919)} &= 2^{0.001033} && \text{(C.513)}\\ &= 1.000716 && \text{(C.514)}\\ &= 0.071628\% && \text{(C.515)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.518919) - 1 && \text{(C.516)}\\ &= 0.037838 && \text{(C.517)}\\ &= 3.783800\% && \text{(C.518)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the United States Leading Economic Indicators executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 3.783800% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 96.216200% will be held in “reserve” with a 51.891900% chance of making twice the 3.783800% back, (and a 48.108100% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.071628%, or a doubling of its rate of revenue returns, (per month,) in 968.054211 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 3.783800% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.071628%, per month, on average.

Note that the metrics presented in this section are representative of the United States Leading Economic Indicators as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 3.783800% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Leading Economic Indicators's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.071628% per month.

As another simple example, a company re-invests 3.783800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 3.783800% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.071628% per month.

As an example of "product portfolio" management, suppose a company re-invests 3.783800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 3.783800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 3.783800$ percent for the second product, implying that the company should diversify its product line[166]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 3.783800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Leading Economic Indicators, as a standard bench mark, then the optimal number will be $\frac{1}{0.037838}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.278, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex

[166] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389 \ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 3.783800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 3.783800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.13.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Leading Economic Indicators, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Leading Economic Indicators time series is 0.000733, and 0.005128respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 27.874555.

If this value seems consistent number of companies in the United States Leading Economic Indicators, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Leading Economic Indicators are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.513537, which would be the value which should be used in Section C.13.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.13.5 is greater than the average Shannon probability for the companies participating in the United States Leading Economic Indicators, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.13.5. The maximum exploitability for the United States Leading Economic Indicators is derived in Section C.13.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Leading Economic Indicators is 0.513537, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.571470 in the United States Leading Economic Indicators. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.519}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Leading Economic Indicators would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.13.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.279. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.278. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Leading Economic Indicators, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.13.5, is derived from the United States Leading Economic Indicators metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.13.9.

An additional exploitable strategy may be time itself. Equations C.496, C.500, and, C.498, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Leading Economic Indicators, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.294, and, C.295 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Leading Economic Indicators for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Leading Economic Indicators. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[167]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.294, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.498, $0.931126^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.496, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.13.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.13.9. Figure C.296 represents a constructional simulation of the time series data presented in Figure C.277. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.278. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.278 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.297 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.280.

[167]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

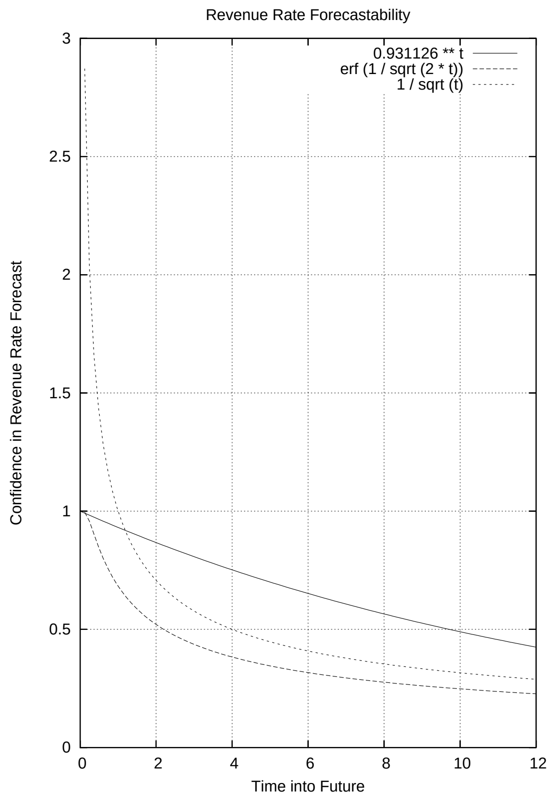

Figure C.294: United States Leading Economic Indicators, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.931126^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

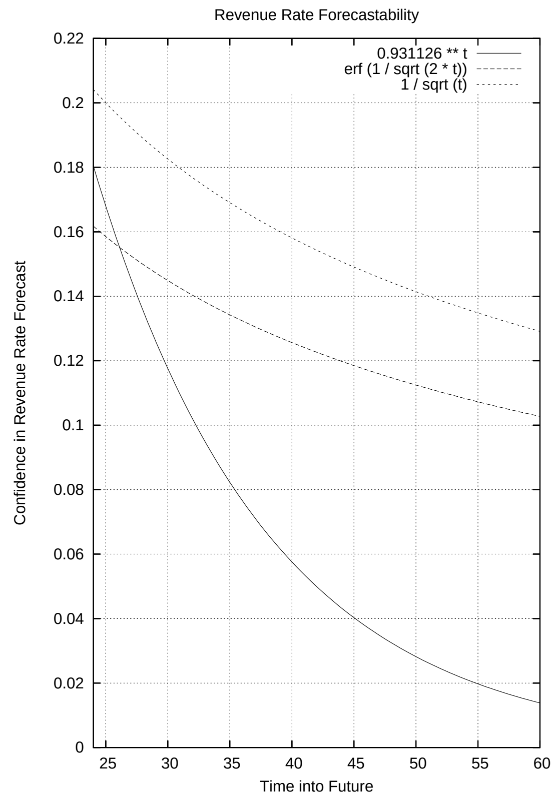

Figure C.295: United States Leading Economic Indicators, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

C.13.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.13.3. One of the issues of analysis, as mentioned in Section C.13.7, is to determine the maximum Shannon probability for the time series presented in Figure C.277. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.298 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.277. Figure C.299 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.277. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.518919, as derived in Section C.13.5 to 0.622222. This process, essentially, extracts the random statistical data from the time series presented in Figure C.277, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new

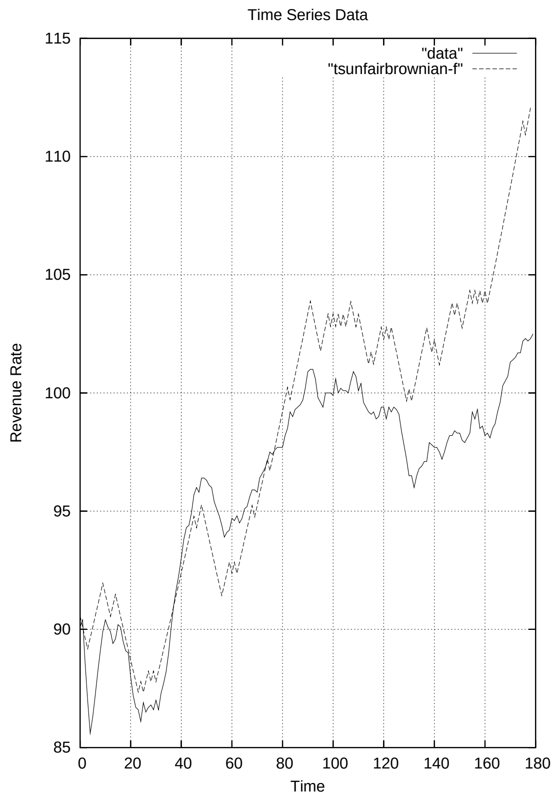

Figure C.296: United States Leading Economic Indicators, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.005128. This data is superimposed on the data presented in Figure C.277.

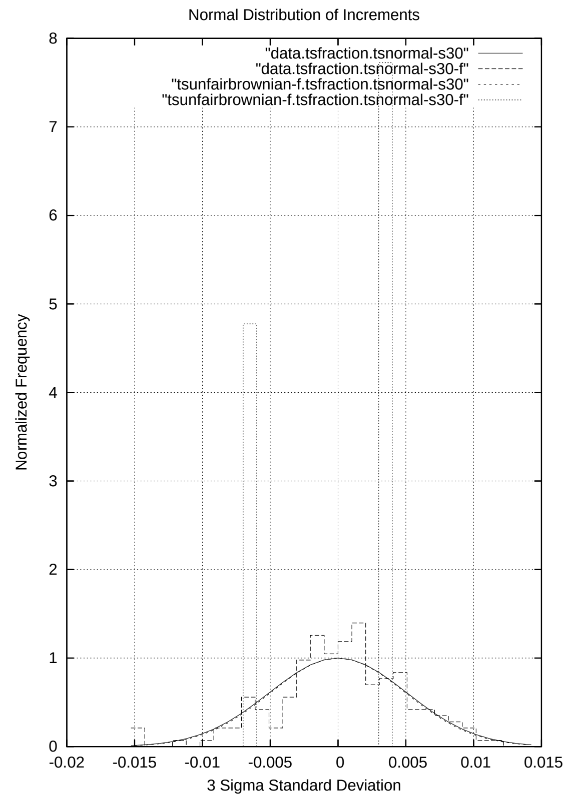

Figure C.297: United States Leading Economic Indicators, normalized histogram of the normalized increments of the time series data shown in Figure C.296, empirical and simulated. The empirical data has a mean of 0.000733, with a standard deviation of 0.005089. By comparison, the simulated data has a mean of 0.001210 with a standard deviation of 0.004997. This data is superimposed on the data presented in Figure C.280. The area under the four curves is identical.

time series as a Brownian fractal with fixed increments. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be “modeled” with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.277, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Leading Economic Indicators movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.620112, as compared with the predicted value from the program *tsshannonmax* of 0.622222.

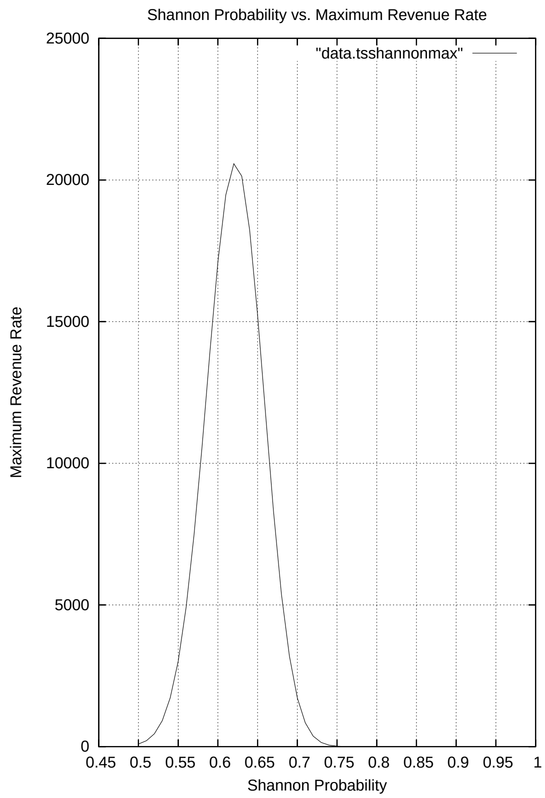

Figure C.298: United States Leading Economic Indicators, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.622222.

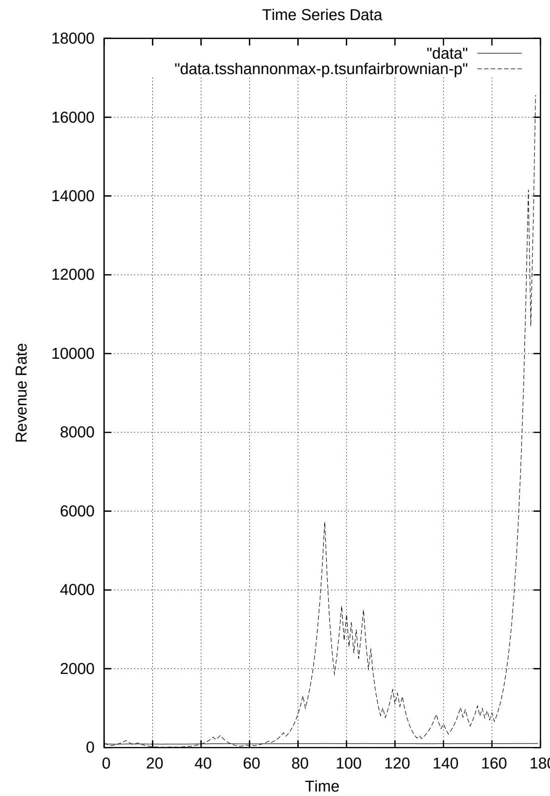

Figure C.299: United States Leading Economic Indicators, maximum rate of revenue returns, per month, at a Shannon probability, of 0.622222, corresponding to a "wager" fraction of 0.244444.

C.13.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.279. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.278. These values will be used in a fixed increment Brownian fractal analysis of the United States Leading Economic Indicators, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.13.6 and D.13.7. As a subjective evaluation of the "quality" of the analysis of the United States Leading Economic Indicators, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.277 from Figure C.278, and the Shannon probability as calculated by counting the total number of months that the United States Leading Economic Indicators movement was positive, as presented in Section C.13.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.520}$$

$$0.620112 \approx \frac{\frac{0.000733}{0.005128} + 1}{2} \tag{C.521}$$
$$0.620112 \approx 0.571470 \tag{C.522}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.620112 \approx 0.571470 \approx 0.622222 \tag{C.523}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.13.5, should be compared. The four methods used were the mean of Figure C.278, the constant in the least squares approximation to Figure C.278, the least squares exponential approximation to Figure C.277, and the logarithmic returns of Figure C.277, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.001057 \approx 0.000985 \approx 0.001105 \approx 0.001033 \tag{C.524}$$

It is implied in Section C.13.5, Subsection C.13.5 and in Section C.13.8 that, a Brownian motion with fixed increments fractal may "model" the United States Leading Economic Indicators. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.525}$$
$$0.005128\,(2 \cdot 0.620112 - 1) \approx \frac{0.005089\,(2 \cdot 0.620112 - 1)}{2\sqrt{0.620112\,(1-0.620112)}} \tag{C.526}$$
$$0.005128 \cdot 0.240223 \approx 0.005089 \cdot 0.247470 \tag{C.527}$$
$$0.001232 \approx 0.001259 \tag{C.528}$$

and, equating to the mean:

$$0.000733 \approx 0.001232 \approx 0.001259 \tag{C.529}$$

where, as in Equation C.522 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.277 from Figure C.278, and the Shannon probability as calculated by counting the total number of months that the United States Leading Economic Indicators movement was positive, as presented in Section C.13.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[168], where the absolute value is presented in Figure C.279, and the root mean square value is presented in Figure C.278:

$$0.003869 \approx 0.005128 \tag{C.530}$$

Note, that if the United States Leading Economic Indicators could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.277 from Figure C.278 should be zero. It is 0.003375.

[168] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

C.14 United States M2

For the analysis, the data was in the directory ../markets/us.m2[169].

The data in this section is presented in tabular form in Section D.14. Note that in this analysis, the rate of revenue returns means the increase or decrease in the United States M2. This is included for comparative purposes. Presumably, the United States M2 represents something of value, or it could be used as a "futures" derivative, and thus, it would be considered that there is a rate of revenue returns.

C.14.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.14.1. Figure C.300 is a graph of the time series data for the United States M2.

Figure C.301 is a graph of the normalized increments of the time series data presented in Figure C.300. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.302 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.301. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[170].

Figure C.303 is the normalized histogram of the normalized increments of the time series data shown in Figure C.301. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.303.

Figure C.304 is the statistical estimate for the data presented in Figure C.301, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.527119$, as derived in Section C.14.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.305 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.301. In principle, if the distribution of the normalized increments presented in Figure C.303 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.306 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.301. In principle, if the distribution of the normalized increments presented in Figure C.303 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.307 is the range of values of the time series shown in Figure C.300. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.307

[169] Data from the United States Federal Reserve Board, 1980—1994, by months, in billions of 1987 dollars, US.

[170] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

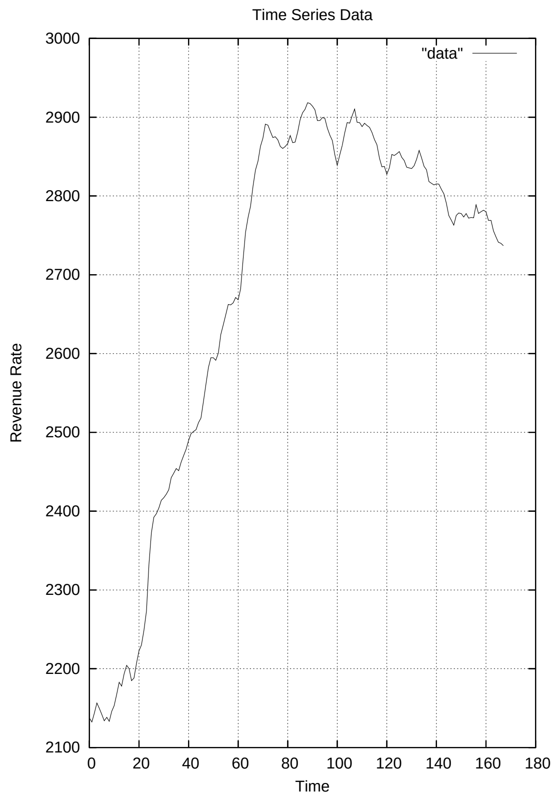

Figure C.300: United States M2, time series data.

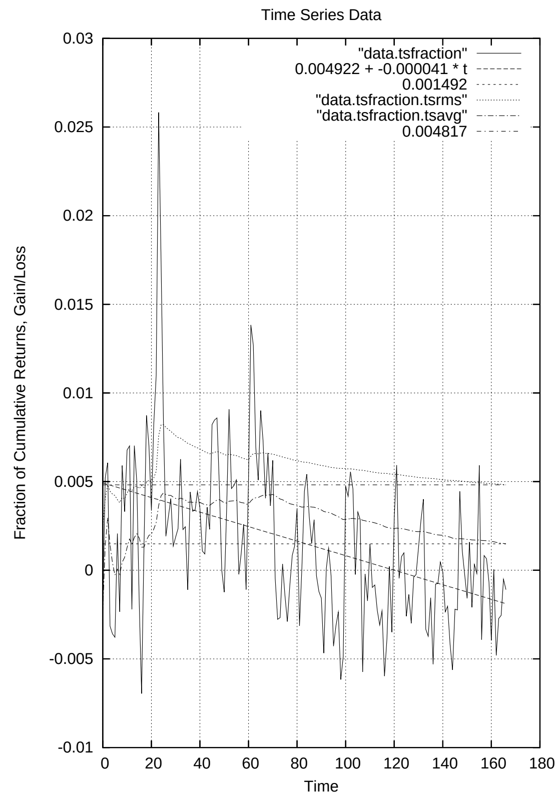

Figure C.301: United States M2, normalized increments of the time series data presented in Figure C.300. The mean is 0.001492 with a standard deviation of 0.004594. The formula for the least squares approximation is $0.004922 + -0.000041t$, and the root mean squared value is 0.004817. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000041, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

would be a square root function[171]. Figure C.308 is the deterministic map of the normalized increments of the time series data shown in Figure C.301. The deterministic map is useful for determining if a time series was created by a

[171]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.307 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

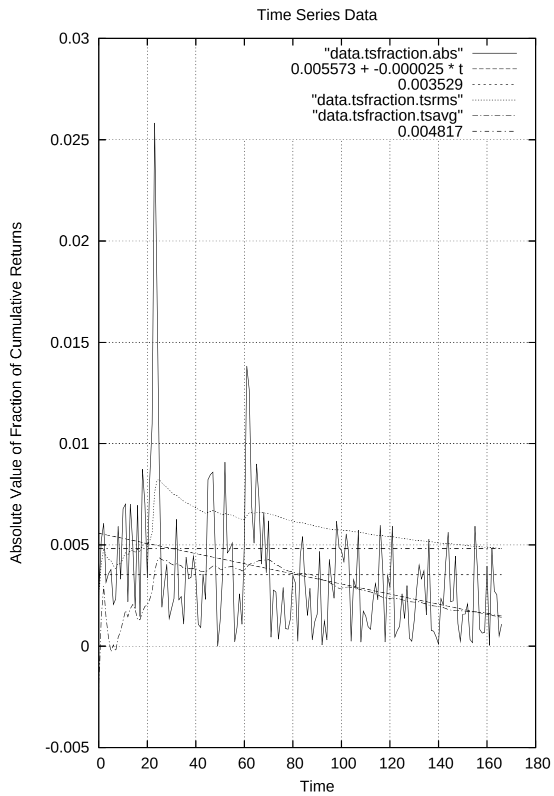

Figure C.302: United States M2, absolute value of the normalized increments of the time series data presented in Figure C.301. The mean is 0.003529 with a standard deviation of 0.003289. The formula for the least squares approximation is $0.005573 + -0.000025t$, and the root mean square value, from Figure C.301, is 0.004817. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.301, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

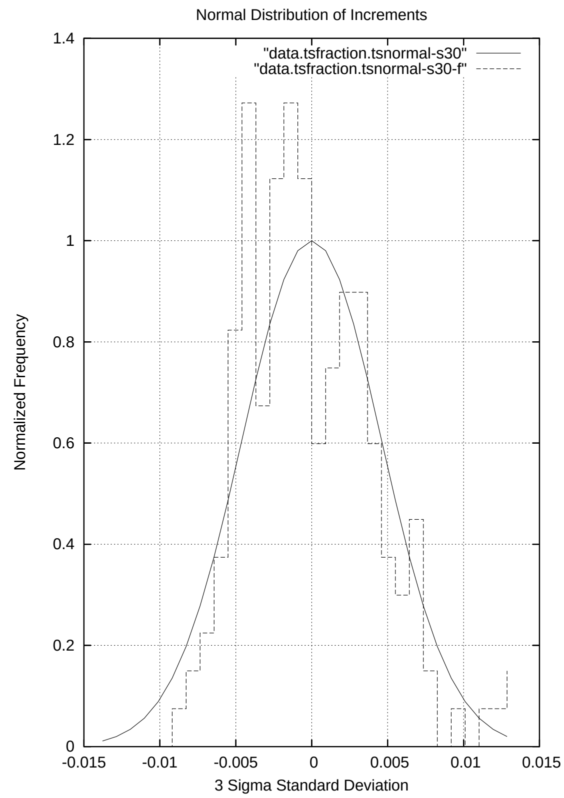

Figure C.303: United States M2, normalized histogram of the normalized increments of the time series data shown in Figure C.301. The data has a mean of 0.001492, with a standard deviation of 0.004594. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 3.176000, with a critical value of 42.557000.

deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

```
For  a mean  of 0.001483,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000148,   2856  samples  would  be  required.
     (With  168  samples,   the  estimated   error  is 0.000611   = 41.227913   percent.)
For  a standard   deviation   of 0.004817,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000482,   136  samples  would  be  required.
     (With  168  samples,   the  estimated   error  is 0.000432   = 8.973412   percent.)
```

Figure C.304: United States M2, statistical estimates of the normalized increments of the time series shown in Figure C.301. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.301.

Observations on the Time Series Increments Analysis

Figure C.303 would seem to indicate that the time series data for the United States M2 represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.14.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[172]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.309 is the instantaneous value of the root mean square of the normalized increments for the United States M2, and Figure C.310 is the instantaneous Shannon probability for the normalized increments.

C.14.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.14.4. Figure C.311 is a graph of the logistic function estimates of the time series data for the United States M2. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[173]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.311 is a graph of the logistic function for the time series data presented in Figure C.300. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters

[172] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[173] For example, in Figures C.311 and C.312, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.14.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

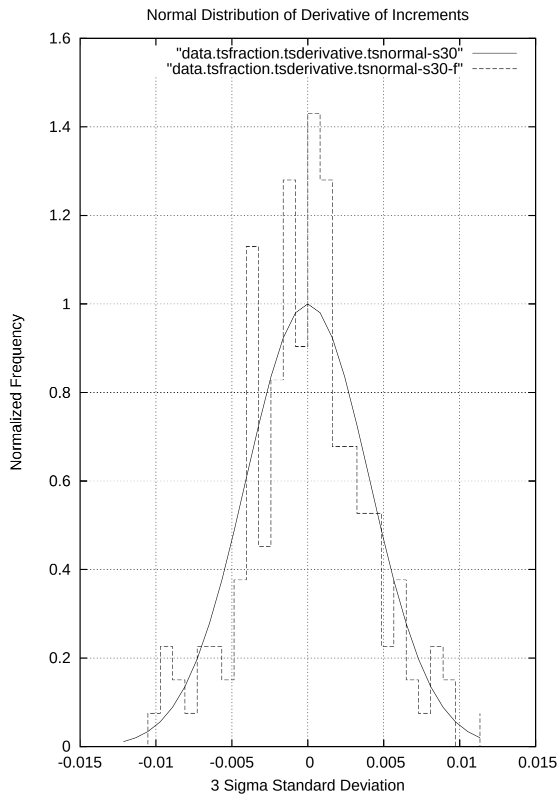

Figure C.305: United States M2, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.301.

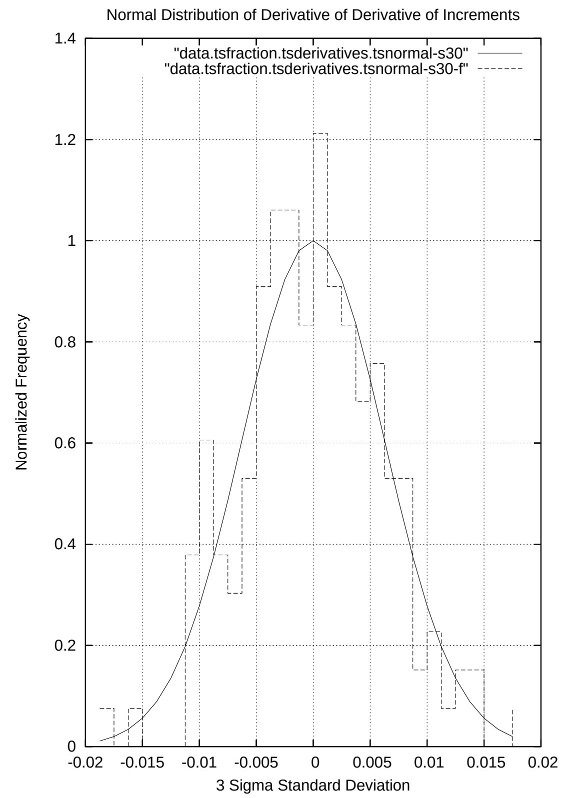

Figure C.306: United States M2, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.301.

extracted from the time series data as suggested in Figure C.301. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.301. Figure C.312 is the same graph, but with the time scale expanded by a factor of two.

C.14.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.14.5. Figure C.313 is a graph of the Hurst coefficient data time series data shown in Figure C.300. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.314 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.301. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.313 implies that the variance of the rate of revenue returns, (per month,) in the United States M2, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional

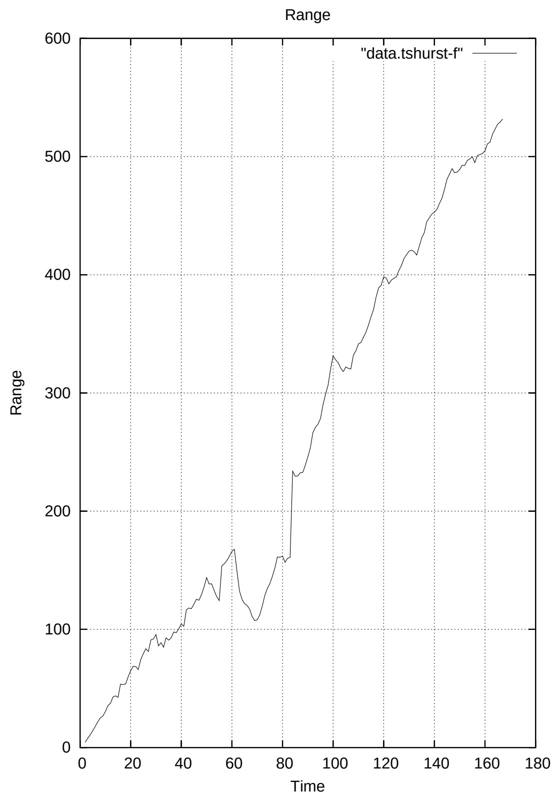

Figure C.307: United States M2, range of the time series data shown in Figure C.300.

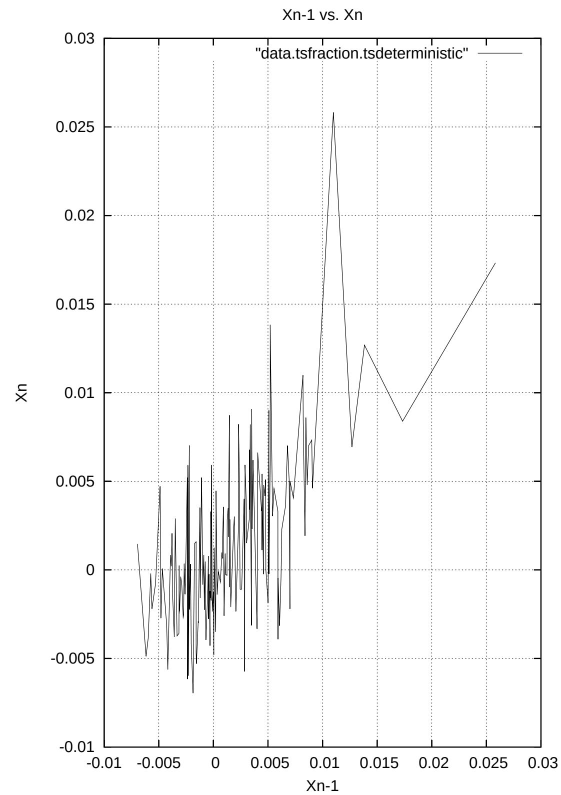

Figure C.308: United States M2, deterministic map of the normalized increments of the time series data shown in Figure C.301.

implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[174], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.317, and, C.318 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States M2 for the near term and far term, respectively [Pet91, pp. 83-84][175]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.313, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.956159, so that:

[174]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[175]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

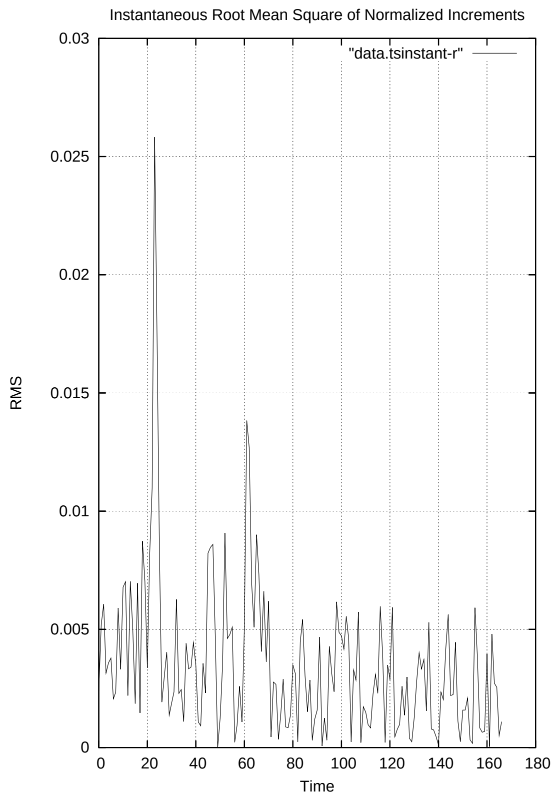

Figure C.309: United States M2, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.300.

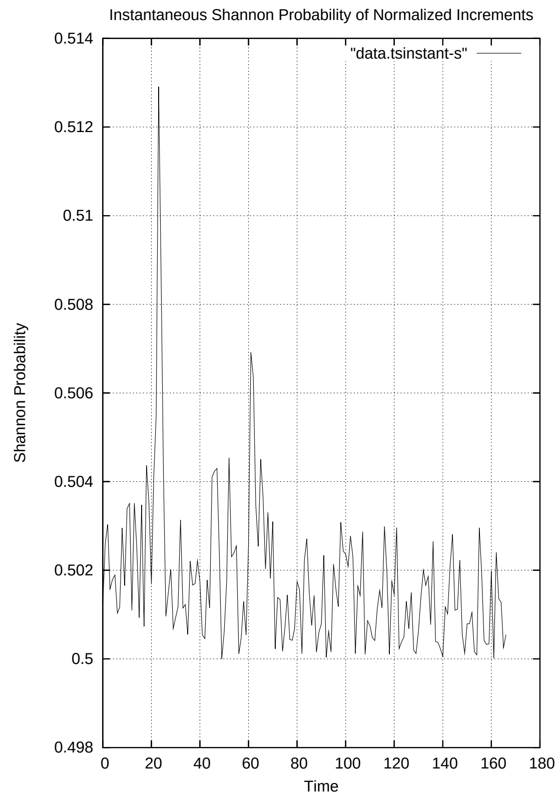

Figure C.310: United States M2, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.300.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.531}$$
$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.956159} \tag{C.532}$$
$$\propto (t_2 - t_1)^{1.912318} \tag{C.533}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[176]. A Hurst coefficient of 0.956159, (for the near future, and 0.917851 for the distant future.) implies that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is

[176] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For

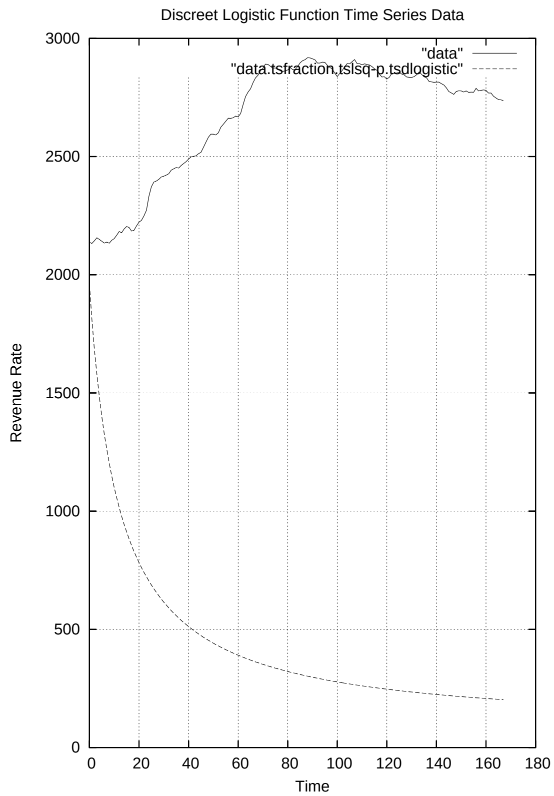

Figure C.311: United States M2, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.301 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

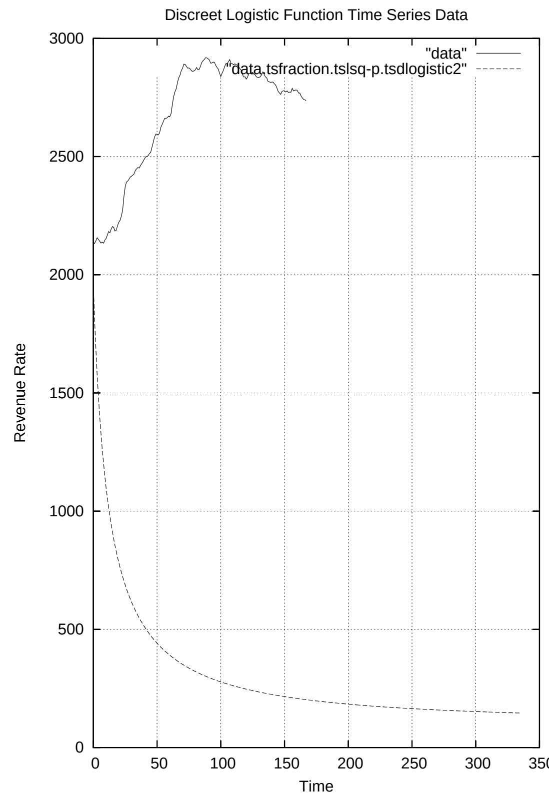

Figure C.312: United States M2, logistic function estimates of Figure C.311 with the time scale expanded by a factor of two.

95.615900% [Pet91, pp. 66] for the near future, and 0.917851 for the distant future. Likewise, there is a 95.615900% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 95.615900% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States M2 are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.534}$$

the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.14.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States M2. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

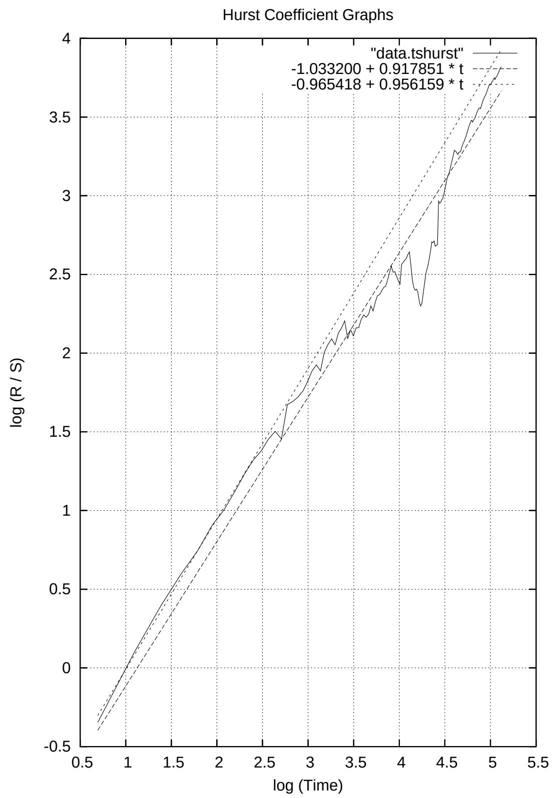

Figure C.313: United States M2, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.301. The slope of the graph is the Hurst coefficient.

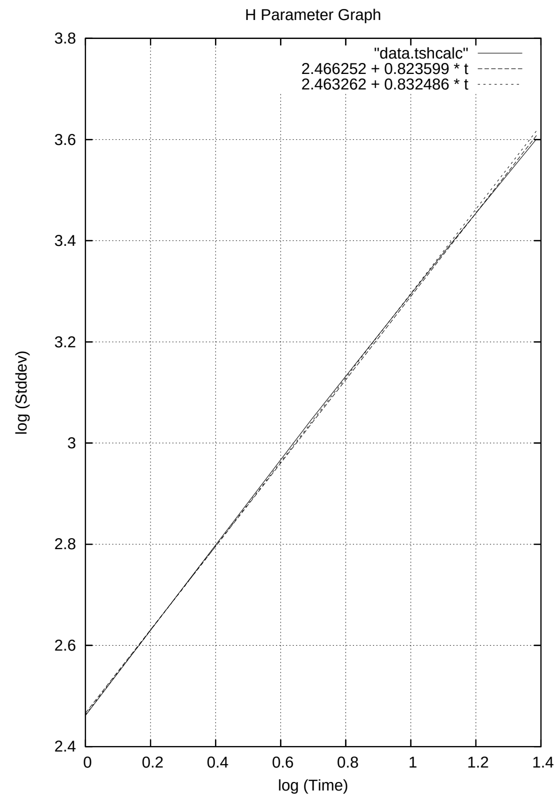

Figure C.314: United States M2, H parameter data for the normalized increments of the time series data shown in Figure C.301 The slope of the graph is the H parameter.

$$= \quad 0.956159^n \tag{C.535}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.301, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.001492 \cdot 100$ percent, on the average, with a standard deviation of $0.004594 \cdot 100$ percent, and a root mean square error value of $0.004817 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^H \tag{C.536}$$

$$\propto \quad (t_2 - t_1)^{0.956159} \tag{C.537}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.537 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.538}$$

$$\propto \sqrt{\left(t_2 - t_1\right)} \tag{C.539}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.540}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[177].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \propto \frac{X\left(rt\right)}{r^{H}} \tag{C.541}$$

$$\propto \frac{X\left(rt\right)}{r^{0.956159}} \tag{C.542}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.314, to provide a least squares approximation to the H parameter for the United States M2. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.832486 for the near future, and 0.823599 for the distant future.

Figures C.313 and C.314 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.301. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.301, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.315 and C.316 was made using the -d option.

C.14.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.14.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.302. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.301. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States M2, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States M2, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

[177] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

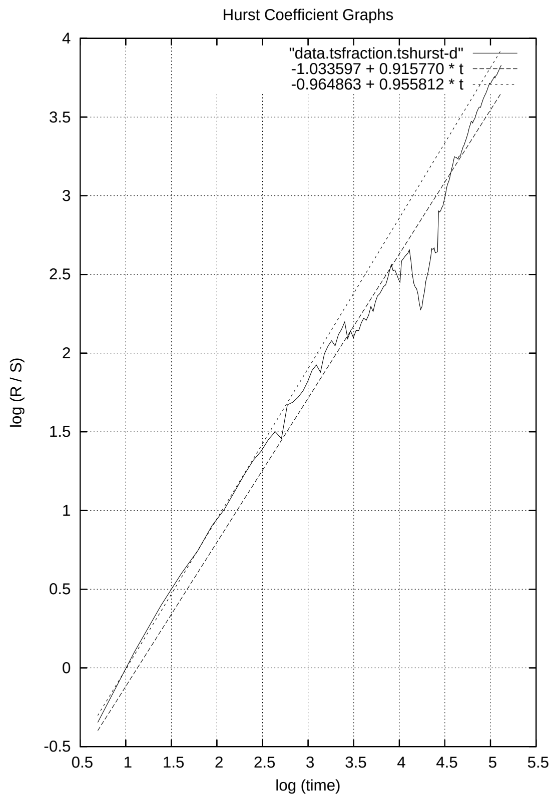

Figure C.315: United States M2, traditional Hurst coefficient data for the time series data shown in Figure C.300. The slope of the graph is the Hurst coefficient, and is 0.955812 for the near term, and 0.915770 for the far term.

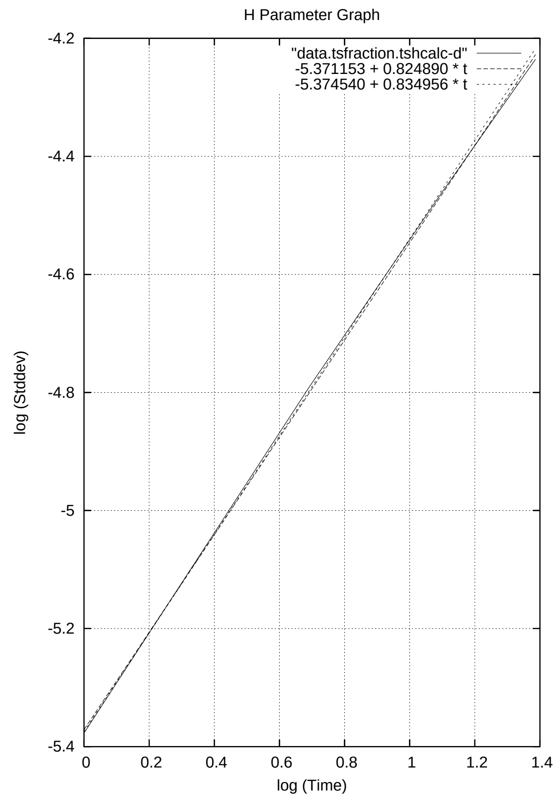

Figure C.316: United States M2, traditional H parameter data for the time series data shown in Figure C.300 The slope of the graph is the H parameter, and is 0.834956 for the near term, and 0.824890 for the far term.

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.301, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.001492 + 1)}{\ln(2)} = 0.002151 \tag{C.543}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.301, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.004922 + 1)}{\ln(2)} = 0.007084 \tag{C.544}$$

Note that if the mean is not constant in Figure C.301, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.300:

$$bits = 0.002294 \tag{C.545}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.300:

$$bits = 0.002123 \tag{C.546}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.14.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.002123t} \tag{C.547}$$

therefore:

$$C(p) = 0.002123 \tag{C.548}$$

and, *tsshannon* 0.002123 gives:

$$C(0.527119) = 0.002123 \tag{C.549}$$

therefore:

$$\begin{aligned} 2^{C(0.527119)} &= 2^{0.002123} && \text{(C.550)} \\ &= 1.001473 && \text{(C.551)} \\ &= 0.147263\% && \text{(C.552)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.527119) - 1 && \text{(C.553)} \\ &= 0.054238 && \text{(C.554)} \\ &= 5.423800\% && \text{(C.555)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the United States M2 executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 5.423800% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 94.576200% will be held in “reserve” with a 52.711900% chance of making twice the 5.423800% back, (and a 47.288100% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.147263%, or a doubling of its rate of revenue returns, (per month,) in 471.031559 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 5.423800% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.147263%, per month, on average.

Note that the metrics presented in this section are representative of the United States M2 as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 5.423800% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States M2's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.147263% per month.

As another simple example, a company re-invests 5.423800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 5.423800% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.147263% per month.

As an example of "product portfolio" management, suppose a company re-invests 5.423800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 5.423800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 5.423800$ percent for the second product, implying that the company should diversify its product line[178]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 5.423800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States M2, as a standard bench mark, then the optimal number will be $\frac{1}{0.054238}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.301, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 5.423800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 5.423800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.14.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States M2, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the

[178]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

normalized increments of the United States M2 time series is 0.001492, and 0.004817respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 64.300675.

If this value seems consistent number of companies in the United States M2, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States M2 are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.519313, which would be the value which should be used in Section C.14.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.14.5 is greater than the average Shannon probability for the companies participating in the United States M2, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.14.5. The maximum exploitability for the United States M2 is derived in Section C.14.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States M2 is 0.519313, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.654868 in the United States M2. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.556}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States M2 would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.14.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.302. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.301. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States M2, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.14.5, is derived from the United States M2 metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.14.9.

An additional exploitable strategy may be time itself. Equations C.533, C.537, and, C.535, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States M2, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.317, and, C.318 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States M2 for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the “average” product or service life cycle in the United States M2. When the “forecastability” of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the “forecastability” of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of “market windows.[179]” The conceptual approximation will probably predict a “conservative” or “pessimistic” value in relation to actual markets.

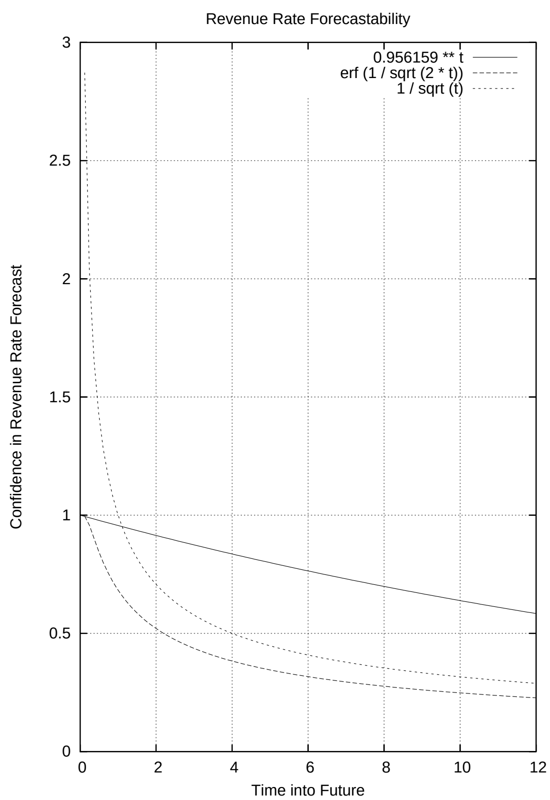

Figure C.317: United States M2, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.956159^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

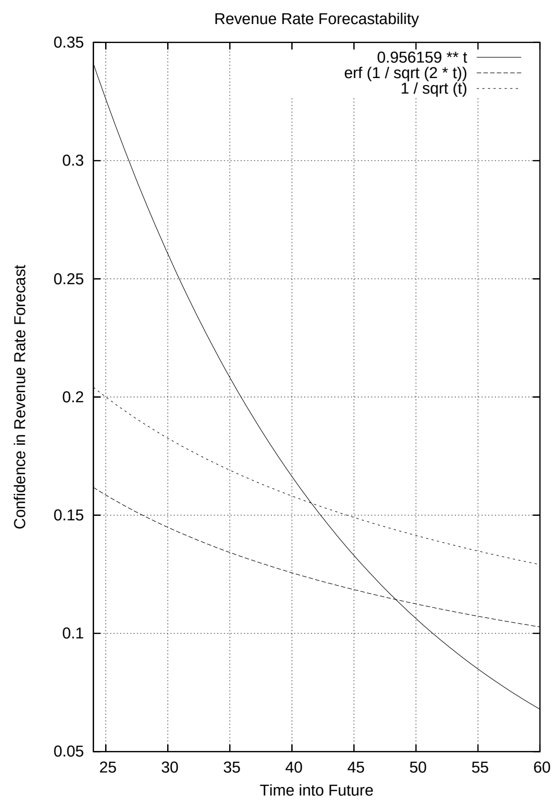

Figure C.318: United States M2, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

As an interesting interpretation of the data presented in Figure C.317, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over

[179]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.535, $0.956159^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.533, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.14.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.14.9. Figure C.319 represents a constructional simulation of the time series data presented in Figure C.300. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.301. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.301 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.320 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.303.

C.14.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.14.3. One of the issues of analysis, as mentioned in Section C.14.7, is to determine the maximum Shannon probability for the time series presented in Figure C.300. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.321 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.300. Figure C.322 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.300. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.527119, as derived in Section C.14.5 to 0.571429. This process, essentially, extracts the random statistical data from the time series presented in Figure C.300, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.300, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States M2 movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.568862, as compared with the predicted value from the program *tsshannonmax* of 0.571429.

C.14.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.302. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.301. These values will be used in a fixed increment Brownian fractal analysis of the United States M2, and may, or may not, provide adequate accuracy for projections.

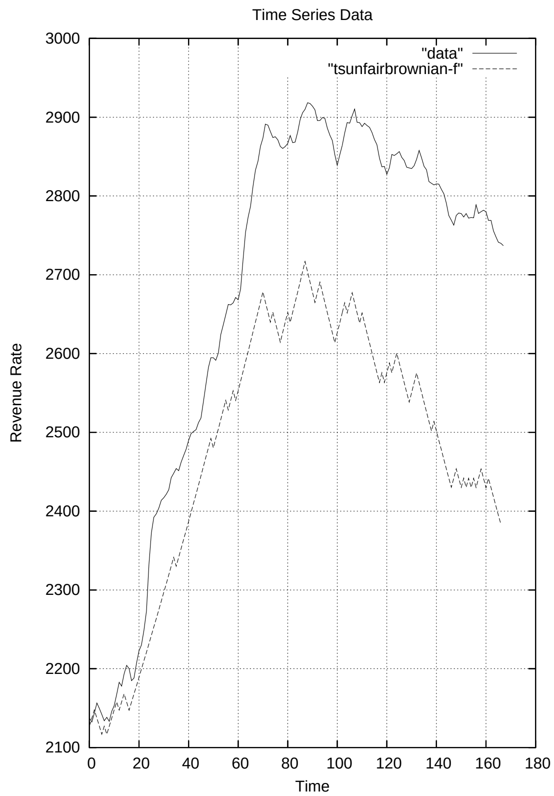

Figure C.319: United States M2, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.004817. This data is superimposed on the data presented in Figure C.300.

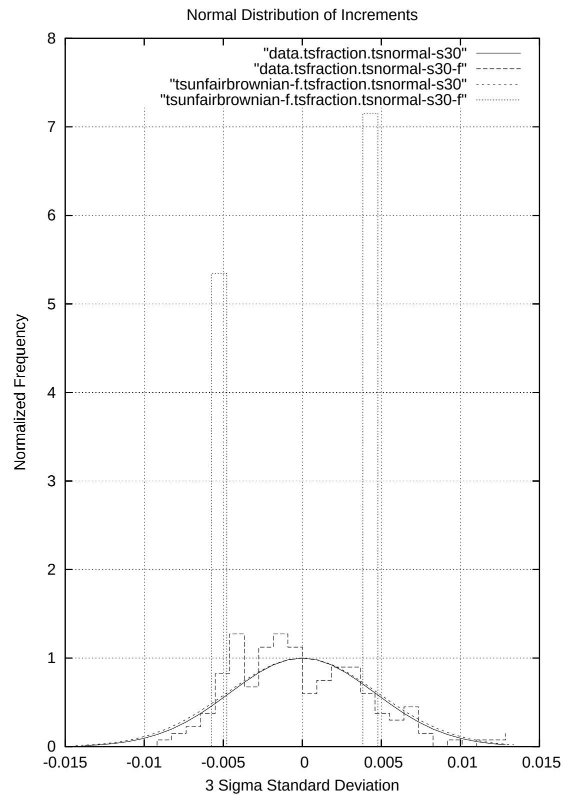

Figure C.320: United States M2, normalized histogram of the normalized increments of the time series data shown in Figure C.319, empirical and simulated. The empirical data has a mean of 0.001492, with a standard deviation of 0.004594. By comparison, the simulated data has a mean of 0.000696 with a standard deviation of 0.004781. This data is superimposed on the data presented in Figure C.303. The area under the four curves is identical.

The data in this section is presented in tabular form in sections D.14.6 and D.14.7. As a subjective evaluation of the "quality" of the analysis of the United States M2, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.300 from Figure C.301, and the Shannon probability as calculated by counting the total number of months that the United States M2 movement was positive, as presented in Section C.14.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.557}$$

$$0.568862 \approx \frac{\frac{0.001492}{0.004817} + 1}{2} \tag{C.558}$$

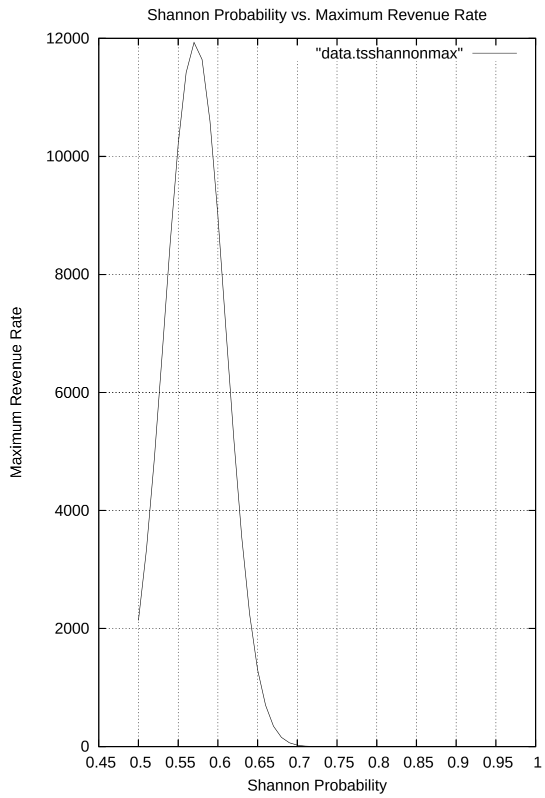

Figure C.321: United States M2, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.571429.

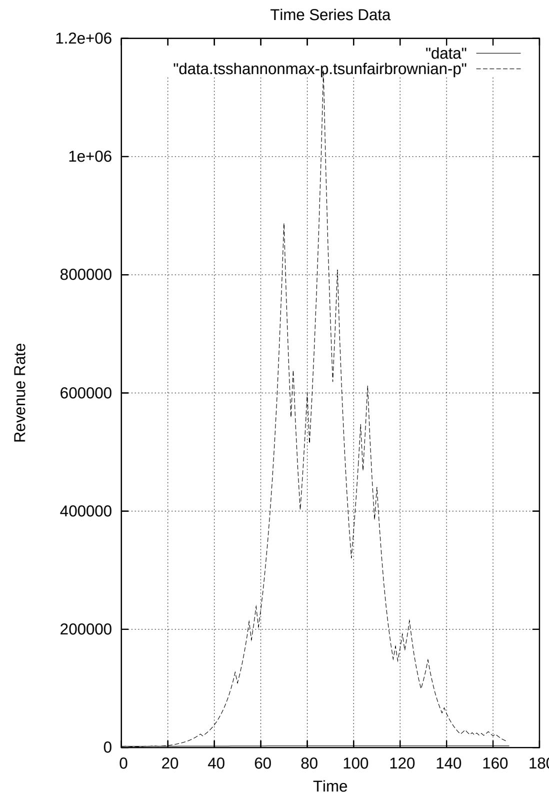

Figure C.322: United States M2, maximum rate of revenue returns, per month, at a Shannon probability, of 0.571429, corresponding to a "wager" fraction of 0.142858.

$$0.568862 \quad \approx \quad 0.654868 \tag{C.559}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.568862 \approx 0.654868 \approx 0.571429 \tag{C.560}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.14.5, should be compared. The four methods used were the mean of Figure C.301, the constant in the least squares approximation to Figure C.301, the least squares exponential approximation to Figure C.300, and the logarithmic returns of Figure C.300, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.002151 \approx 0.007084 \approx 0.002294 \approx 0.002123 \tag{C.561}$$

It is implied in Section C.14.5, Subsection C.14.5 and in Section C.14.8 that, a Brownian motion with fixed increments fractal may "model" the United States M2. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.562}$$

$$0.004817\,(2\cdot 0.568862-1) \approx \frac{0.004594\,(2\cdot 0.568862-1)}{2\sqrt{0.568862\,(1-0.568862)}} \tag{C.563}$$

$$0.004817\cdot 0.137725 \approx 0.004594\cdot 0.139050 \tag{C.564}$$

$$0.000663 \approx 0.000639 \tag{C.565}$$

and, equating to the mean:

$$0.001492 \approx 0.000663 \approx 0.000639 \tag{C.566}$$

where, as in Equation C.559 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.300 from Figure C.301, and the Shannon probability as calculated by counting the total number of months that the United States M2 movement was positive, as presented in Section C.14.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[180], where the absolute value is presented in Figure C.302, and the root mean square value is presented in Figure C.301:

$$0.003529 \approx 0.004817 \tag{C.567}$$

Note, that if the United States M2 could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.300 from Figure C.301 should be zero. It is 0.003289.

C.15 United States Treasury Bill Returns

For the analysis, the data was in the directory ../markets/us.tbill[181].

The data in this section is presented in tabular form in Section D.15. Note that in this analysis, the rate of revenue returns means the increase or decrease in the United States Treasury Bill Returns. This is included for comparative purposes. The data file actually represents how the value of an investment in United States Treasury Bill Returns Returns has increased, over the years.

C.15.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.15.1. Figure C.323 is a graph of the time series data for the United States Treasury Bill Returns.

Figure C.324 is a graph of the normalized increments of the time series data presented in Figure C.323. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

[180] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[181] Data from the United States Federal Reserve Board, 1980—1994, by months, in percent. The time series, which was Treasury Bill rate of returns, in percent per year, was converted to cumulative growth per month by converting each element in the time series to a fraction, dividing by 12, and adding 1. The previous value of cumulative returns was multiplied by this number for the next value of cumulative returns.

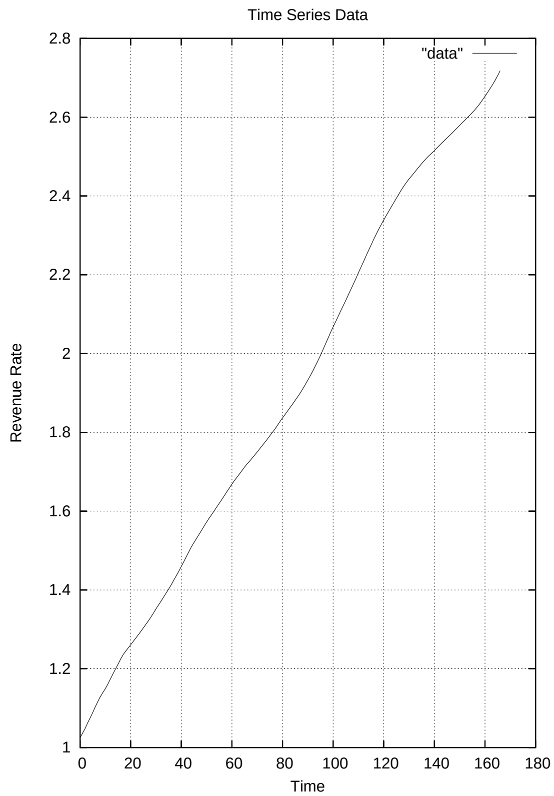

Figure C.323: United States Treasury Bill Returns, time series data.

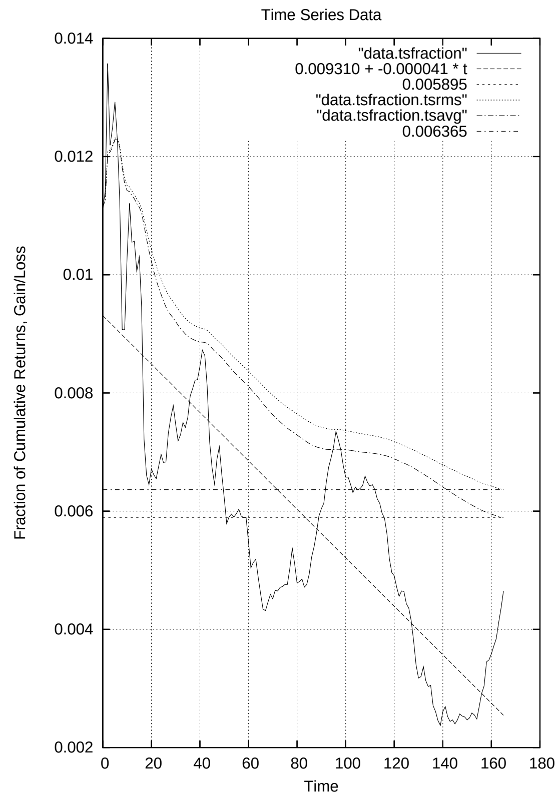

Figure C.324: United States Treasury Bill Returns, normalized increments of the time series data presented in Figure C.323. The mean is 0.005895 with a standard deviation of 0.002409. The formula for the least squares approximation is $0.009310 + -0.000041t$, and the root mean squared value is 0.006365. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000041, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.325 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.324. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[182].

Figure C.326 is the normalized histogram of the normalized increments of the time series data shown in Figure C.324. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.326.

Figure C.327 is the statistical estimate for the data presented in Figure C.324, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.553983$, as derived in Section C.15.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.328 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.324. In principle, if the distribution of the normalized increments presented in Figure C.326 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.329 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.324. In principle, if the distribution of the normalized increments presented in Figure C.326 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.330 is the range of values of the time series shown in Figure C.323. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.330 would be a square root function[183]. Figure C.331 is the deterministic map of the normalized increments of the time series data shown in Figure C.324. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Observations on the Time Series Increments Analysis

Figure C.326 would seem to indicate that the time series data for the United States Treasury Bill Returns represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.15.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[184]. Squaring this value is the average of the instantaneous fraction

[182] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[183] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.330 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[184] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

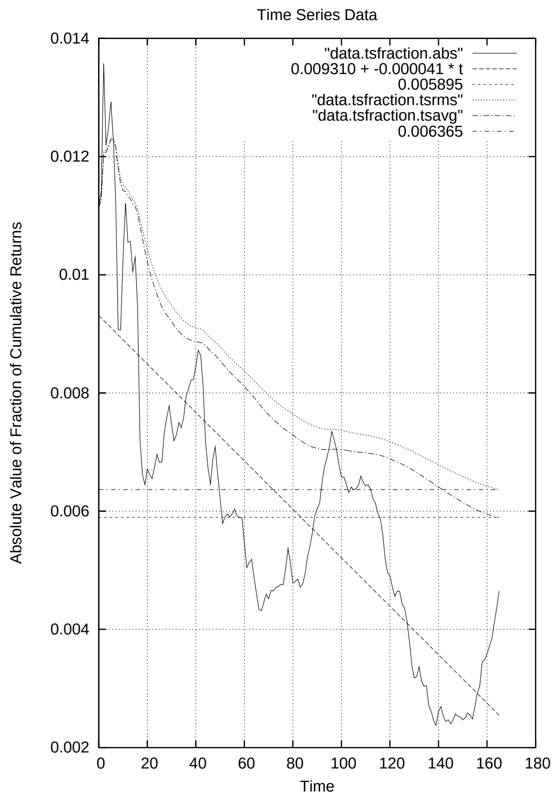

Figure C.325: United States Treasury Bill Returns, absolute value of the normalized increments of the time series data presented in Figure C.324. The mean is 0.005895 with a standard deviation of 0.002409. The formula for the least squares approximation is $0.009310 + -0.000041t$, and the root mean square value, from Figure C.324, is 0.006365. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.324, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

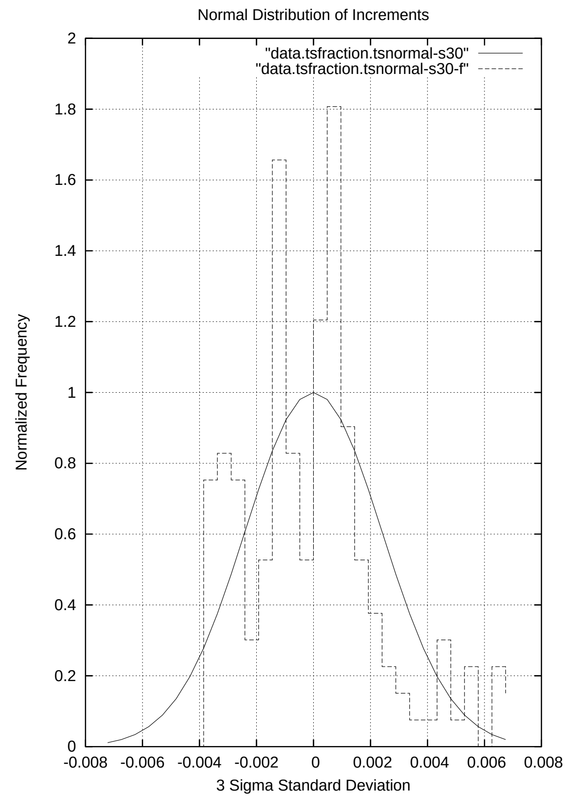

Figure C.326: United States Treasury Bill Returns, normalized histogram of the normalized increments of the time series data shown in Figure C.324. The data has a mean of 0.005895, with a standard deviation of 0.002409. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 7.453000, with a critical value of 42.557000.

of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.332 is the instantaneous value of the root mean square of the normalized increments for the United States Treasury Bill Returns, and Figure C.333 is the instantaneous Shannon probability for the normalized increments.

```
For  a mean  of 0.005859,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000586,   320  samples  would  be  required.
     (With  167  samples,   the  estimated   error  is 0.000810   = 13.826739   percent.)
For  a standard   deviation   of 0.006365,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.000637,   136  samples  would  be  required.
     (With  167  samples,   the  estimated   error  is 0.000573   = 9.000239   percent.)
```

Figure C.327: United States Treasury Bill Returns, statistical estimates of the normalized increments of the time series shown in Figure C.324. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.324.

C.15.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.15.4. Figure C.334 is a graph of the logistic function estimates of the time series data for the United States Treasury Bill Returns. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[185]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.334 is a graph of the logistic function for the time series data presented in Figure C.323. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.324. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.324. Figure C.335 is the same graph, but with the time scale expanded by a factor of two.

C.15.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.15.5. Figure C.336 is a graph of the Hurst coefficient data time series data shown in Figure C.323. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.337 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.324. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.336 implies that the variance of the rate of revenue returns, (per month,) in the United States Treasury Bill Returns, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[186], t, $p(t) = erf(1/\sqrt{2t})$ which

[185]For example, in Figures C.334 and C.335, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.15.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

[186]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$

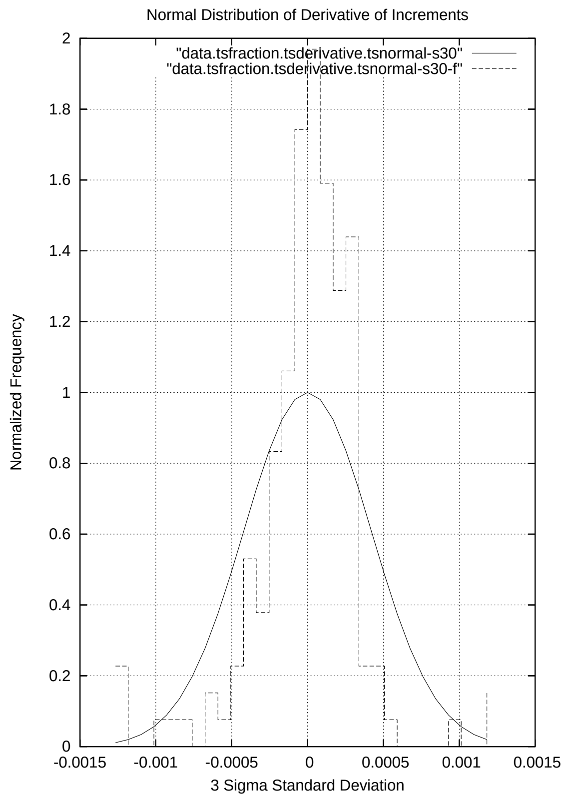

Figure C.328: United States Treasury Bill Returns, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.324.

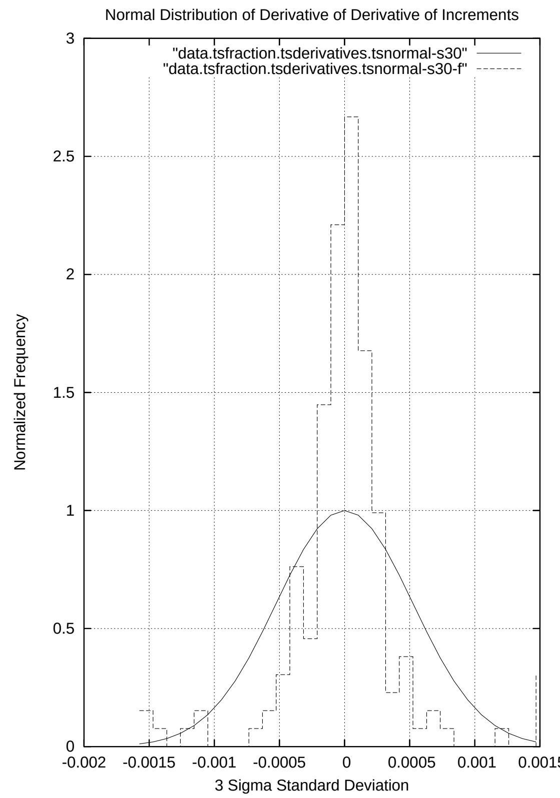

Figure C.329: United States Treasury Bill Returns, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.324.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.340, and, C.341 compare methods of approximation of the "forecastability" of the rate of revenue returns in the United States Treasury Bill Returns for the near term and far term, respectively [Pet91, pp. 83-84][187]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.336, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 1.086831, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \quad \text{(C.568)}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 1.086831} \quad \text{(C.569)}$$

characteristic.)

[187]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

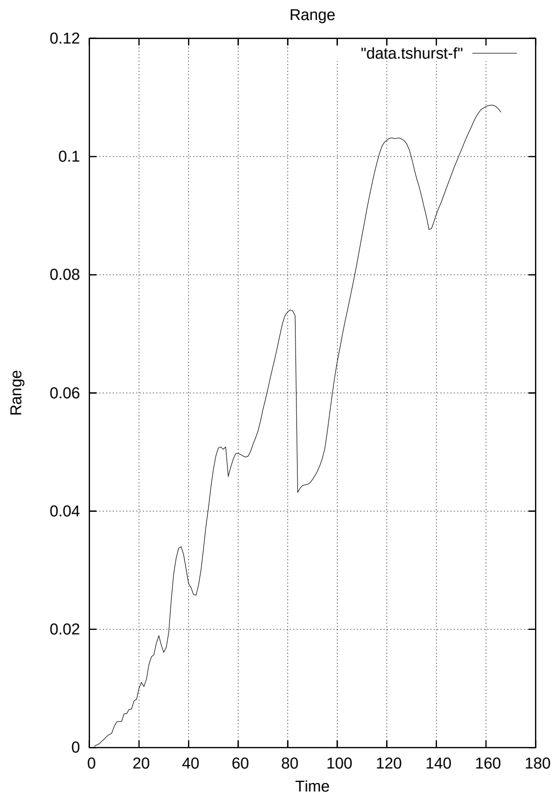

Figure C.330: United States Treasury Bill Returns, range of the time series data shown in Figure C.323.

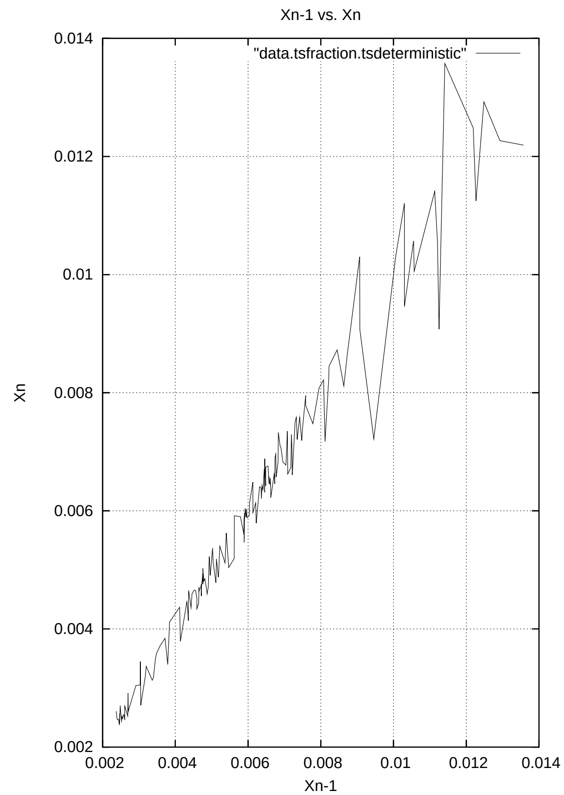

Figure C.331: United States Treasury Bill Returns, deterministic map of the normalized increments of the time series data shown in Figure C.324.

$$\propto \quad (t_2 - t_1)^{2.173662} \tag{C.570}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[188]. A Hurst coefficient of 1.086831, (for the near future, and 0.918814 for the distant future.) implies that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 108.683100% [Pet91, pp. 66] for the near future, and 0.918814 for the distant future. Likewise, there is a 108.683100% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in

[188] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.15.5 for the particulars on using Hurst Coefficient to sum fractal process' for the United States Treasury Bill Returns. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

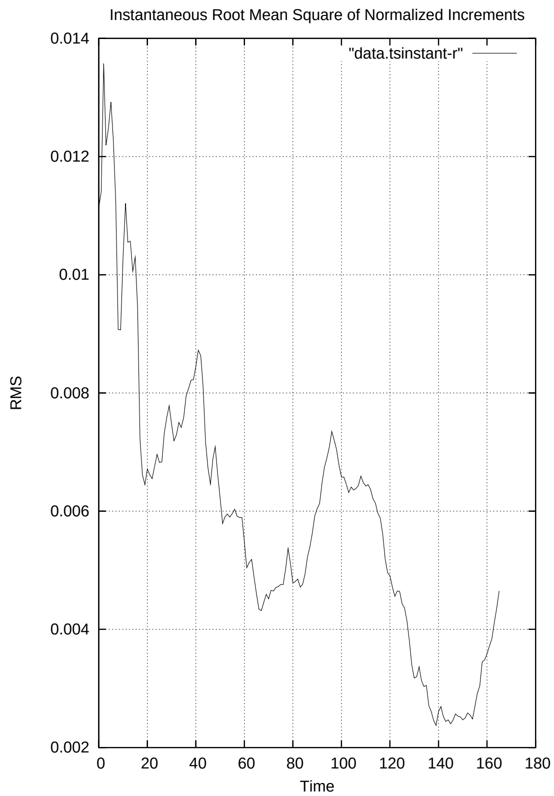

Figure C.332: United States Treasury Bill Returns, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.323.

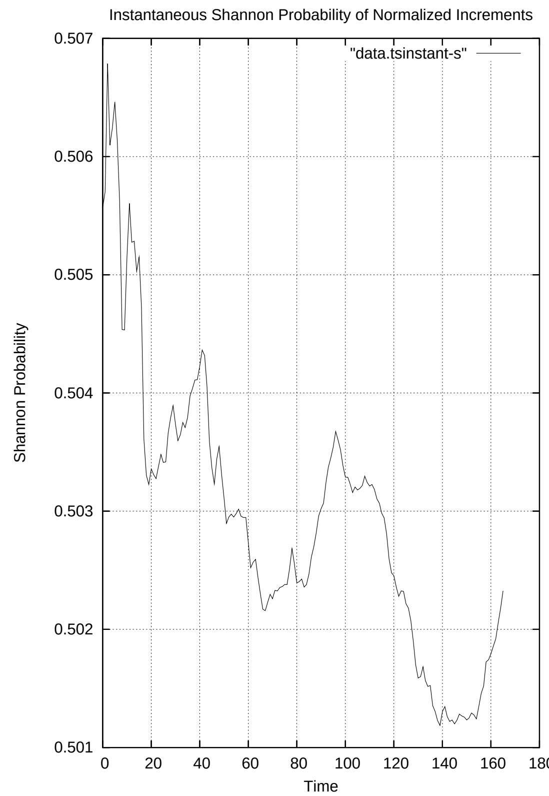

Figure C.333: United States Treasury Bill Returns, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.323.

a given month, the rate of revenue returns, (per month,) is increasing, there is a 108.683100% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the United States Treasury Bill Returns are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.571}$$

$$= 1.086831^n \tag{C.572}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.324, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.005895 \cdot 100$ percent, on the average, with a standard deviation of $0.002409 \cdot 100$ percent, and a root mean square error value of $0.006365 \cdot 100$ percent—small values for such a simple

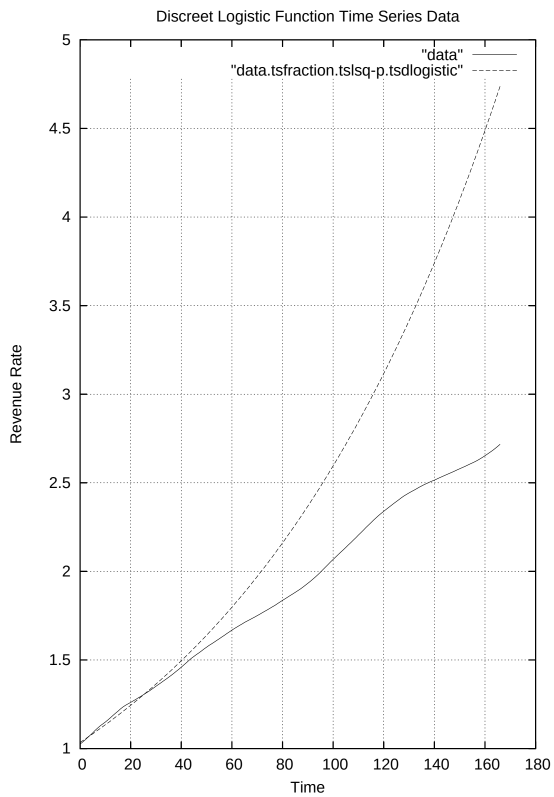

Figure C.334: United States Treasury Bill Returns, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.324 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

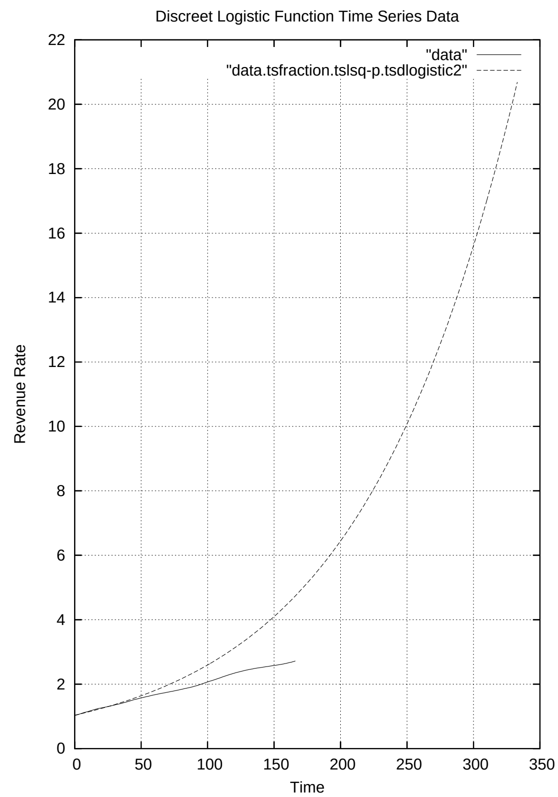

Figure C.335: United States Treasury Bill Returns, logistic function estimates of Figure C.334 with the time scale expanded by a factor of two.

forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^H \tag{C.573}$$

$$\propto\; (t_2 - t_1)^{1.086831} \tag{C.574}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.574 reduces to, [Sch91, pp. 129]:

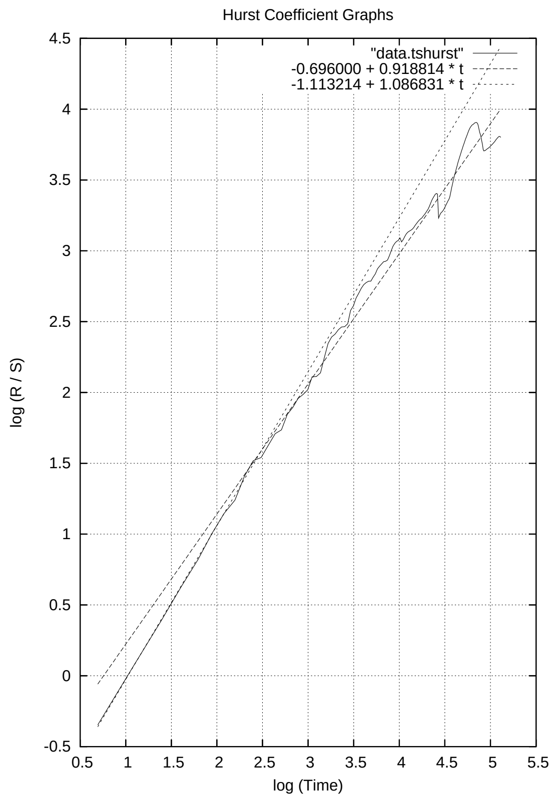

Figure C.336: United States Treasury Bill Returns, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.324. The slope of the graph is the Hurst coefficient.

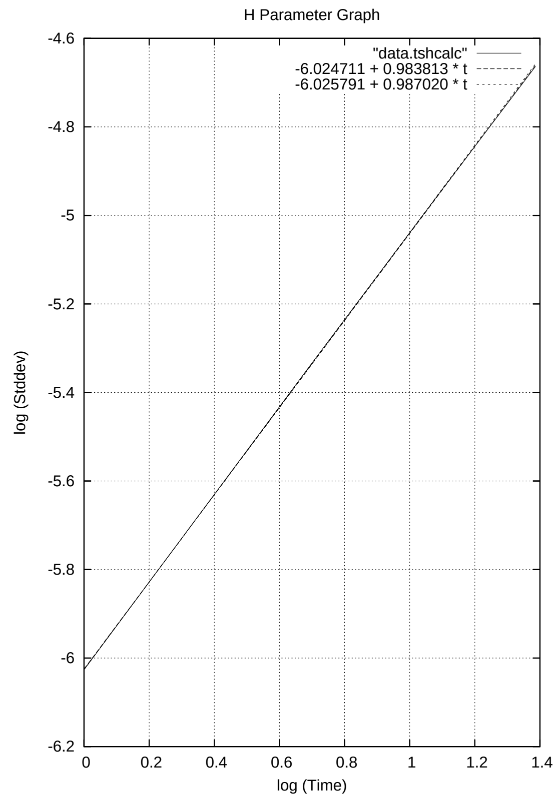

Figure C.337: United States Treasury Bill Returns, H parameter data for the normalized increments of the time series data shown in Figure C.324 The slope of the graph is the H parameter.

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.575}$$
$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.576}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.577}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[189].

[189] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.578}$$

$$\propto \frac{X(rt)}{r^{1.086831}} \tag{C.579}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.337, to provide a least squares approximation to the H parameter for the United States Treasury Bill Returns. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.987020 for the near future, and 0.983813 for the distant future.

Figures C.336 and C.337 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.324. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.324, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.338 and C.339 was made using the -d option.

C.15.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.15.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.325. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.324. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Treasury Bill Returns, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the United States Treasury Bill Returns, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.324, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.005895 + 1)}{\ln(2)} = 0.008480 \tag{C.580}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.324, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.009310 + 1)}{\ln(2)} = 0.013369 \tag{C.581}$$

Note that if the mean is not constant in Figure C.324, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.323:

$$bits = 0.008215 \tag{C.582}$$

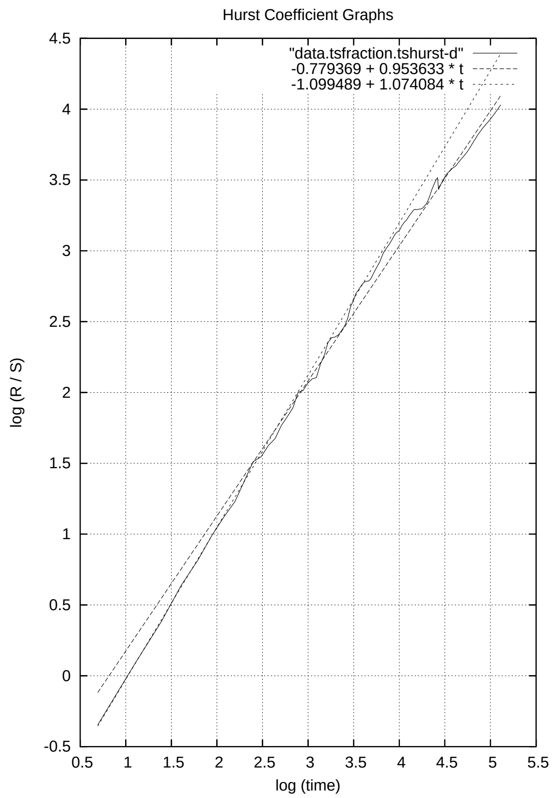

Figure C.338: United States Treasury Bill Returns, traditional Hurst coefficient data for the time series data shown in Figure C.323. The slope of the graph is the Hurst coefficient, and is 1.074084 for the near term, and 0.953633 for the far term.

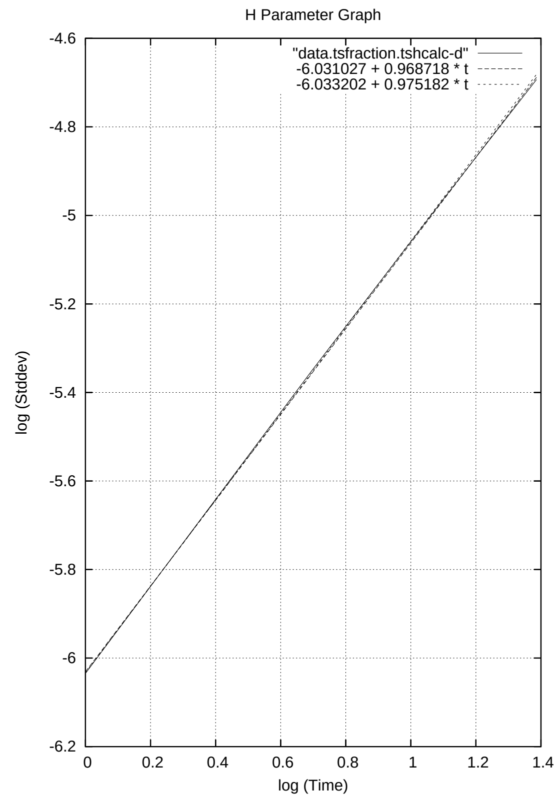

Figure C.339: United States Treasury Bill Returns, traditional H parameter data for the time series data shown in Figure C.323 The slope of the graph is the H parameter, and is 0.975182 for the near term, and 0.968718 for the far term.

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.323:

$$bits = 0.008425 \tag{C.583}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.15.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.008425t} \tag{C.584}$$

therefore:

$$C(p) = 0.008425 \tag{C.585}$$

and, *tsshannon* 0.008425 gives:

$$C\,(0.553983) = 0.008425 \tag{C.586}$$

therefore:

$$2^{C(0.553983)} = 2^{0.008425} \tag{C.587}$$
$$= 1.005857 \tag{C.588}$$
$$= 0.585685\% \tag{C.589}$$

and:

$$2p - 1 = (2 \cdot 0.553983) - 1 \tag{C.590}$$
$$= 0.107966 \tag{C.591}$$
$$= 10.796600\% \tag{C.592}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the United States Treasury Bill Returns executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 10.796600% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 89.203400% will be held in "reserve" with a 55.398300% chance of making twice the 10.796600% back, (and a 44.601700% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.585685%, or a doubling of its rate of revenue returns, (per month,) in 118.694362 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 10.796600% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.585685%, per month, on average.

Note that the metrics presented in this section are representative of the United States Treasury Bill Returns as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 10.796600% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the United States Treasury Bill Returns's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.585685% per month.

As another simple example, a company re-invests 10.796600% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 10.796600% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.585685% per month.

As an example of "product portfolio" management, suppose a company re-invests 10.796600% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 10.796600$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 10.796600$ percent for the second product, implying that the company should diversify its product line[190]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 10.796600%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65-1)}{(2 \cdot 0.55-1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the United States Treasury Bill Returns, as a standard bench mark, then the optimal number will be $\frac{1}{0.107966}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.324, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 10.796600% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 10.796600% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.15.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the United States Treasury Bill Returns, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the United States Treasury Bill Returns time series is 0.005895, and 0.006365respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 145.508041.

If this value seems consistent number of companies in the United States Treasury Bill Returns, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the United States Treasury Bill Returns are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.538389, which would be the value which should be used in Section C.15.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.15.5 is greater than the average Shannon probability for the companies participating in the United States Treasury Bill Returns, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.15.5. The maximum exploitability for the United States Treasury Bill Returns is derived in Section C.15.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the United States Treasury Bill Returns is 0.538389, with several alternative solutions listed in the previous paragraph. However, these should be

[190] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

contrasted to the Shannon probability that maximizes a company's P&L which is 0.963079 in the United States Treasury Bill Returns. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.593}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the United States Treasury Bill Returns would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.15.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.325. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.324. These values will be used in a fixed increment Brownian fractal analysis and simulation of the United States Treasury Bill Returns, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.15.5, is derived from the United States Treasury Bill Returns metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.15.9.

An additional exploitable strategy may be time itself. Equations C.570, C.574, and, C.572, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the United States Treasury Bill Returns, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.340, and, C.341 compare methods of approximation of the "forecastability" of rate of revenue returns in the United States Treasury Bill Returns for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the United States Treasury Bill Returns. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[191]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.340, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory

[191] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

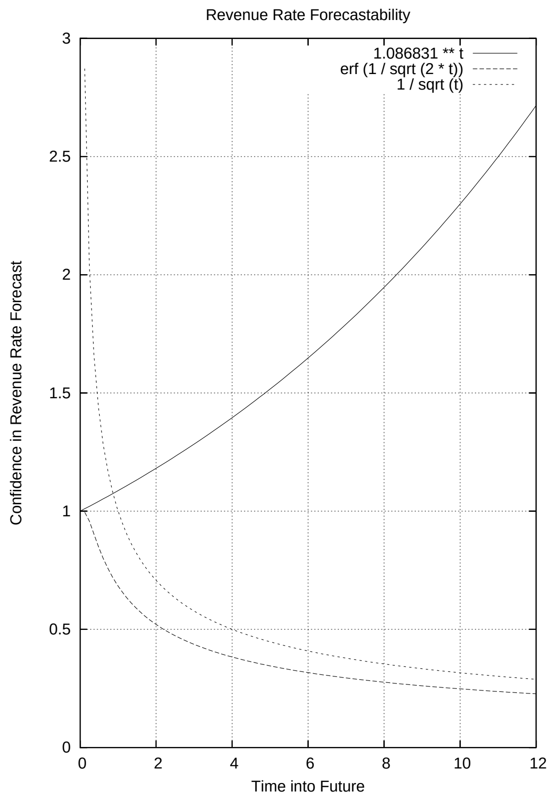

Figure C.340: United States Treasury Bill Returns, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 1.086831^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

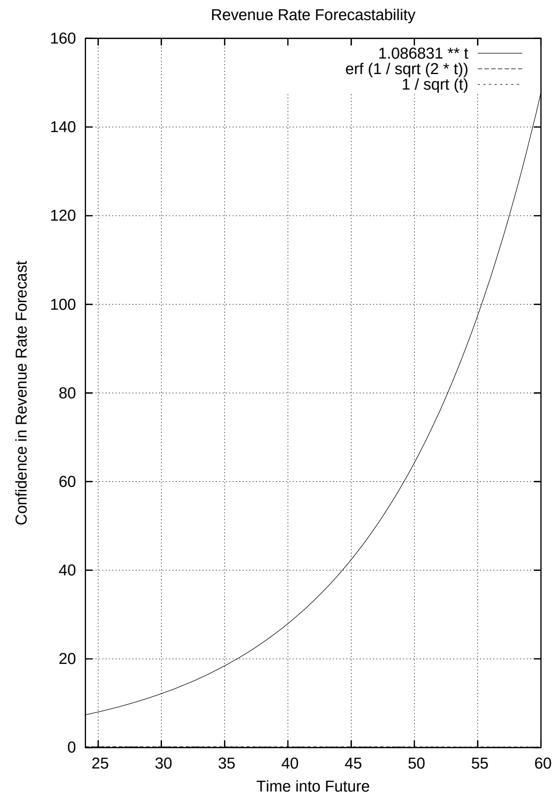

Figure C.341: United States Treasury Bill Returns, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

levels that do not exceed, from Equation C.572, $1.086831^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.570, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.15.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.15.9. Figure C.342 represents a constructional simulation of the time series data presented in Figure C.323. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented

in Figure C.324. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.324 represent a relatively complex process, that may not be “modeled” with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.343 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.326.

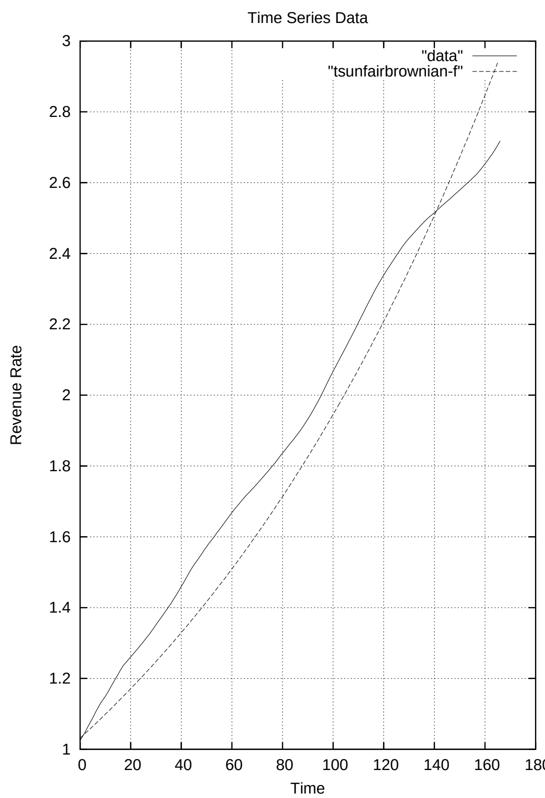

Figure C.342: United States Treasury Bill Returns, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.006365. This data is superimposed on the data presented in Figure C.323.

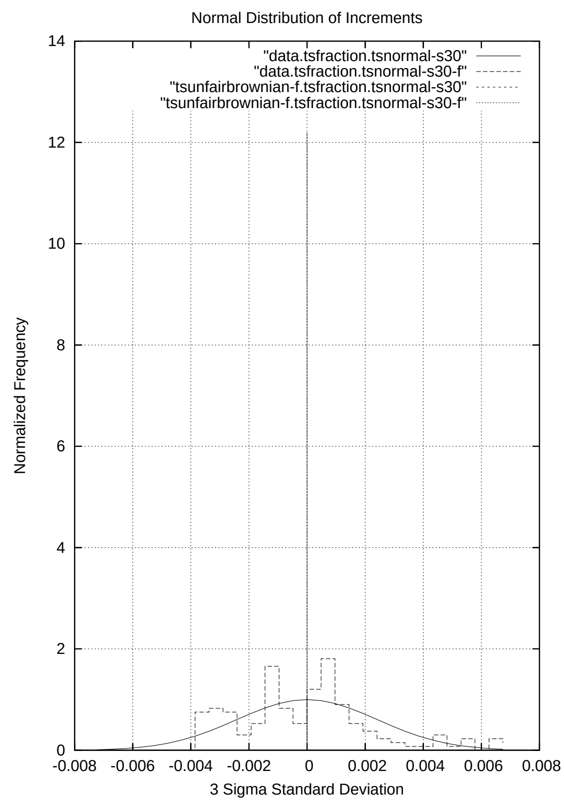

Figure C.343: United States Treasury Bill Returns, normalized histogram of the normalized increments of the time series data shown in Figure C.342, empirical and simulated. The empirical data has a mean of 0.005895, with a standard deviation of 0.002409. By comparison, the simulated data has a mean of 0.006365 with a standard deviation of 0.000000. This data is superimposed on the data presented in Figure C.326. The area under the four curves is identical.

C.15.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.15.3. One of the issues of analysis, as mentioned in Section C.15.7, is to determine the maximum Shannon probability for the time series presented in Figure C.323. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.344 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.323. Figure C.345 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.323. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.553983, as derived in Section C.15.5 to 1.000000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.323, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.323, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the United States Treasury Bill Returns movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.999990, as compared with the predicted value from the program *tsshannonmax* of 1.000000.

C.15.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.325. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.324. These values will be used in a fixed increment Brownian fractal analysis of the United States Treasury Bill Returns, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.15.6 and D.15.7. As a subjective evaluation of the "quality" of the analysis of the United States Treasury Bill Returns, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.323 from Figure C.324, and the Shannon probability as calculated by counting the total number of months that the United States Treasury Bill Returns movement was positive, as presented in Section C.15.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.594}$$

$$0.999990 \approx \frac{\frac{0.005895}{0.006365} + 1}{2} \tag{C.595}$$

$$0.999990 \approx 0.963079 \tag{C.596}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.999990 \approx 0.963079 \approx 1.000000 \tag{C.597}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.15.5, should be compared. The four methods used were the mean of Figure C.324, the constant in the least squares approximation to Figure C.324, the least squares exponential approximation to Figure C.323, and the logarithmic returns of Figure C.323,

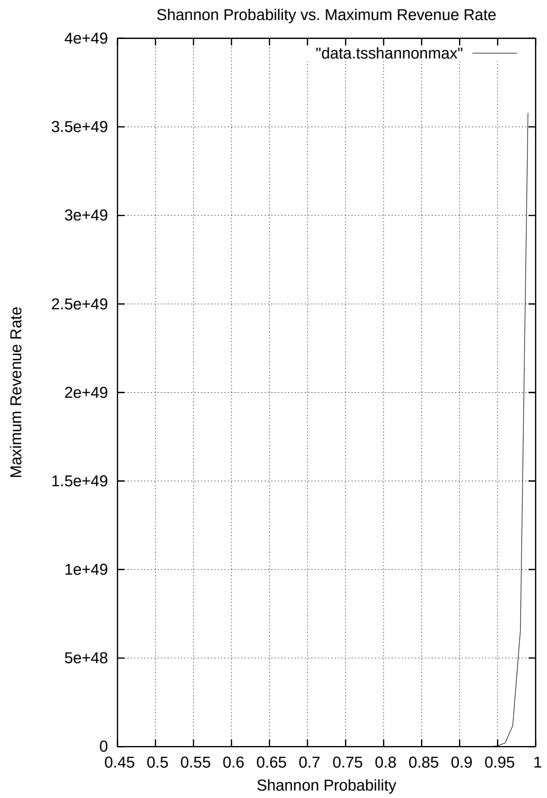

Figure C.344: United States Treasury Bill Returns, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 1.000000.

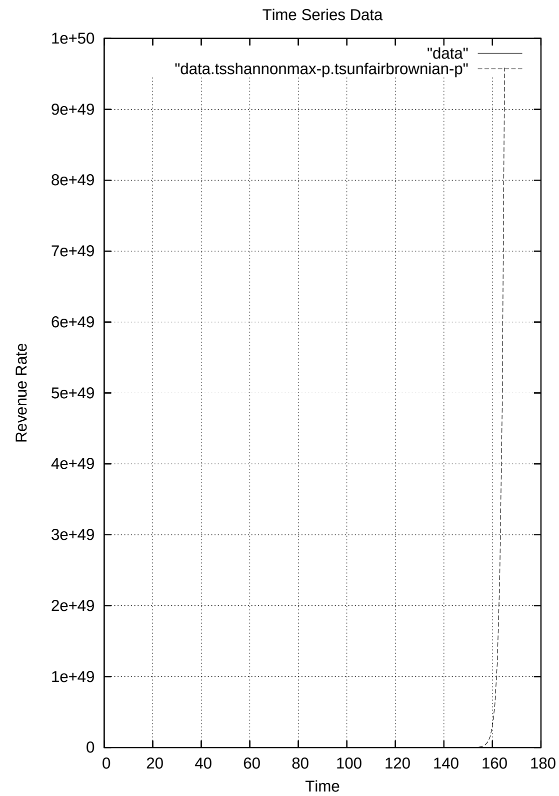

Figure C.345: United States Treasury Bill Returns, maximum rate of revenue returns, per month, at a Shannon probability, of 1.000000, corresponding to a "wager" fraction of 1.000000.

derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.008480 \approx 0.013369 \approx 0.008215 \approx 0.008425 \tag{C.598}$$

It is implied in Section C.15.5, Subsection C.15.5 and in Section C.15.8 that, a Brownian motion with fixed increments fractal may "model" the United States Treasury Bill Returns. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.599}$$

$$0.006365\,(2\cdot 0.999990-1) \approx \frac{0.002409\,(2\cdot 0.999990-1)}{2\sqrt{0.999990\,(1-0.999990)}} \tag{C.600}$$

$$0.006365\cdot 0.999980 \approx 0.002409\cdot 158.111511 \tag{C.601}$$

$$0.006365 \quad \approx \quad 0.380891 \tag{C.602}$$

and, equating to the mean:

$$0.005895 \approx 0.006365 \approx 0.380891 \tag{C.603}$$

where, as in Equation C.596 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.323 from Figure C.324, and the Shannon probability as calculated by counting the total number of months that the United States Treasury Bill Returns movement was positive, as presented in Section C.15.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[192], where the absolute value is presented in Figure C.325, and the root mean square value is presented in Figure C.324:

$$0.005895 \approx 0.006365 \tag{C.604}$$

Note, that if the United States Treasury Bill Returns could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.323 from Figure C.324 should be zero. It is 0.002409.

C.16 Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin[193].

The data in this section is presented in tabular form in Section D.16. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Coin Tossing Game. This is included for "theoretical" comparative purposes.

C.16.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.16.1. Figure C.346 is a graph of the time series data for the Coin Tossing Game.

Figure C.347 is a graph of the normalized increments of the time series data presented in Figure C.346. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.348 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.347. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[194].

[192] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[193] As a simulation model, the program *tscoin* was run to make a time series data file, with the following parameters:

```
tscoin -p 0.6 300 > data
```

to make a time series of 300 elements, with a Shannon probability of 0.6. The data is by tosses.

[194] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

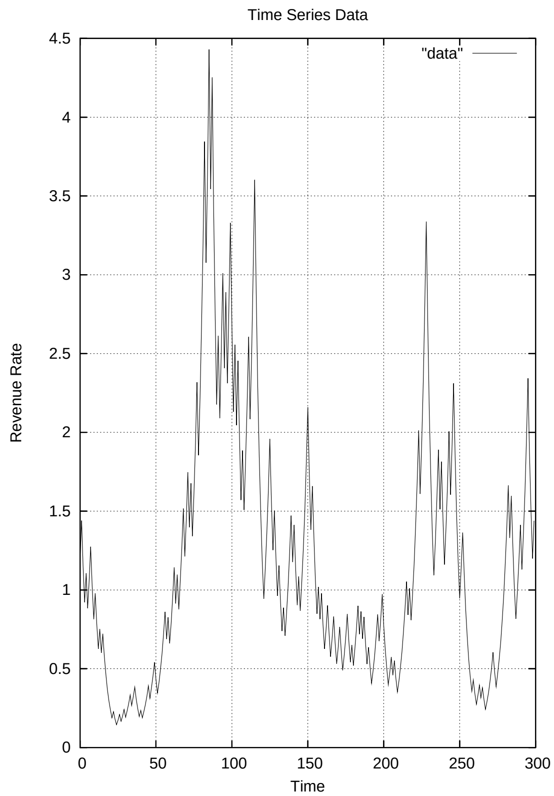

Figure C.346: Coin Tossing Game, time series data.

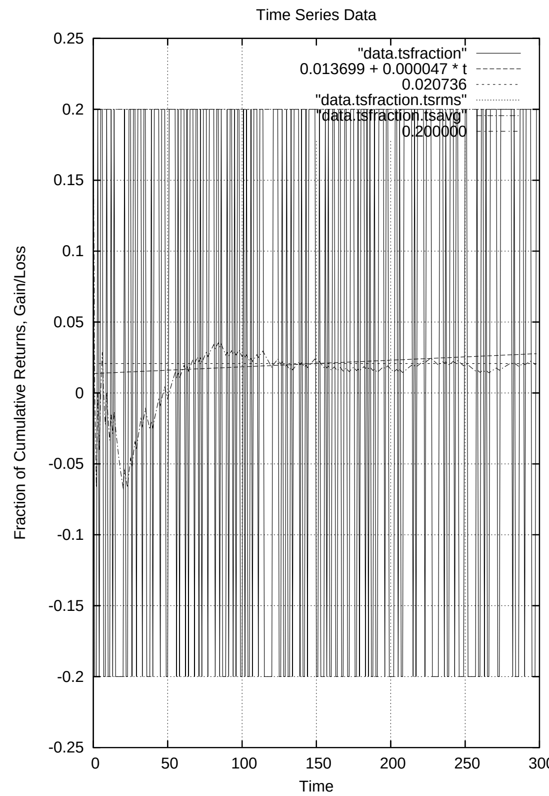

Figure C.347: Coin Tossing Game, normalized increments of the time series data presented in Figure C.346. The mean is 0.020736 with a standard deviation of 0.199256. The formula for the least squares approximation is $0.013699 + 0.000047t$, and the root mean squared value is 0.200000. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000047, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.349 is the normalized histogram of the normalized increments of the time series data shown in Figure C.347. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.349.

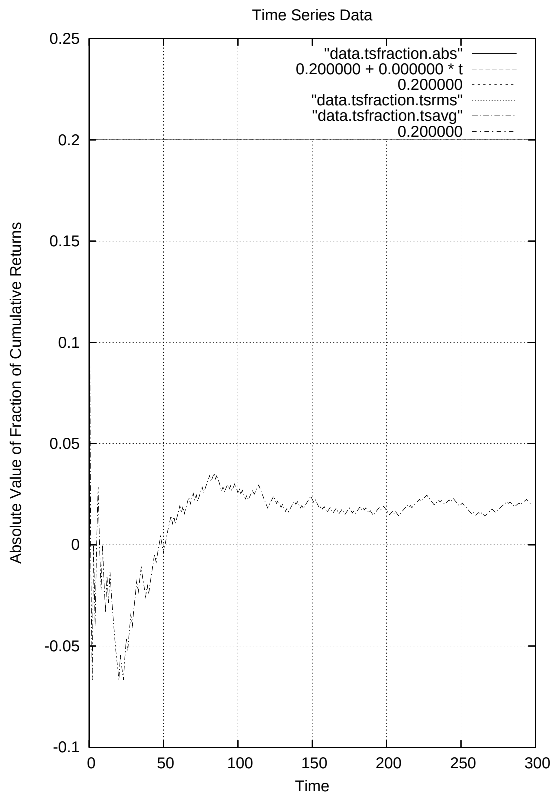

Figure C.348: Coin Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.347. The mean is 0.200000 with a standard deviation of 0.000001. The formula for the least squares approximation is $0.200000 + 0.000000t$, and the root mean square value, from Figure C.347, is 0.200000. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments presented in Figure C.347, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

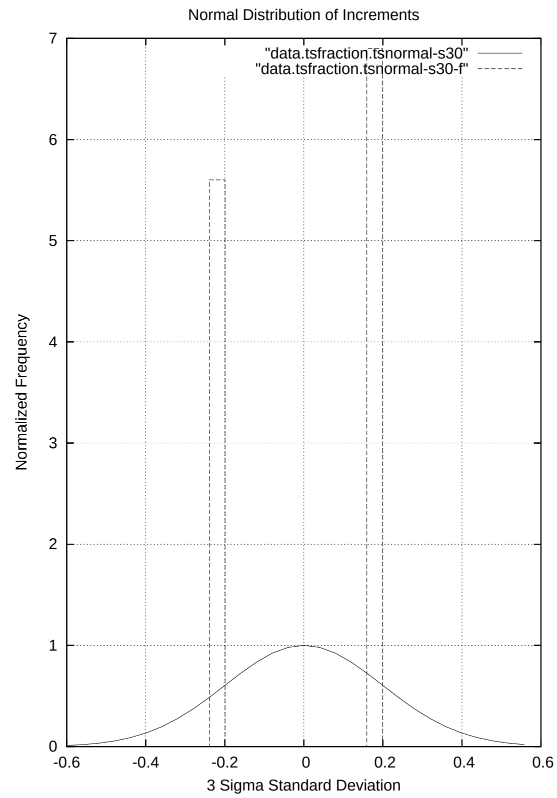

Figure C.349: Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.347. The data has a mean of 0.020736, with a standard deviation of 0.199256. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 117.483000, with a critical value of 42.557000.

Figure C.350 is the statistical estimate for the data presented in Figure C.347, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.517402$, as derived in Section C.16.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude

```
For  a mean  of 0.020667,   with  a confidence   level  of 0.900000
     that  the error  did not exceed  0.002067,   25339  samples  would  be required.
     (With  300  samples,   the estimated   error  is 0.018993   = 91.902275   percent.)
For  a standard   deviation   of 0.200000,   with  a confidence   level  of 0.900000
     that  the error  did not exceed  0.020000,   136  samples  would  be required.
     (With  300  samples,   the estimated   error  is 0.013430   = 6.715087   percent.)
```

Figure C.350: Coin Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.347. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.347.

too high.

Figure C.351 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.347. In principle, if the distribution of the normalized increments presented in Figure C.349 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.352 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.347. In principle, if the distribution of the normalized increments presented in Figure C.349 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.353 is the range of values of the time series shown in Figure C.346. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.353 would be a square root function[195]. Figure C.354 is the deterministic map of the normalized increments of the time series data shown in Figure C.347. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

C.16.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[196]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.355 is the instantaneous value of the root mean square of the normalized increments for the Coin Tossing Game, and Figure C.356 is the instantaneous Shannon probability for the normalized increments.

[195]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.353 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[196]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

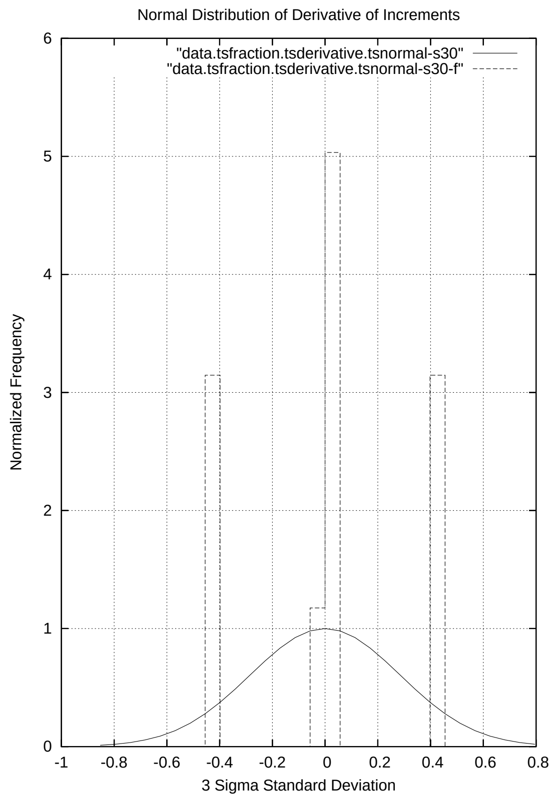

Figure C.351: Coin Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.347.

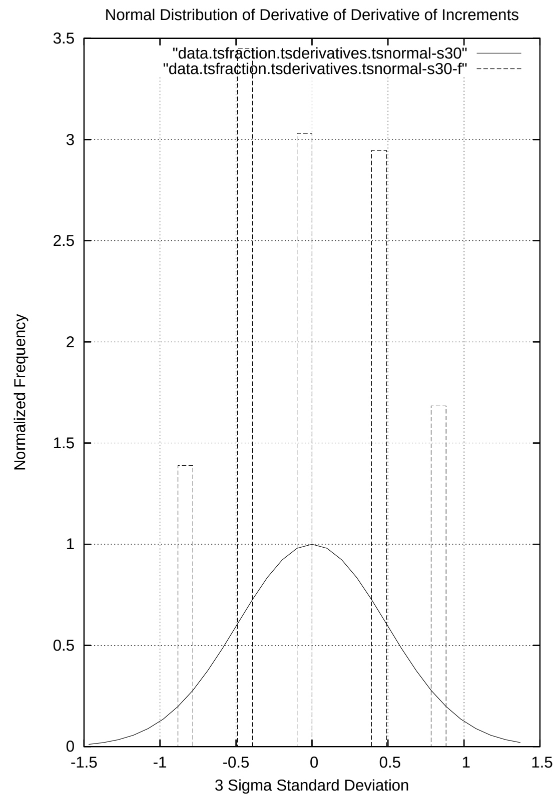

Figure C.352: Coin Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.347.

C.16.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.16.4. Figure C.357 is a graph of the logistic function estimates of the time series data for the Coin Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[197]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.357 is a graph of the logistic function for the time series data presented in Figure C.346. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.347. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.347. Figure C.358 is the same graph, but with the time scale expanded by a factor of two.

[197]For example, in Figures C.357 and C.358, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.16.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

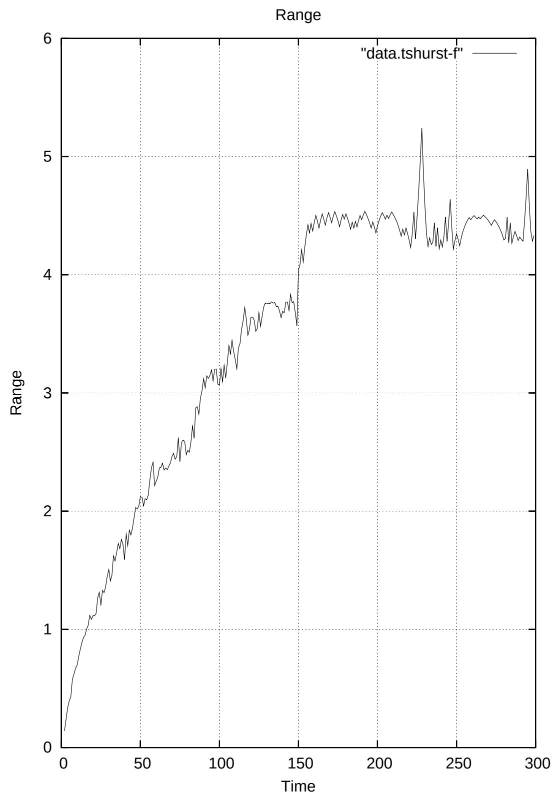

Figure C.353: Coin Tossing Game, range of the time series data shown in Figure C.346.

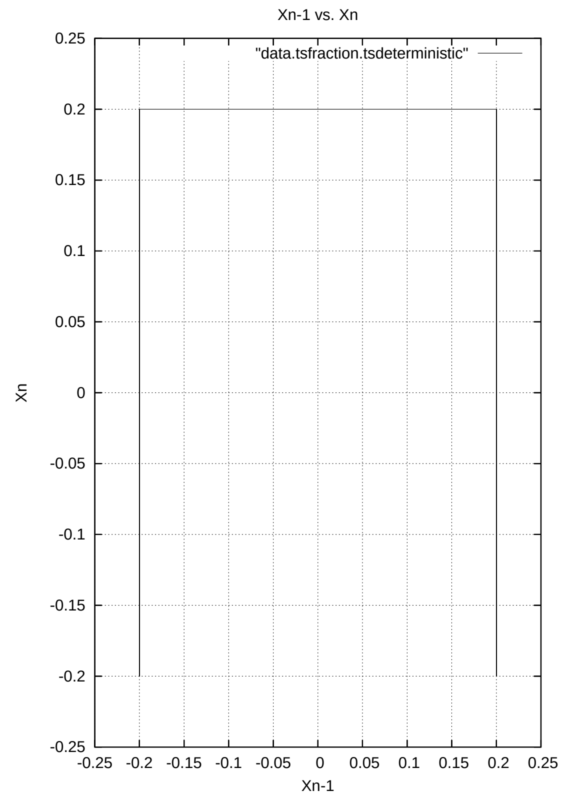

Figure C.354: Coin Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.347.

C.16.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.16.5. Figure C.359 is a graph of the Hurst coefficient data time series data shown in Figure C.346. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.360 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.347. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.359 implies that the variance of the rate of revenue returns, (per tosses,) in the Coin Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[198], t, $p(t) = erf(1/\sqrt{2t})$ which

[198]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp.

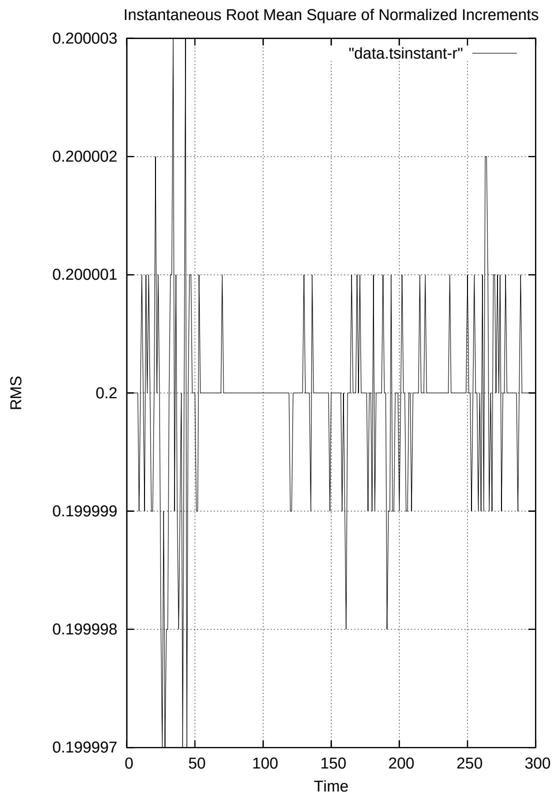

Figure C.355: Coin Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.346.

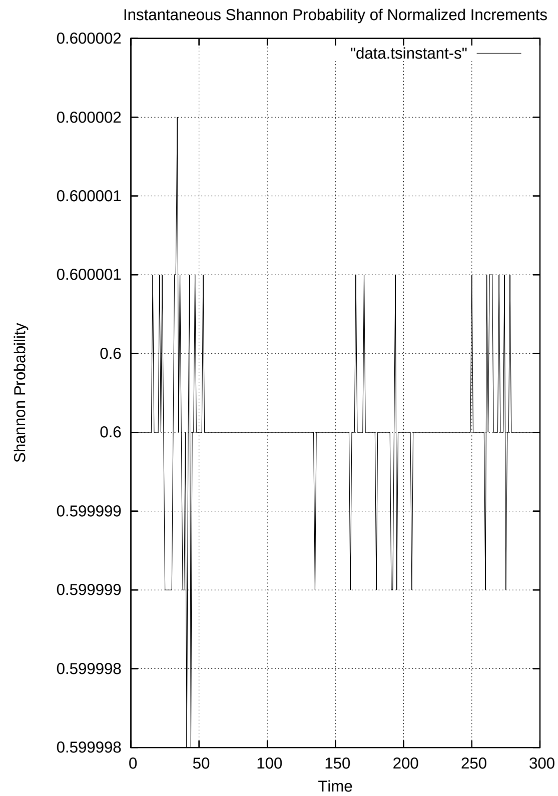

Figure C.356: Coin Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.346.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.363, and, C.364 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Coin Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][199]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.359, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.853212, so that:

$$V\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.605}$$

153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[199]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

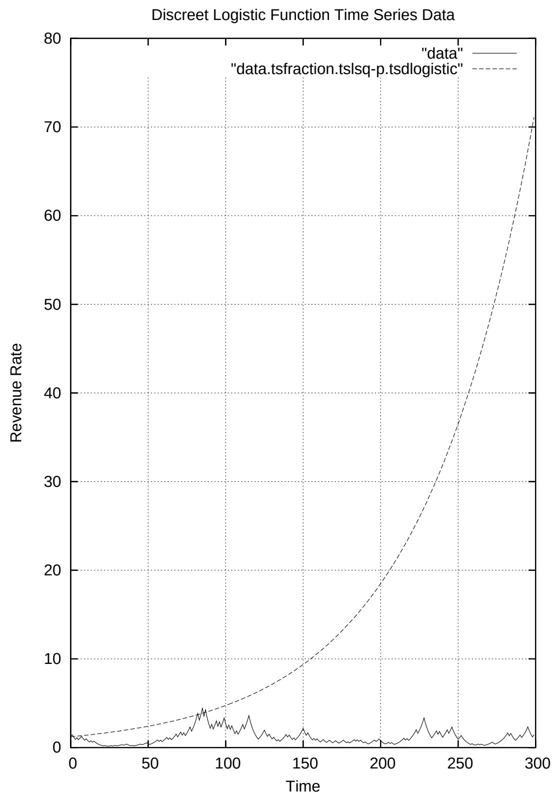

Figure C.357: Coin Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.347 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

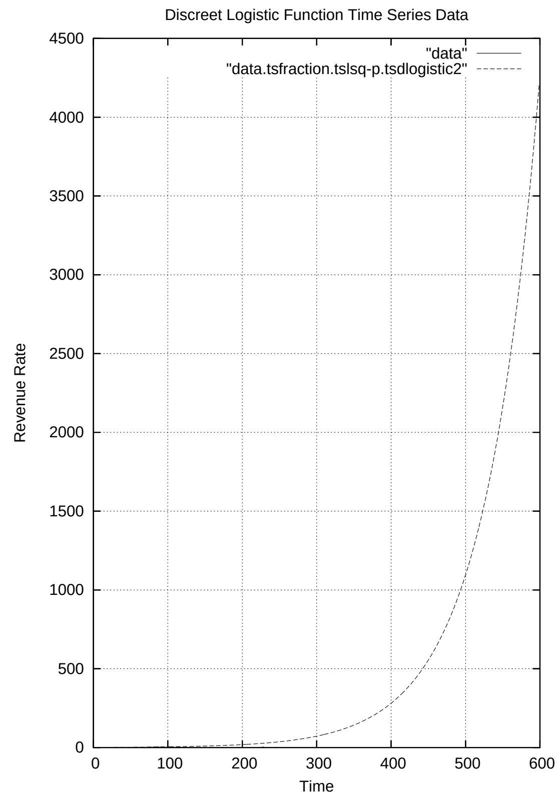

Figure C.358: Coin Tossing Game, logistic function estimates of Figure C.357 with the time scale expanded by a factor of two.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.853212} \quad \text{(C.606)}$$

$$\propto (t_2 - t_1)^{1.706424} \quad \text{(C.607)}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[200]. A Hurst coefficient of 0.853212, (for the near future, and 0.506256 for the distant future.) implies

[200] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.16.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Coin Tossing Game. See also [Pet91, pp. 67, 83-84]

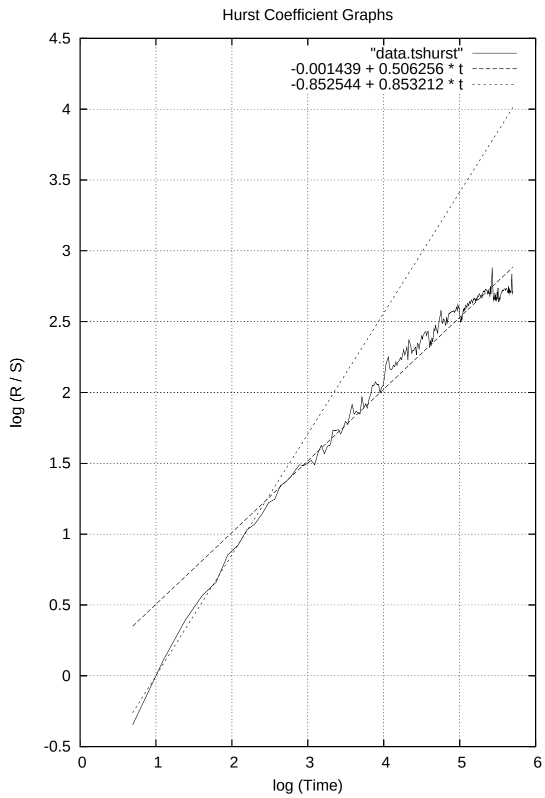

Figure C.359: Coin Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.347. The slope of the graph is the Hurst coefficient.

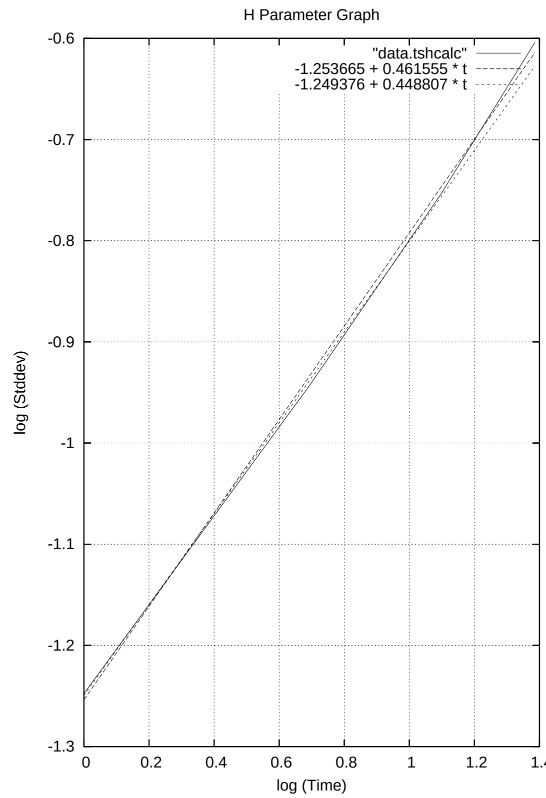

Figure C.360: Coin Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.347 The slope of the graph is the H parameter.

that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is 85.321200% [Pet91, pp. 66] for the near future, and 0.506256 for the distant future. Likewise, there is a 85.321200% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 85.321200% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Coin Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \quad \text{(C.608)}$$
$$= 0.853212^n \quad \text{(C.609)}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.347, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that

and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.020736 \cdot 100$ percent, on the average, with a standard deviation of $0.199256 \cdot 100$ percent, and a root mean square error value of $0.200000 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^H \tag{C.610}$$

$$\propto \left(t_2 - t_1\right)^{0.853212} \tag{C.611}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.611 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.612}$$

$$\propto \sqrt{\left(t_2 - t_1\right)} \tag{C.613}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.614}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[201].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \propto \frac{X\left(rt\right)}{r^H} \tag{C.615}$$

$$\propto \frac{X\left(rt\right)}{r^{0.853212}} \tag{C.616}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.360, to provide a least squares approximation to the H parameter for the Coin Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.448807 for the near future, and 0.461555 for the distant future.

Figures C.359 and C.360 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.347. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.347, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.361 and C.362 was made using the -d option.

[201]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

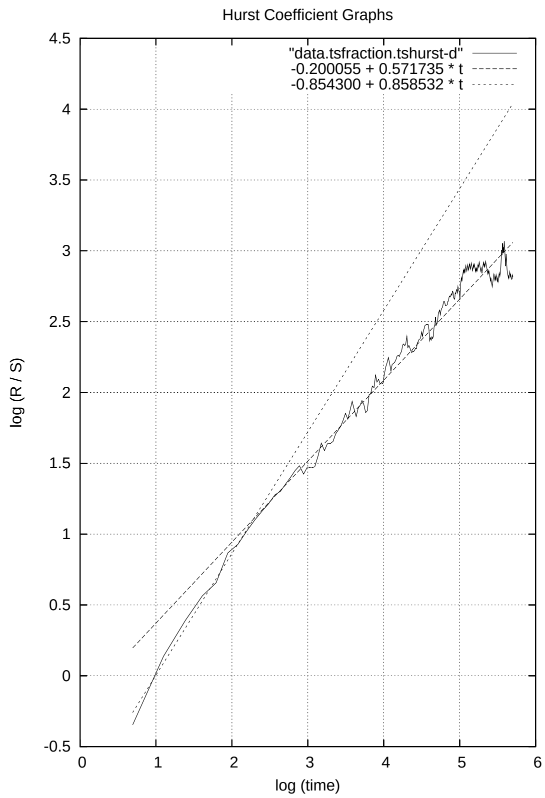

Figure C.361: Coin Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.346. The slope of the graph is the Hurst coefficient, and is 0.858532 for the near term, and 0.571735 for the far term.

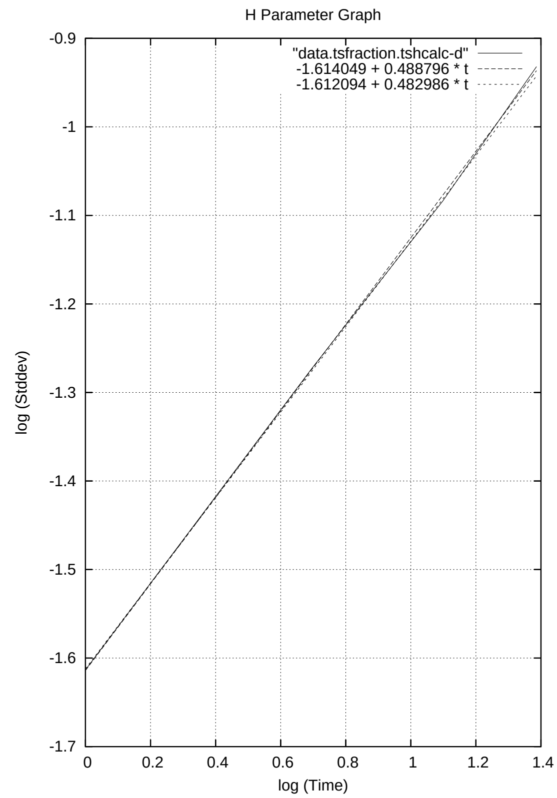

Figure C.362: Coin Tossing Game, traditional H parameter data for the time series data shown in Figure C.346 The slope of the graph is the H parameter, and is 0.482986 for the near term, and 0.488796 for the far term.

C.16.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.16.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.348. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.347. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Coin Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in

Chapter B, and is presented in Figure C.347, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.020736 + 1)}{\ln(2)} = 0.029610 \tag{C.617}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.347, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.013699 + 1)}{\ln(2)} = 0.019629 \tag{C.618}$$

Note that if the mean is not constant in Figure C.347, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.346:

$$bits = 0.001451 \tag{C.619}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.346:

$$bits = 0.000874 \tag{C.620}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.16.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.000874t} \tag{C.621}$$

therefore:

$$C(p) = 0.000874 \tag{C.622}$$

and, *tsshannon* 0.000874 gives:

$$C(0.517402) = 0.000874 \tag{C.623}$$

therefore:

$$\begin{aligned} 2^{C(0.517402)} &= 2^{0.000874} && \text{(C.624)} \\ &= 1.000606 && \text{(C.625)} \\ &= 0.060599\% && \text{(C.626)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.517402) - 1 && \text{(C.627)} \\ &= 0.034804 && \text{(C.628)} \\ &= 3.480400\% && \text{(C.629)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Coin Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 3.480400% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 96.519600% will be held in "reserve" with a 51.740200% chance of making twice the 3.480400% back, (and a 48.259800% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 0.060599%, or a doubling of its rate of revenue returns, (per tosses,) in 1144.164760 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 3.480400% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.060599%, per tosses, on average.

Note that the metrics presented in this section are representative of the Coin Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 3.480400% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Coin Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.060599% per tosses.

As another simple example, a company re-invests 3.480400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 3.480400% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.060599% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 3.480400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 3.480400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 3.480400$ percent for the second product, implying that the company should diversify its product line[202]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 3.480400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Coin Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.034804}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.347, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of

[202] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

product mix. In this interpretation, 3.480400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 3.480400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.16.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Coin Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Coin Tossing Game time series is 0.020736, and 0.200000respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.518400.

If this value seems consistent number of companies in the Coin Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Coin Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.572000, which would be the value which should be used in Section C.16.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.16.5 is greater than the average Shannon probability for the companies participating in the Coin Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.16.5. The maximum exploitability for the Coin Tossing Game is derived in Section C.16.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Coin Tossing Game is 0.572000, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.551840 in the Coin Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.630}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Coin Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.16.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.348. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.347. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.16.5, is derived from the Coin Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.16.9.

An additional exploitable strategy may be time itself. Equations C.607, C.611, and, C.609, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Coin Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.363, and, C.364 compare methods of approximation of the "forecastability" of rate of revenue returns in the Coin Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Coin Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[203]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.363, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.609, $0.853212^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.607, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.16.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.16.9. Figure C.365 represents a constructional simulation of the time series data presented in Figure C.346. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.347. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.347 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.366 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.349.

C.16.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.16.3. One of the issues of analysis, as mentioned in Section C.16.7, is to determine the maximum Shannon probability for the time series presented in Figure C.346. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.367 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum

[203]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

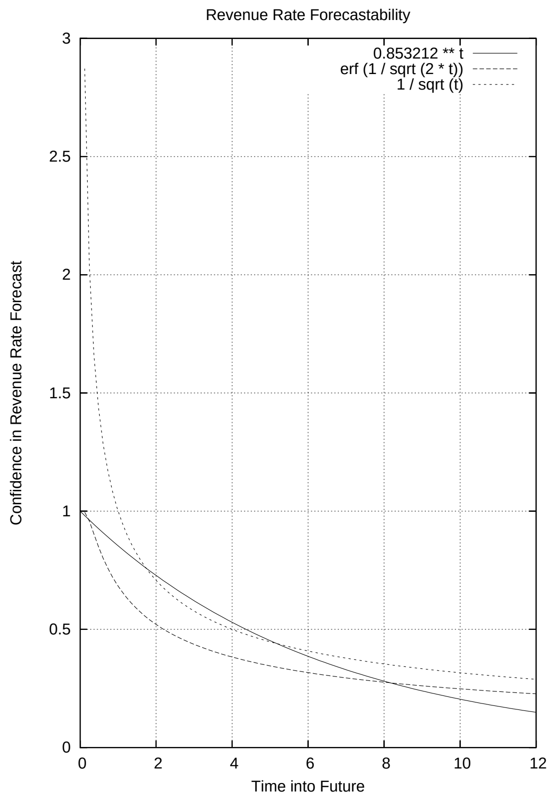

Figure C.363: Coin Tossing Game, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.853212^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

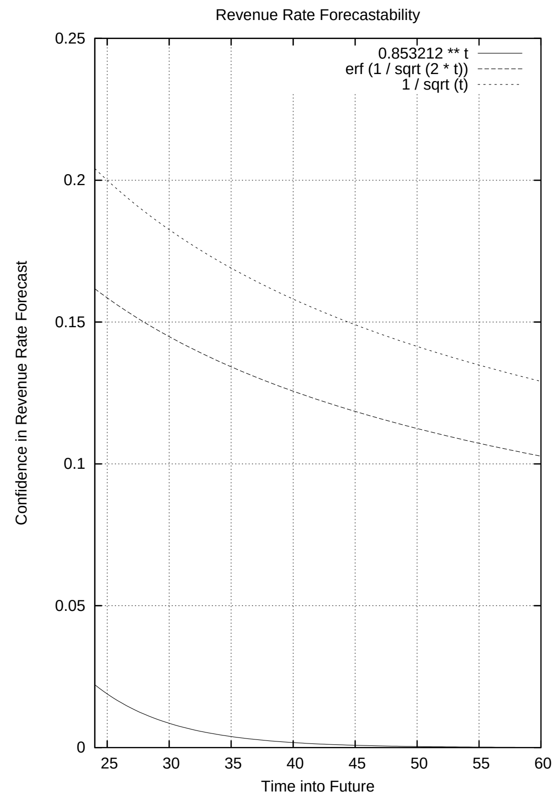

Figure C.364: Coin Tossing Game, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

Shannon probability for the time series data presented in Figure C.346. Figure C.368 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.346. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.517402, as derived in Section C.16.5 to 0.553333. This process, essentially, extracts the random statistical data from the time series presented in Figure C.346, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.346, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Coin Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.551839, as compared with the predicted value from the program *tsshannonmax* of 0.553333.

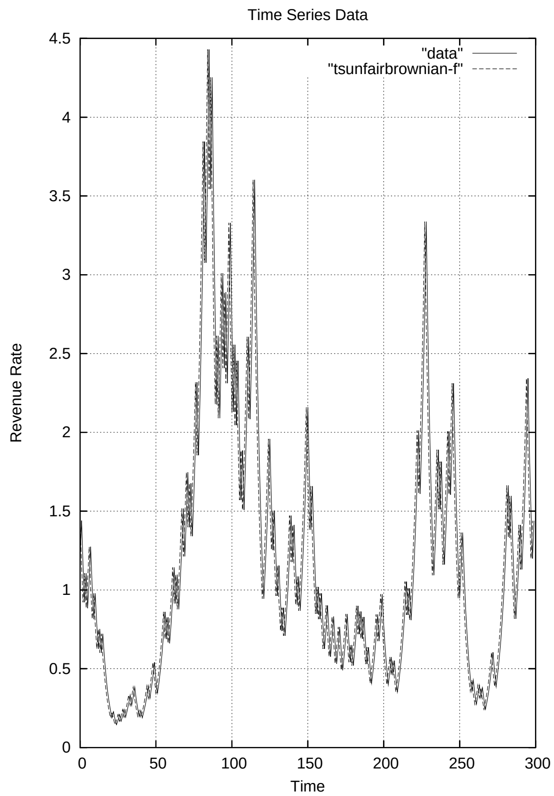

Figure C.365: Coin Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.200000. This data is superimposed on the data presented in Figure C.346.

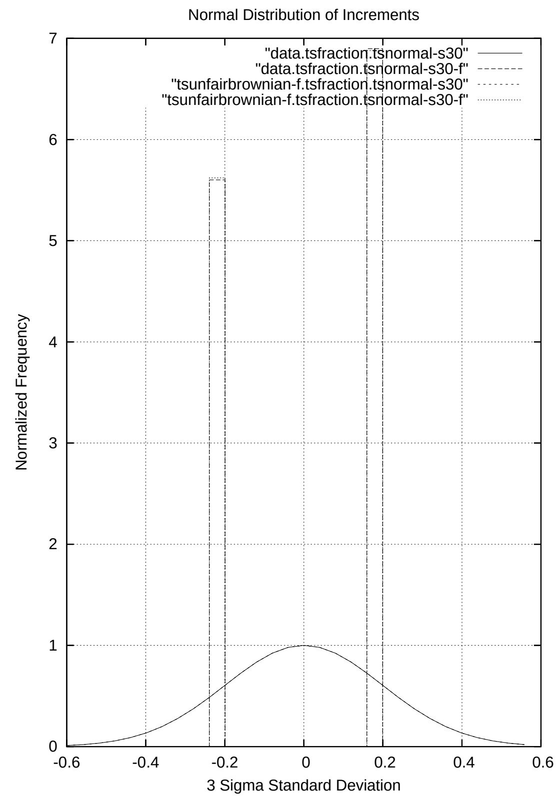

Figure C.366: Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.365, empirical and simulated. The empirical data has a mean of 0.020736, with a standard deviation of 0.199256. By comparison, the simulated data has a mean of 0.020134 with a standard deviation of 0.199319. This data is superimposed on the data presented in Figure C.349. The area under the four curves is identical.

C.16.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the “average” of the normalized increments presented in Figure C.348. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.347. These values will be used in a fixed increment Brownian fractal analysis of the Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.16.6 and D.16.7. As a subjective evaluation of the “quality” of the analysis of the Coin Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.346 from Figure C.347, and the Shannon probability as calculated by counting the total number of tossess that the Coin Tossing Game movement was positive, as presented in Section C.16.9:

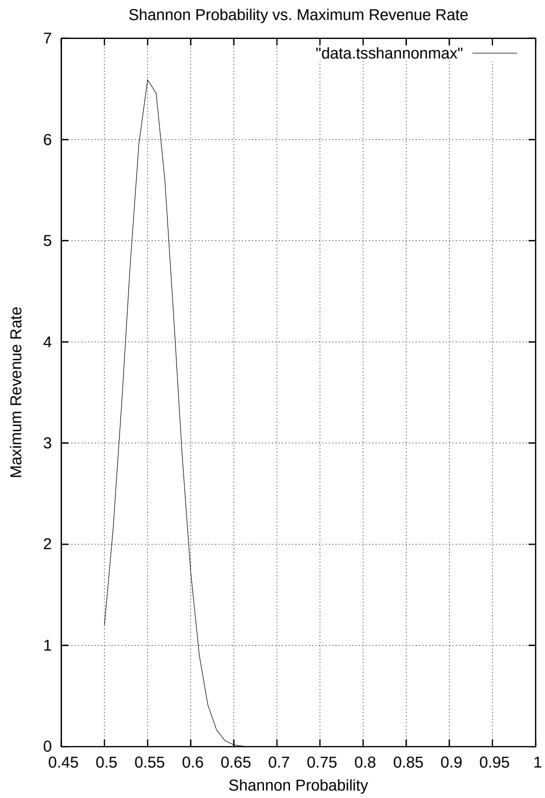

Figure C.367: Coin Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.553333.

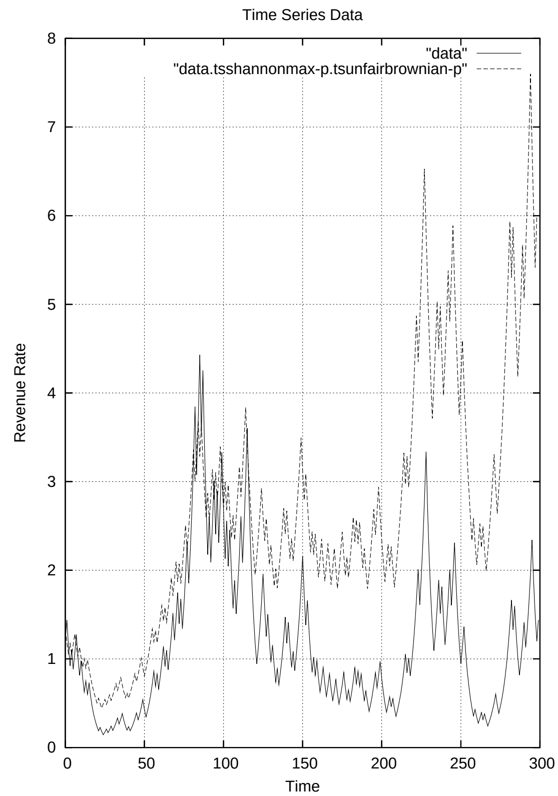

Figure C.368: Coin Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.553333, corresponding to a “wager” fraction of 0.106666.

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.631}$$

$$0.551839 \approx \frac{\frac{0.020736}{0.200000} + 1}{2} \tag{C.632}$$

$$0.551839 \approx 0.551840 \tag{C.633}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.551839 \approx 0.551840 \approx 0.553333 \tag{C.634}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.16.5, should be compared. The four methods used were the mean of Figure C.347, the constant in the least squares approximation to

Figure C.347, the least squares exponential approximation to Figure C.346, and the logarithmic returns of Figure C.346, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.029610 \approx 0.019629 \approx 0.001451 \approx 0.000874 \tag{C.635}$$

It is implied in Section C.16.5, Subsection C.16.5 and in Section C.16.8 that, a Brownian motion with fixed increments fractal may "model" the Coin Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.636}$$

$$0.200000\,(2 \cdot 0.551839 - 1) \approx \frac{0.199256\,(2 \cdot 0.551839 - 1)}{2\sqrt{0.551839\,(1 - 0.551839)}} \tag{C.637}$$

$$0.200000 \cdot 0.103679 \approx 0.199256 \cdot 0.104241 \tag{C.638}$$

$$0.020736 \approx 0.020771 \tag{C.639}$$

and, equating to the mean:

$$0.020736 \approx 0.020736 \approx 0.020771 \tag{C.640}$$

where, as in Equation C.633 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.346 from Figure C.347, and the Shannon probability as calculated by counting the total number of tossess that the Coin Tossing Game movement was positive, as presented in Section C.16.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[204], where the absolute value is presented in Figure C.348, and the root mean square value is presented in Figure C.347:

$$0.200000 \approx 0.200000 \tag{C.641}$$

Note, that if the Coin Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.346 from Figure C.347 should be zero. It is 0.000001.

C.17 Non-optimal Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tsunfairbrownian[205].

The data in this section is presented in tabular form in Section D.17. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Non-optimal Coin Tossing Game. This is included for "theoretical" comparative purposes.

[204] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[205] As a simulation model, the program *tscoin* was run to make a time series data file, with the following parameters:

```
tscoin -p 0.7 300 > data.1
```

to make a time series of 300 elements, with a Shannon probability of 0.7. In addition, the program *tsunfairbrownian* was run on the data file with the following parameters:

```
tsunfairbrownian -f 0.03 data.1 > data
```

to make a time series with a known non-optimal investment strategy. The data is by tosses.

C.17.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.17.1. Figure C.369 is a graph of the time series data for the Non-optimal Coin Tossing Game.

Figure C.370 is a graph of the normalized increments of the time series data presented in Figure C.369. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.371 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.370. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[206].

Figure C.372 is the normalized histogram of the normalized increments of the time series data shown in Figure C.370. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.372.

Figure C.373 is the statistical estimate for the data presented in Figure C.370, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.574001$, as derived in Section C.17.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.374 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.370. In principle, if the distribution of the normalized increments presented in Figure C.372 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.375 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.370. In principle, if the distribution of the normalized increments presented in Figure C.372 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.376 is the range of values of the time series shown in Figure C.369. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.376 would be a square root function[207]. Figure C.377 is the deterministic map of the normalized increments of the time series data shown in Figure C.370. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

C.17.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series,

[206] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[207] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.376 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

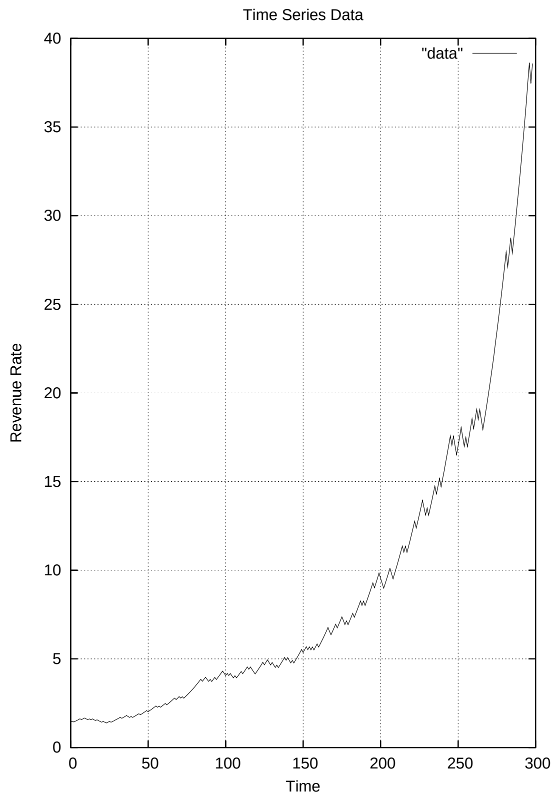

Figure C.369: Non-optimal Coin Tossing Game, time series data.

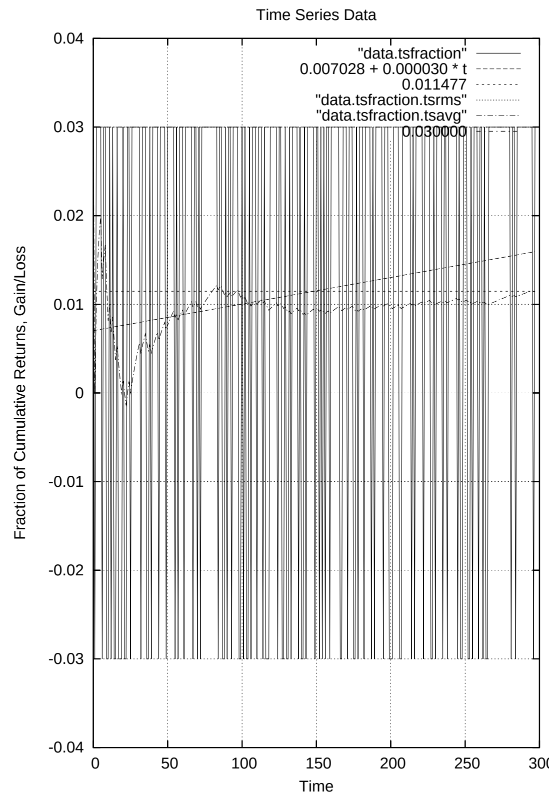

Figure C.370: Non-optimal Coin Tossing Game, normalized increments of the time series data presented in Figure C.369. The mean is 0.011477 with a standard deviation of 0.027765. The formula for the least squares approximation is $0.007028+0.000030t$, and the root mean squared value is 0.030000. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000030, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

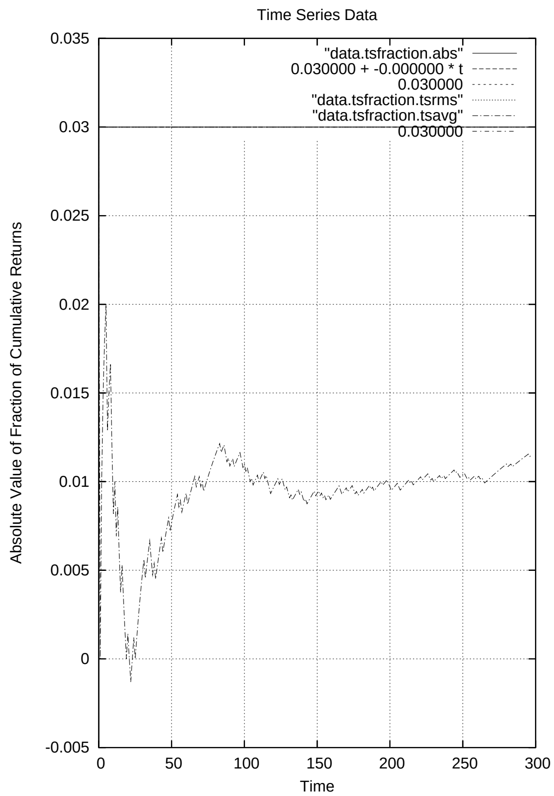

Figure C.371: Non-optimal Coin Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.370. The mean is 0.030000 with a standard deviation of 0.000000. The formula for the least squares approximation is $0.030000 + 0.000000t$, and the root mean square value, from Figure C.370, is 0.030000. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.370, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

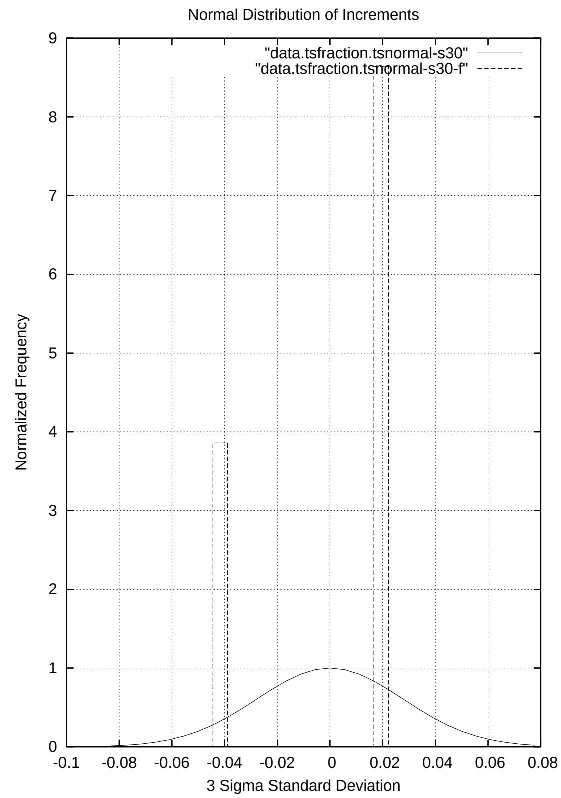

Figure C.372: Non-optimal Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.370. The data has a mean of 0.011477, with a standard deviation of 0.027765. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 130.435000, with a critical value of 42.557000.

mean square of the instantaneous fraction of change[208]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

[208]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

```
For  a mean  of 0.011438,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.001144,   1862  samples  would  be required.
     (With  299  samples,   the estimated  error  is 0.002854  = 24.949283  percent.)
For  a standard  deviation  of 0.030000,  with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.003000,   136  samples  would  be required.
     (With  299  samples,   the estimated  error  is 0.002018  = 6.726307  percent.)
```

Figure C.373: Non-optimal Coin Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.370. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.370.

Figure C.378 is the instantaneous value of the root mean square of the normalized increments for the Non-optimal Coin Tossing Game, and Figure C.379 is the instantaneous Shannon probability for the normalized increments.

C.17.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.17.4. Figure C.380 is a graph of the logistic function estimates of the time series data for the Non-optimal Coin Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[209]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.380 is a graph of the logistic function for the time series data presented in Figure C.369. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.370. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.370. Figure C.381 is the same graph, but with the time scale expanded by a factor of two.

C.17.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.17.5. Figure C.382 is a graph of the Hurst coefficient data time series data shown in Figure C.369. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.383 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.370. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.382 implies that the variance of the rate of revenue returns, (per tosses,) in the Non-optimal Coin Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the

[209] For example, in Figures C.380 and C.381, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.17.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

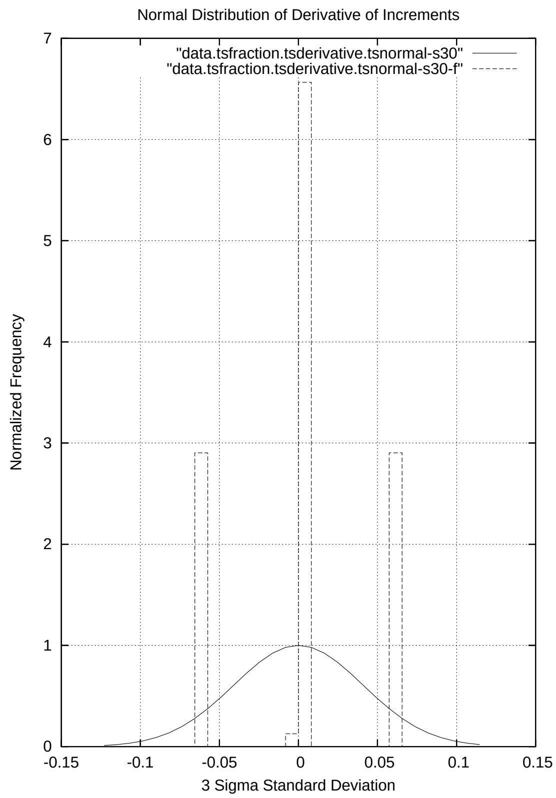

Figure C.374: Non-optimal Coin Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.370.

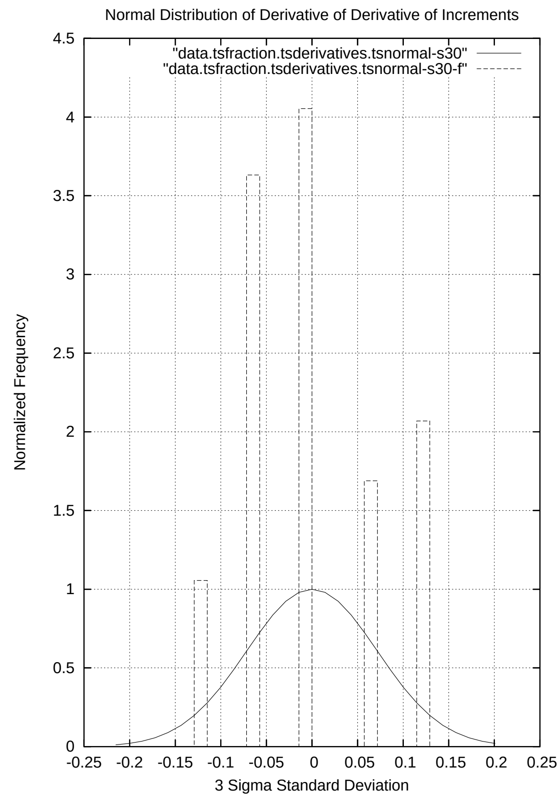

Figure C.375: Non-optimal Coin Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.370.

state of affairs repeating sometime in the future goes down with increasing time[210], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.386, and, C.387 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Non-optimal Coin Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][211]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.382, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of

[210]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[211]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

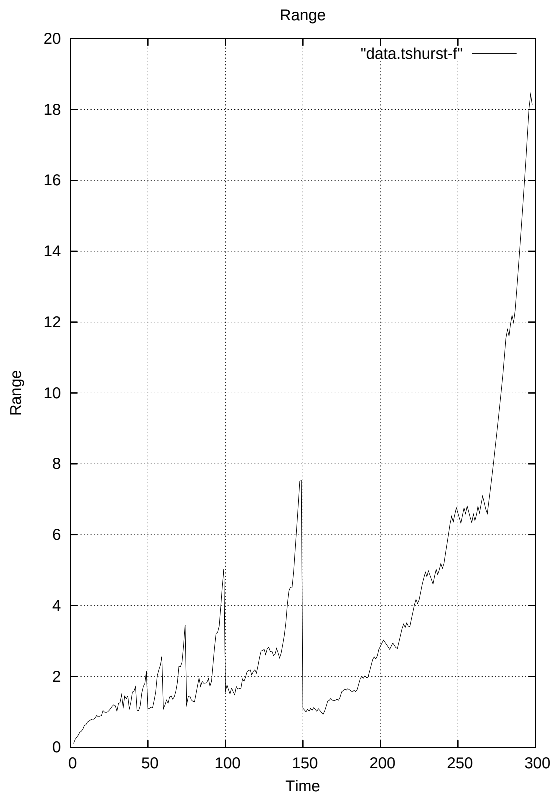

Figure C.376: Non-optimal Coin Tossing Game, range of the time series data shown in Figure C.369.

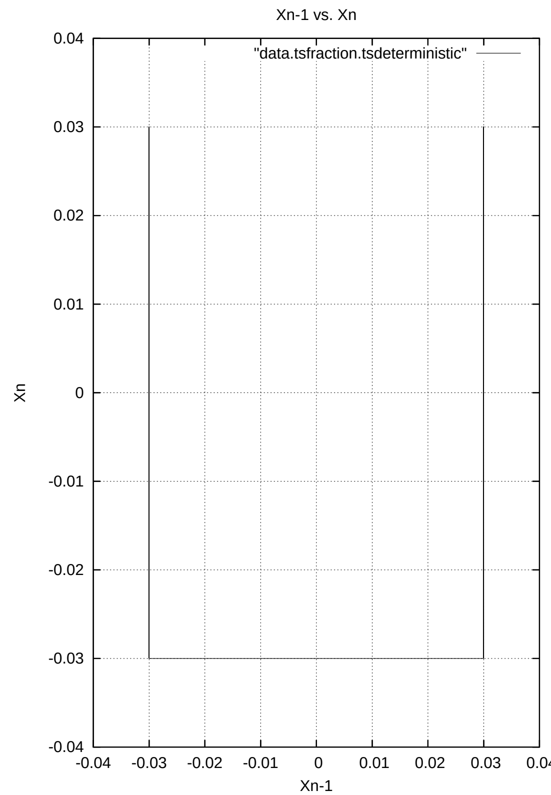

Figure C.377: Non-optimal Coin Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.370.

0.836828, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.642}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.836828} \tag{C.643}$$

$$\propto (t_2 - t_1)^{1.673656} \tag{C.644}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[212]. A Hurst coefficient of 0.836828, (for the near future, and 0.737672 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is

[212]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian

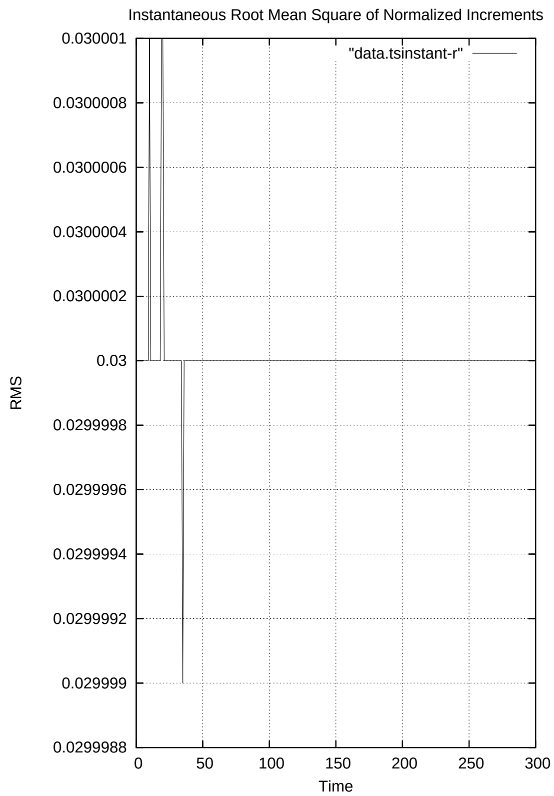

Figure C.378: Non-optimal Coin Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.369.

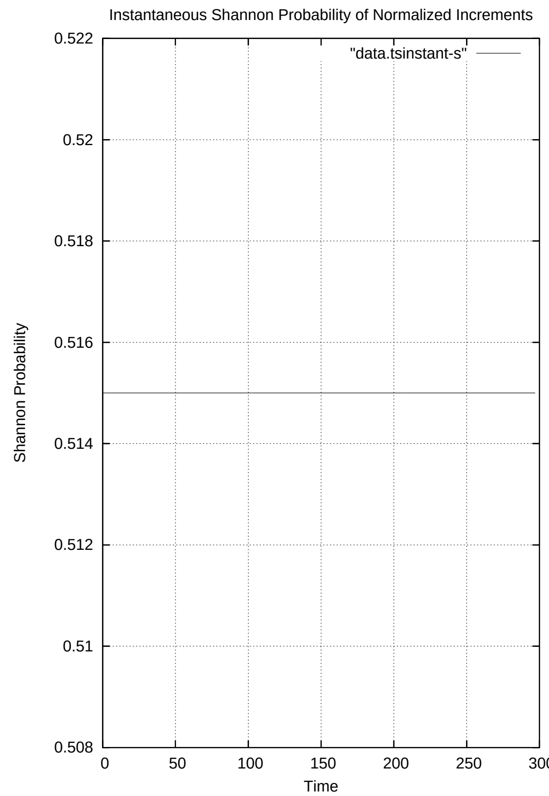

Figure C.379: Non-optimal Coin Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.369.

83.682800% [Pet91, pp. 66] for the near future, and 0.737672 for the distant future. Likewise, there is a 83.682800% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 83.682800% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Non-optimal Coin Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.645}$$

$$= 0.836828^n \tag{C.646}$$

motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.17.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Non-optimal Coin Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

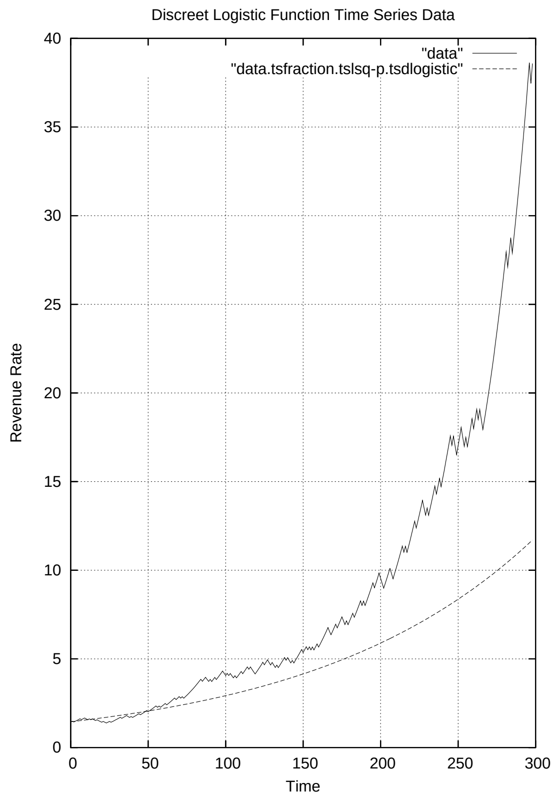

Figure C.380: Non-optimal Coin Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.370 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

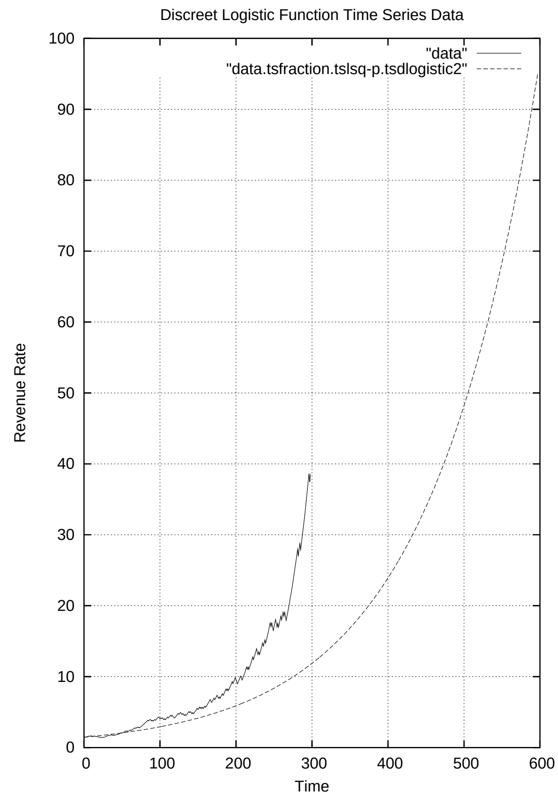

Figure C.381: Non-optimal Coin Tossing Game, logistic function estimates of Figure C.380 with the time scale expanded by a factor of two.

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.370, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.011477 \cdot 100$ percent, on the average, with a standard deviation of $0.027765 \cdot 100$ percent, and a root mean square error value of $0.030000 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^H \tag{C.647}$$

$$\propto\; (t_2 - t_1)^{0.836828} \tag{C.648}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate

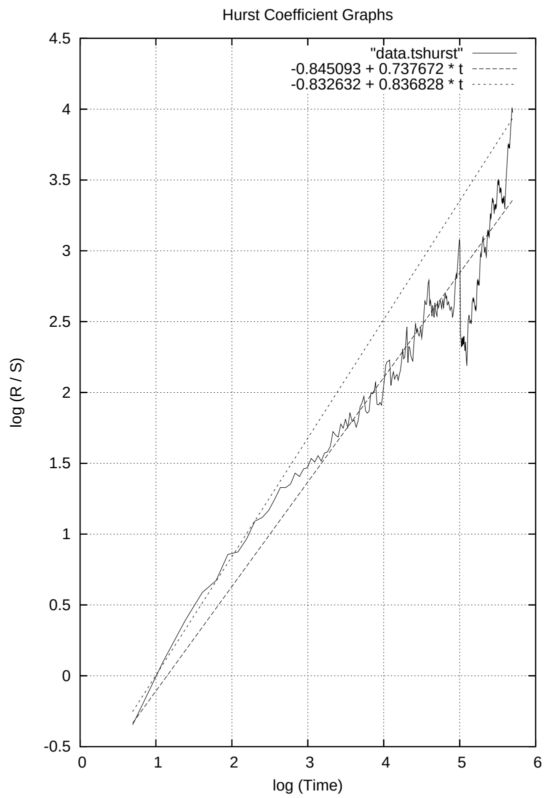

Figure C.382: Non-optimal Coin Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.370. The slope of the graph is the Hurst coefficient.

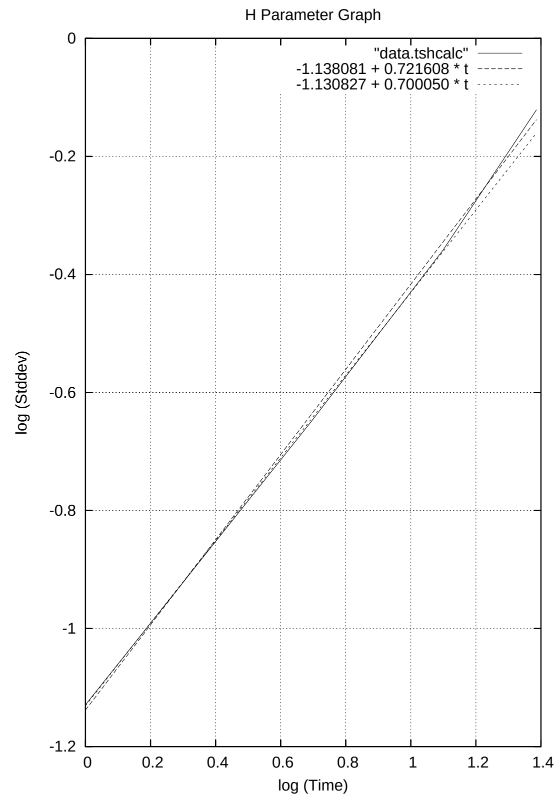

Figure C.383: Non-optimal Coin Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.370 The slope of the graph is the H parameter.

of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.648 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.649}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.650}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.651}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[213].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.652}$$

$$\propto \frac{X(rt)}{r^{0.836828}} \tag{C.653}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.383, to provide a least squares approximation to the H parameter for the Non-optimal Coin Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.700050 for the near future, and 0.721608 for the distant future.

Figures C.382 and C.383 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.370. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.370, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.384 and C.385 was made using the -d option.

C.17.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.17.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.371. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.370. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Non-optimal Coin Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.370, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.011477 + 1)}{\ln(2)} = 0.016464 \tag{C.654}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.370, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.007028 + 1)}{\ln(2)} = 0.010104 \tag{C.655}$$

Note that if the mean is not constant in Figure C.370, this method will not provide accurate results.

[213]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

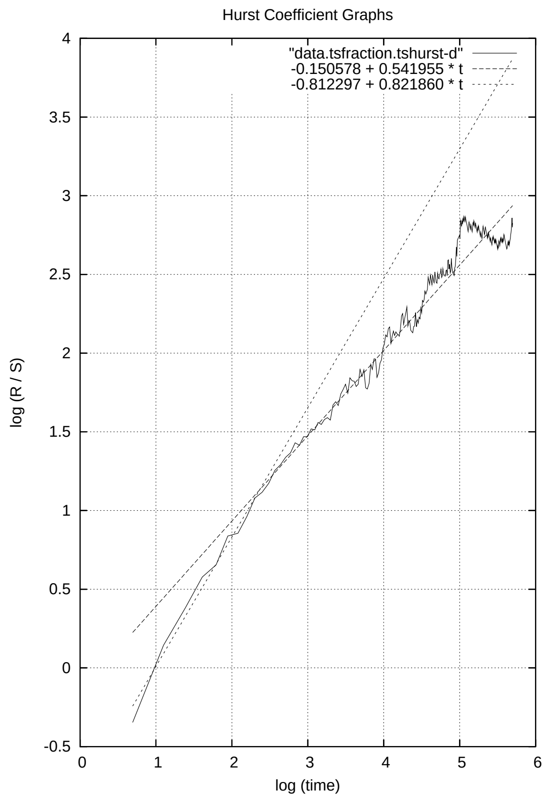

Figure C.384: Non-optimal Coin Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.369. The slope of the graph is the Hurst coefficient, and is 0.821860 for the near term, and 0.541955 for the far term.

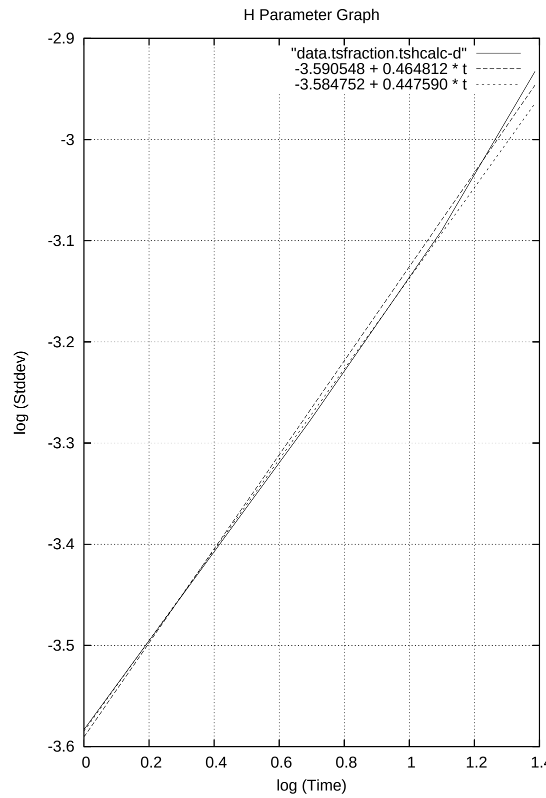

Figure C.385: Non-optimal Coin Tossing Game, traditional H parameter data for the time series data shown in Figure C.369 The slope of the graph is the H parameter, and is 0.447590 for the near term, and 0.464812 for the far term.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.369:

$$bits = 0.014992 \tag{C.656}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.369:

$$bits = 0.015859 \tag{C.657}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.17.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.015859t} \tag{C.658}$$

therefore:

$$C(p) = 0.015859 \tag{C.659}$$

and, *tsshannon* 0.015859 gives:

$$C(0.574001) = 0.015859 \tag{C.660}$$

therefore:

$$\begin{aligned} 2^{C(0.574001)} &= 2^{0.015859} && \text{(C.661)} \\ &= 1.011053 && \text{(C.662)} \\ &= 1.105326\% && \text{(C.663)} \end{aligned}$$

and:

$$\begin{aligned} 2p-1 &= (2 \cdot 0.574001) - 1 && \text{(C.664)} \\ &= 0.148002 && \text{(C.665)} \\ &= 14.800200\% && \text{(C.666)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Non-optimal Coin Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 14.800200% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 85.199800% will be held in "reserve" with a 57.400100% chance of making twice the 14.800200% back, (and a 42.599900% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 1.105326%, or a doubling of its rate of revenue returns, (per tosses,) in 63.055678 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 14.800200% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.105326%, per tosses, on average.

Note that the metrics presented in this section are representative of the Non-optimal Coin Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 14.800200% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Non-optimal Coin Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.105326% per tosses.

As another simple example, a company re-invests 14.800200% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 14.800200% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.105326% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 14.800200% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 14.800200$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 14.800200$ percent for the second product, implying that the company should diversify its product line[214]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 14.800200%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Non-optimal Coin Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.148002}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.370, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 14.800200% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 14.800200% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.17.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Non-optimal Coin Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Non-optimal Coin Tossing Game time series is 0.011477, and 0.030000respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 12.752222.

If this value seems consistent number of companies in the Non-optimal Coin Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Non-optimal Coin Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.553565, which would be the value which should be used in Section C.17.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.17.5 is greater than the average Shannon probability for the companies participating in the Non-optimal Coin Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.17.5. The maximum exploitability for the Non-optimal Coin Tossing Game is derived in Section C.17.9, but it is probably of doubtful practicality.

[214]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Non-optimal Coin Tossing Game is 0.553565, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.691283 in the Non-optimal Coin Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.667}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Non-optimal Coin Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.17.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.371. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.370. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.17.5, is derived from the Non-optimal Coin Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.17.9.

An additional exploitable strategy may be time itself. Equations C.644, C.648, and, C.646, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Non-optimal Coin Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.386, and, C.387 compare methods of approximation of the "forecastability" of rate of revenue returns in the Non-optimal Coin Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Non-optimal Coin Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[215]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

[215]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

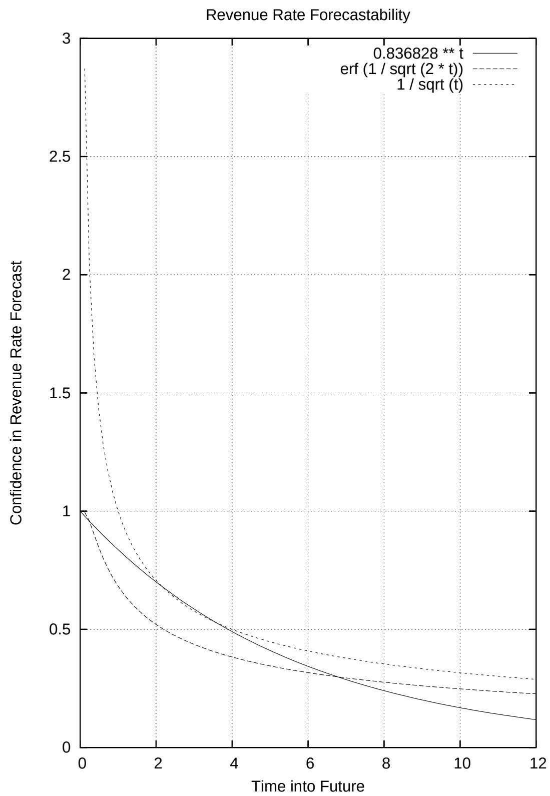

Figure C.386: Non-optimal Coin Tossing Game, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.836828^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

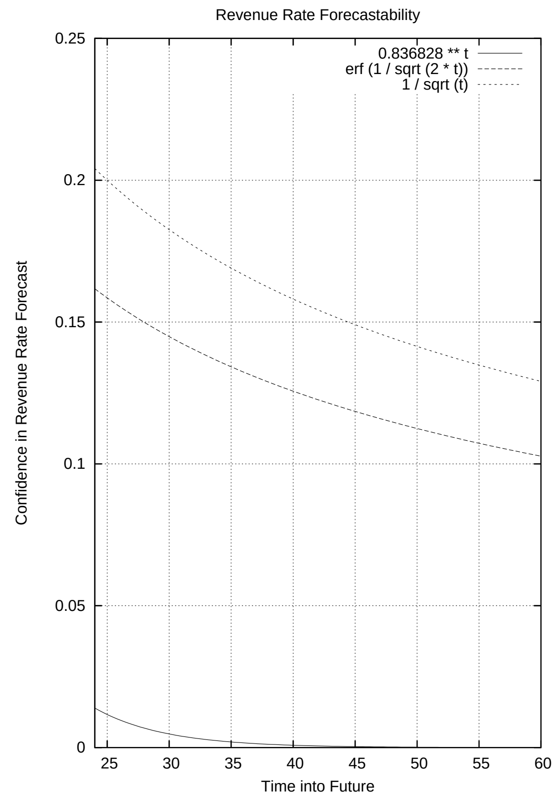

Figure C.387: Non-optimal Coin Tossing Game, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

As an interesting interpretation of the data presented in Figure C.386, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.646, $0.836828^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.644, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.17.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.17.9. Figure C.388 represents a constructional simulation of the time series data presented in Figure C.369. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.370. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.370 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.389 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.372.

C.17.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.17.3. One of the issues of analysis, as mentioned in Section C.17.7, is to determine the maximum Shannon probability for the time series presented in Figure C.369. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.390 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.369. Figure C.391 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.369. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.574001, as derived in Section C.17.5 to 0.692308. This process, essentially, extracts the random statistical data from the time series presented in Figure C.369, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.369, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Non-optimal Coin Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.691275, as compared with the predicted value from the program *tsshannonmax* of 0.692308.

C.17.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.371. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.370. These values will be used in a fixed increment Brownian fractal analysis of the Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.17.6 and D.17.7. As a subjective evaluation of the "quality" of the analysis of the Non-optimal Coin Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.369 from Figure C.370, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Coin Tossing Game movement was positive, as presented in Section C.17.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \quad \text{(C.668)}$$

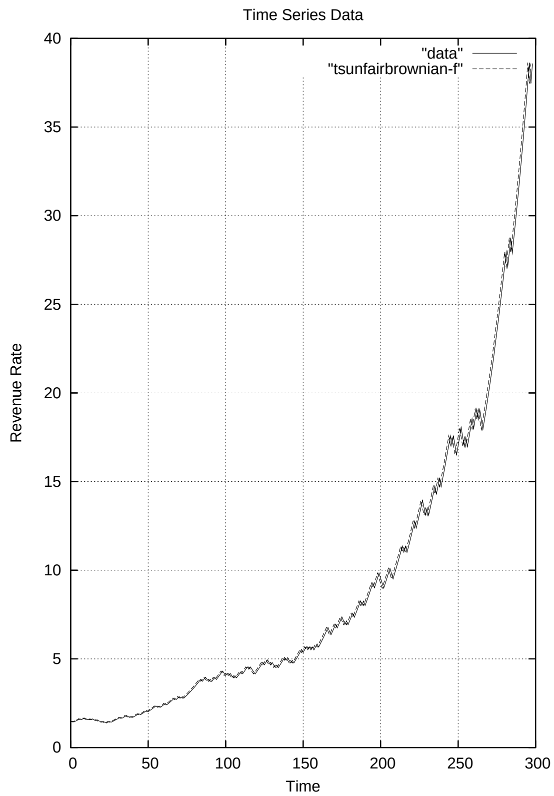

Figure C.388: Non-optimal Coin Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.030000. This data is superimposed on the data presented in Figure C.369.

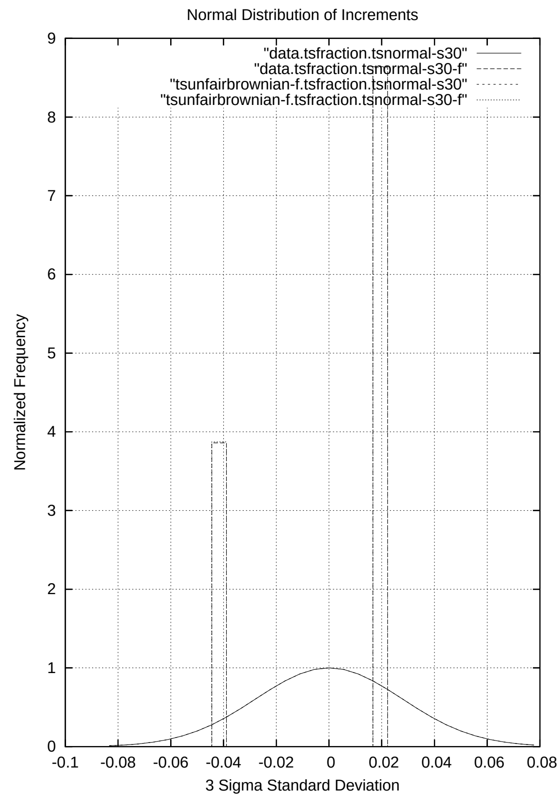

Figure C.389: Non-optimal Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.388, empirical and simulated. The empirical data has a mean of 0.011477, with a standard deviation of 0.027765. By comparison, the simulated data has a mean of 0.011414 with a standard deviation of 0.027791. This data is superimposed on the data presented in Figure C.372. The area under the four curves is identical.

$$0.691275 \approx \frac{\frac{0.011477}{0.030000}+1}{2} \quad \text{(C.669)}$$

$$0.691275 \approx 0.691283 \quad \text{(C.670)}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.691275 \approx 0.691283 \approx 0.692308 \quad \text{(C.671)}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.17.5, should be compared. The four methods used were the mean of Figure C.370, the constant in the least squares approximation to

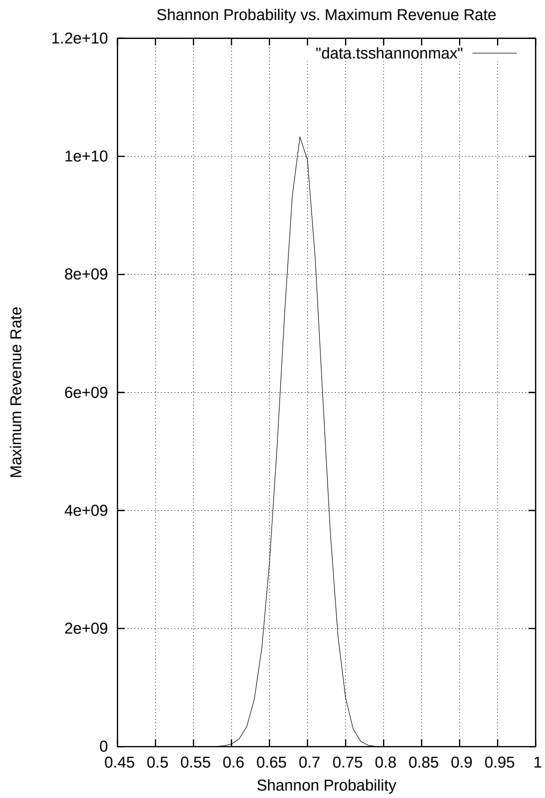

Figure C.390: Non-optimal Coin Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.692308.

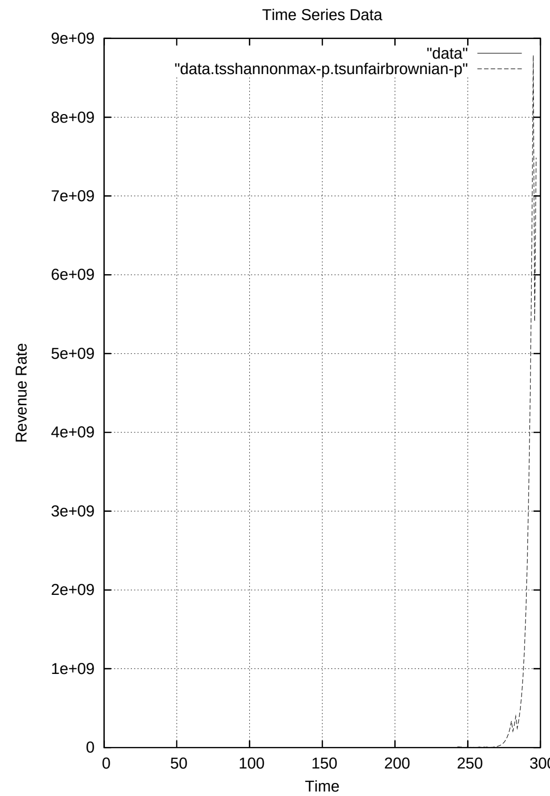

Figure C.391: Non-optimal Coin Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.692308, corresponding to a "wager" fraction of 0.384616.

Figure C.370, the least squares exponential approximation to Figure C.369, and the logarithmic returns of Figure C.369, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.016464 \approx 0.010104 \approx 0.014992 \approx 0.015859 \tag{C.672}$$

It is implied in Section C.17.5, Subsection C.17.5 and in Section C.17.8 that, a Brownian motion with fixed increments fractal may "model" the Non-optimal Coin Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.673}$$

$$0.030000\,(2 \cdot 0.691275 - 1) \approx \frac{0.027765\,(2 \cdot 0.691275 - 1)}{2\sqrt{0.691275\,(1 - 0.691275)}} \tag{C.674}$$

$$0.030000 \cdot 0.382550 \approx 0.027765 \cdot 0.414045 \quad \text{(C.675)}$$
$$0.011477 \approx 0.011496 \quad \text{(C.676)}$$

and, equating to the mean:

$$0.011477 \approx 0.011477 \approx 0.011496 \quad \text{(C.677)}$$

where, as in Equation C.670 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.369 from Figure C.370, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Coin Tossing Game movement was positive, as presented in Section C.17.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[216], where the absolute value is presented in Figure C.371, and the root mean square value is presented in Figure C.370:

$$0.030000 \approx 0.030000 \quad \text{(C.678)}$$

Note, that if the Non-optimal Coin Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.369 from Figure C.370 should be zero. It is 0.000000.

C.18 Time Sampled Non-optimal Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tsunfairbrownian.tssample[217].

The data in this section is presented in tabular form in Section D.18. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Time Sampled Non-optimal Coin Tossing Game. This is included for "theoretical" comparative purposes.

C.18.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.18.1. Figure C.392 is a graph of the time series data for the Time Sampled Non-optimal Coin Tossing Game.

Figure C.393 is a graph of the normalized increments of the time series data presented in Figure C.392. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described

[216] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of of "fit" to a Gaussian distribution.

[217] As a simulation model, the program *tscoin* was run to make a time series data file, with the following parameters:

```
tscoin -p 0.70 1500 > data.1
```

to make a time series of 1500 elements, with a Shannon probability of 0.70. In addition, the program *tsunfairbrownian* was run on the data file with the following parameters:

```
tsunfairbrownian -f 0.0894 data.1 > data.2
```

to make a time series with a known non-optimal investment strategy. The value, 0.0894 was calculated by reducing the desired value, 0.2, by a factor of $\frac{1}{\sqrt{5}}$, where the sampling occurs every fifth time series element. Then the program *tssample* was run with the following parameters:

```
tssample -i 5 data.2 > data
```

to time sample every fifth element in the time series to make a time sampled time series with a known non optimal investment strategy. The data is by tosses.

briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

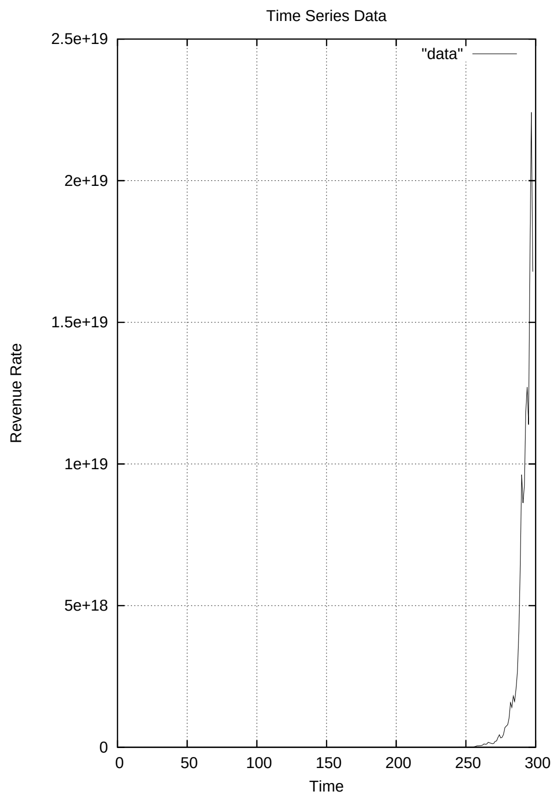

Figure C.392: Time Sampled Non-optimal Coin Tossing Game, time series data.

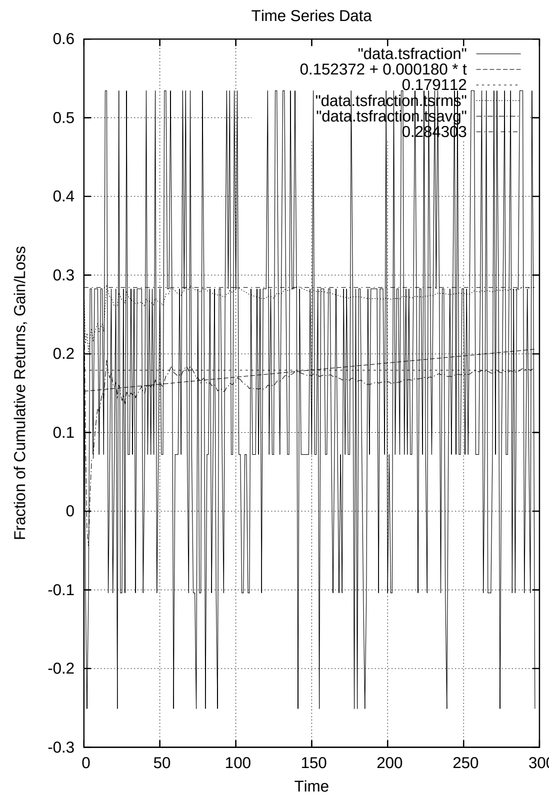

Figure C.393: Time Sampled Non-optimal Coin Tossing Game, normalized increments of the time series data presented in Figure C.392. The mean is 0.179112 with a standard deviation of 0.221159. The formula for the least squares approximation is $0.152372 + 0.000180t$, and the root mean squared value is 0.284303. The graph, labeled “data-.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000180, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.394 is a graph of the absolute value of the normalized increments of the time series data presented in

Figure C.393. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[218].

Figure C.395 is the normalized histogram of the normalized increments of the time series data shown in Figure C.393. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.395.

Figure C.396 is the statistical estimate for the data presented in Figure C.393, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.763464$, as derived in Section C.18.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.397 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.393. In principle, if the distribution of the normalized increments presented in Figure C.395 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.398 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.393. In principle, if the distribution of the normalized increments presented in Figure C.395 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.399 is the range of values of the time series shown in Figure C.392. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.399 would be a square root function[219]. Figure C.400 is the deterministic map of the normalized increments of the time series data shown in Figure C.393. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

C.18.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[220]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.401 is the instantaneous value of the root mean square of the normalized increments for the Time Sampled Non-optimal Coin Tossing Game, and Figure C.402 is the instantaneous Shannon probability for the normalized

[218]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[219]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.399 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[220]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

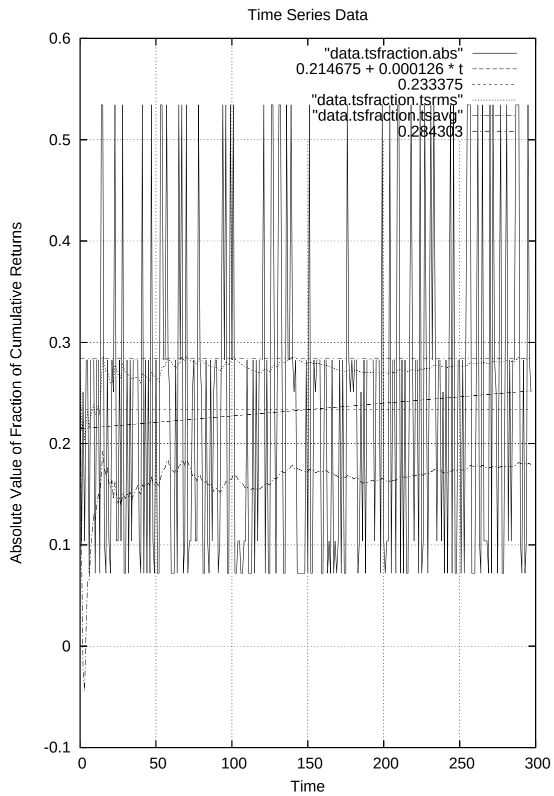

Figure C.394: Time Sampled Non-optimal Coin Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.393. The mean is 0.233375 with a standard deviation of 0.162645. The formula for the least squares approximation is $0.214675 + 0.000126t$, and the root mean square value, from Figure C.393, is 0.284303. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.393, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

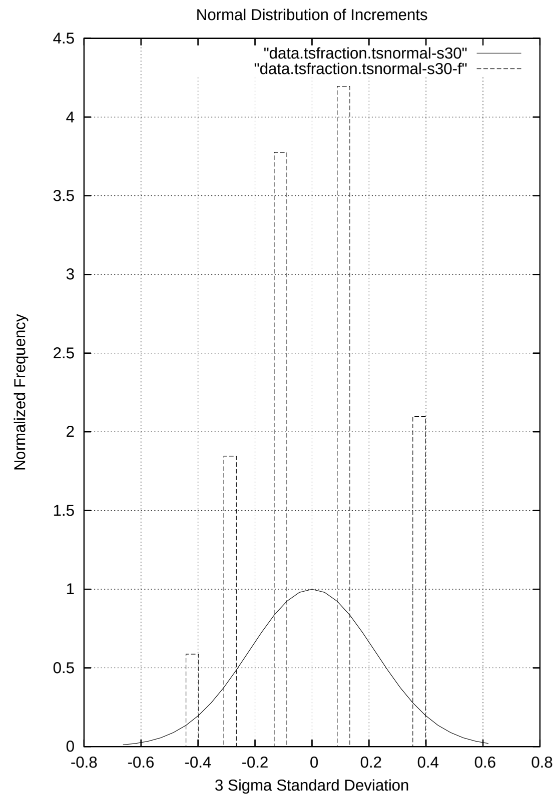

Figure C.395: Time Sampled Non-optimal Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.393. The data has a mean of 0.179112, with a standard deviation of 0.221159. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 51.060000, with a critical value of 42.557000.

increments.

C.18.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.18.4. Figure C.403 is a graph of the logistic function estimates of the time series data for the Time Sampled Non-optimal Coin Tossing Game. The reader is cautioned

```
For  a mean  of 0.178513,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.017851,   687 samples  would  be required.
     (With  299 samples,   the estimated   error  is 0.027044   = 15.149664   percent.)
For  a standard   deviation   of 0.284303,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.028430,   136 samples  would  be required.
     (With  299 samples,   the estimated   error  is 0.019123   = 6.726307   percent.)
```

Figure C.396: Time Sampled Non-optimal Coin Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.393. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.393.

that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[221]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.403 is a graph of the logistic function for the time series data presented in Figure C.392. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.393. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.393. Figure C.404 is the same graph, but with the time scale expanded by a factor of two.

C.18.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.18.5. Figure C.405 is a graph of the Hurst coefficient data time series data shown in Figure C.392. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.406 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.393. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.405 implies that the variance of the rate of revenue returns, (per tosses,) in the Time Sampled Non-optimal Coin Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[222], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.409, and, C.410 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Time Sampled Non-optimal Coin Tossing Game for the near

[221]For example, in Figures C.403 and C.404, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.18.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

[222]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

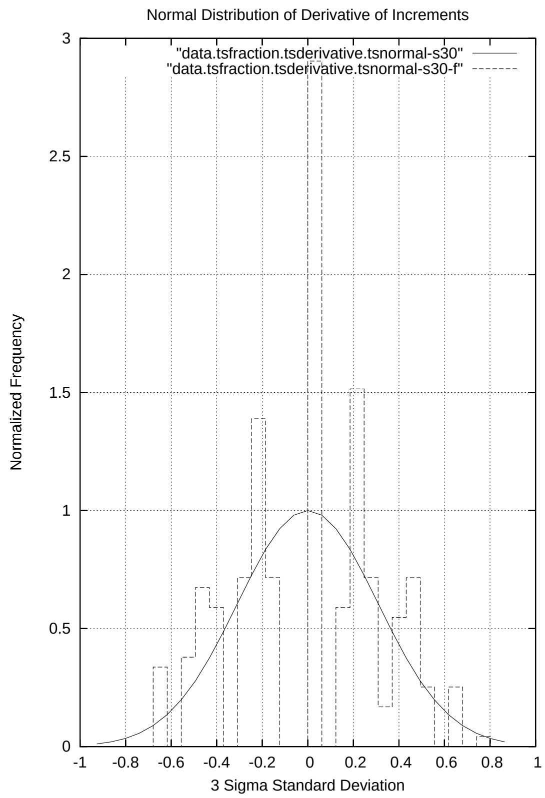

Figure C.397: Time Sampled Non-optimal Coin Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.393.

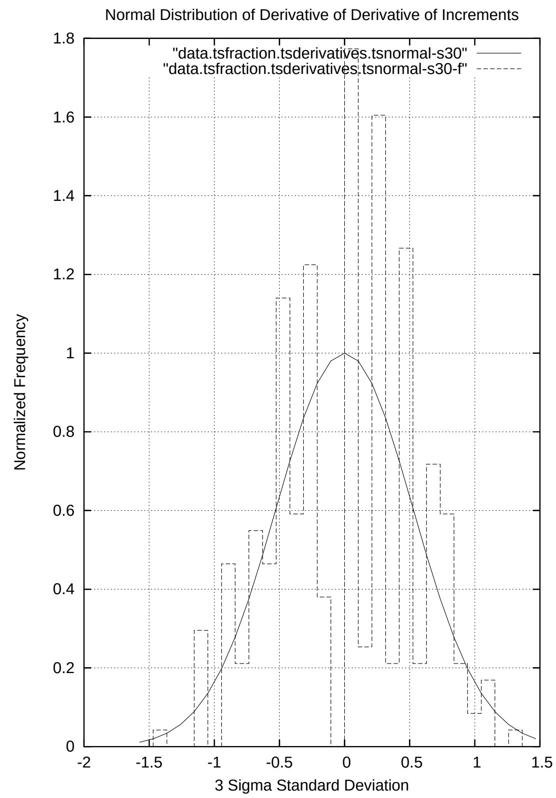

Figure C.398: Time Sampled Non-optimal Coin Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.393.

term and far term, respectively [Pet91, pp. 83-84][223]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.405, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.869484, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.679}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.869484} \tag{C.680}$$

$$\propto (t_2 - t_1)^{1.738968} \tag{C.681}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval

[223] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

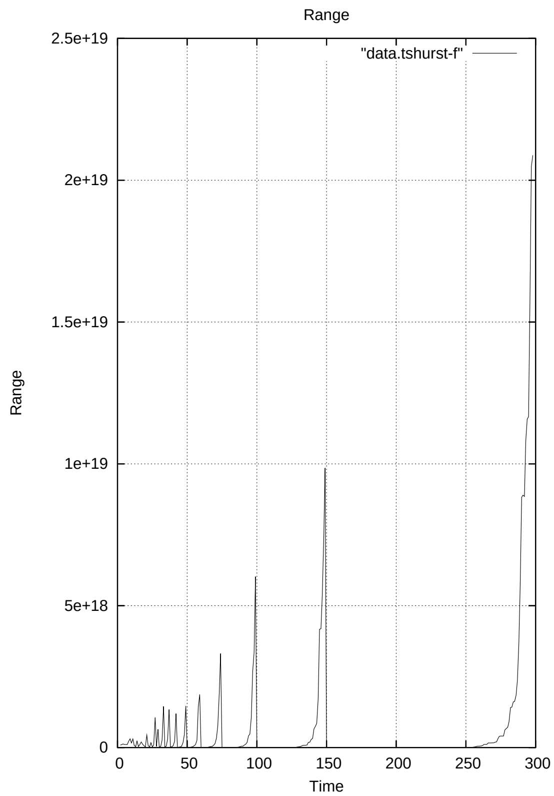

Figure C.399: Time Sampled Non-optimal Coin Tossing Game, range of the time series data shown in Figure C.392.

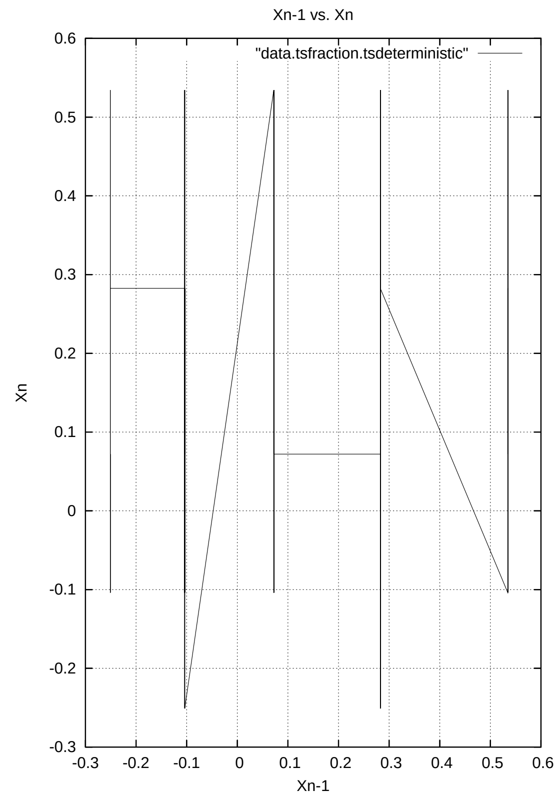

Figure C.400: Time Sampled Non-optimal Coin Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.393.

$t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[224]. A Hurst coefficient of 0.869484, (for the near future, and 0.734095 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is 86.948400% [Pet91, pp. 66] for the near future, and 0.734095 for the distant future. Likewise, there is a 86.948400% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 86.948400% that the rate of revenue

[224]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.18.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Time Sampled Non-optimal Coin Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

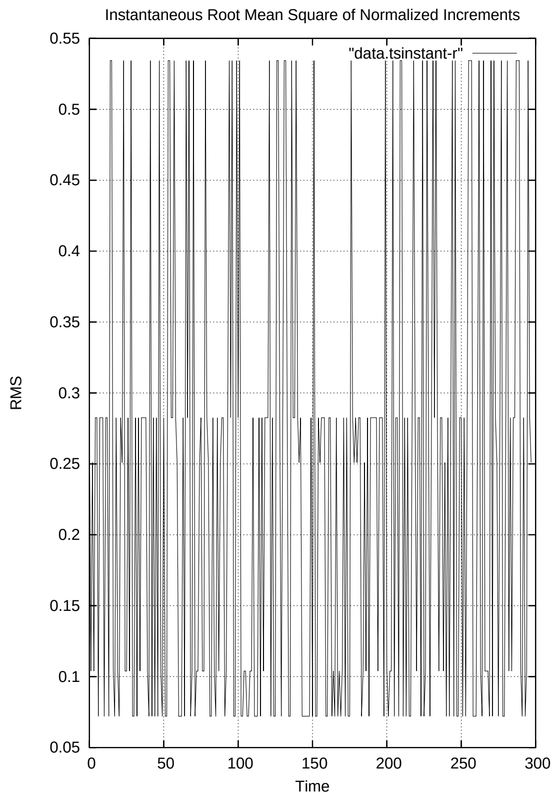

Figure C.401: Time Sampled Non-optimal Coin Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.392.

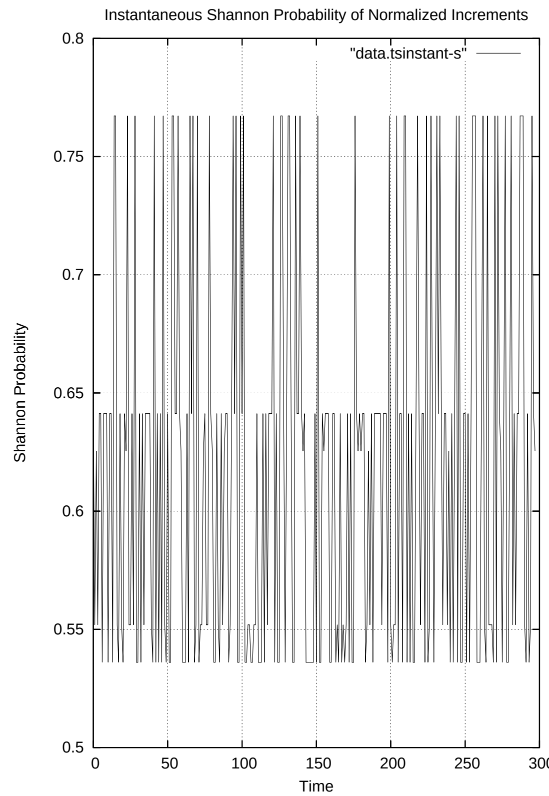

Figure C.402: Time Sampled Non-optimal Coin Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.392.

returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Time Sampled Non-optimal Coin Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \quad \text{(C.682)}$$

$$= 0.869484^n \quad \text{(C.683)}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.393, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.179112 \cdot 100$ percent, on the average, with a standard deviation of $0.221159 \cdot 100$ percent, and a root mean square error value of $0.284303 \cdot 100$ percent—small values for such a simple

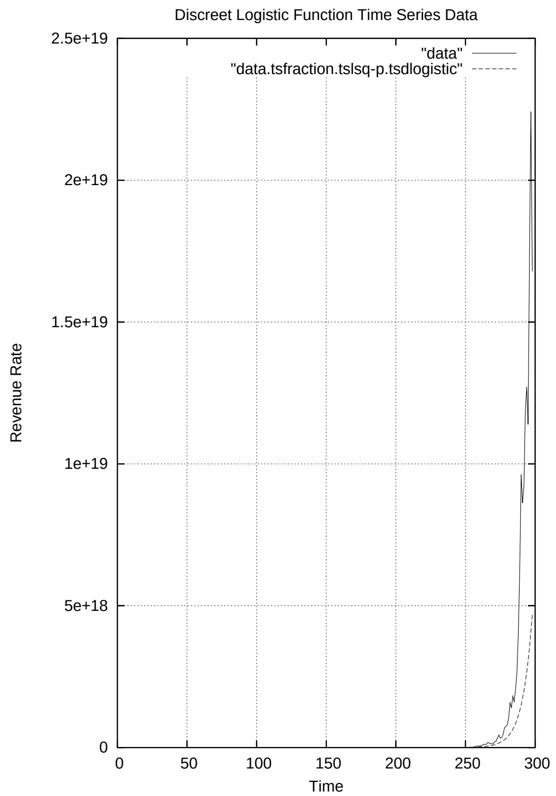

Figure C.403: Time Sampled Non-optimal Coin Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.393 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

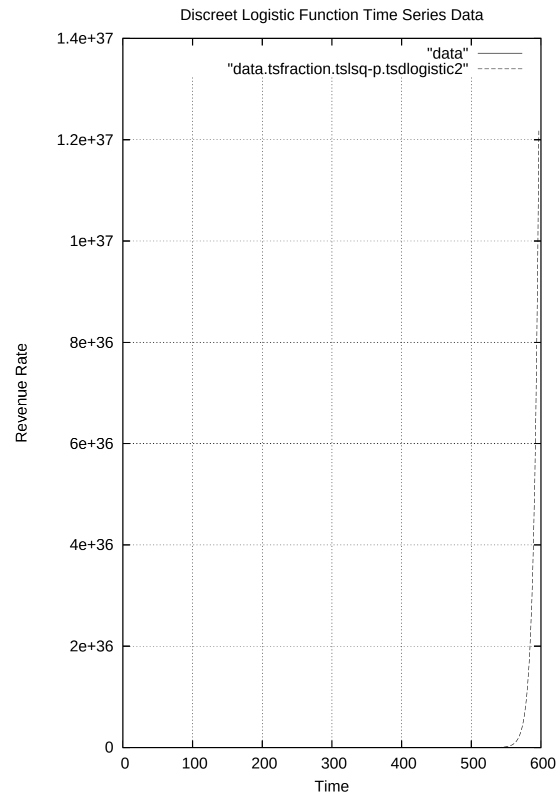

Figure C.404: Time Sampled Non-optimal Coin Tossing Game, logistic function estimates of Figure C.403 with the time scale expanded by a factor of two.

forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^H \qquad \text{(C.684)}$$
$$\propto\; (t_2 - t_1)^{0.869484} \qquad \text{(C.685)}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

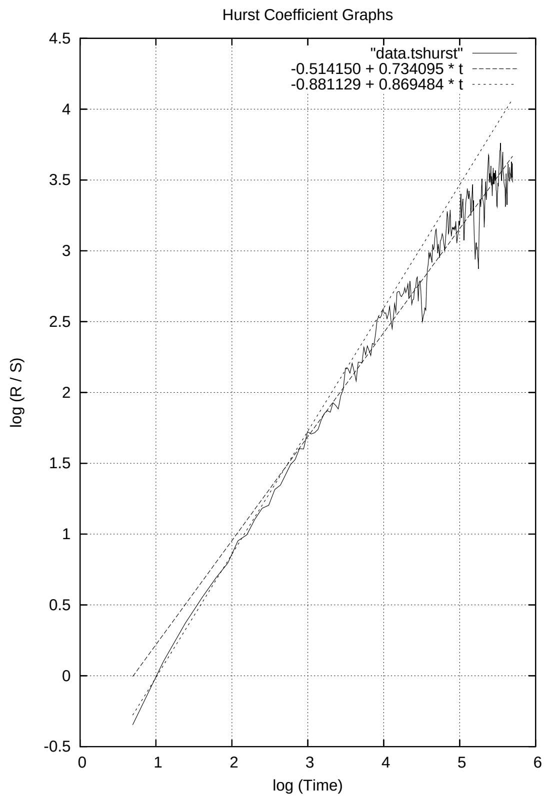

Figure C.405: Time Sampled Non-optimal Coin Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.393. The slope of the graph is the Hurst coefficient.

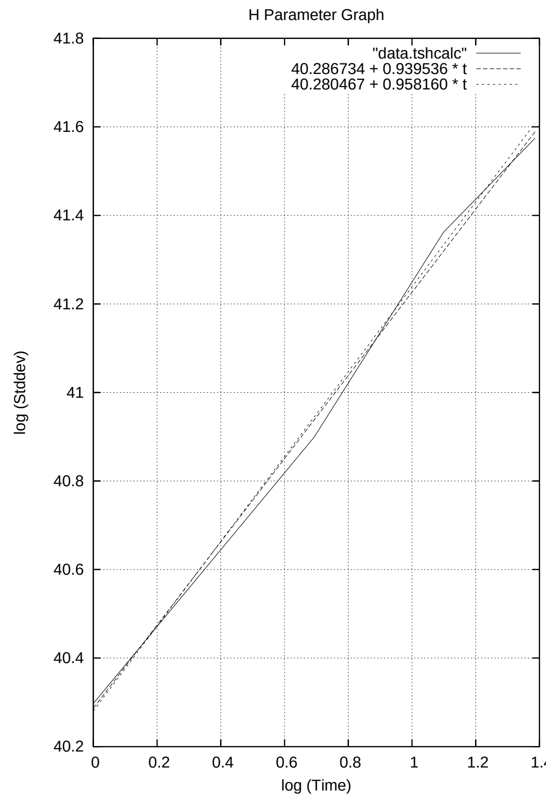

Figure C.406: Time Sampled Non-optimal Coin Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.393 The slope of the graph is the H parameter.

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.685 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.686}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.687}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.688}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[225].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.689}$$

$$\propto \frac{X(rt)}{r^{0.869484}} \tag{C.690}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.406, to provide a least squares approximation to the H parameter for the Time Sampled Non-optimal Coin Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.958160 for the near future, and 0.939536 for the distant future.

Figures C.405 and C.406 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.393. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.393, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.407 and C.408 was made using the -d option.

C.18.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.18.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.394. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.393. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Time Sampled Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Time Sampled Non-optimal Coin Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.393, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.179112 + 1)}{\ln(2)} = 0.237701 \tag{C.691}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.393, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.152372 + 1)}{\ln(2)} = 0.204607 \tag{C.692}$$

Note that if the mean is not constant in Figure C.393, this method will not provide accurate results.

[225] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

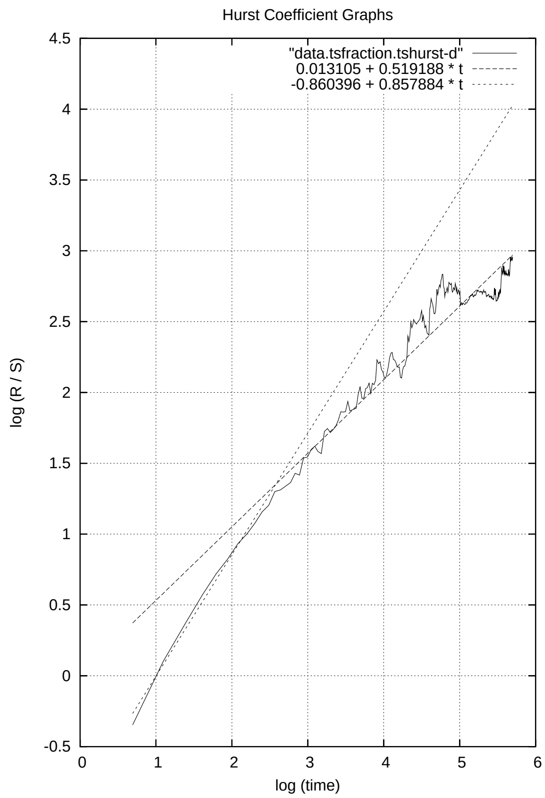

Figure C.407: Time Sampled Non-optimal Coin Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.392. The slope of the graph is the Hurst coefficient, and is 0.857884 for the near term, and 0.519188 for the far term.

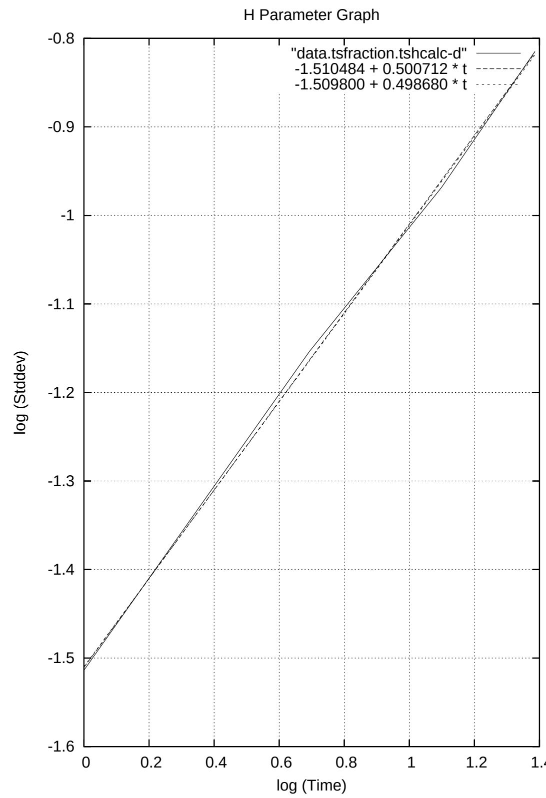

Figure C.408: Time Sampled Non-optimal Coin Tossing Game, traditional H parameter data for the time series data shown in Figure C.392 The slope of the graph is the H parameter, and is 0.498680 for the near term, and 0.500712 for the far term.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.392:

$$bits = 0.210487 \tag{C.693}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.392:

$$bits = 0.210768 \tag{C.694}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.18.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.210768t} \tag{C.695}$$

therefore:

$$C\left(p\right) = 0.210768 \tag{C.696}$$

and, *tsshannon* 0.210768 gives:

$$C\left(0.763464\right) = 0.210768 \tag{C.697}$$

therefore:

$$2^{C(0.763464)} = 2^{0.210768} \tag{C.698}$$
$$= 1.157304 \tag{C.699}$$
$$= 15.730410\% \tag{C.700}$$

and:

$$2p - 1 = (2 \cdot 0.763464) - 1 \tag{C.701}$$
$$= 0.526928 \tag{C.702}$$
$$= 52.692800\% \tag{C.703}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Time Sampled Non-optimal Coin Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 52.692800% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 47.307200% will be held in "reserve" with a 76.346400% chance of making twice the 52.692800% back, (and a 23.653600% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 15.730410%, or a doubling of its rate of revenue returns, (per tosses,) in 4.744553 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 52.692800% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 15.730410%, per tosses, on average.

Note that the metrics presented in this section are representative of the Time Sampled Non-optimal Coin Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 52.692800% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Time Sampled Non-optimal Coin Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 15.730410% per tosses.

As another simple example, a company re-invests 52.692800% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 52.692800% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 15.730410% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 52.692800% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 52.692800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 52.692800$ percent for the second product, implying that the company should diversify its product line[226]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 52.692800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Time Sampled Non-optimal Coin Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.526928}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.393, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 52.692800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 52.692800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.18.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Time Sampled Non-optimal Coin Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Time Sampled Non-optimal Coin Tossing Game time series is 0.179112, and 0.284303respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 2.215959.

If this value seems consistent number of companies in the Time Sampled Non-optimal Coin Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Time Sampled Non-optimal Coin Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.711608, which would be the value which should be used in Section C.18.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.18.5 is greater than the average Shannon probability for the companies participating in the Time Sampled

[226]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Non-optimal Coin Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.18.5. The maximum exploitability for the Time Sampled Non-optimal Coin Tossing Game is derived in Section C.18.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Time Sampled Non-optimal Coin Tossing Game is 0.711608, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.815002 in the Time Sampled Non-optimal Coin Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.704}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Time Sampled Non-optimal Coin Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.18.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.394. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.393. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Time Sampled Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.18.5, is derived from the Time Sampled Non-optimal Coin Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.18.9.

An additional exploitable strategy may be time itself. Equations C.681, C.685, and, C.683, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Time Sampled Non-optimal Coin Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.409, and, C.410 compare methods of approximation of the "forecastability" of rate of revenue returns in the Time Sampled Non-optimal Coin Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Time Sampled Non-optimal Coin Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[227]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

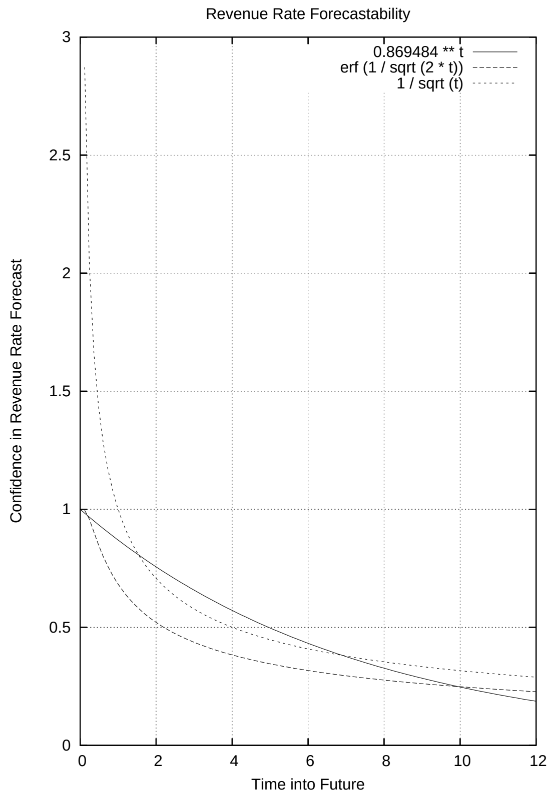

Figure C.409: Time Sampled Non-optimal Coin Tossing Game, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.869484^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

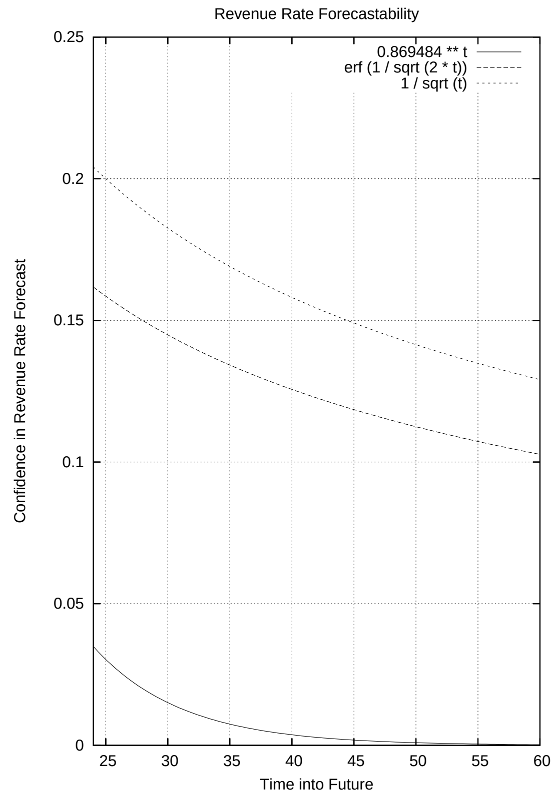

Figure C.410: Time Sampled Non-optimal Coin Tossing Game, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

As an interesting interpretation of the data presented in Figure C.409, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.683, $0.869484^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.681, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing

[227]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.18.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.18.9. Figure C.411 represents a constructional simulation of the time series data presented in Figure C.392. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.393. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.393 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.412 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.395.

C.18.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.18.3. One of the issues of analysis, as mentioned in Section C.18.7, is to determine the maximum Shannon probability for the time series presented in Figure C.392. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.413 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.392. Figure C.414 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.392. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.763464, as derived in Section C.18.5 to 0.806020. This process, essentially, extracts the random statistical data from the time series presented in Figure C.392, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.392, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Time Sampled Non-optimal Coin Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.805369, as compared with the predicted value from the program *tsshannonmax* of 0.806020.

C.18.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.394. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.393. These values will be used in a fixed increment Brownian fractal analysis of the Time Sampled Non-optimal Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.18.6 and D.18.7. As a subjective evaluation of the "quality" of the analysis of the Time Sampled Non-optimal Coin Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.392 from Figure C.393, and the Shannon probability as calculated by counting the total number of tossess that the Time Sampled Non-optimal Coin Tossing Game movement was positive, as presented in Section C.18.9:

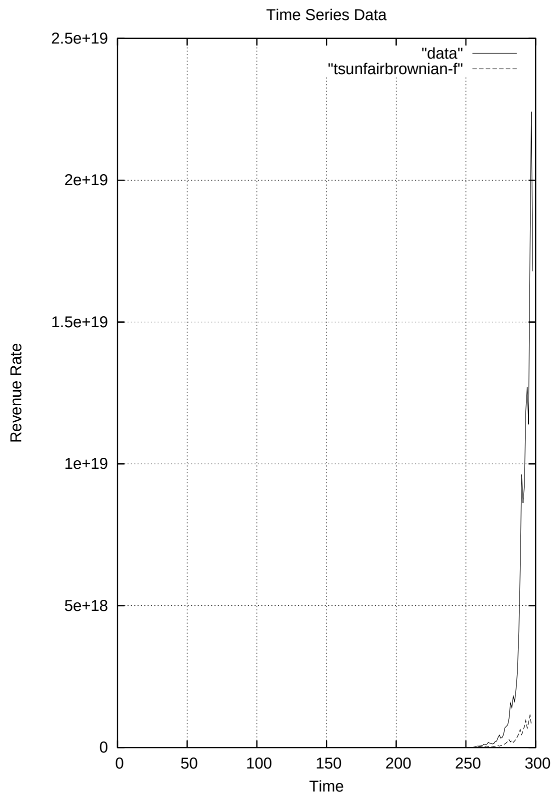

Figure C.411: Time Sampled Non-optimal Coin Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.284303. This data is superimposed on the data presented in Figure C.392.

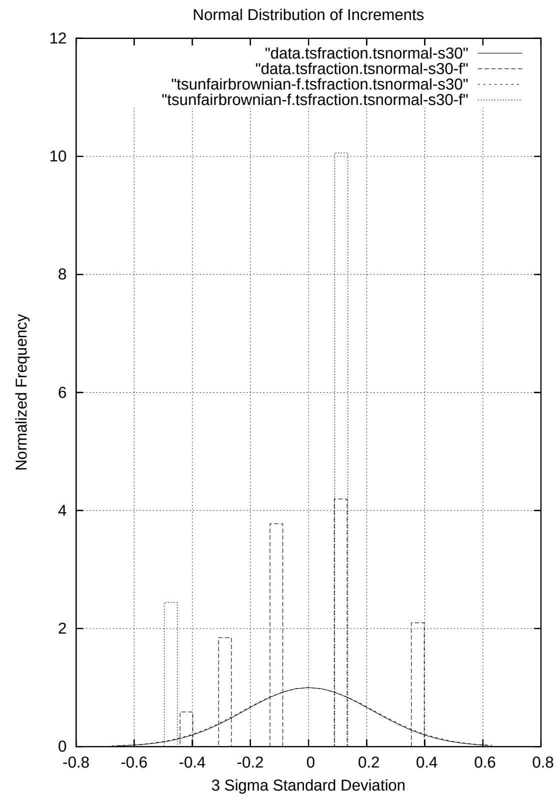

Figure C.412: Time Sampled Non-optimal Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.411, empirical and simulated. The empirical data has a mean of 0.179112, with a standard deviation of 0.221159. By comparison, the simulated data has a mean of 0.173262 with a standard deviation of 0.225788. This data is superimposed on the data presented in Figure C.395. The area under the four curves is identical.

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.705}$$

$$0.805369 \approx \frac{\frac{0.179112}{0.284303} + 1}{2} \tag{C.706}$$

$$0.805369 \approx 0.815002 \tag{C.707}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

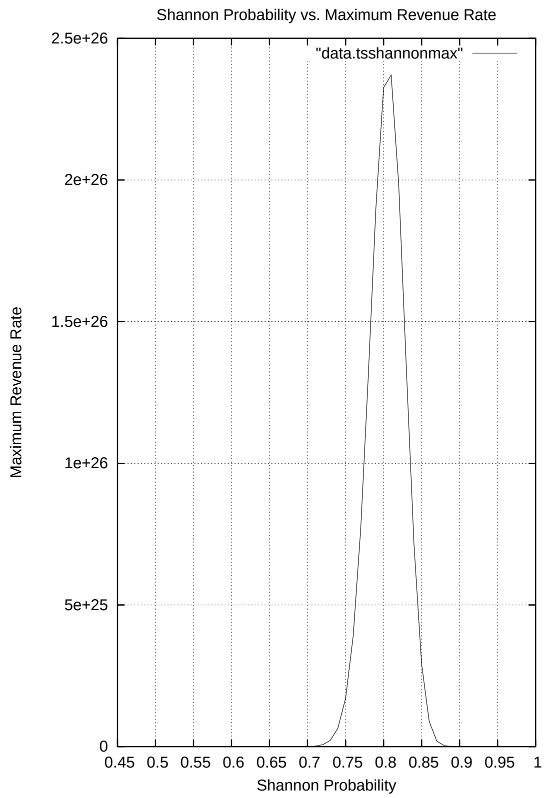

Figure C.413: Time Sampled Non-optimal Coin Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.806020.

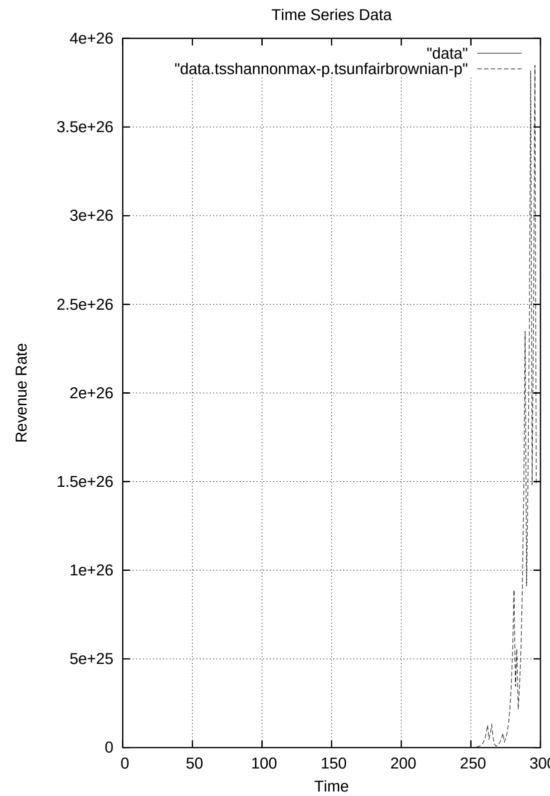

Figure C.414: Time Sampled Non-optimal Coin Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.806020, corresponding to a "wager" fraction of 0.612040.

$$0.805369 \approx 0.815002 \approx 0.806020 \tag{C.708}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.18.5, should be compared. The four methods used were the mean of Figure C.393, the constant in the least squares approximation to Figure C.393, the least squares exponential approximation to Figure C.392, and the logarithmic returns of Figure C.392, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.237701 \approx 0.204607 \approx 0.210487 \approx 0.210768 \tag{C.709}$$

It is implied in Section C.18.5, Subsection C.18.5 and in Section C.18.8 that, a Brownian motion with fixed increments fractal may "model" the Time Sampled Non-optimal Coin Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.710}$$

$$0.284303\,(2\cdot 0.805369-1) \approx \frac{0.221159\,(2\cdot 0.805369-1)}{2\sqrt{0.805369\,(1-0.805369)}} \tag{C.711}$$

$$0.284303\cdot 0.610738 \approx 0.221159\cdot 0.771297 \tag{C.712}$$

$$0.173635 \approx 0.170579 \tag{C.713}$$

and, equating to the mean:

$$0.179112 \approx 0.173635 \approx 0.170579 \tag{C.714}$$

where, as in Equation C.707 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.392 from Figure C.393, and the Shannon probability as calculated by counting the total number of tossess that the Time Sampled Non-optimal Coin Tossing Game movement was positive, as presented in Section C.18.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[228], where the absolute value is presented in Figure C.394, and the root mean square value is presented in Figure C.393:

$$0.233375 \approx 0.284303 \tag{C.715}$$

Note, that if the Time Sampled Non-optimal Coin Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.392 from Figure C.393 should be zero. It is 0.162645.

C.19 Time Sampled Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tssample[229].

The data in this section is presented in tabular form in Section D.19. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Time Sampled Coin Tossing Game. This is included for "theoretical" comparative purposes.

C.19.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.19.1. Figure C.415 is a graph of the time series data for the Time Sampled Coin Tossing Game.

[228]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[229]As a simulation model, the program *tscoin* was run to make a time series data file, with the following parameters:

```
tscoin -p 0.5447 1500 > data.1
```

to make a time series of 1500 elements, with a Shannon probability of 0.5447. Since $f = 2P - 1$, where the desired Shannon probability, P, is 0.6, f must be reduced by a factor of $\frac{1}{\sqrt{5}}$. Reducing f from 0.2 to 0.0894, and recalculating P to be 0.5447. Then the program *tssample* was run with the following parameters:

```
tssample -i 5 data.1 > data
```

to time sample every fifth element in the time series to make a time sampled time series. The data is by tosses.

Figure C.416 is a graph of the normalized increments of the time series data presented in Figure C.415. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.417 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.416. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[230].

Figure C.418 is the normalized histogram of the normalized increments of the time series data shown in Figure C.416. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.418.

Figure C.419 is the statistical estimate for the data presented in Figure C.416, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.557377$, as derived in Section C.19.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.420 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.416. In principle, if the distribution of the normalized increments presented in Figure C.418 is Gaussian in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.421 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.416. In principle, if the distribution of the normalized increments presented in Figure C.418 is an integrated Gaussian distribution in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43.

Figure C.422 is the range of values of the time series shown in Figure C.415. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.422 would be a square root function[231]. Figure C.423 is the deterministic map of the normalized increments of the time series data shown in Figure C.416. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

C.19.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

[230]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[231]Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.422 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

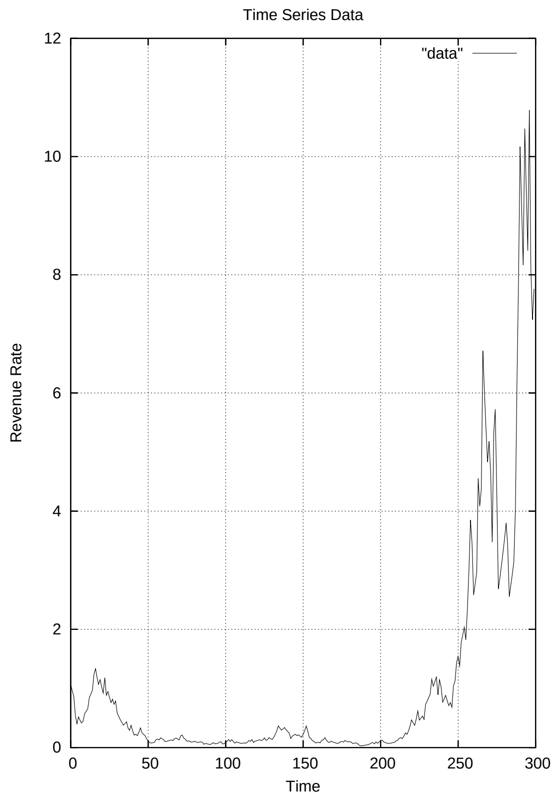

Figure C.415: Time Sampled Coin Tossing Game, time series data.

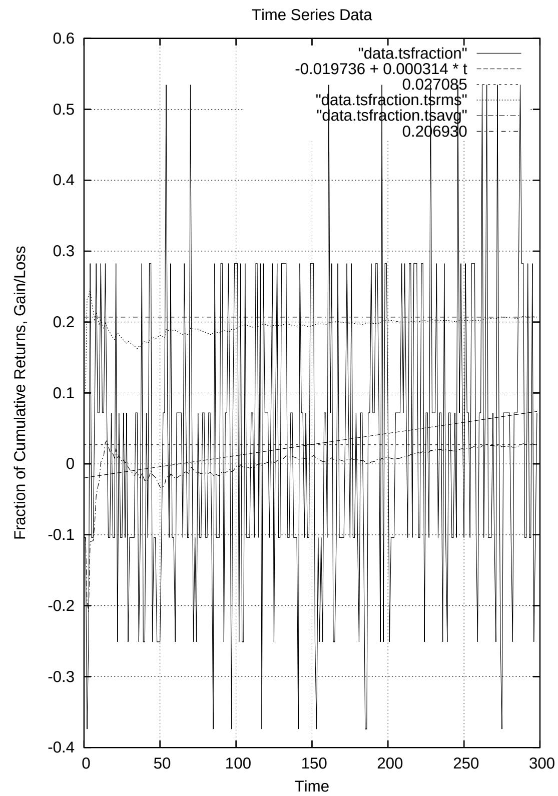

Figure C.416: Time Sampled Coin Tossing Game, normalized increments of the time series data presented in Figure C.415. The mean is 0.027085 with a standard deviation of 0.205494. The formula for the least squares approximation is $-0.019736 + 0.000314t$, and the root mean squared value is 0.206930. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000314, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

mean square of the instantaneous fraction of change[232]. Squaring this value is the average of the instantaneous fraction

[232] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

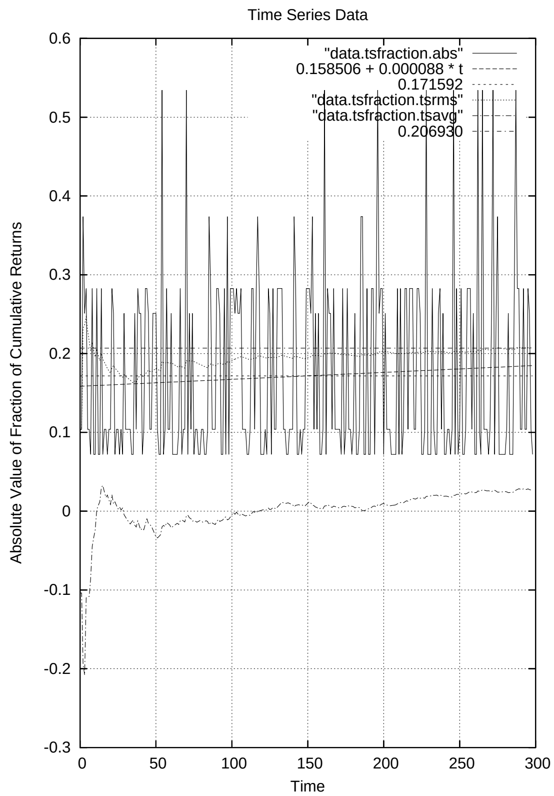

Figure C.417: Time Sampled Coin Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.416. The mean is 0.171592 with a standard deviation of 0.115850. The formula for the least squares approximation is $0.158506 + 0.000088t$, and the root mean square value, from Figure C.416, is 0.206930. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.416, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

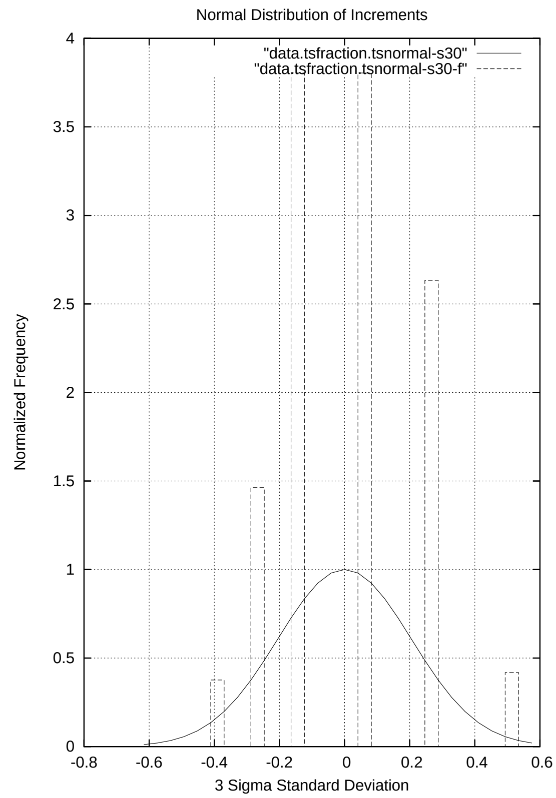

Figure C.418: Time Sampled Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.416. The data has a mean of 0.027085, with a standard deviation of 0.205494. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 46.304000, with a critical value of 42.557000.

of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.424 is the instantaneous value of the root mean square of the normalized increments for the Time Sampled Coin Tossing Game, and Figure C.425 is the instantaneous Shannon probability for the normalized increments.

```
For  a mean  of 0.026994,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.002699,   15899  samples  would  be  required.
     (With  300  samples,   the  estimated   error  is 0.019651  = 72.797624   percent.)
For  a standard   deviation   of 0.206930,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.020693,   136  samples  would  be  required.
     (With  300  samples,   the  estimated   error  is 0.013896  = 6.715087   percent.)
```

Figure C.419: Time Sampled Coin Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.416. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.416.

C.19.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.19.4. Figure C.426 is a graph of the logistic function estimates of the time series data for the Time Sampled Coin Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[233]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.426 is a graph of the logistic function for the time series data presented in Figure C.415. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.416. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.416. Figure C.427 is the same graph, but with the time scale expanded by a factor of two.

C.19.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.19.5. Figure C.428 is a graph of the Hurst coefficient data time series data shown in Figure C.415. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.429 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.416. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.428 implies that the variance of the rate of revenue returns, (per tosses,) in the Time Sampled Coin Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[234], t, $p(t) = erf(1/\sqrt{2t})$ which

[233]For example, in Figures C.426 and C.427, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.19.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

[234]It can be shown that the number of expected market “high” and “low” transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's “high” or “low” exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's “high” or “low” exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the “zero free” run-lengths of a market being “high” or “low,” over a long time, would have a $1/t^{3/2}$

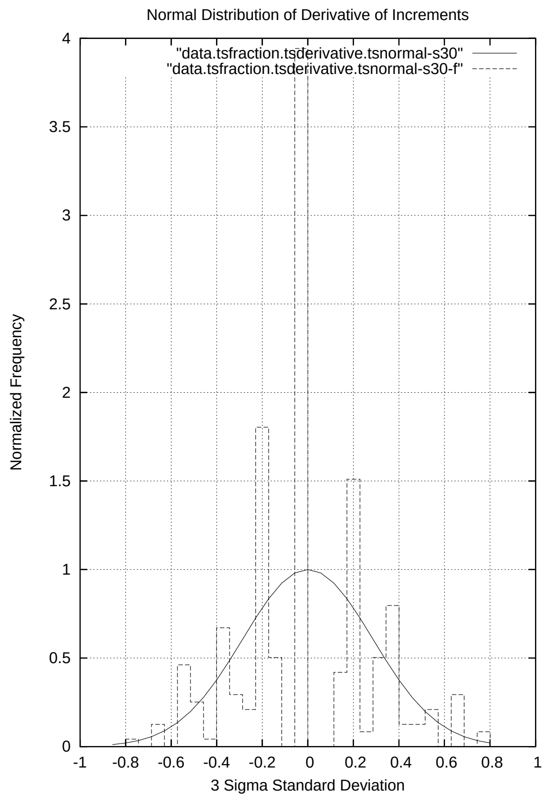

Figure C.420: Time Sampled Coin Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.416.

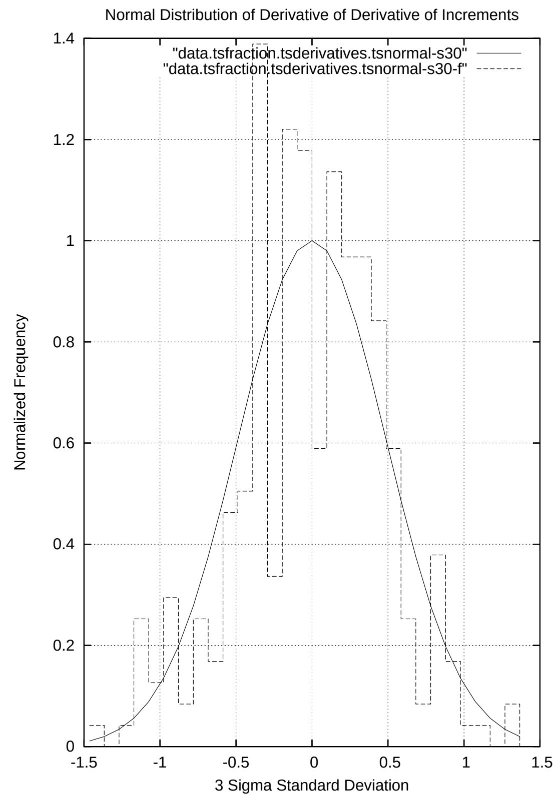

Figure C.421: Time Sampled Coin Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.416.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.432, and, C.433 compare methods of approximation of the “forecastability” of the rate of revenue returns in the Time Sampled Coin Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][235]. This seems to be a quantitative statement concerning “windows of opportunity” in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.428, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.835189, so that:

$$V\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{2 \cdot H} \tag{C.716}$$

$$V\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{2 \cdot 0.835189} \tag{C.717}$$

characteristic.)

[235]The author is not comfortable with Peters’ interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on “white noise” which, by definition, never has any correlations, the short term Hurst coefficient, and thus the “forecastability,” is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

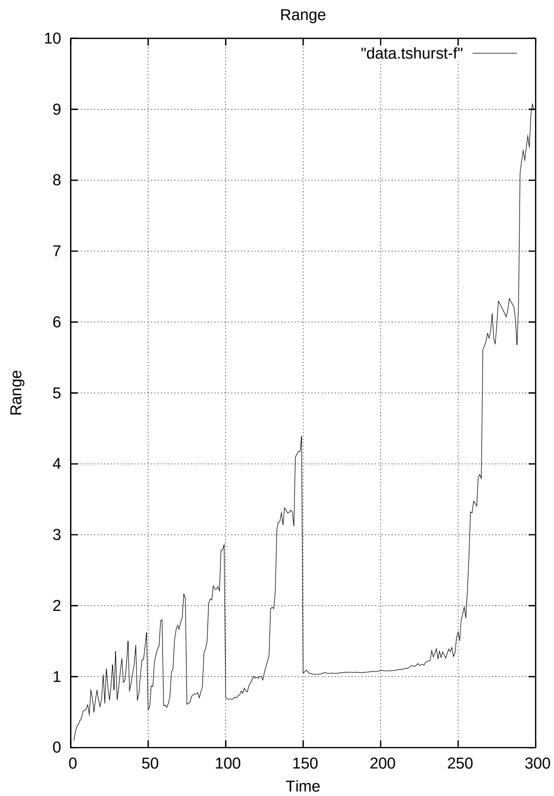

Figure C.422: Time Sampled Coin Tossing Game, range of the time series data shown in Figure C.415.

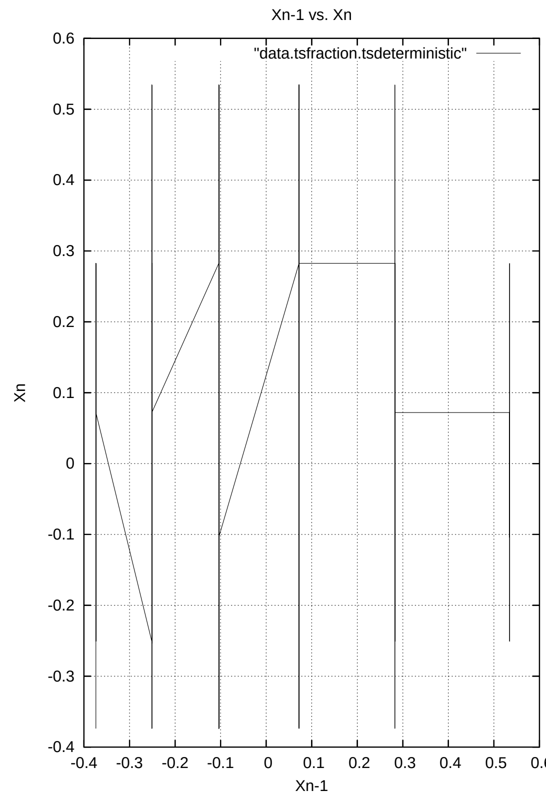

Figure C.423: Time Sampled Coin Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.416.

$$\propto \quad (t_2 - t_1)^{1.670378} \tag{C.718}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[236]. A Hurst coefficient of 0.835189, (for the near future, and 0.604532 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is 83.518900% [Pet91, pp. 66] for the near future, and 0.604532 for the distant future. Likewise, there is a 83.518900% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in

[236] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.19.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Time Sampled Coin Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

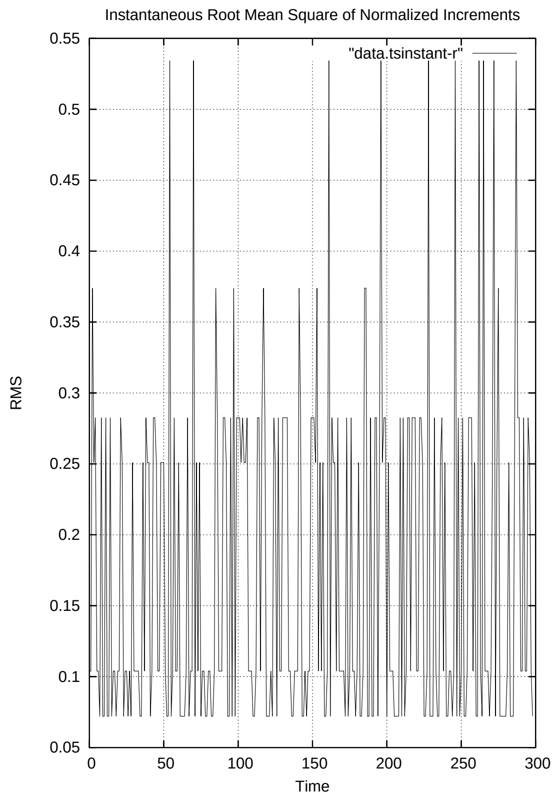

Figure C.424: Time Sampled Coin Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.415.

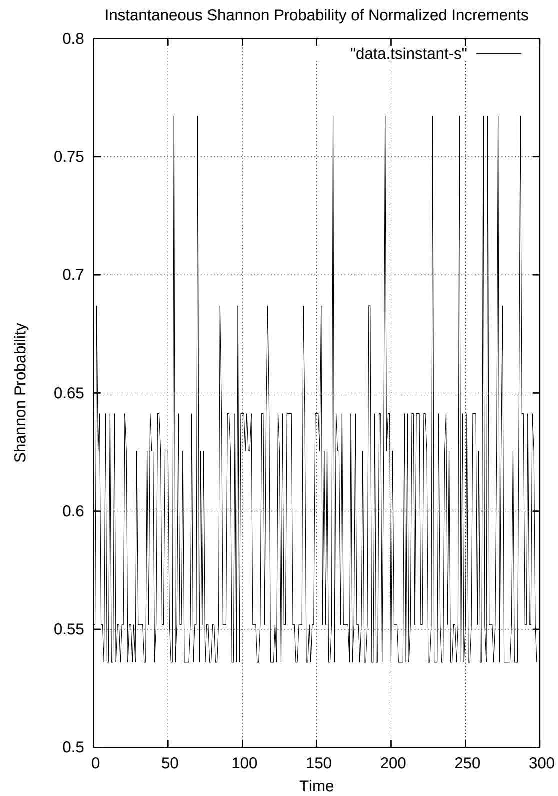

Figure C.425: Time Sampled Coin Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.415.

a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 83.518900% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Time Sampled Coin Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.719}$$

$$= 0.835189^n \tag{C.720}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.416, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.027085 \cdot 100$ percent, on the average, with a standard deviation of $0.205494 \cdot 100$ percent, and a root mean square error value of $0.206930 \cdot 100$ percent—small values for such a simple

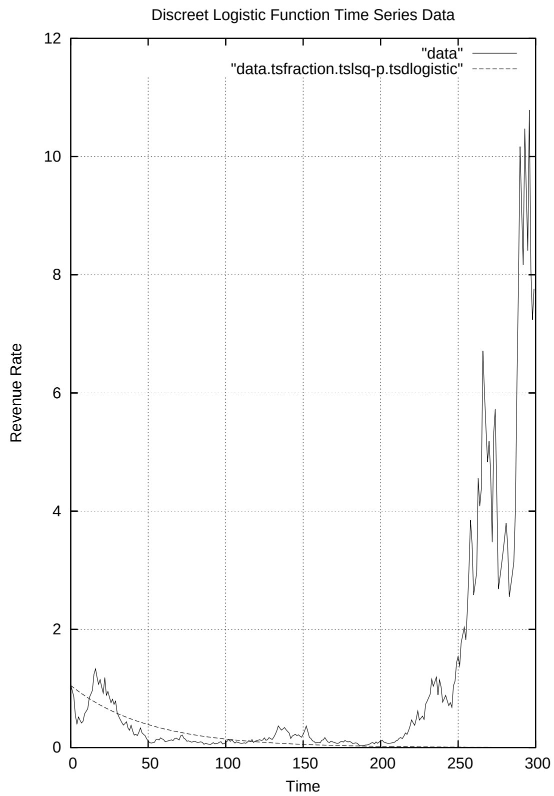

Figure C.426: Time Sampled Coin Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.416 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

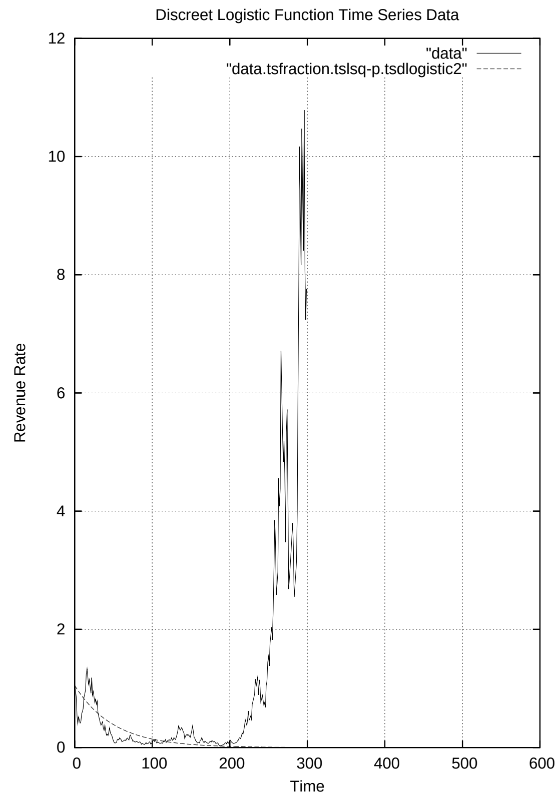

Figure C.427: Time Sampled Coin Tossing Game, logistic function estimates of Figure C.426 with the time scale expanded by a factor of two.

forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.721}$$

$$\propto (t_2 - t_1)^{0.835189} \tag{C.722}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.722 reduces to, [Sch91, pp. 129]:

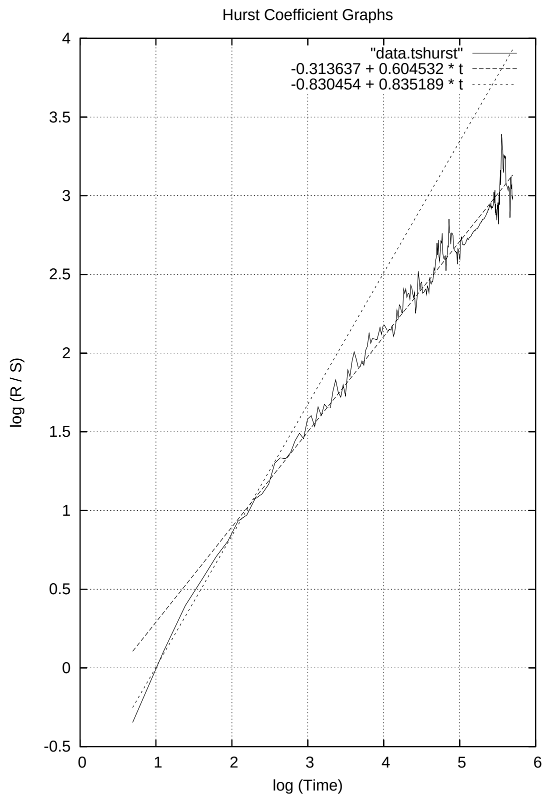

Figure C.428: Time Sampled Coin Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.416. The slope of the graph is the Hurst coefficient.

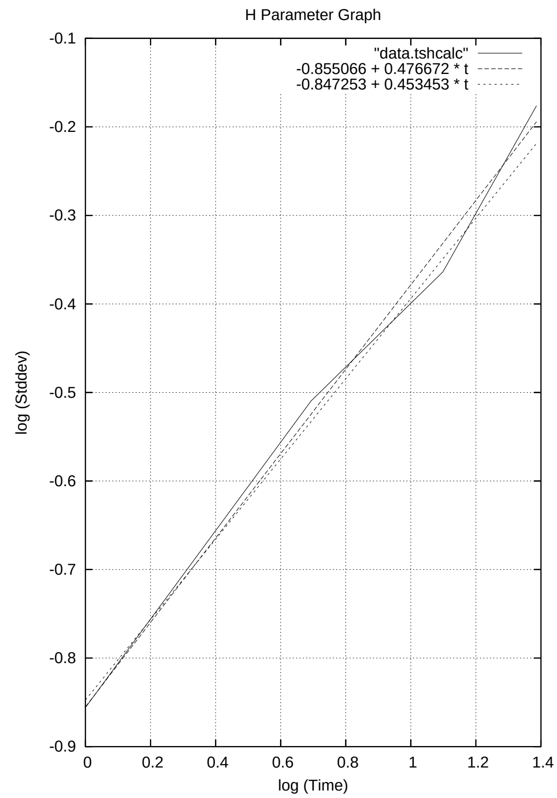

Figure C.429: Time Sampled Coin Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.416 The slope of the graph is the H parameter.

$$
\begin{aligned}
R\,(t_2 - t_1) &\propto (t_2 - t_1)^{\frac{1}{2}} && \text{(C.723)} \\
&\propto \sqrt{(t_2 - t_1)} && \text{(C.724)}
\end{aligned}
$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \qquad \text{(C.725)}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[237].

[237]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.726}$$
$$\propto \frac{X(rt)}{r^{0.835189}} \tag{C.727}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.429, to provide a least squares approximation to the H parameter for the Time Sampled Coin Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.453453 for the near future, and 0.476672 for the distant future.

Figures C.428 and C.429 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.416. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.416, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.430 and C.431 was made using the -d option.

C.19.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.19.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.417. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.416. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Time Sampled Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Time Sampled Coin Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.416, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.027085 + 1)}{\ln(2)} = 0.038556 \tag{C.728}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.416, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(-0.019736 + 1)}{\ln(2)} = -0.028758 \tag{C.729}$$

Note that if the mean is not constant in Figure C.416, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.415:

$$bits = 0.011235 \tag{C.730}$$

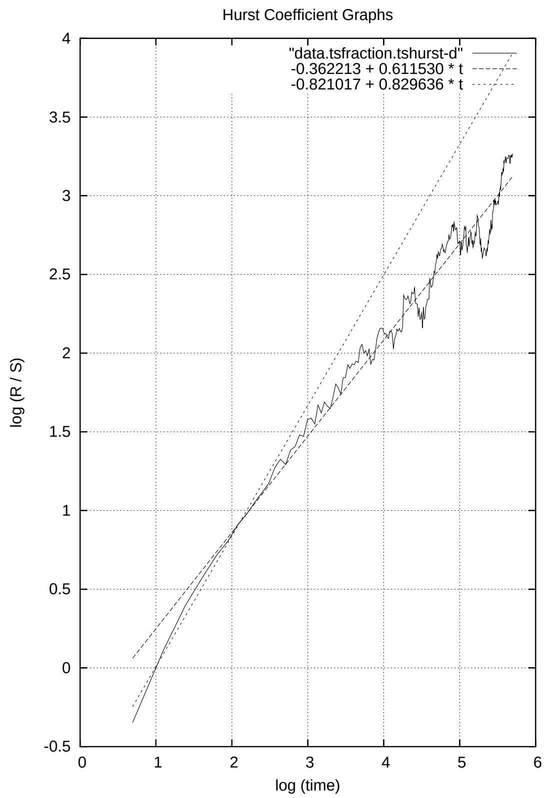

Figure C.430: Time Sampled Coin Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.415. The slope of the graph is the Hurst coefficient, and is 0.829636 for the near term, and 0.611530 for the far term.

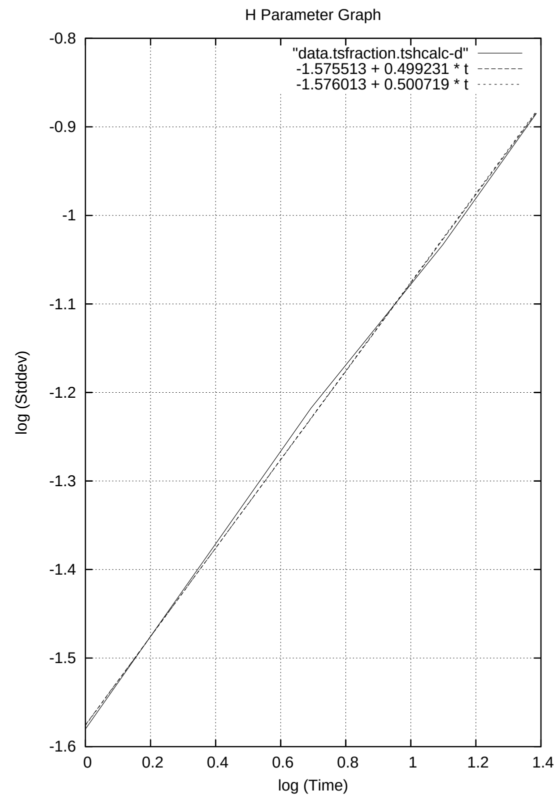

Figure C.431: Time Sampled Coin Tossing Game, traditional H parameter data for the time series data shown in Figure C.415 The slope of the graph is the H parameter, and is 0.500719 for the near term, and 0.499231 for the far term.

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.415:

$$bits = 0.009520 \tag{C.731}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.19.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.009520t} \tag{C.732}$$

therefore:

$$C(p) = 0.009520 \tag{C.733}$$

and, *tsshannon* 0.009520 gives:

$$C\,(0.557377) = 0.009520 \tag{C.734}$$

therefore:

$$\begin{aligned} 2^{C(0.557377)} &= 2^{0.009520} && \text{(C.735)} \\ &= 1.006621 && \text{(C.736)} \\ &= 0.662058\% && \text{(C.737)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.557377) - 1 && \text{(C.738)} \\ &= 0.114754 && \text{(C.739)} \\ &= 11.475400\% && \text{(C.740)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the Time Sampled Coin Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 11.475400% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 88.524600% will be held in “reserve” with a 55.737700% chance of making twice the 11.475400% back, (and a 44.262300% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 0.662058%, or a doubling of its rate of revenue returns, (per tosses,) in 105.042017 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 11.475400% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.662058%, per tosses, on average.

Note that the metrics presented in this section are representative of the Time Sampled Coin Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 11.475400% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Time Sampled Coin Tossing Game’s environment, the company’s rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company’s revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.662058% per tosses.

As another simple example, a company re-invests 11.475400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 11.475400% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.662058% per tosses.

As an example of “product portfolio” management, suppose a company re-invests 11.475400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 11.475400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 11.475400$ percent for the second product, implying that the company should diversify its product line[238]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 11.475400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Time Sampled Coin Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.114754}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.416, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 11.475400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 11.475400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.19.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Time Sampled Coin Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Time Sampled Coin Tossing Game time series is 0.027085, and 0.206930respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.632531.

If this value seems consistent number of companies in the Time Sampled Coin Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Time Sampled Coin Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.582288, which would be the value which should be used in Section C.19.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.19.5 is greater than the average Shannon probability for the companies participating in the Time Sampled Coin Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.19.5. The maximum exploitability for the Time Sampled Coin Tossing Game is derived in Section C.19.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Time Sampled Coin Tossing Game is 0.582288, with several alternative solutions listed in the previous paragraph. However, these should be

[238]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

contrasted to the Shannon probability that maximizes a company's P&L which is 0.565445 in the Time Sampled Coin Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.741}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Time Sampled Coin Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.19.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.417. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.416. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Time Sampled Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.19.5, is derived from the Time Sampled Coin Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.19.9.

An additional exploitable strategy may be time itself. Equations C.718, C.722, and, C.720, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Time Sampled Coin Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.432, and, C.433 compare methods of approximation of the "forecastability" of rate of revenue returns in the Time Sampled Coin Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Time Sampled Coin Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[239]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.432, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory

[239]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

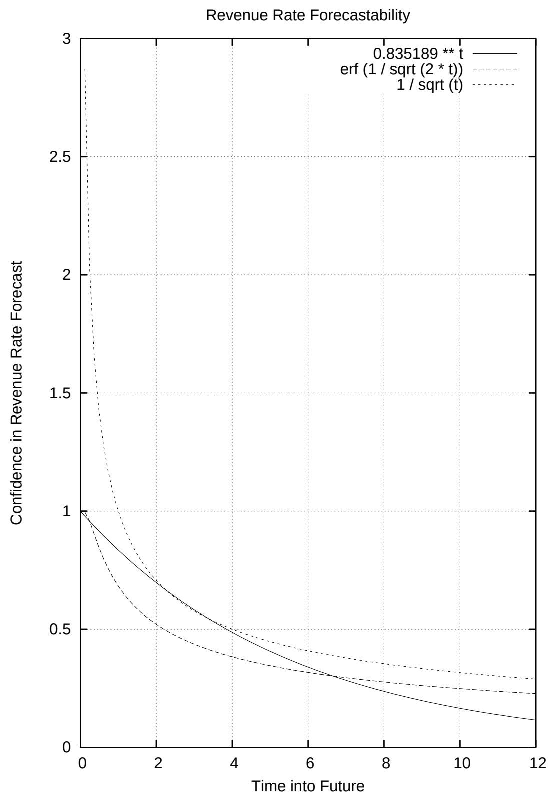

Figure C.432: Time Sampled Coin Tossing Game, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.835189^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

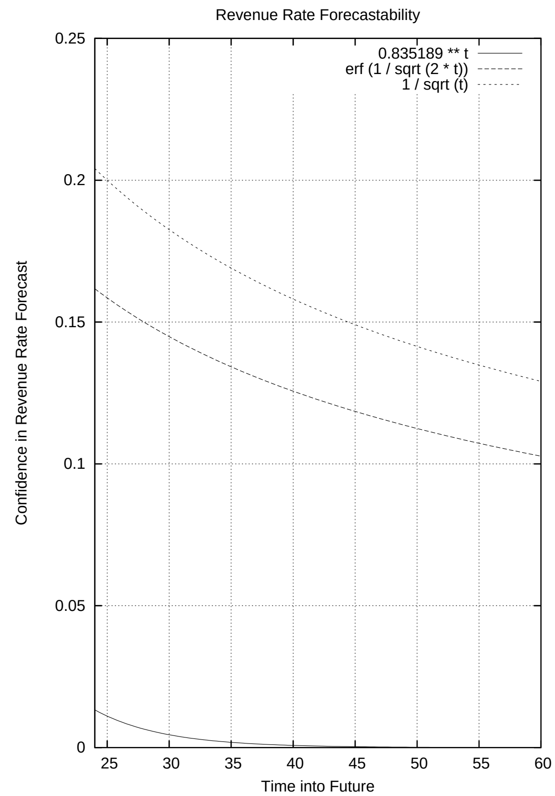

Figure C.433: Time Sampled Coin Tossing Game, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

levels that do not exceed, from Equation C.720, $0.835189^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.718, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.19.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.19.9. Figure C.434 represents a constructional simulation of the time series data presented in Figure C.415. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented

in Figure C.416. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.416 represent a relatively complex process, that may not be “modeled” with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.435 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.418.

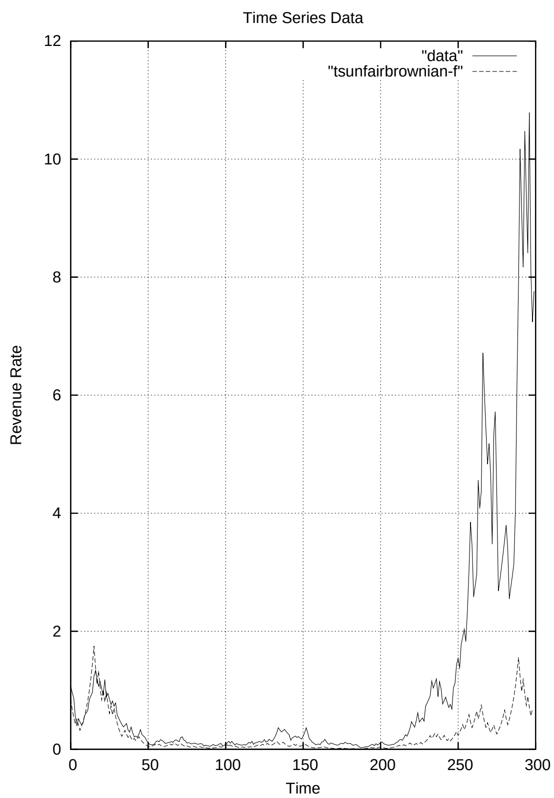

Figure C.434: Time Sampled Coin Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.206930. This data is superimposed on the data presented in Figure C.415.

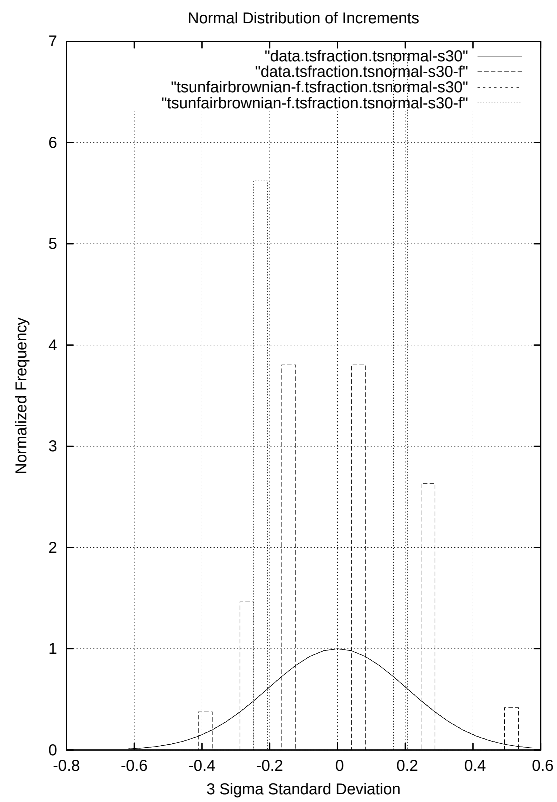

Figure C.435: Time Sampled Coin Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.434, empirical and simulated. The empirical data has a mean of 0.027085, with a standard deviation of 0.205494. By comparison, the simulated data has a mean of 0.020832 with a standard deviation of 0.206224. This data is superimposed on the data presented in Figure C.418. The area under the four curves is identical.

C.19.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.19.3. One of the issues of analysis, as mentioned in Section C.19.7, is to determine the maximum Shannon probability for the time series presented in Figure C.415. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.436 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.415. Figure C.437 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.415. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.557377, as derived in Section C.19.5 to 0.550000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.415, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.415, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Time Sampled Coin Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.548495, as compared with the predicted value from the program *tsshannonmax* of 0.550000.

C.19.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.417. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.416. These values will be used in a fixed increment Brownian fractal analysis of the Time Sampled Coin Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.19.6 and D.19.7. As a subjective evaluation of the "quality" of the analysis of the Time Sampled Coin Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.415 from Figure C.416, and the Shannon probability as calculated by counting the total number of tossess that the Time Sampled Coin Tossing Game movement was positive, as presented in Section C.19.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.742}$$

$$0.548495 \approx \frac{\frac{0.027085}{0.206930} + 1}{2} \tag{C.743}$$

$$0.548495 \approx 0.565445 \tag{C.744}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.548495 \approx 0.565445 \approx 0.550000 \tag{C.745}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.19.5, should be compared. The four methods used were the mean of Figure C.416, the constant in the least squares approximation to Figure C.416, the least squares exponential approximation to Figure C.415, and the logarithmic returns of Figure C.415,

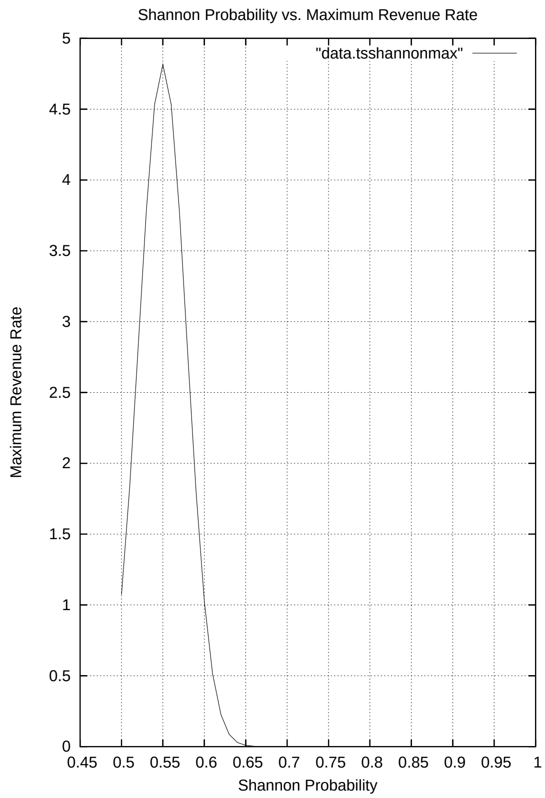

Figure C.436: Time Sampled Coin Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.550000.

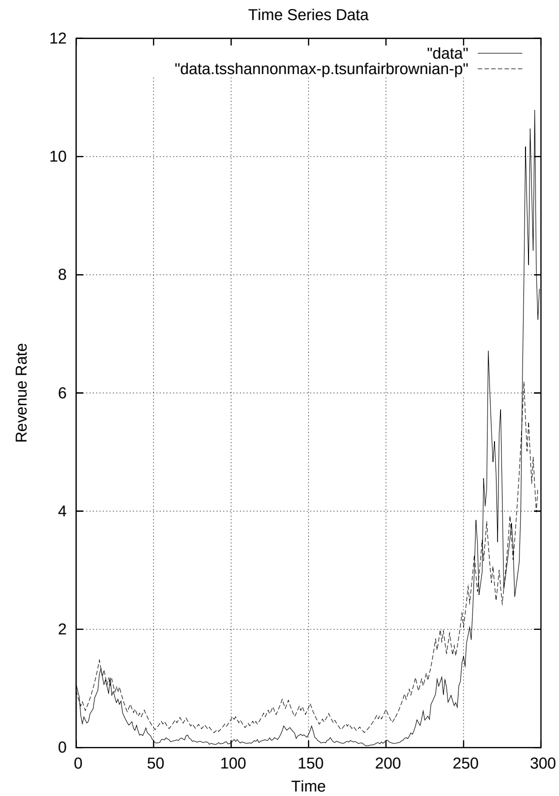

Figure C.437: Time Sampled Coin Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.550000, corresponding to a "wager" fraction of 0.100000.

derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.038556 \approx -0.028758 \approx 0.011235 \approx 0.009520 \tag{C.746}$$

It is implied in Section C.19.5, Subsection C.19.5 and in Section C.19.8 that, a Brownian motion with fixed increments fractal may "model" the Time Sampled Coin Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.747}$$

$$0.206930\,(2 \cdot 0.548495 - 1) \approx \frac{0.205494\,(2 \cdot 0.548495 - 1)}{2\sqrt{0.548495\,(1-0.548495)}} \tag{C.748}$$

$$0.206930 \cdot 0.096990 \approx 0.205494 \cdot 0.097449 \tag{C.749}$$

$$0.020070 \quad \approx \quad 0.020025 \tag{C.750}$$

and, equating to the mean:

$$0.027085 \approx 0.020070 \approx 0.020025 \tag{C.751}$$

where, as in Equation C.744 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.415 from Figure C.416, and the Shannon probability as calculated by counting the total number of tossess that the Time Sampled Coin Tossing Game movement was positive, as presented in Section C.19.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[240], where the absolute value is presented in Figure C.417, and the root mean square value is presented in Figure C.416:

$$0.171592 \approx 0.206930 \tag{C.752}$$

Note, that if the Time Sampled Coin Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.415 from Figure C.416 should be zero. It is 0.115850.

C.20 Simulated Shannon Probability of 0.6 Game

For the analysis, the data was in the directory ../markets/tsunfairbrownian.exponential[241].

[240]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[241]As a simulation model, the program *tsunfairbrownian* was run on the time series, "data.original," constructed with a text editor, by replicating the following fragment 1000 times:

```
0.2
-0.2
0.2
-0.2
0.2
```

to produce a time series data file of 5000 records that "oscillates," on a period of 5, with a Shannon probability of 3 / 5 = 0.6. A data file was made by running:

```
tsunfairbrownian -d -i 1.0 -f 0.2 data.original > data
```

since $f = 2P - 1$, where $P = 0.6$, $f = 0.2$. An i of 1.0 was used simulate an exponential beginning with e^0. After running the *tsunfairbrownian* program to make the data time series, and the program *tsfraction*, the sequence will be:

```
0.2
-0.2
0.2
-0.2
0.2
```

Note that there are $3 +$ 0.2's for every $2 -$ .2s in 5 time unitss, for an average of $+0.2/5 = 0.04$. The rationale for the numbers, $+0.2$ and -0.2, is that it is the optimum for a Shannon probability of $P = 0.6$, since $0.2 = 2P - 1$, (which also equals $P - (1 - P)$,) where $2 \cdot 0.6 - 1 = 0.2$, which is the optimal amount of the cumulative returns to wager with an unfair coin that has a probability of 0.6 of a win, ie., 3 out of 5. If the $n - 1$'th value in the time series is subtracted from the n'th value, and the value of this subtraction is then divided by the $n - 1$'th value, then this quotient should be either $+0.2$ or -0.2 depending on the whether the wager was won or lost.
Under this scenario, $P = 0.6$, and the returns are:

The data in this section is presented in tabular form in Section D.20. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Simulated Shannon Probability of 0.6 Game. This is included for "theoretical" comparative purposes.

C.20.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.20.1. Figure C.438 is a graph of the time series data for the Simulated Shannon Probability of 0.6 Game.

Figure C.439 is a graph of the normalized increments of the time series data presented in Figure C.438. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.440 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.439. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[242].

Figure C.441 is the normalized histogram of the normalized increments of the time series data shown in Figure C.439. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.441.

Figure C.442 is the statistical estimate for the data presented in Figure C.439, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.599910$, as derived in Section C.20.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.443 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.439. In principle, if the distribution of the normalized increments presented in Figure C.441 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.444 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.439. In principle, if the distribution of the normalized increments presented in Figure C.441 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.445 is the range of values of the time series shown in Figure C.438. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.445

$$2^{0.029049406} = e^{0.020135514} \tag{C.753}$$

which can be verified with the program *tsshannon,* and are consistent with [Sch91, pp. 128].

Using tsunfairbrownian -f 0.2 will construct an exponential data time series that is known to be optimum, ie., a Shannon probability of 0.6 with an optimal wager fraction of 0.2, with an "approximate" Brownian motion noise content-albeit not random. For an analytical insight, see appendix A, Section 2.3. It is useful for evaluating methodologies. The data is by time units.

[242]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

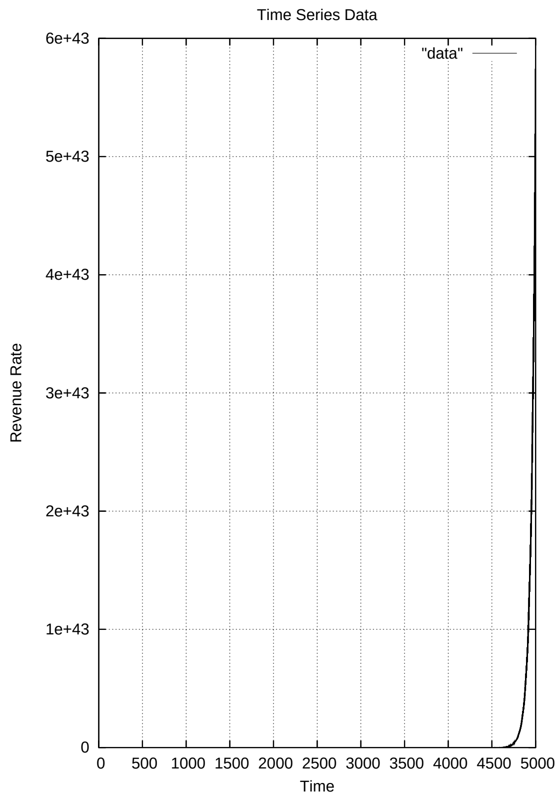

Figure C.438: Simulated Shannon Probability of 0.6 Game, time series data.

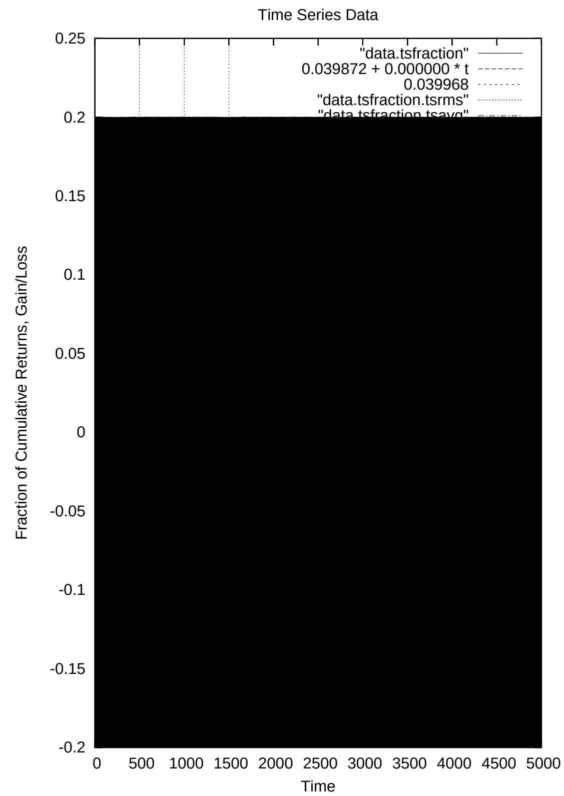

Figure C.439: Simulated Shannon Probability of 0.6 Game, normalized increments of the time series data presented in Figure C.438. The mean is 0.039968 with a standard deviation of 0.195985. The formula for the least squares approximation is $0.039872 + 0.000000t$, and the root mean squared value is 0.200000. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000000, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

would be a square root function[243]. Figure C.446 is the deterministic map of the normalized increments of the time series data shown in Figure C.439. The deterministic map is useful for determining if a time series was created by a

[243] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.445 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

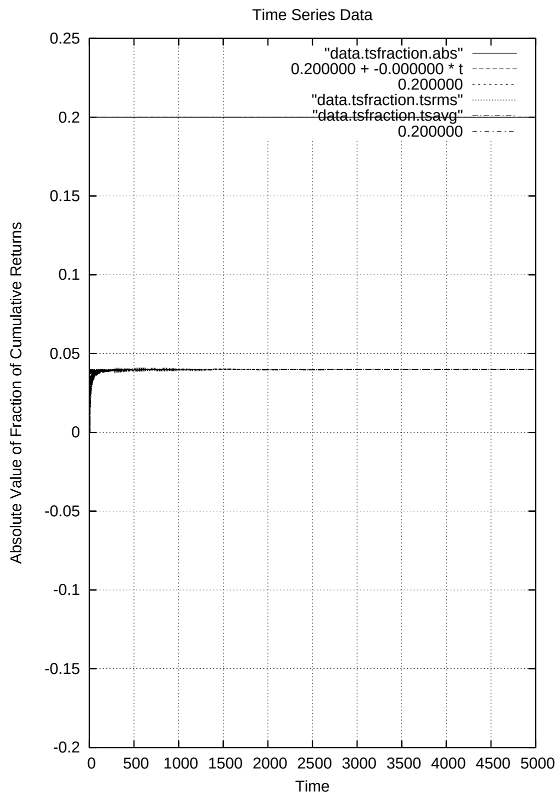

Figure C.440: Simulated Shannon Probability of 0.6 Game, absolute value of the normalized increments of the time series data presented in Figure C.439. The mean is 0.200000 with a standard deviation of 0.000000. The formula for the least squares approximation is $0.200000 + 0.000000t$, and the root mean square value, from Figure C.439, is 0.200000. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments presented in Figure C.439, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

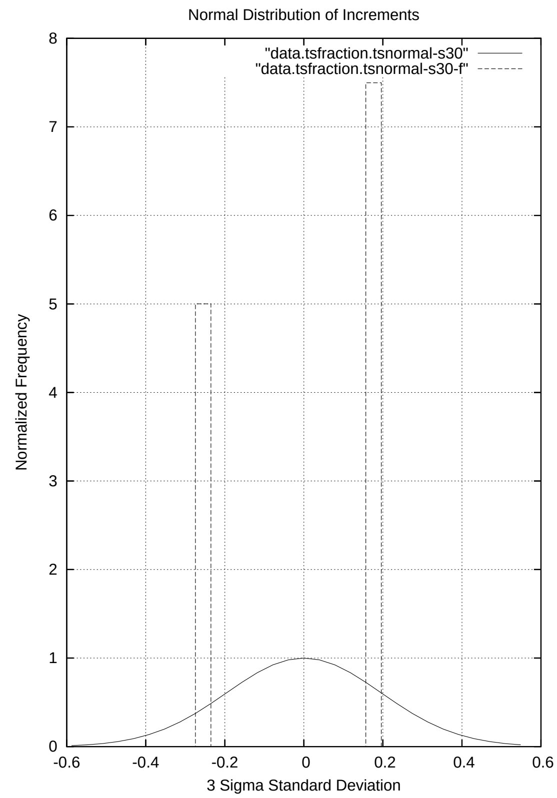

Figure C.441: Simulated Shannon Probability of 0.6 Game, normalized histogram of the normalized increments of the time series data shown in Figure C.439. The data has a mean of 0.039968, with a standard deviation of 0.195985. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 131.562000, with a critical value of 42.557000.

deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

```
For  a mean  of 0.039960,   with  a confidence   level  of  0.900000
     that  the  error  did  not  exceed  0.003996,   6778  samples  would  be required.
     (With  5000  samples,   the  estimated   error  is 0.004652   = 11.642514   percent.)
For  a standard  deviation   of 0.200000,   with  a confidence   level  of  0.900000
     that  the  error  did  not  exceed  0.020000,   136  samples  would  be required.
     (With  5000  samples,   the  estimated   error  is 0.003290   = 1.644854   percent.)
```

Figure C.442: Simulated Shannon Probability of 0.6 Game, statistical estimates of the normalized increments of the time series shown in Figure C.439. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.439.

C.20.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[244]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.447 is the instantaneous value of the root mean square of the normalized increments for the Simulated Shannon Probability of 0.6 Game, and Figure C.448 is the instantaneous Shannon probability for the normalized increments.

C.20.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.20.4. Figure C.449 is a graph of the logistic function estimates of the time series data for the Simulated Shannon Probability of 0.6 Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[245]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.449 is a graph of the logistic function for the time series data presented in Figure C.438. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.439. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.439. Figure C.450 is the same graph, but with the time scale expanded by a factor of two.

[244]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[245]For example, in Figures C.449 and C.450, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.20.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

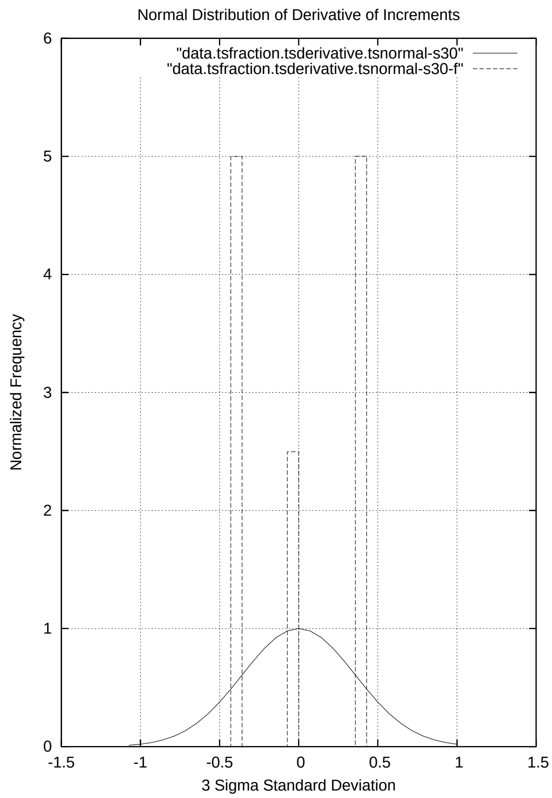

Figure C.443: Simulated Shannon Probability of 0.6 Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.439.

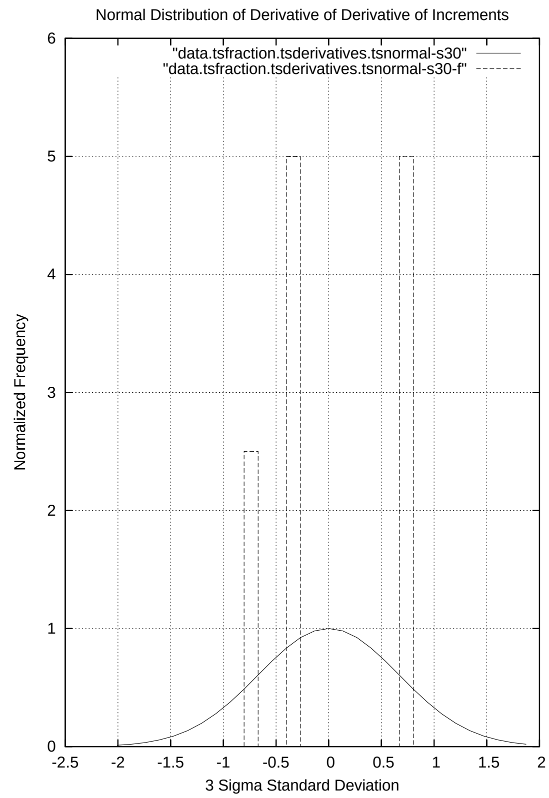

Figure C.444: Simulated Shannon Probability of 0.6 Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.439.

C.20.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.20.5. Figure C.451 is a graph of the Hurst coefficient data time series data shown in Figure C.438. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.452 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.439. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.451 implies that the variance of the rate of revenue returns, (per time units,) in the Simulated Shannon Probability of 0.6 Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[246], t, $p(t) = erf(1/\sqrt{2t})$ which

[246]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional

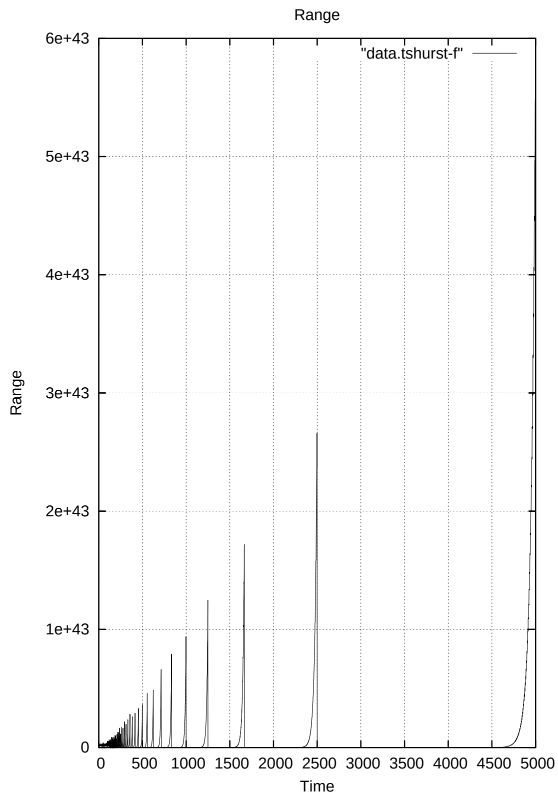

Figure C.445: Simulated Shannon Probability of 0.6 Game, range of the time series data shown in Figure C.438.

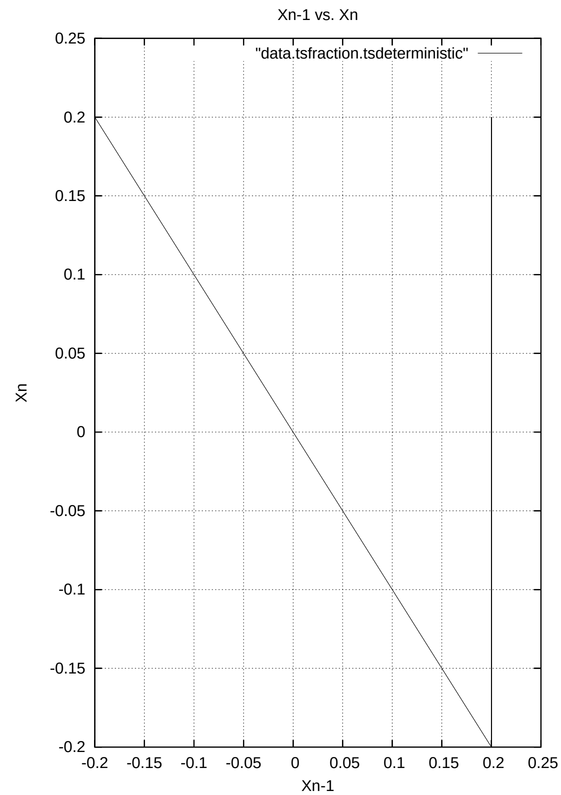

Figure C.446: Simulated Shannon Probability of 0.6 Game, deterministic map of the normalized increments of the time series data shown in Figure C.439.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.455, and, C.456 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Simulated Shannon Probability of 0.6 Game for the near term and far term, respectively [Pet91, pp. 83-84][247]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per time units.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.451, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.539904, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.754}$$

to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[247] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

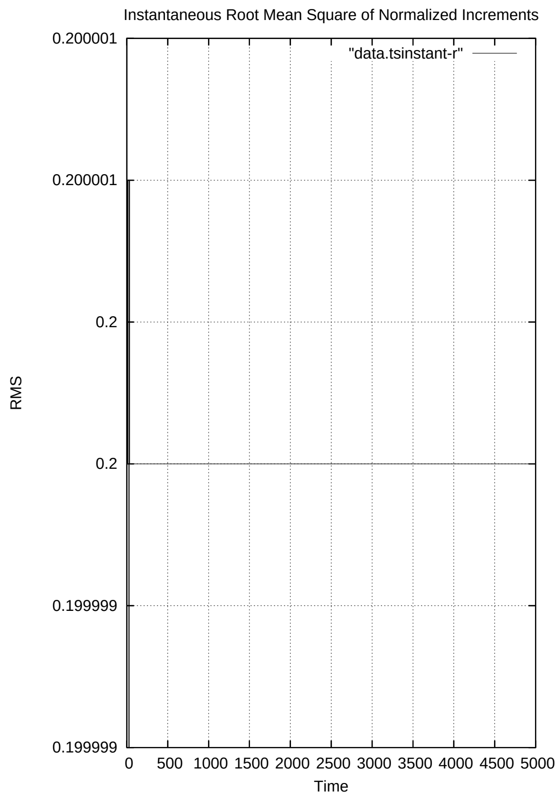

Figure C.447: Simulated Shannon Probability of 0.6 Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.438.

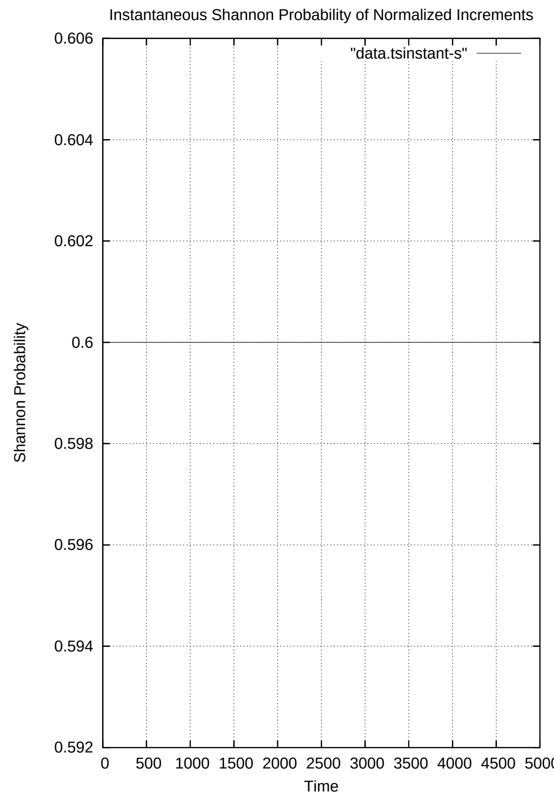

Figure C.448: Simulated Shannon Probability of 0.6 Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.438.

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.539904} \tag{C.755}$$

$$\propto (t_2 - t_1)^{1.079808} \tag{C.756}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per time units,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[248]. A Hurst coefficient of 0.539904, (for the near future, and 0.668253 for the distant future.) implies that

[248] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.20.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Simulated Shannon Probability of 0.6 Game. See

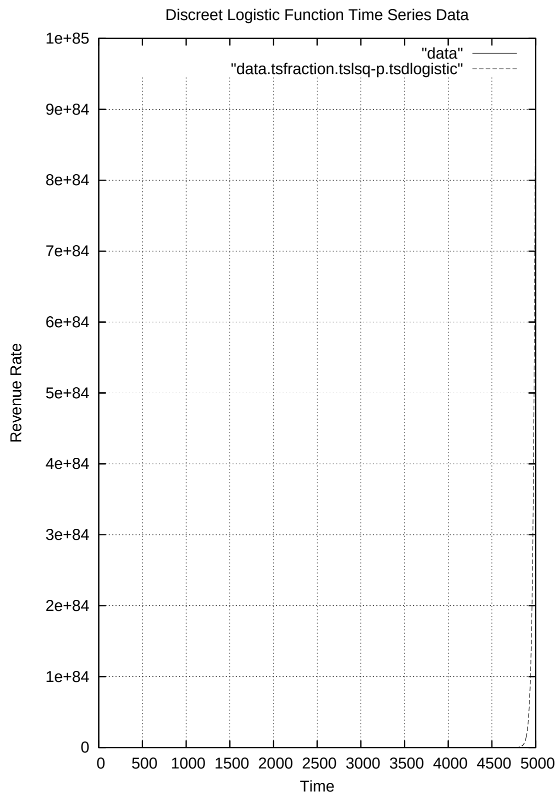

Figure C.449: Simulated Shannon Probability of 0.6 Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.439 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

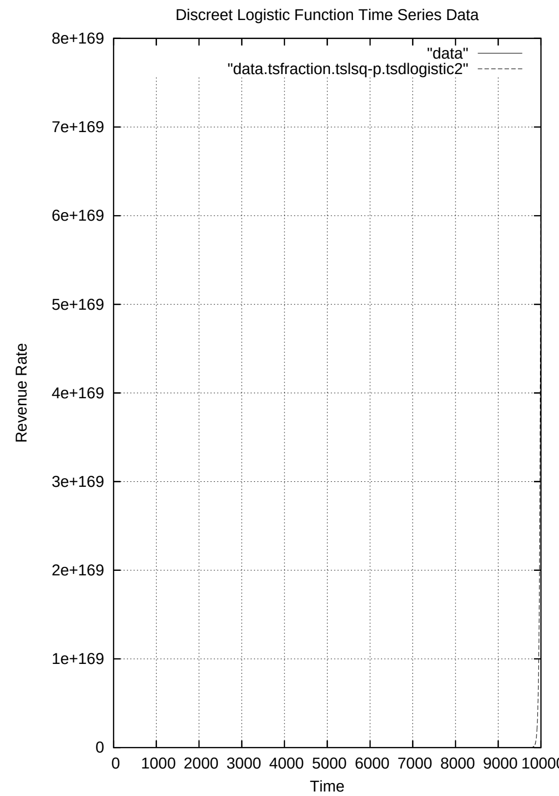

Figure C.450: Simulated Shannon Probability of 0.6 Game, logistic function estimates of Figure C.449 with the time scale expanded by a factor of two.

the likelihood of the rate of revenue returns, (per time units,) for any two consecutive time unitss being the same is 53.990400% [Pet91, pp. 66] for the near future, and 0.668253 for the distant future. Likewise, there is a 53.990400% chance of the rate of revenue returns, (per time units,) movements being the same in consecutive time periods—ie., if, in a given time units, the rate of revenue returns, (per time units,) is increasing, there is a 53.990400% that the rate of revenue returns, (per time units,) will increase in the following period, also. In some sense, this is a quantitative statement on how “predictable,” or “forecastable” the rate of revenue returns, (per time units,) for the Simulated Shannon Probability of 0.6 Game are over time, since the probability of having n many consecutive time unitss of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many time unitss of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.757}$$

also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

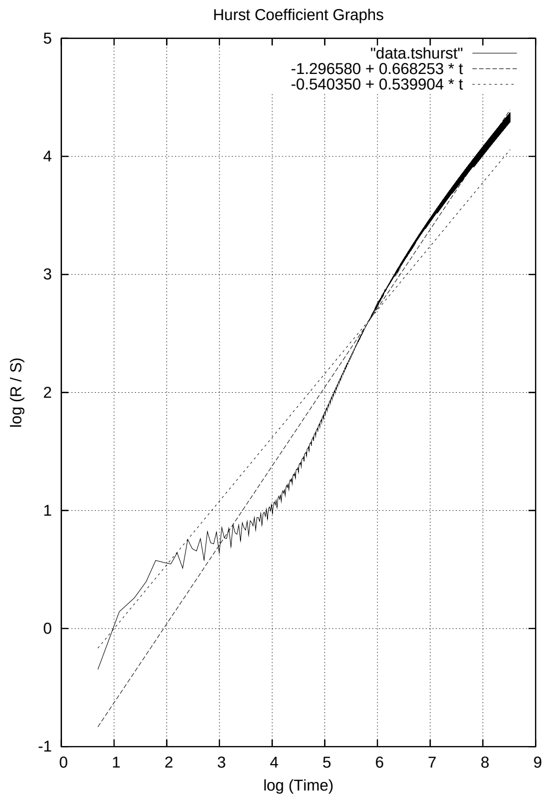

Figure C.451: Simulated Shannon Probability of 0.6 Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.439. The slope of the graph is the Hurst coefficient.

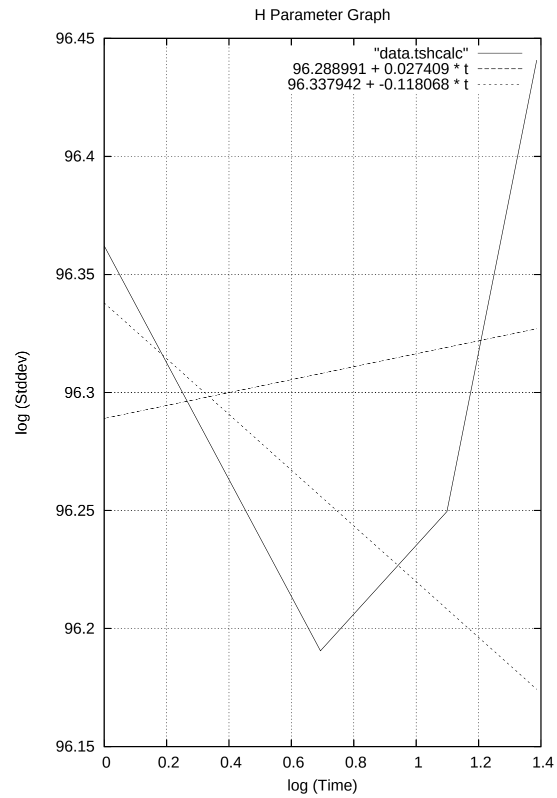

Figure C.452: Simulated Shannon Probability of 0.6 Game, H parameter data for the normalized increments of the time series data shown in Figure C.439 The slope of the graph is the H parameter.

$$= \quad 0.539904^n \tag{C.758}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.439, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next time units's rate of revenue returns would be the same as the current time units's revenue rate. Interestingly, it is $0.039968 \cdot 100$ percent, on the average, with a standard deviation of $0.195985 \cdot 100$ percent, and a root mean square error value of $0.200000 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per time units,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^H \tag{C.759}$$

$$\propto \quad (t_2 - t_1)^{0.539904} \tag{C.760}$$

where R is the range of values in the increments of the rate of revenue returns, (per time units.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per time units,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per time units) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.760 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.761}$$

$$\propto \sqrt{\left(t_2 - t_1\right)} \tag{C.762}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per time units,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.763}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per time units,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[249].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \propto \frac{X\left(rt\right)}{r^H} \tag{C.764}$$

$$\propto \frac{X\left(rt\right)}{r^{0.539904}} \tag{C.765}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.452, to provide a least squares approximation to the H parameter for the Simulated Shannon Probability of 0.6 Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is -0.118068 for the near future, and 0.027409 for the distant future.

Figures C.451 and C.452 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.439. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.439, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.453 and C.454 was made using the -d option.

Observations on the Hurst Coefficient Analysis

Note that both the Hurst coefficient and H parameter graphs indicate that the time series data set does not contain a random process—which is to be anticipated, since the data set is periodic.

[249] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

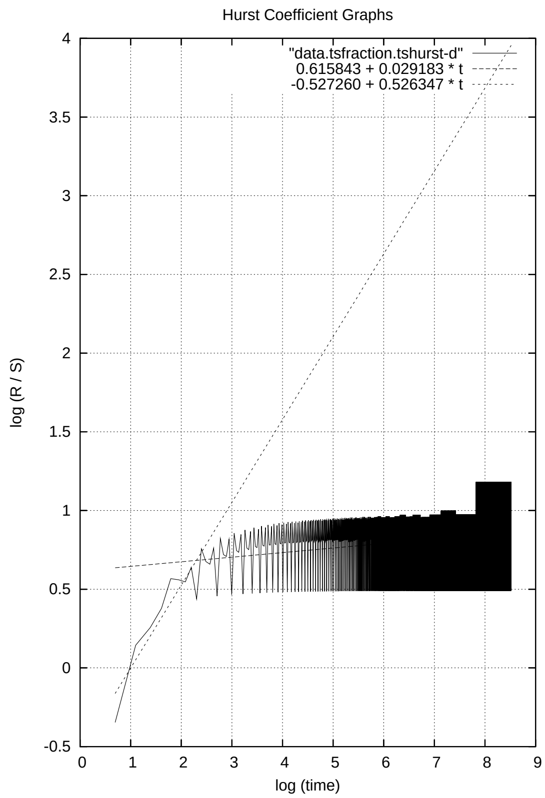

Figure C.453: Simulated Shannon Probability of 0.6 Game, traditional Hurst coefficient data for the time series data shown in Figure C.438. The slope of the graph is the Hurst coefficient, and is 0.526347 for the near term, and 0.029183 for the far term.

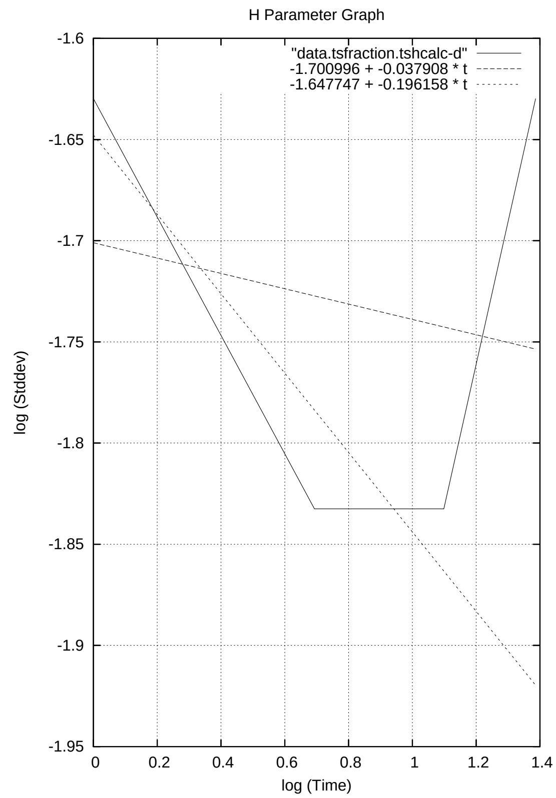

Figure C.454: Simulated Shannon Probability of 0.6 Game, traditional H parameter data for the time series data shown in Figure C.438 The slope of the graph is the H parameter, and is -0.196158 for the near term, and -0.037908 for the far term.

C.20.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.20.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.440. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.439. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Shannon Probability of 0.6 Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Simulated Shannon Probability of 0.6 Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.439, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.039968 + 1)}{\ln(2)} = 0.056539 \tag{C.766}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.439, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.039872 + 1)}{\ln(2)} = 0.056406 \tag{C.767}$$

Note that if the mean is not constant in Figure C.439, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.438:

$$bits = 0.029049 \tag{C.768}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.438:

$$bits = 0.028997 \tag{C.769}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.20.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.028997t} \tag{C.770}$$

therefore:

$$C(p) = 0.028997 \tag{C.771}$$

and, *tsshannon* 0.028997 gives:

$$C(0.599910) = 0.028997 \tag{C.772}$$

therefore:

$$2^{C(0.599910)} = 2^{0.028997} \tag{C.773}$$

$$= 1.020303 \tag{C.774}$$

$$= 2.030254\% \tag{C.775}$$

and:

$$2p - 1 = (2 \cdot 0.599910) - 1 \tag{C.776}$$

$$= 0.199820 \tag{C.777}$$

$$= 19.982000\% \tag{C.778}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Simulated Shannon Probability of 0.6 Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every time units, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 19.982000% of its rate of revenue returns, (per time units.) As a conceptual model, the remaining 80.018000% will be held in "reserve" with a 59.991000% chance of making twice the 19.982000% back, (and a 40.009000% chance of making 0.0,) in one time units, on the average, for an average growth in its rate of revenue returns, (per time units,) of 2.030254%, or a doubling of its rate of revenue returns, (per time units,) in 34.486326 time unitss.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 19.982000% per time units of the rate of revenue returns, (per time units,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 2.030254%, per time units, on average.

Note that the metrics presented in this section are representative of the Simulated Shannon Probability of 0.6 Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 19.982000% of its rate of revenue returns, (per time units,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some time unitss, depending on the Simulated Shannon Probability of 0.6 Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other time unitss, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 2.030254% per time units.

As another simple example, a company re-invests 19.982000% of its rate of revenue returns, (per time units,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 19.982000% per time units investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 2.030254% per time units.

As an example of "product portfolio" management, suppose a company re-invests 19.982000% of its rate of revenue returns, (per time units,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 19.982000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 19.982000$ percent for the second product, implying that the company should diversify its product line[250]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 19.982000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Simulated Shannon Probability of 0.6 Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.199820}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk

[250]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

that many of the products could suffer a “down market” concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal “hedging of bets,” in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.439, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of “product portfolio” management, consider the issue of product mix. In this interpretation, 19.982000% of the product manufactured should be “proprietary,” while the rest is “industry standard.” As yet another possibility, 19.982000% of the product manufactured should be predatory into new markets, and the remainder in markets that are “traditional” for the company.

C.20.6 Number of Companies

This section evaluates the approximate, or “average,” number of companies in the Simulated Shannon Probability of 0.6 Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Simulated Shannon Probability of 0.6 Game time series is 0.039968, and 0.200000respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.999200.

If this value seems consistent number of companies in the Simulated Shannon Probability of 0.6 Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Simulated Shannon Probability of 0.6 Game are operating optimally, and the “average” Shannon probability, P for each participating company would be, using Equation 2.110, 0.599960, which would be the value which should be used in Section C.20.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.20.5 is greater than the average Shannon probability for the companies participating in the Simulated Shannon Probability of 0.6 Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.20.5. The maximum exploitability for the Simulated Shannon Probability of 0.6 Game is derived in Section C.20.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company’s market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company’s P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Simulated Shannon Probability of 0.6 Game is 0.599960, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company’s P&L which is 0.599920 in the Simulated Shannon Probability of 0.6 Game. In all cases, the fraction of the P&L that should be “wagered” on the future, f, should be:

$$f = 2P - 1 \tag{C.779}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Simulated Shannon Probability of 0.6 Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of “increasing returns.”

C.20.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the “average” of the normalized increments presented in Figure C.440. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.439. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Shannon Probability of 0.6 Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.20.5, is derived from the Simulated Shannon Probability of 0.6 Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.20.9.

An additional exploitable strategy may be time itself. Equations C.756, C.760, and, C.758, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Simulated Shannon Probability of 0.6 Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.455, and, C.456 compare methods of approximation of the "forecastability" of rate of revenue returns in the Simulated Shannon Probability of 0.6 Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Simulated Shannon Probability of 0.6 Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[251]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.455, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.758, $0.539904^n = 0.5$ time unitss of operations. Since the optimal amount of inventory and, from Equation C.756, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.20.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.20.9. Figure C.457 represents a constructional simulation of the time series data presented in Figure C.438. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.439. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.439 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

[251]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

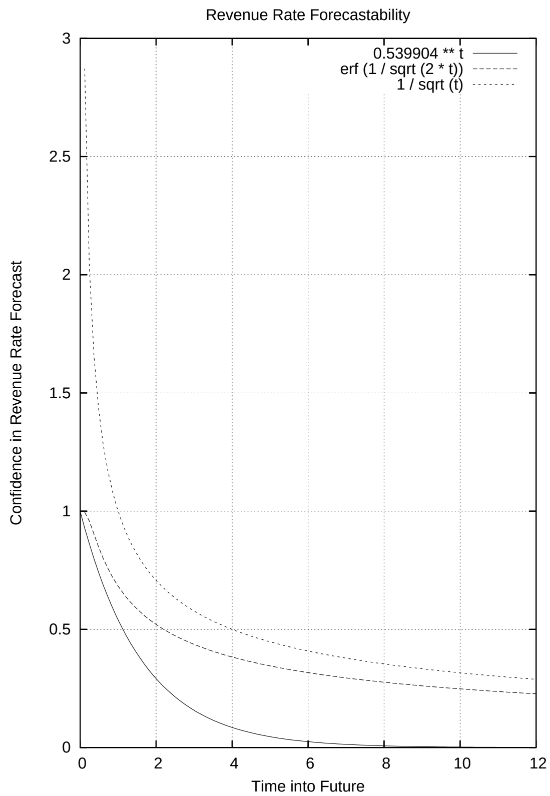

Figure C.455: Simulated Shannon Probability of 0.6 Game, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.539904^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

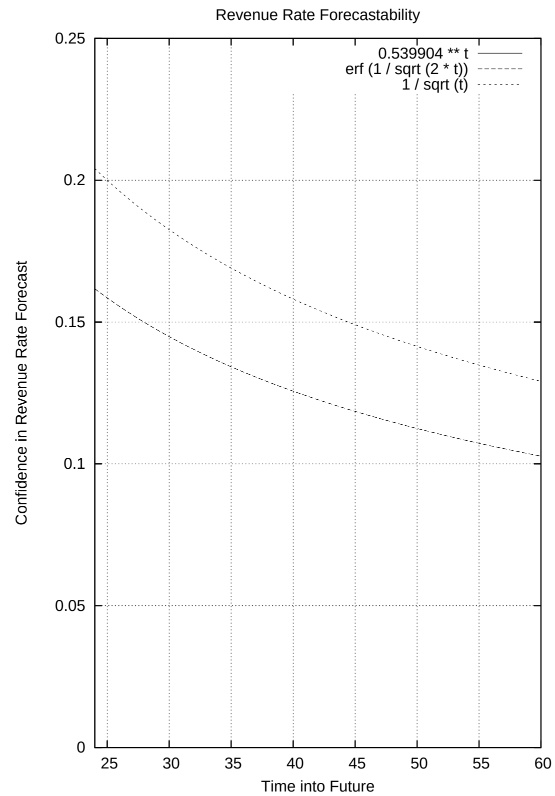

Figure C.456: Simulated Shannon Probability of 0.6 Game, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

As a further comparison of the the constructional simulation with the original time series data, Figure C.458 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.441.

C.20.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.20.3. One of the issues of analysis, as mentioned in Section C.20.7, is to determine the maximum Shannon probability for the time series presented in Figure C.438. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.459 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.438. Figure C.460 was constructed using *tsunfair-brownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series

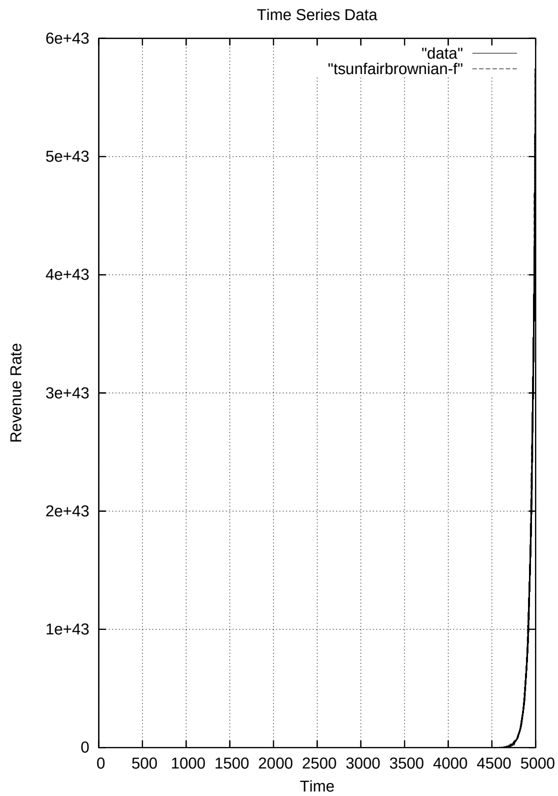

Figure C.457: Simulated Shannon Probability of 0.6 Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.200000. This data is superimposed on the data presented in Figure C.438.

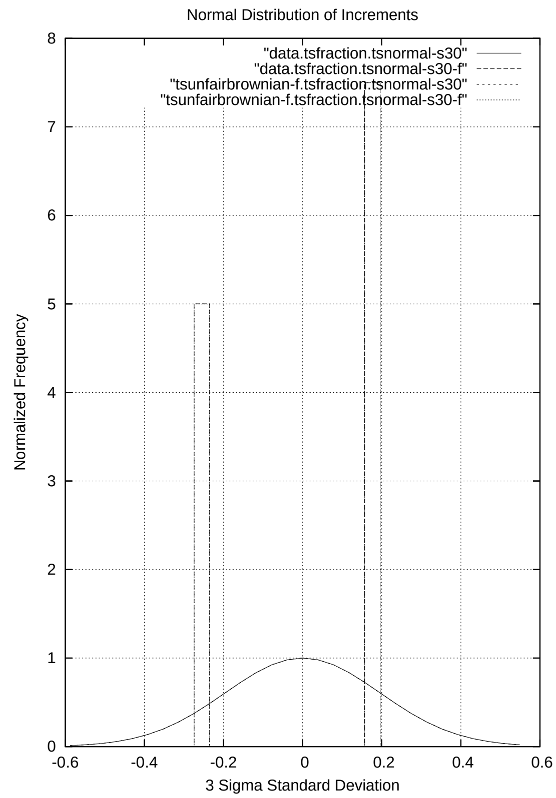

Figure C.458: Simulated Shannon Probability of 0.6 Game, normalized histogram of the normalized increments of the time series data shown in Figure C.457, empirical and simulated. The empirical data has a mean of 0.039968, with a standard deviation of 0.195985. By comparison, the simulated data has a mean of 0.040016 with a standard deviation of 0.195976. This data is superimposed on the data presented in Figure C.441. The area under the four curves is identical.

data presented in Figure C.438. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.599910, as derived in Section C.20.5 to 0.600000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.438, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.438, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of time unitss that the Simulated Shannon

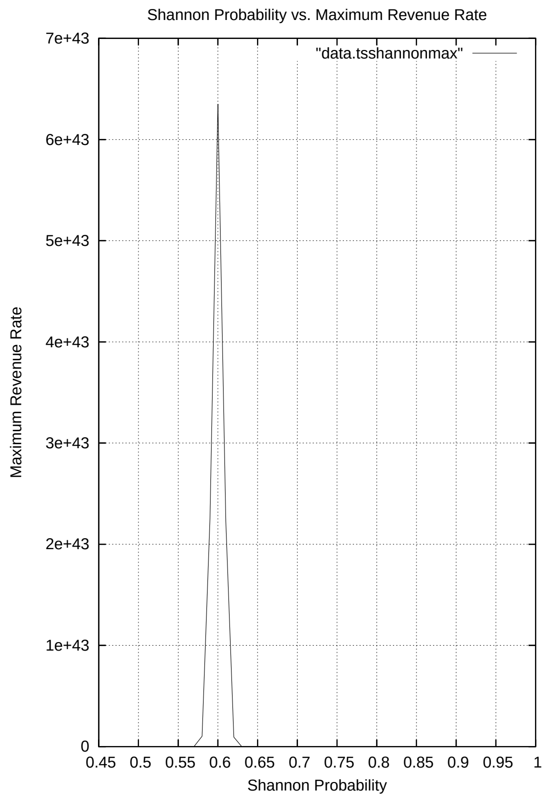

Figure C.459: Simulated Shannon Probability of 0.6 Game, maximum rate of revenue returns, per time units, vs. Shannon probability. The maximum rate of revenue returns, per time units, occurs at a Shannon probability of 0.600000.

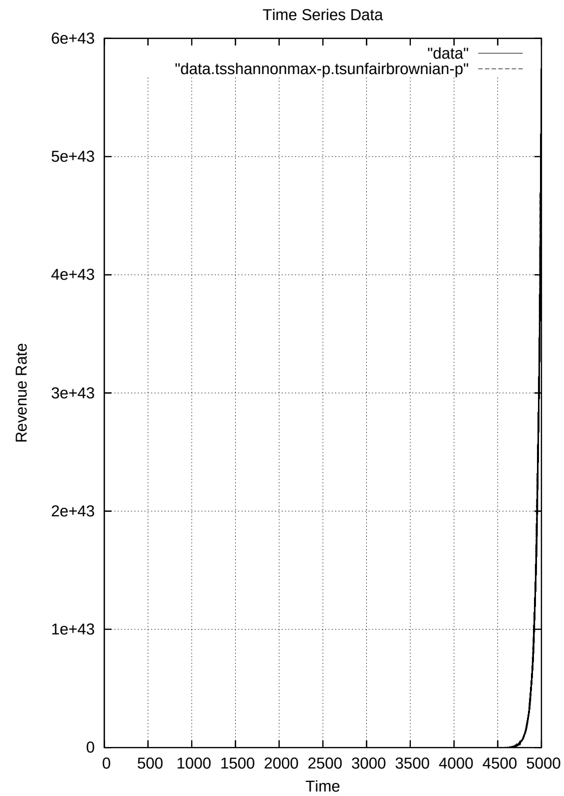

Figure C.460: Simulated Shannon Probability of 0.6 Game, maximum rate of revenue returns, per time units, at a Shannon probability, of 0.600000, corresponding to a “wager” fraction of 0.200000.

Probability of 0.6 Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.599920, as compared with the predicted value from the program *tsshannonmax* of 0.600000.

C.20.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the “average” of the normalized increments presented in Figure C.440. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.439. These values will be used in a fixed increment Brownian fractal analysis of the Simulated Shannon Probability of 0.6 Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.20.6 and D.20.7. As a subjective evaluation of the “quality” of the analysis of the Simulated Shannon Probability of 0.6 Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.438 from Figure C.439, and the Shannon probability as calculated by counting the total number of time unitss that the Simulated Shannon Probability of 0.6 Game movement was positive, as presented in Section C.20.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.780}$$

$$0.599920 \approx \frac{\frac{0.039968}{0.200000} + 1}{2} \tag{C.781}$$

$$0.599920 \approx 0.599920 \tag{C.782}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.599920 \approx 0.599920 \approx 0.600000 \tag{C.783}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.20.5, should be compared. The four methods used were the mean of Figure C.439, the constant in the least squares approximation to Figure C.439, the least squares exponential approximation to Figure C.438, and the logarithmic returns of Figure C.438, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.056539 \approx 0.056406 \approx 0.029049 \approx 0.028997 \tag{C.784}$$

It is implied in Section C.20.5, Subsection C.20.5 and in Section C.20.8 that, a Brownian motion with fixed increments fractal may "model" the Simulated Shannon Probability of 0.6 Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.785}$$

$$0.200000\,(2 \cdot 0.599920 - 1) \approx \frac{0.195985\,(2 \cdot 0.599920 - 1)}{2\sqrt{0.599920\,(1 - 0.599920)}} \tag{C.786}$$

$$0.200000 \cdot 0.199840 \approx 0.195985 \cdot 0.203954 \tag{C.787}$$

$$0.039968 \approx 0.039972 \tag{C.788}$$

and, equating to the mean:

$$0.039968 \approx 0.039968 \approx 0.039972 \tag{C.789}$$

where, as in Equation C.782 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.438 from Figure C.439, and the Shannon probability as calculated by counting the total number of time unitss that the Simulated Shannon Probability of 0.6 Game movement was positive, as presented in Section C.20.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[252], where the absolute value is presented in Figure C.440, and the root mean square value is presented in Figure C.439:

$$0.200000 \approx 0.200000 \tag{C.790}$$

Note, that if the Simulated Shannon Probability of 0.6 Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.438 from Figure C.439 should be zero. It is 0.000000.

[252] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

C.21 Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins[253].

The data in this section is presented in tabular form in Section D.21. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Coins Tossing Game. This is included for "theoretical" comparative purposes.

C.21.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.21.1. Figure C.461 is a graph of the time series data for the Coins Tossing Game.

Figure C.462 is a graph of the normalized increments of the time series data presented in Figure C.461. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.463 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.462. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[254].

Figure C.464 is the normalized histogram of the normalized increments of the time series data shown in Figure C.462. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.464.

Figure C.465 is the statistical estimate for the data presented in Figure C.462, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.645287$, as derived in Section C.21.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.466 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.462. In principle, if the distribution of the normalized increments presented in Figure C.464 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.467 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.462. In principle, if the distribution of the normalized increments presented in Figure C.464 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

[253] As a simulation model, the program *tscoins* was run to make a time series data file, with the following parameters:

```
tscoins -p 0.6 300 > data
```

to make a time series of 300 elements, with a Shannon probability of 0.6. The data is by tosses.

[254] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

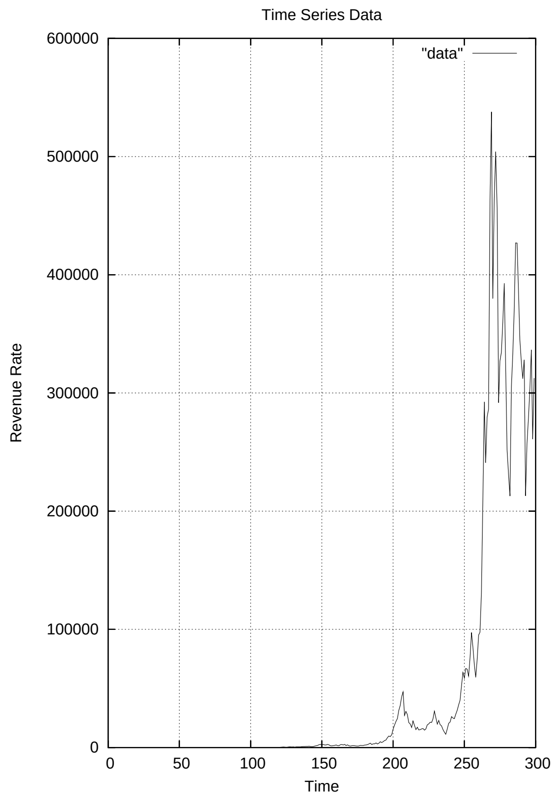

Figure C.461: Coins Tossing Game, time series data.

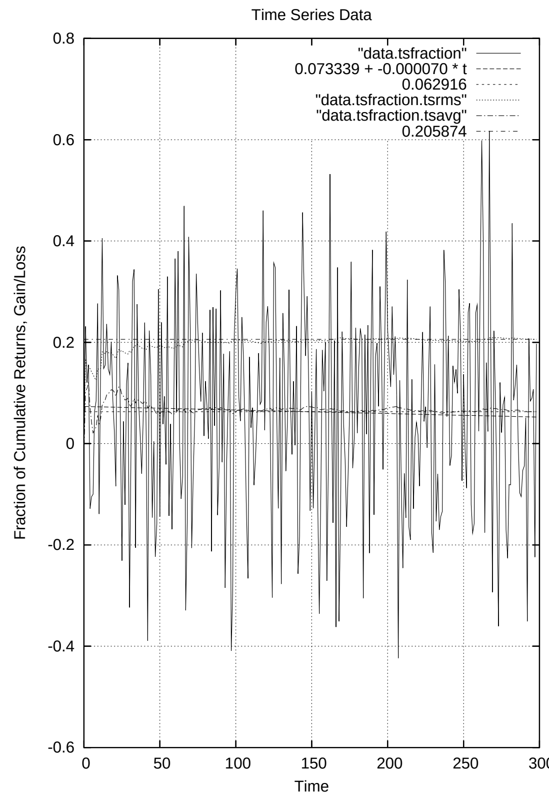

Figure C.462: Coins Tossing Game, normalized increments of the time series data presented in Figure C.461. The mean is 0.062916 with a standard deviation of 0.196353. The formula for the least squares approximation is $0.073339 + -0.000070t$, and the root mean squared value is 0.205874. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000070, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.468 is the range of values of the time series shown in Figure C.461. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.468 would be a square root function[255]. Figure C.469 is the deterministic map of the normalized increments of the time

[255]Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.468 are a computational artifact—caused by not using the -m

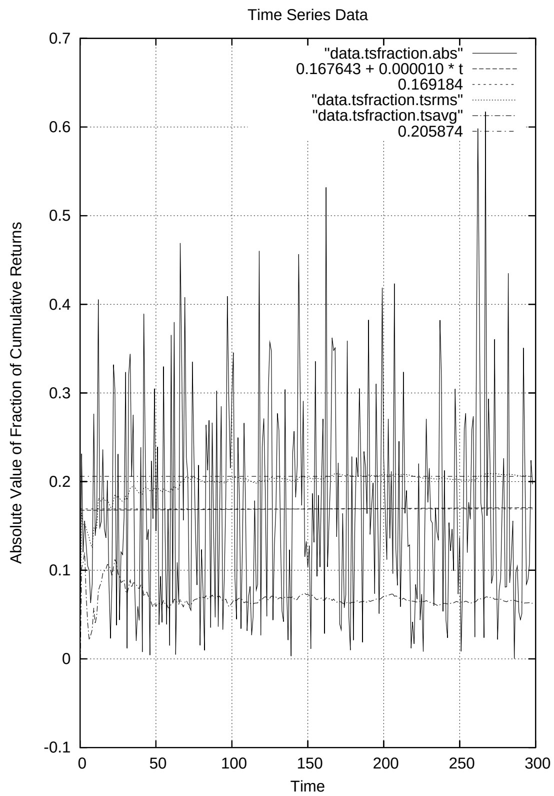

Figure C.463: Coins Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.462. The mean is 0.169184 with a standard deviation of 0.117504. The formula for the least squares approximation is $0.167643 + 0.000010t$, and the root mean square value, from Figure C.462, is 0.205874. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments presented in Figure C.462, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

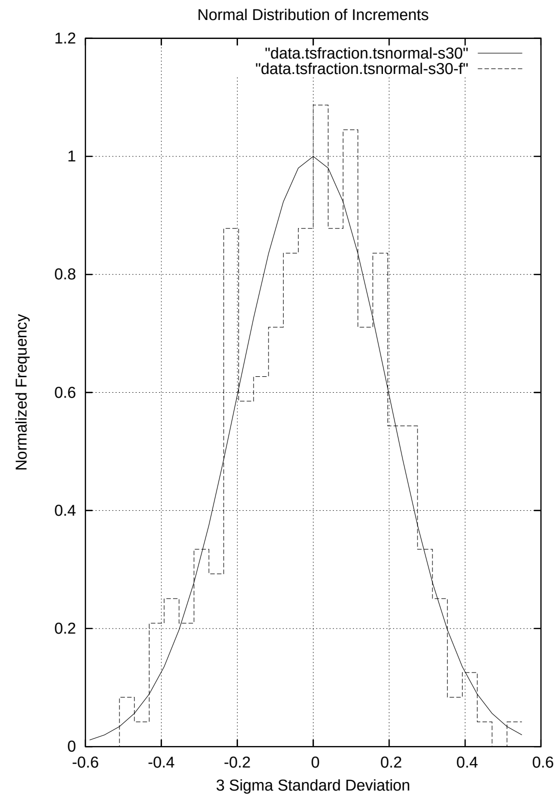

Figure C.464: Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.462. The data has a mean of 0.062916, with a standard deviation of 0.196353. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 1.028000, with a critical value of 42.557000.

series data shown in Figure C.462. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.464 would seem to indicate that the time series data for the Coins Tossing Game represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property

option to the program *tshurst*, which is computationally inefficient.

```
For a mean of 0.062707, with a confidence level of 0.900000
    that the error did not exceed 0.006271, 2917 samples would be required.
    (With 300 samples, the estimated error is 0.019551 = 31.178531 percent.)
For a standard deviation of 0.205874, with a confidence level of 0.900000
    that the error did not exceed 0.020587, 136 samples would be required.
    (With 300 samples, the estimated error is 0.013825 = 6.715087 percent.)
```

Figure C.465: Coins Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.462. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.462.

of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.21.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[256]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.470 is the instantaneous value of the root mean square of the normalized increments for the Coins Tossing Game, and Figure C.471 is the instantaneous Shannon probability for the normalized increments.

C.21.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.21.4. Figure C.472 is a graph of the logistic function estimates of the time series data for the Coins Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[257]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.472 is a graph of the logistic function for the time series data presented in Figure C.461. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.462. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.462. Figure C.473 is the same graph, but with the time scale expanded by a factor of two.

[256]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

[257]For example, in Figures C.472 and C.473, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.21.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

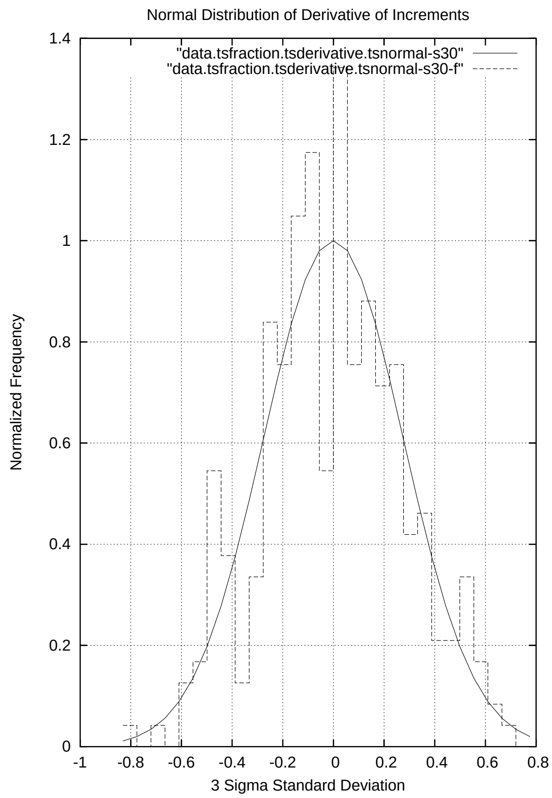

Figure C.466: Coins Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.462.

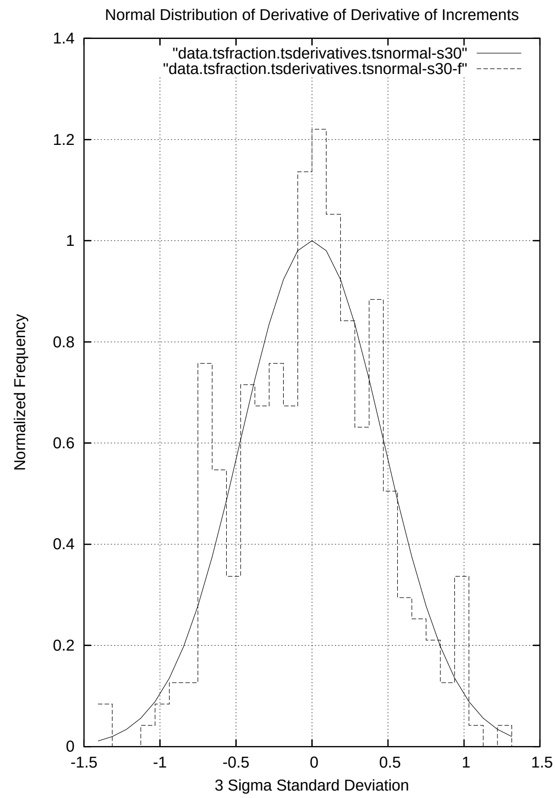

Figure C.467: Coins Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.462.

C.21.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.21.5. Figure C.474 is a graph of the Hurst coefficient data time series data shown in Figure C.461. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.475 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.462. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.474 implies that the variance of the rate of revenue returns, (per tosses,) in the Coins Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[258], t, $p(t) = erf(1/\sqrt{2t})$ which

[258]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or

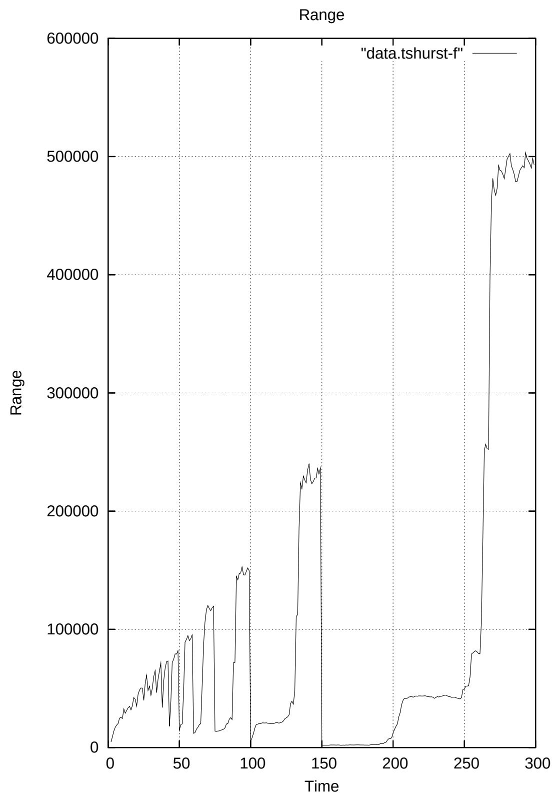

Figure C.468: Coins Tossing Game, range of the time series data shown in Figure C.461.

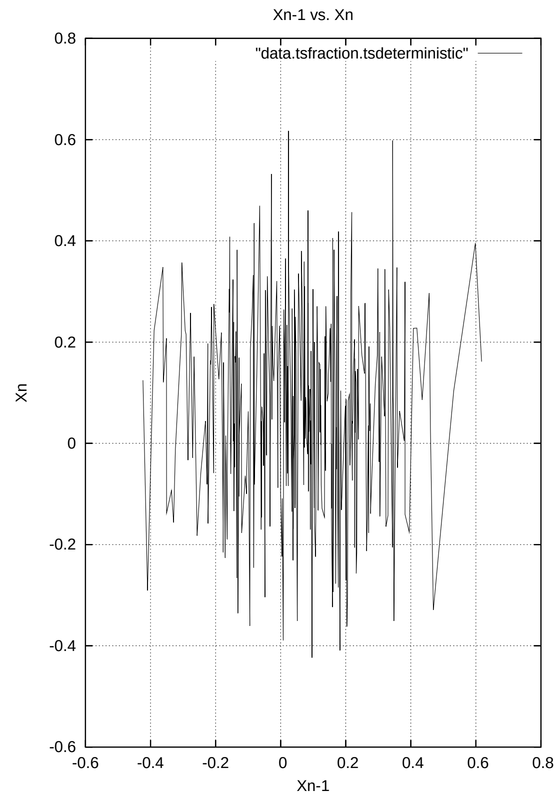

Figure C.469: Coins Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.462.

is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.478, and, C.479 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Coins Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][259]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.474, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.856676, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.791}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.856676} \tag{C.792}$$

"low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[259] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

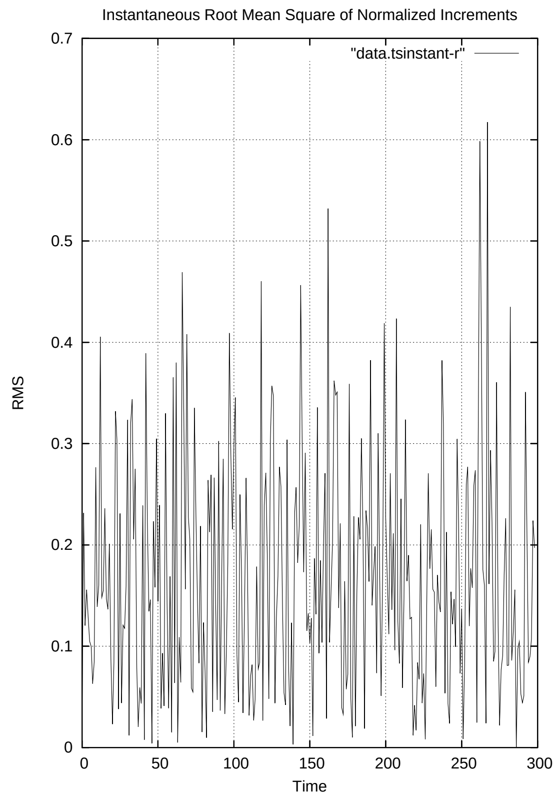

Figure C.470: Coins Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.461.

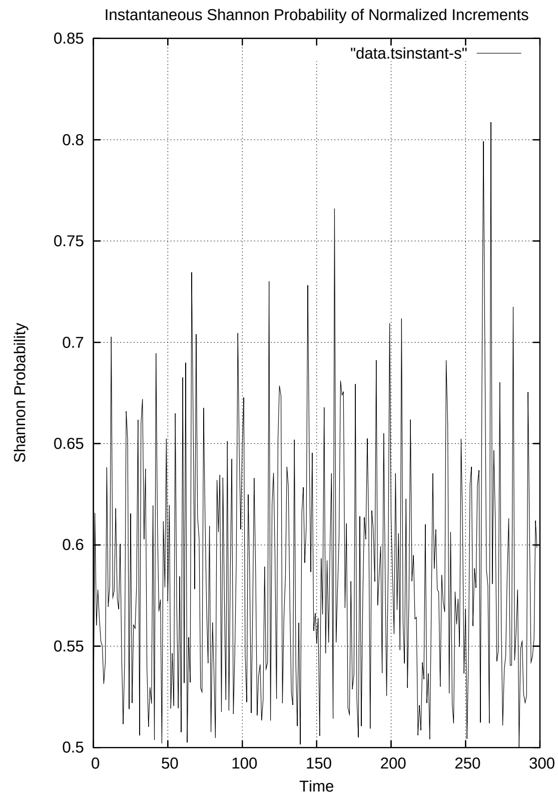

Figure C.471: Coins Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.461.

$$\propto \quad (t_2 - t_1)^{1.713352} \tag{C.793}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[260]. A Hurst coefficient of 0.856676, (for the near future, and 0.642211 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is

[260] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.21.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Coins Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

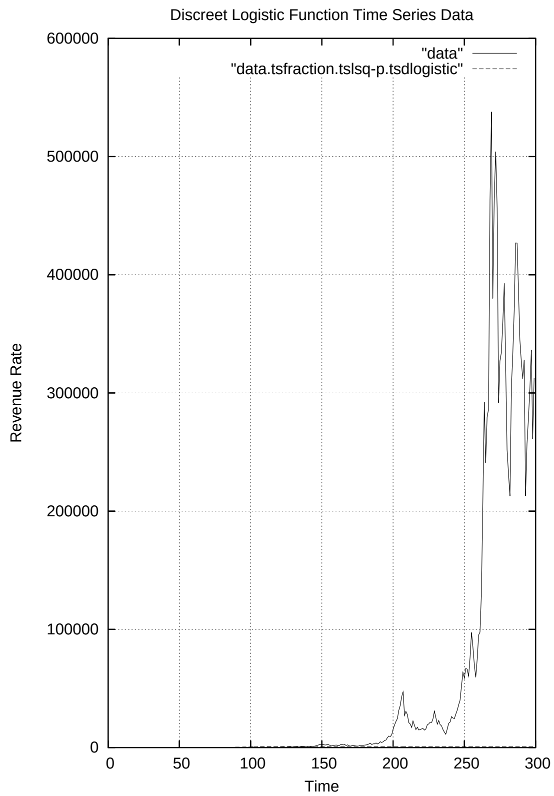

Figure C.472: Coins Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.462 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

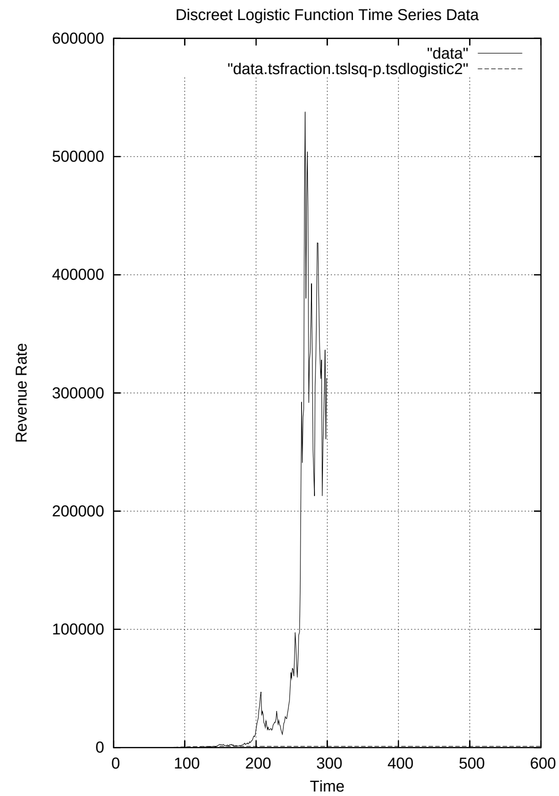

Figure C.473: Coins Tossing Game, logistic function estimates of Figure C.472 with the time scale expanded by a factor of two.

85.667600% [Pet91, pp. 66] for the near future, and 0.642211 for the distant future. Likewise, there is a 85.667600% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 85.667600% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Coins Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.794}$$

$$= 0.856676^n \tag{C.795}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.462, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the

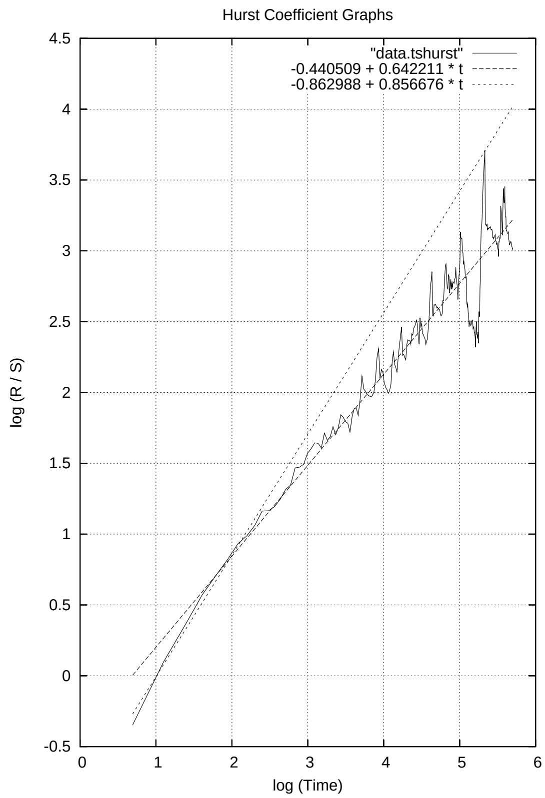

Figure C.474: Coins Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.462. The slope of the graph is the Hurst coefficient.

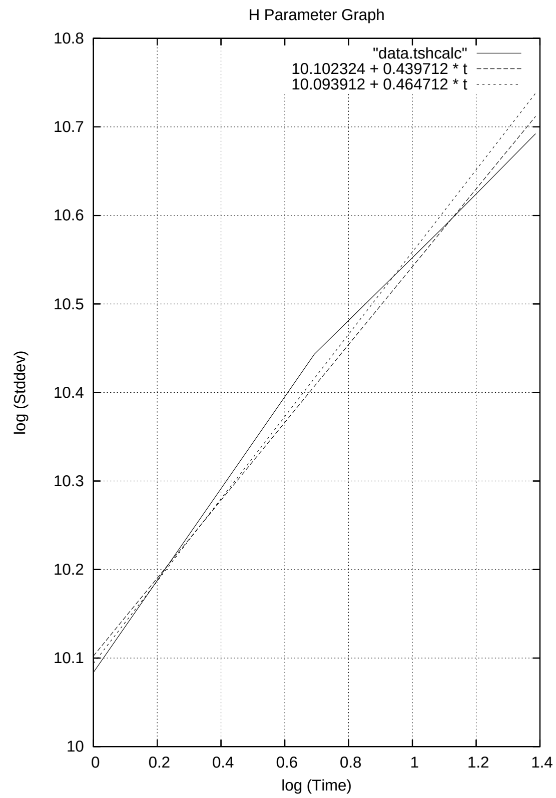

Figure C.475: Coins Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.462 The slope of the graph is the H parameter.

current tosses's revenue rate. Interestingly, it is $0.062916 \cdot 100$ percent, on the average, with a standard deviation of $0.196353 \cdot 100$ percent, and a root mean square error value of $0.205874 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \;\propto\; (t_2 - t_1)^H \tag{C.796}$$

$$\propto\; (t_2 - t_1)^{0.856676} \tag{C.797}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.797 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.798}$$

$$\propto \sqrt{\left(t_2 - t_1\right)} \tag{C.799}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.800}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[261].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \propto \frac{X\left(rt\right)}{r^H} \tag{C.801}$$

$$\propto \frac{X\left(rt\right)}{r^{0.856676}} \tag{C.802}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.475, to provide a least squares approximation to the H parameter for the Coins Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.464712 for the near future, and 0.439712 for the distant future.

Figures C.474 and C.475 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.462. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.462, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.476 and C.477 was made using the -d option.

C.21.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.21.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.463. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.462. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Coins Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

[261] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

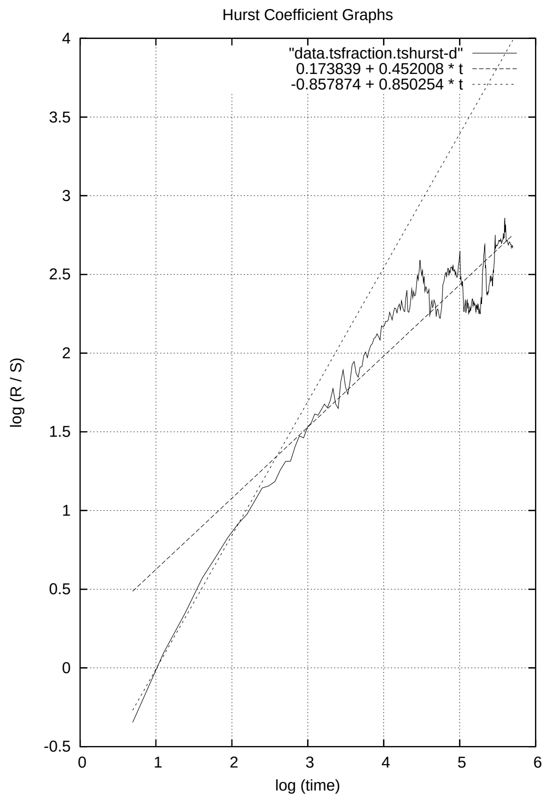

Figure C.476: Coins Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.461. The slope of the graph is the Hurst coefficient, and is 0.850254 for the near term, and 0.452008 for the far term.

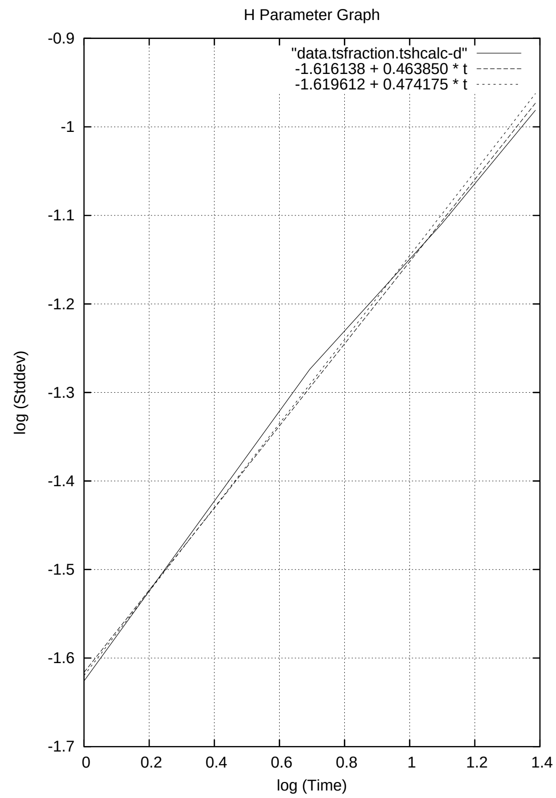

Figure C.477: Coins Tossing Game, traditional H parameter data for the time series data shown in Figure C.461 The slope of the graph is the H parameter, and is 0.474175 for the near term, and 0.463850 for the far term.

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.462, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.062916 + 1)}{\ln(2)} = 0.088028 \tag{C.803}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.462, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.073339 + 1)}{\ln(2)} = 0.102106 \tag{C.804}$$

Note that if the mean is not constant in Figure C.462, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.461:

$$bits = 0.063132 \tag{C.805}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.461:

$$bits = 0.061793 \tag{C.806}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.21.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.061793t} \tag{C.807}$$

therefore:

$$C(p) = 0.061793 \tag{C.808}$$

and, *tsshannon* 0.061793 gives:

$$C(0.645287) = 0.061793 \tag{C.809}$$

therefore:

$$\begin{aligned} 2^{C(0.645287)} &= 2^{0.061793} && \text{(C.810)} \\ &= 1.043762 && \text{(C.811)} \\ &= 4.376216\% && \text{(C.812)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.645287) - 1 && \text{(C.813)} \\ &= 0.290574 && \text{(C.814)} \\ &= 29.057400\% && \text{(C.815)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the Coins Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 29.057400% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 70.942600% will be held in “reserve” with a 64.528700% chance of making twice the 29.057400% back, (and a 35.471300% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 4.376216%, or a doubling of its rate of revenue returns, (per tosses,) in 16.183063 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 29.057400% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 4.376216%, per tosses, on average.

Note that the metrics presented in this section are representative of the Coins Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 29.057400% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Coins Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 4.376216% per tosses.

As another simple example, a company re-invests 29.057400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 29.057400% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 4.376216% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 29.057400% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 29.057400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 29.057400$ percent for the second product, implying that the company should diversify its product line[262]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 29.057400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Coins Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.290574}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.462, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 29.057400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 29.057400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.21.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Coins Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the

[262] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

normalized increments of the Coins Tossing Game time series is 0.062916, and 0.205874respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 1.484424.

If this value seems consistent number of companies in the Coins Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Coins Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.625415, which would be the value which should be used in Section C.21.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.21.5 is greater than the average Shannon probability for the companies participating in the Coins Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.21.5. The maximum exploitability for the Coins Tossing Game is derived in Section C.21.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Coins Tossing Game is 0.625415, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.652802 in the Coins Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.816}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Coins Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.21.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.463. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.462. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.21.5, is derived from the Coins Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.21.9.

An additional exploitable strategy may be time itself. Equations C.793, C.797, and, C.795, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Coins Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.478, and, C.479 compare methods of approximation of the "forecastability" of rate of revenue returns in the Coins Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Coins Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[263]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

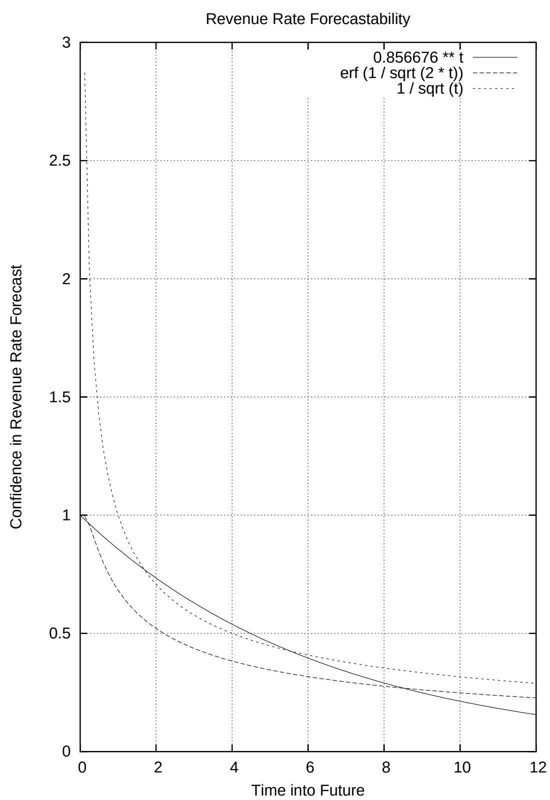

Figure C.478: Coins Tossing Game, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.856676^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

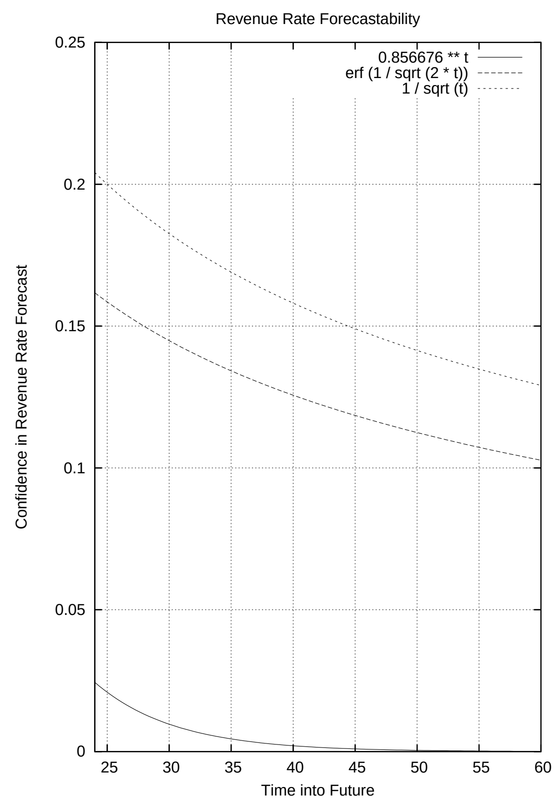

Figure C.479: Coins Tossing Game, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

As an interesting interpretation of the data presented in Figure C.478, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over

[263]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.795, $0.856676^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.793, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.21.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.21.9. Figure C.480 represents a constructional simulation of the time series data presented in Figure C.461. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.462. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.462 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.481 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.464.

C.21.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.21.3. One of the issues of analysis, as mentioned in Section C.21.7, is to determine the maximum Shannon probability for the time series presented in Figure C.461. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.482 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.461. Figure C.483 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.461. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.645287, as derived in Section C.21.5 to 0.646667. This process, essentially, extracts the random statistical data from the time series presented in Figure C.461, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.461, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Coins Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.645485, as compared with the predicted value from the program *tsshannonmax* of 0.646667.

C.21.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.463. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.462. These values will be used in a fixed increment Brownian fractal analysis of the Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

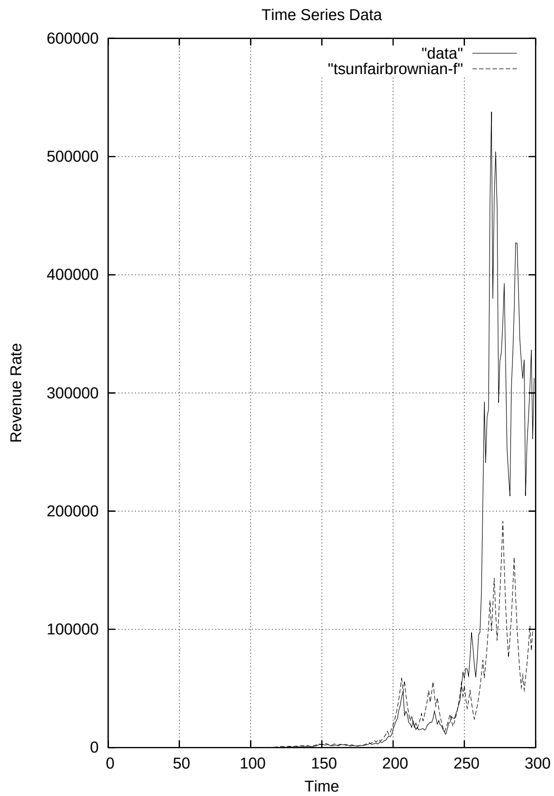

Figure C.480: Coins Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.205874. This data is superimposed on the data presented in Figure C.461.

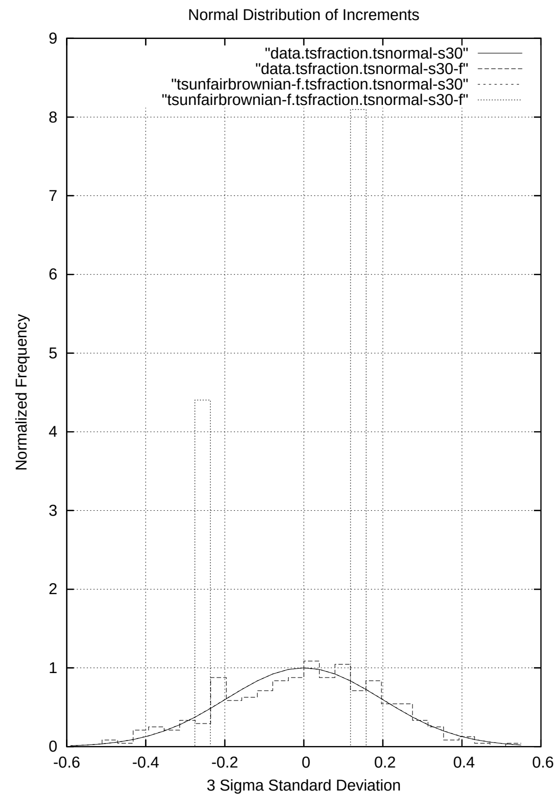

Figure C.481: Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.480, empirical and simulated. The empirical data has a mean of 0.062916, with a standard deviation of 0.196353. By comparison, the simulated data has a mean of 0.060795 with a standard deviation of 0.197024. This data is superimposed on the data presented in Figure C.464. The area under the four curves is identical.

The data in this section is presented in tabular form in sections D.21.6 and D.21.7. As a subjective evaluation of the "quality" of the analysis of the Coins Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.461 from Figure C.462, and the Shannon probability as calculated by counting the total number of tossess that the Coins Tossing Game movement was positive, as presented in Section C.21.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.817}$$

$$0.645485 \approx \frac{\frac{0.062916}{0.205874} + 1}{2} \tag{C.818}$$

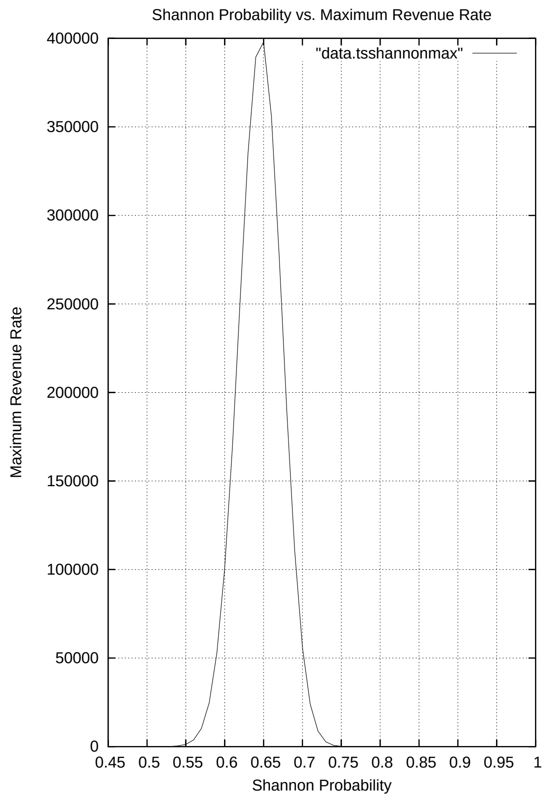

Figure C.482: Coins Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.646667.

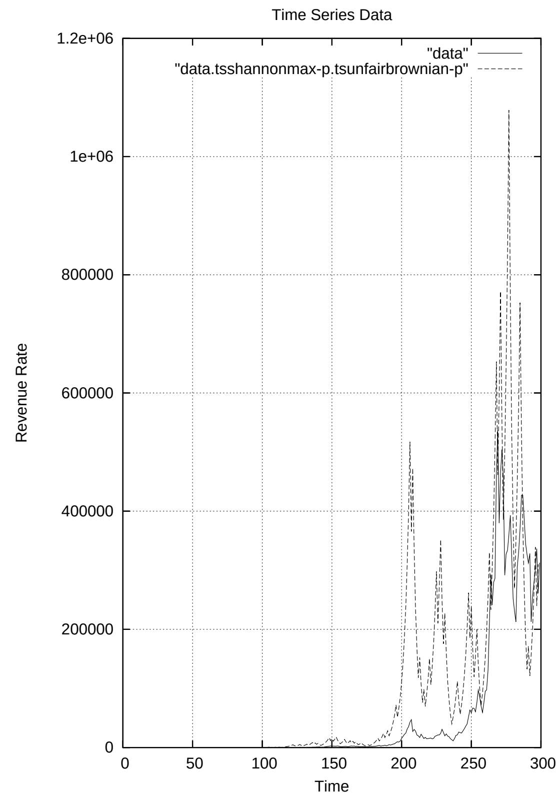

Figure C.483: Coins Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.646667, corresponding to a "wager" fraction of 0.293334.

$$0.645485 \quad \approx \quad 0.652802 \tag{C.819}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.645485 \approx 0.652802 \approx 0.646667 \tag{C.820}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.21.5, should be compared. The four methods used were the mean of Figure C.462, the constant in the least squares approximation to Figure C.462, the least squares exponential approximation to Figure C.461, and the logarithmic returns of Figure C.461, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.088028 \approx 0.102106 \approx 0.063132 \approx 0.061793 \tag{C.821}$$

It is implied in Section C.21.5, Subsection C.21.5 and in Section C.21.8 that, a Brownian motion with fixed increments fractal may "model" the Coins Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$
\begin{aligned}
rms\,(2P-1) &\approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} && \text{(C.822)}\\
0.205874\,(2\cdot 0.645485-1) &\approx \frac{0.196353\,(2\cdot 0.645485-1)}{2\sqrt{0.645485\,(1-0.645485)}} && \text{(C.823)}\\
0.205874\cdot 0.290970 &\approx 0.196353\cdot 0.304129 && \text{(C.824)}\\
0.059903 &\approx 0.059717 && \text{(C.825)}
\end{aligned}
$$

and, equating to the mean:

$$0.062916 \approx 0.059903 \approx 0.059717 \qquad \text{(C.826)}$$

where, as in Equation C.819 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.461 from Figure C.462, and the Shannon probability as calculated by counting the total number of tossess that the Coins Tossing Game movement was positive, as presented in Section C.21.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[264], where the absolute value is presented in Figure C.463, and the root mean square value is presented in Figure C.462:

$$0.169184 \approx 0.205874 \qquad \text{(C.827)}$$

Note, that if the Coins Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.461 from Figure C.462 should be zero. It is 0.117504.

C.22 Non-optimal Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins-f[265].

The data in this section is presented in tabular form in Section D.22. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Non-optimal Coins Tossing Game. This is included for "theoretical" comparative purposes.

C.22.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.22.1. Figure C.484 is a graph of the time series data for the Non-optimal Coins Tossing Game.

Figure C.485 is a graph of the normalized increments of the time series data presented in Figure C.484. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

[264]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[265]As a simulation model, the program *tscoins* was run to make a time series data file, with the following parameters:

```
tscoins -p 0.6 -f 0.03 300 > data
```

to make a time series of 300 elements, with a Shannon probability of 0.6 and a known non-optimal investment strategy. The data is by tosses.

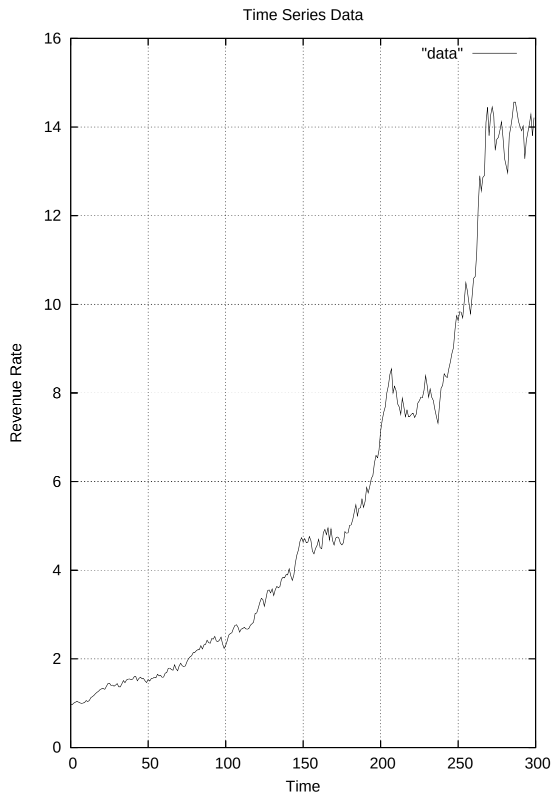

Figure C.484: Non-optimal Coins Tossing Game, time series data.

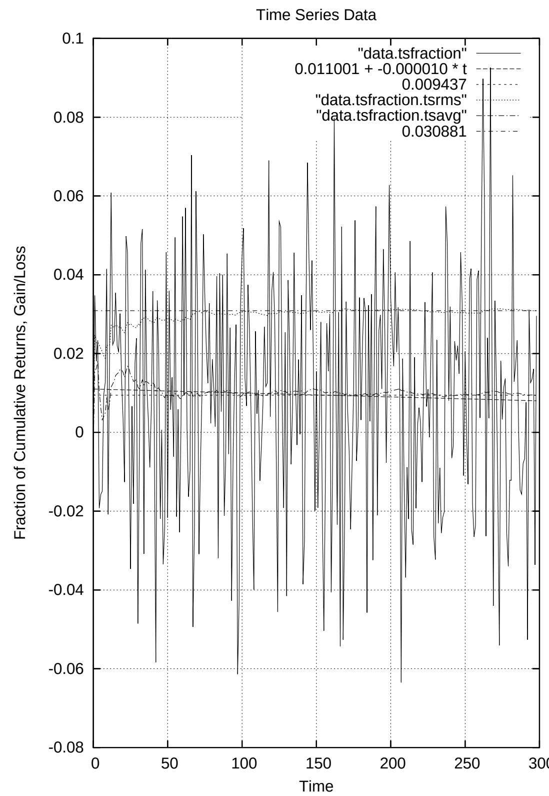

Figure C.485: Non-optimal Coins Tossing Game, normalized increments of the time series data presented in Figure C.484. The mean is 0.009437 with a standard deviation of 0.029453. The formula for the least squares approximation is $0.011001 + -0.000010t$, and the root mean squared value is 0.030881. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000010, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.486 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.485. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[266].

Figure C.487 is the normalized histogram of the normalized increments of the time series data shown in Figure C.485. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.487.

Figure C.488 is the statistical estimate for the data presented in Figure C.485, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.566751$, as derived in Section C.22.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.489 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.485. In principle, if the distribution of the normalized increments presented in Figure C.487 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.490 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.485. In principle, if the distribution of the normalized increments presented in Figure C.487 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.491 is the range of values of the time series shown in Figure C.484. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.491 would be a square root function[267]. Figure C.492 is the deterministic map of the normalized increments of the time series data shown in Figure C.485. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.487 would seem to indicate that the time series data for the Non-optimal Coins Tossing Game represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.22.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[268]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

[266] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[267] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.491 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[268] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

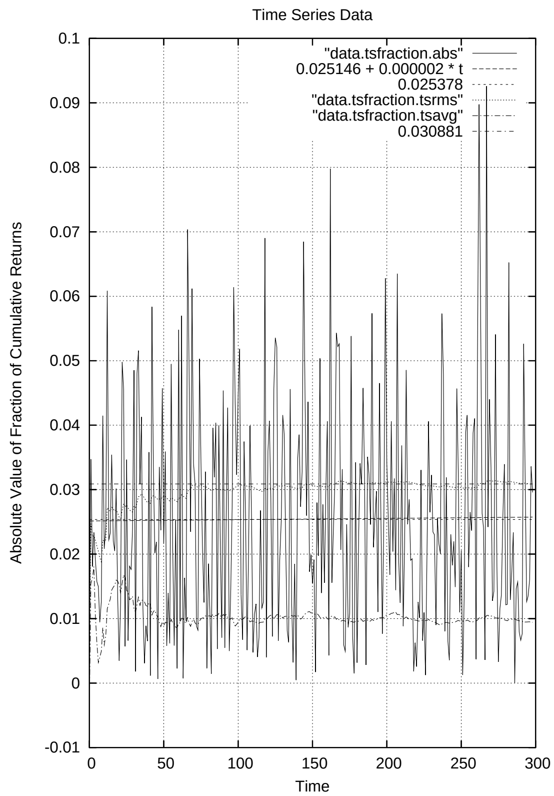

Figure C.486: Non-optimal Coins Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.485. The mean is 0.025378 with a standard deviation of 0.017626. The formula for the least squares approximation is $0.025146 + 0.000002t$, and the root mean square value, from Figure C.485, is 0.030881. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.485, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

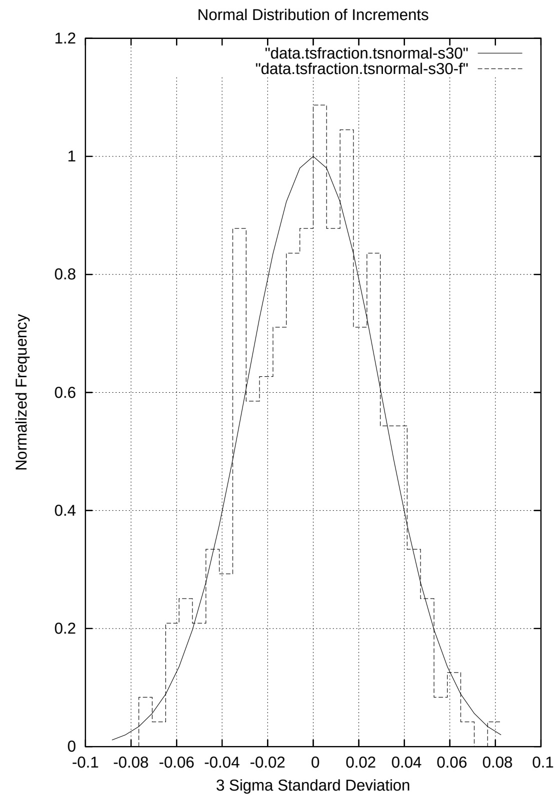

Figure C.487: Non-optimal Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.485. The data has a mean of 0.009437, with a standard deviation of 0.029453. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 1.028000, with a critical value of 42.557000.

Figure C.493 is the instantaneous value of the root mean square of the normalized increments for the Non-optimal Coins Tossing Game, and Figure C.494 is the instantaneous Shannon probability for the normalized increments.

```
For a mean of 0.009406, with a confidence level of 0.900000
    that the error did not exceed 0.000941, 2917 samples would be required.
    (With 300 samples, the estimated error is 0.002933 = 31.178526 percent.)
For a standard deviation of 0.030881, with a confidence level of 0.900000
    that the error did not exceed 0.003088, 136 samples would be required.
    (With 300 samples, the estimated error is 0.002074 = 6.715087 percent.)
```

Figure C.488: Non-optimal Coins Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.485. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.485.

C.22.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.22.4. Figure C.495 is a graph of the logistic function estimates of the time series data for the Non-optimal Coins Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[269]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.495 is a graph of the logistic function for the time series data presented in Figure C.484. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.485. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.485. Figure C.496 is the same graph, but with the time scale expanded by a factor of two.

C.22.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.22.5. Figure C.497 is a graph of the Hurst coefficient data time series data shown in Figure C.484. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.498 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.485. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.497 implies that the variance of the rate of revenue returns, (per tosses,) in the Non-optimal Coins Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[270], t, $p(t) = erf(1/\sqrt{2t})$ which is

[269] For example, in Figures C.495 and C.496, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.22.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

[270] It can be shown that the number of expected market “high” and “low” transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's “high” or “low” exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's “high” or “low” exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the “zero free” run-lengths of a market being “high” or “low,” over a long time, would have a $1/t^{3/2}$

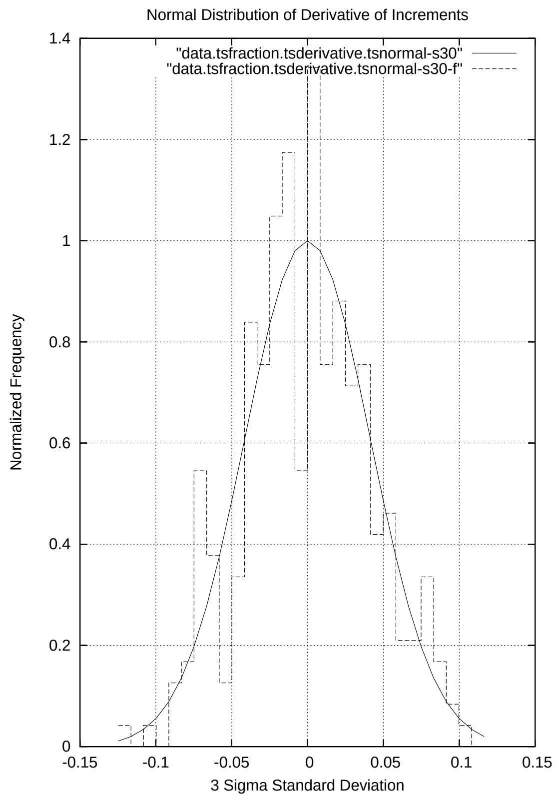

Figure C.489: Non-optimal Coins Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.485.

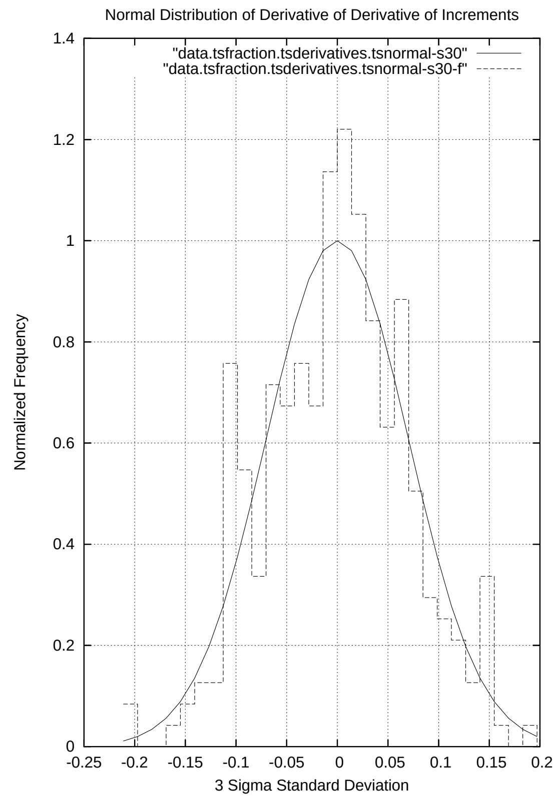

Figure C.490: Non-optimal Coins Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.485.

approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.501, and, C.502 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Non-optimal Coins Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][271]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.497, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.849887, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.828}$$
$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.849887} \tag{C.829}$$

characteristic.)

[271]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

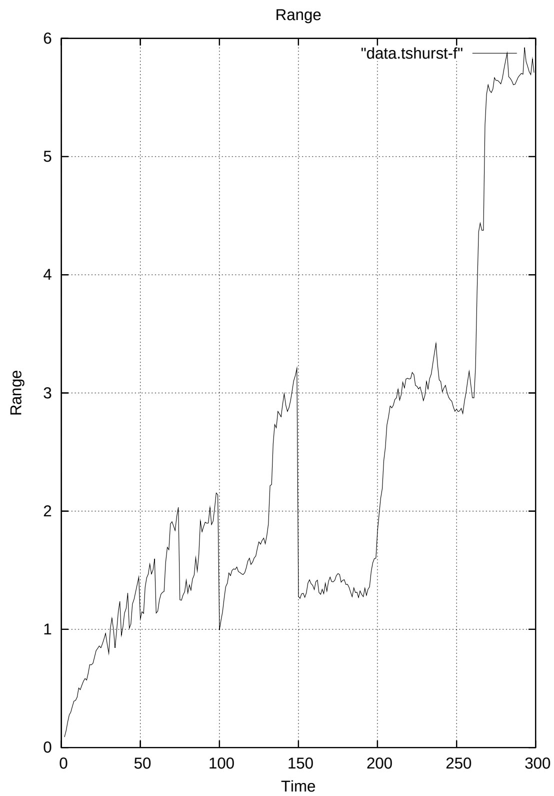

Figure C.491: Non-optimal Coins Tossing Game, range of the time series data shown in Figure C.484.

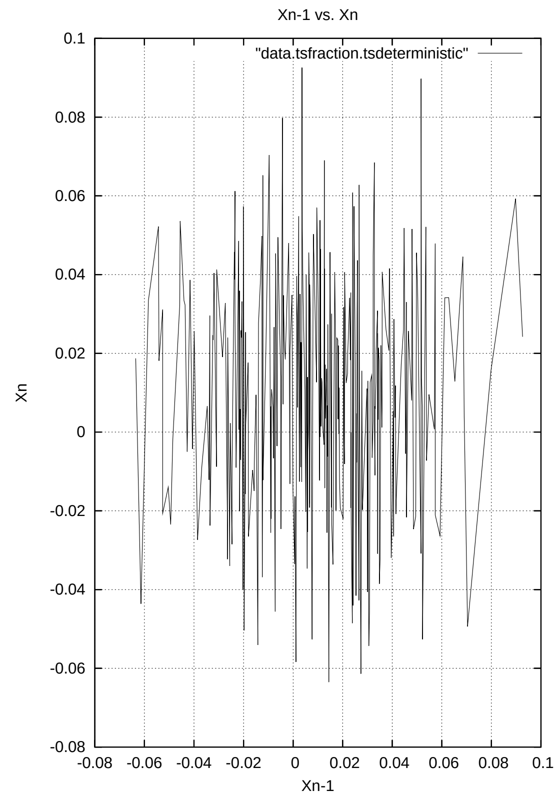

Figure C.492: Non-optimal Coins Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.485.

$$\propto \quad (t_2 - t_1)^{1.699774} \tag{C.830}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[272]. A Hurst coefficient of 0.849887, (for the near future, and 0.680509 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is 84.988700% [Pet91, pp. 66] for the near future, and 0.680509 for the distant future. Likewise, there is a 84.988700% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in

[272] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.22.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Non-optimal Coins Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

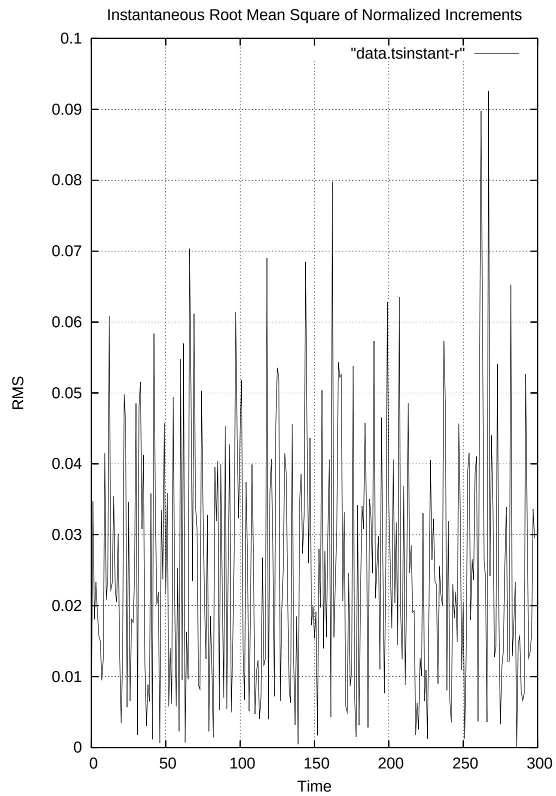

Figure C.493: Non-optimal Coins Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.484.

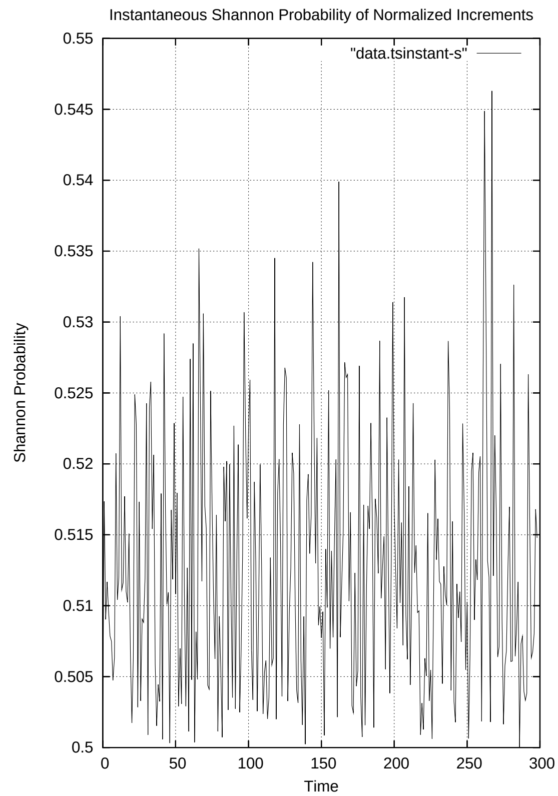

Figure C.494: Non-optimal Coins Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.484.

a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 84.988700% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Non-optimal Coins Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.831}$$

$$= 0.849887^n \tag{C.832}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.485, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.009437 \cdot 100$ percent, on the average, with a standard deviation of $0.029453 \cdot 100$ percent, and a root mean square error value of $0.030881 \cdot 100$ percent—small values for such a simple

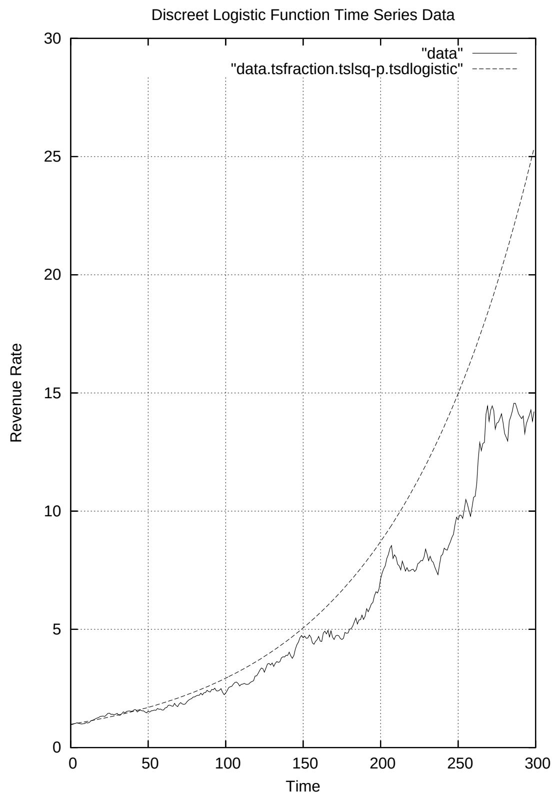

Figure C.495: Non-optimal Coins Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.485 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

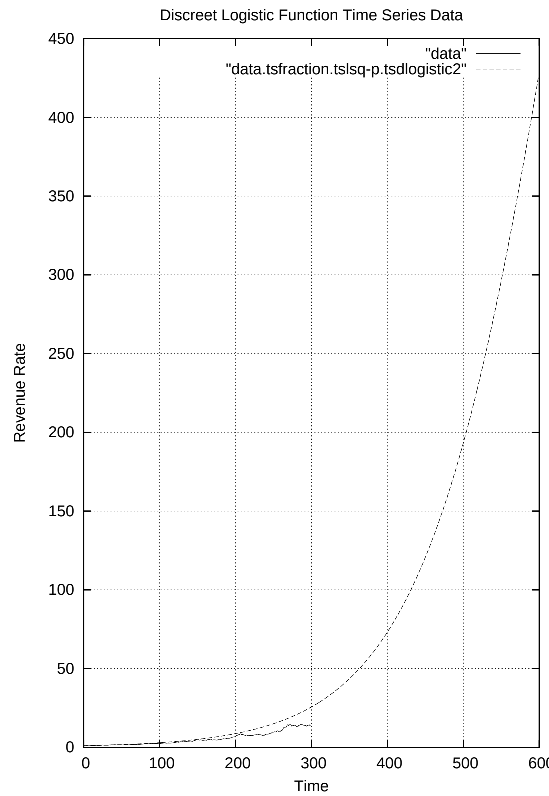

Figure C.496: Non-optimal Coins Tossing Game, logistic function estimates of Figure C.495 with the time scale expanded by a factor of two.

forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^H \tag{C.833}$$

$$\propto \quad (t_2 - t_1)^{0.849887} \tag{C.834}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.834 reduces to, [Sch91, pp. 129]:

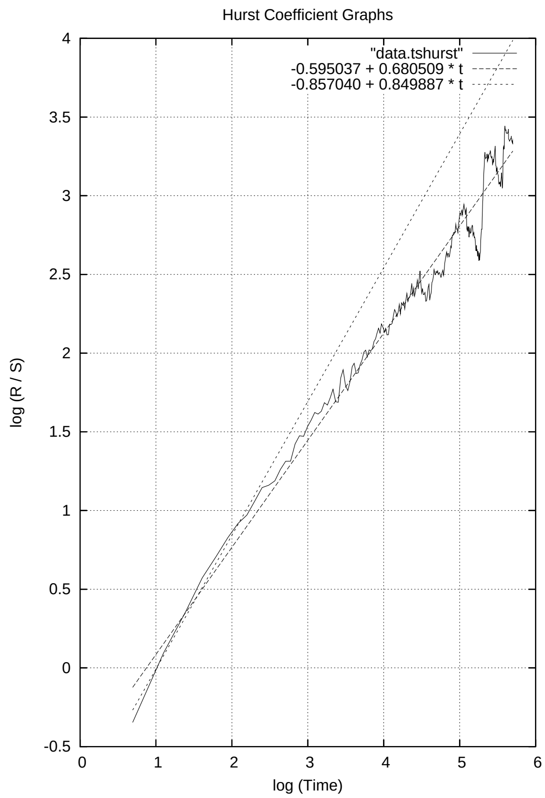

Figure C.497: Non-optimal Coins Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.485. The slope of the graph is the Hurst coefficient.

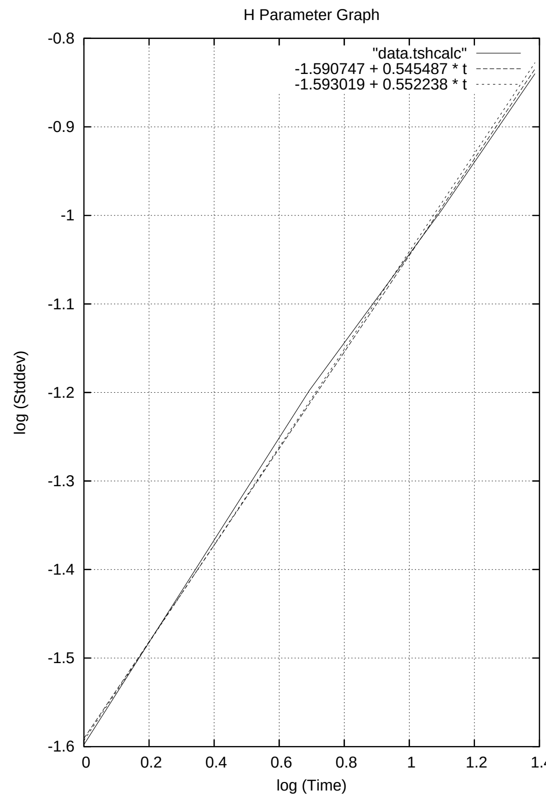

Figure C.498: Non-optimal Coins Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.485 The slope of the graph is the H parameter.

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.835}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.836}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\,(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.837}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[273].

[273]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.838}$$

$$\propto \frac{X(rt)}{r^{0.849887}} \tag{C.839}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.498, to provide a least squares approximation to the H parameter for the Non-optimal Coins Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.552238 for the near future, and 0.545487 for the distant future.

Figures C.497 and C.498 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.485. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.485, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.499 and C.500 was made using the -d option.

C.22.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.22.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.486. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.485. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Non-optimal Coins Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.485, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.009437 + 1)}{\ln(2)} = 0.013551 \tag{C.840}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.485, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.011001 + 1)}{\ln(2)} = 0.015784 \tag{C.841}$$

Note that if the mean is not constant in Figure C.485, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.484:

$$bits = 0.013218 \tag{C.842}$$

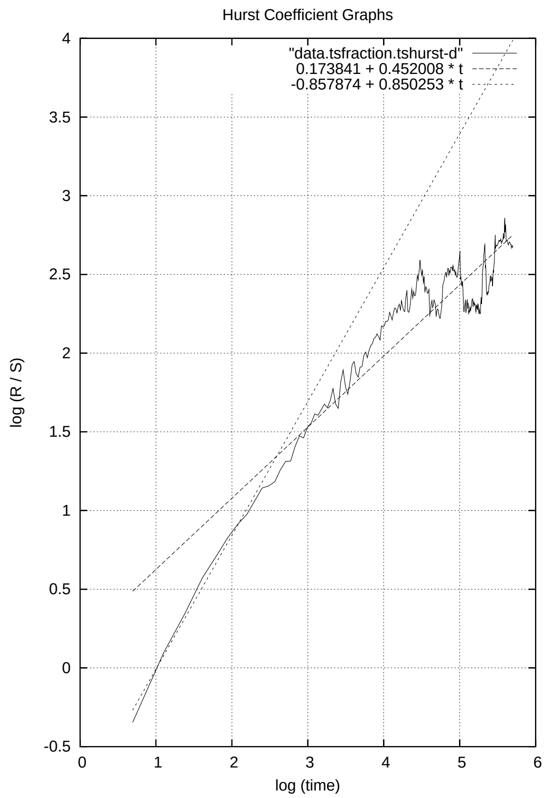

Figure C.499: Non-optimal Coins Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.484. The slope of the graph is the Hurst coefficient, and is 0.850253 for the near term, and 0.452008 for the far term.

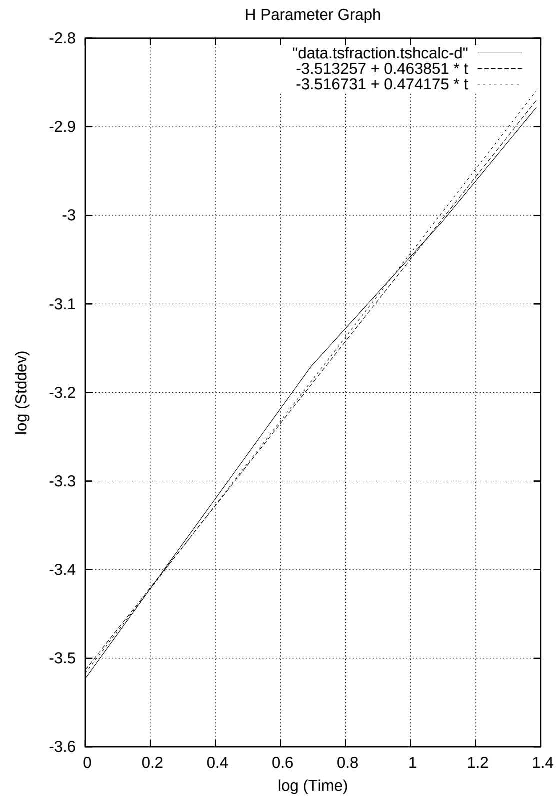

Figure C.500: Non-optimal Coins Tossing Game, traditional H parameter data for the time series data shown in Figure C.484 The slope of the graph is the H parameter, and is 0.474175 for the near term, and 0.463851 for the far term.

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.484:

$$bits = 0.012895 \tag{C.843}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.22.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.012895t} \tag{C.844}$$

therefore:

$$C(p) = 0.012895 \tag{C.845}$$

and, *tsshannon* 0.012895 gives:

$$C\,(0.566751) = 0.012895 \tag{C.846}$$

therefore:

$$
\begin{aligned}
2^{C(0.566751)} &= 2^{0.012895} && \text{(C.847)}\\
&= 1.008978 && \text{(C.848)}\\
&= 0.897820\% && \text{(C.849)}
\end{aligned}
$$

and:

$$
\begin{aligned}
2p - 1 &= (2 \cdot 0.566751) - 1 && \text{(C.850)}\\
&= 0.133502 && \text{(C.851)}\\
&= 13.350200\% && \text{(C.852)}
\end{aligned}
$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Non-optimal Coins Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 13.350200% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 86.649800% will be held in "reserve" with a 56.675100% chance of making twice the 13.350200% back, (and a 43.324900% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 0.897820%, or a doubling of its rate of revenue returns, (per tosses,) in 77.549438 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 13.350200% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.897820%, per tosses, on average.

Note that the metrics presented in this section are representative of the Non-optimal Coins Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 13.350200% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Non-optimal Coins Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.897820% per tosses.

As another simple example, a company re-invests 13.350200% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 13.350200% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.897820% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 13.350200% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 13.350200$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 13.350200$ percent for the second product, implying that the company should diversify its product line[274]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 13.350200%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Non-optimal Coins Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.133502}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.485, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 13.350200% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 13.350200% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.22.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Non-optimal Coins Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Non-optimal Coins Tossing Game time series is 0.009437, and 0.030881respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 9.895808.

If this value seems consistent number of companies in the Non-optimal Coins Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Non-optimal Coins Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.548572, which would be the value which should be used in Section C.22.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.22.5 is greater than the average Shannon probability for the companies participating in the Non-optimal Coins Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.22.5. The maximum exploitability for the Non-optimal Coins Tossing Game is derived in Section C.22.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Non-optimal Coins Tossing Game is 0.548572, with several alternative solutions listed in the previous paragraph. However, these should be

[274]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P^H_{total} = P^H_1 + P^H_2 + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

contrasted to the Shannon probability that maximizes a company's P&L which is 0.652796 in the Non-optimal Coins Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.853}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Non-optimal Coins Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.22.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.486. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.485. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.22.5, is derived from the Non-optimal Coins Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.22.9.

An additional exploitable strategy may be time itself. Equations C.830, C.834, and, C.832, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Non-optimal Coins Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.501, and, C.502 compare methods of approximation of the "forecastability" of rate of revenue returns in the Non-optimal Coins Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Non-optimal Coins Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[275]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.501, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory

[275] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

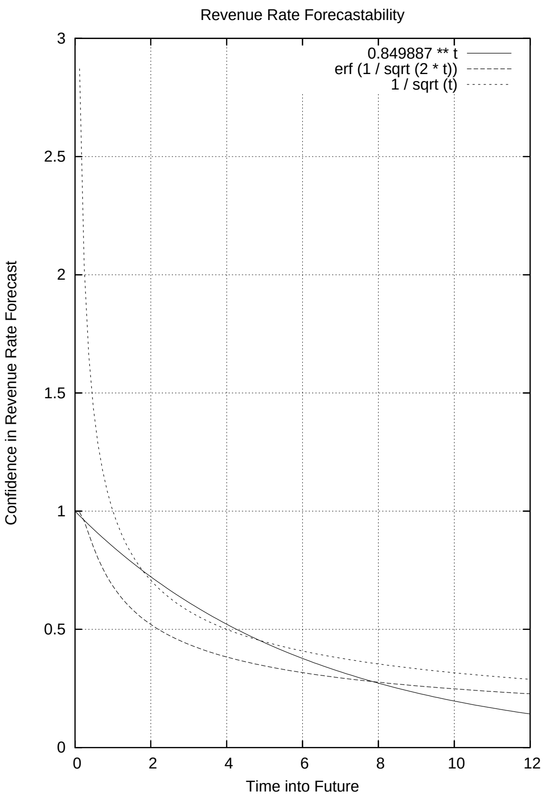

Figure C.501: Non-optimal Coins Tossing Game, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.849887^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

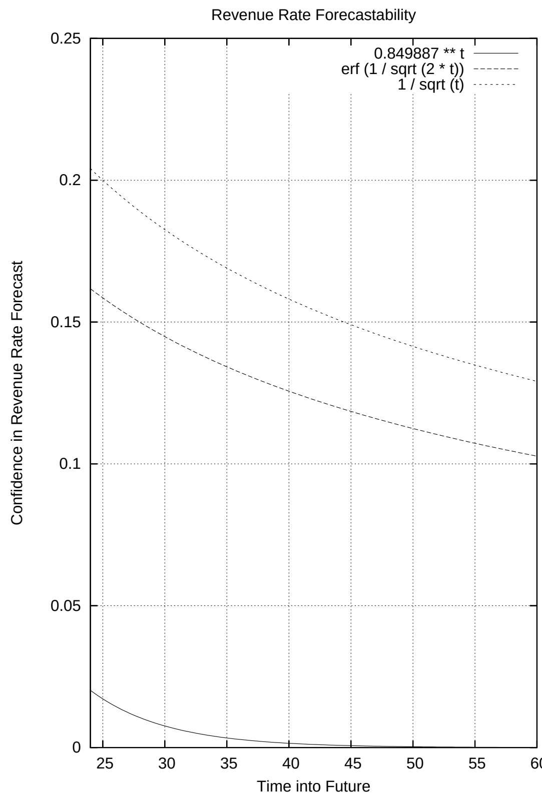

Figure C.502: Non-optimal Coins Tossing Game, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

levels that do not exceed, from Equation C.832, $0.849887^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.830, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.22.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.22.9. Figure C.503 represents a constructional simulation of the time series data presented in Figure C.484. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented

in Figure C.485. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.485 represent a relatively complex process, that may not be “modeled” with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.504 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.487.

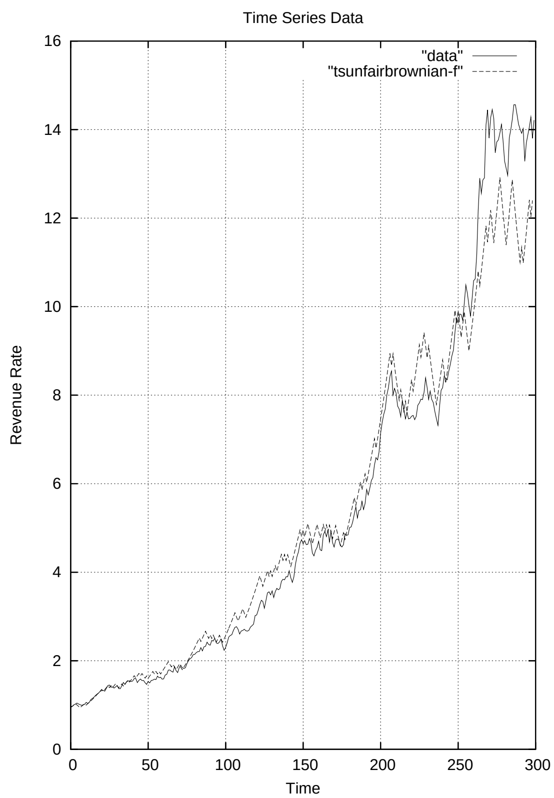

Figure C.503: Non-optimal Coins Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.030881. This data is superimposed on the data presented in Figure C.484.

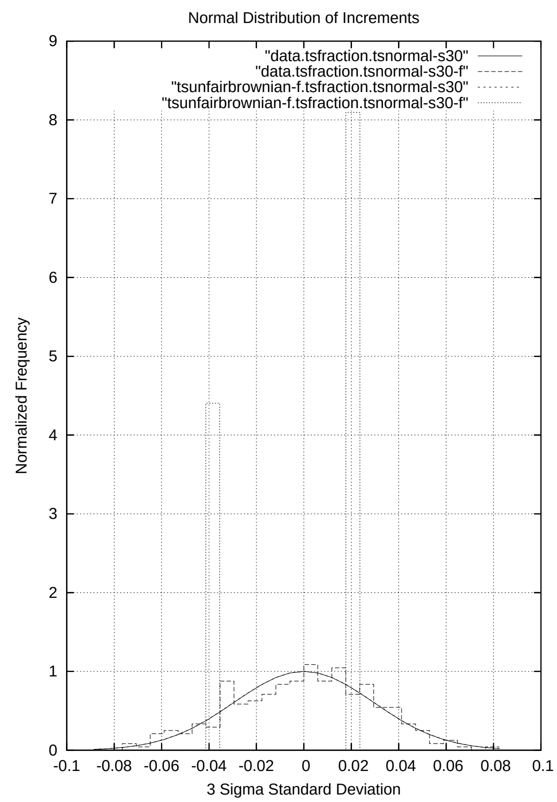

Figure C.504: Non-optimal Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.503, empirical and simulated. The empirical data has a mean of 0.009437, with a standard deviation of 0.029453. By comparison, the simulated data has a mean of 0.009119 with a standard deviation of 0.029553. This data is superimposed on the data presented in Figure C.487. The area under the four curves is identical.

C.22.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.22.3. One of the issues of analysis, as mentioned in Section C.22.7, is to determine the maximum Shannon probability for the time series presented in Figure C.484. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.505 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.484. Figure C.506 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.484. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.566751, as derived in Section C.22.5 to 0.646667. This process, essentially, extracts the random statistical data from the time series presented in Figure C.484, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.484, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Non-optimal Coins Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.645485, as compared with the predicted value from the program *tsshannonmax* of 0.646667.

C.22.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.486. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.485. These values will be used in a fixed increment Brownian fractal analysis of the Non-optimal Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.22.6 and D.22.7. As a subjective evaluation of the "quality" of the analysis of the Non-optimal Coins Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.484 from Figure C.485, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Coins Tossing Game movement was positive, as presented in Section C.22.9:

$$P \approx \frac{\frac{avg}{rms}+1}{2} \tag{C.854}$$

$$0.645485 \approx \frac{\frac{0.009437}{0.030881}+1}{2} \tag{C.855}$$

$$0.645485 \approx 0.652796 \tag{C.856}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.645485 \approx 0.652796 \approx 0.646667 \tag{C.857}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.22.5, should be compared. The four methods used were the mean of Figure C.485, the constant in the least squares approximation to Figure C.485, the least squares exponential approximation to Figure C.484, and the logarithmic returns of Figure C.484,

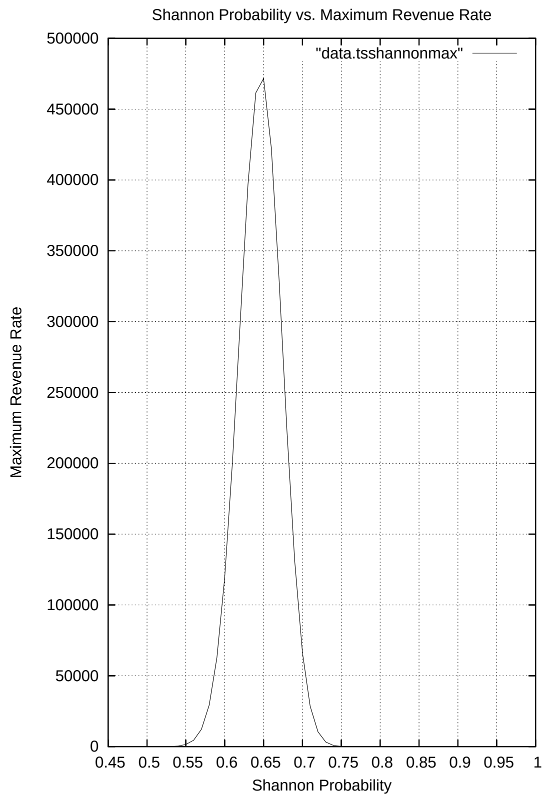

Figure C.505: Non-optimal Coins Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.646667.

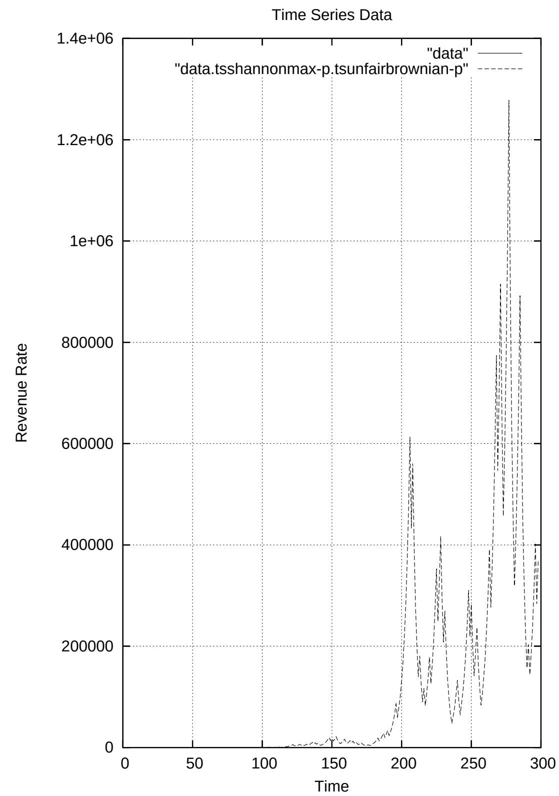

Figure C.506: Non-optimal Coins Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.646667, corresponding to a "wager" fraction of 0.293334.

derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.013551 \approx 0.015784 \approx 0.013218 \approx 0.012895 \tag{C.858}$$

It is implied in Section C.22.5, Subsection C.22.5 and in Section C.22.8 that, a Brownian motion with fixed increments fractal may "model" the Non-optimal Coins Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.859}$$

$$0.030881\,(2 \cdot 0.645485 - 1) \approx \frac{0.029453\,(2 \cdot 0.645485 - 1)}{2\sqrt{0.645485\,(1 - 0.645485)}} \tag{C.860}$$

$$0.030881 \cdot 0.290970 \approx 0.029453 \cdot 0.304129 \tag{C.861}$$

$$0.008985 \approx 0.008958 \tag{C.862}$$

and, equating to the mean:

$$0.009437 \approx 0.008985 \approx 0.008958 \tag{C.863}$$

where, as in Equation C.856 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.484 from Figure C.485, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Coins Tossing Game movement was positive, as presented in Section C.22.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[276], where the absolute value is presented in Figure C.486, and the root mean square value is presented in Figure C.485:

$$0.025378 \approx 0.030881 \tag{C.864}$$

Note, that if the Non-optimal Coins Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.484 from Figure C.485 should be zero. It is 0.017626.

C.23 Non-optimal Logistic Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins-b[277].

The data in this section is presented in tabular form in Section D.23. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Non-optimal Logistic Coins Tossing Game. This is included for "theoretical" comparative purposes.

C.23.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.23.1. Figure C.507 is a graph of the time series data for the Non-optimal Logistic Coins Tossing Game.

Figure C.508 is a graph of the normalized increments of the time series data presented in Figure C.507. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.509 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.508. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the

[276]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[277]As a simulation model, the program *tscoins* was run to make a time series data file, with the following parameters:

```
tscoins -p -b 0.00000005 0.6 -b 0.03 1000 > data
```

to make a time series of 1000 elements, with a Shannon probability of 0.6 and a known non-optimal investment strategy. The non-linearity term of the logistic function is 0.00000005. Otherwise, the first 300 elements of the simulation is approximately the same as in Section C.22. Note that there is some possibility that the analytical techniques used could be used to determine the maturity of an industrial market. See Chapter 2, Section 2.8. The data is by tosses.

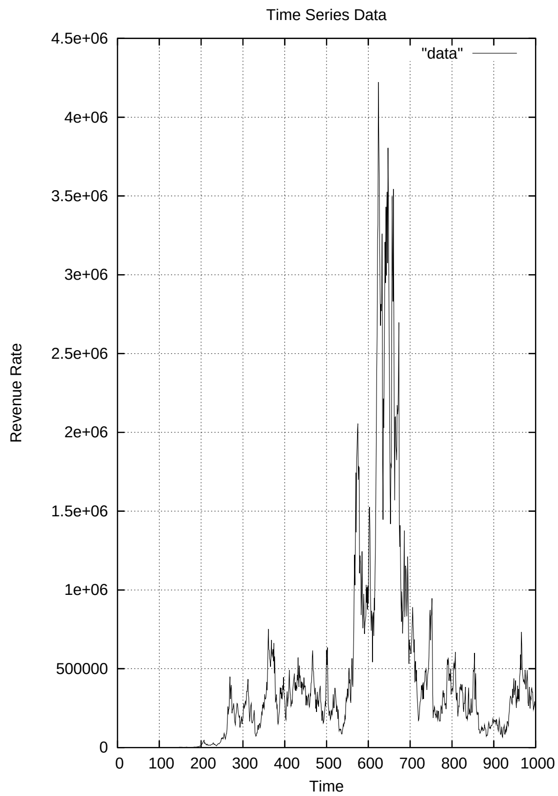

Figure C.507: Non-optimal Logistic Coins Tossing Game, time series data.

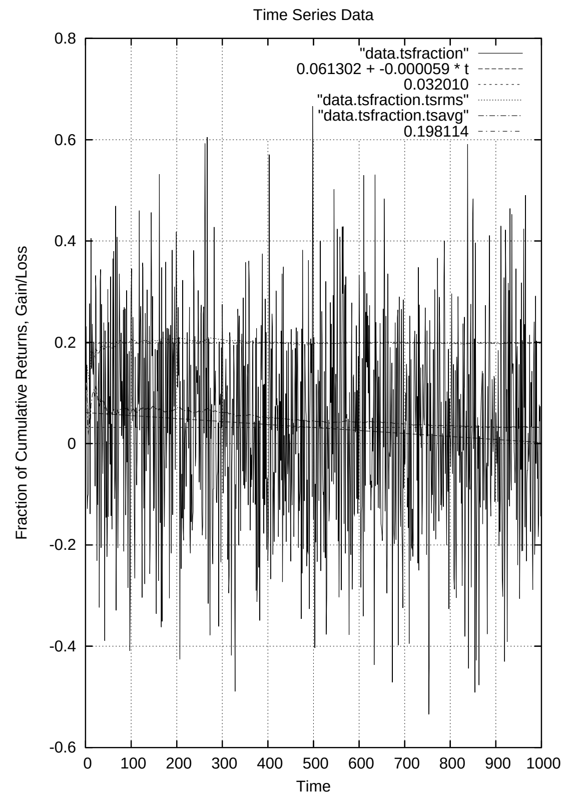

Figure C.508: Non-optimal Logistic Coins Tossing Game, normalized increments of the time series data presented in Figure C.507. The mean is 0.032010 with a standard deviation of 0.195609. The formula for the least squares approximation is $0.061302 + -0.000059t$, and the root mean squared value is 0.198114. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction-.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000059, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[278].

[278]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8

Figure C.510 is the normalized histogram of the normalized increments of the time series data shown in Figure C.508. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

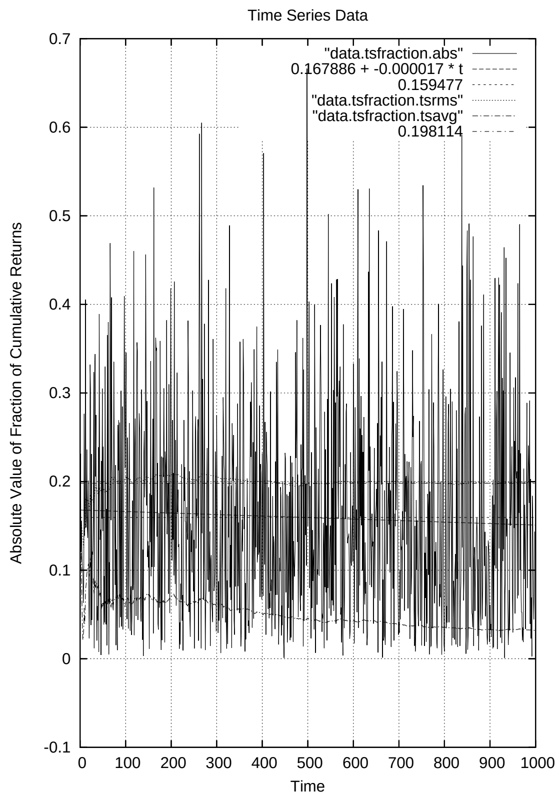

Figure C.509: Non-optimal Logistic Coins Tossing Game, absolute value of the normalized increments of the time series data presented in Figure C.508. The mean is 0.159477 with a standard deviation of 0.117600. The formula for the least squares approximation is $0.167886 + -0.000017t$, and the root mean square value, from Figure C.508, is 0.198114. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments presented in Figure C.508, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

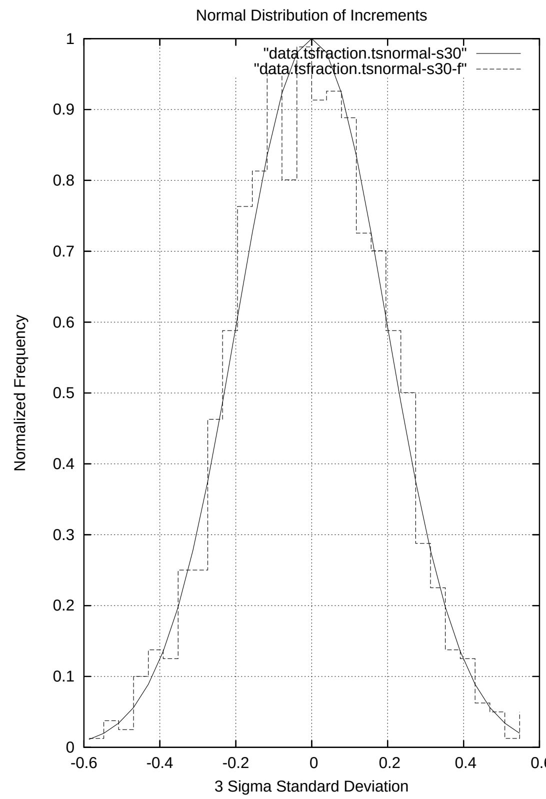

Figure C.510: Non-optimal Logistic Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.508. The data has a mean of 0.032010, with a standard deviation of 0.195609. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 0.367000, with a critical value of 42.557000.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.510.

depending on the accuracy of of “fit” to a Gaussian distribution.

Figure C.511 is the statistical estimate for the data presented in Figure C.508, as derived by the program *tsstatest*, which is briefly described in appendix B.

```
For a mean of 0.031978, with a confidence level of 0.900000
    that the error did not exceed 0.003198, 10385 samples would be required.
    (With 1000 samples, the estimated error is 0.010305 = 32.225289 percent.)
For a standard deviation of 0.198114, with a confidence level of 0.900000
    that the error did not exceed 0.019811, 136 samples would be required.
    (With 1000 samples, the estimated error is 0.007287 = 3.678005 percent.)
```

Figure C.511: Non-optimal Logistic Coins Tossing Game, statistical estimates of the normalized increments of the time series shown in Figure C.508. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.508.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.579290$, as derived in Section C.23.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.512 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.508. In principle, if the distribution of the normalized increments presented in Figure C.510 is Gaussian in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.513 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.508. In principle, if the distribution of the normalized increments presented in Figure C.510 is an integrated Gaussian distribution in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43.

Figure C.514 is the range of values of the time series shown in Figure C.507. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.514 would be a square root function[279]. Figure C.515 is the deterministic map of the normalized increments of the time series data shown in Figure C.508. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.510 would seem to indicate that the time series data for the Non-optimal Logistic Coins Tossing Game represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.23.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

[279] Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.514 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

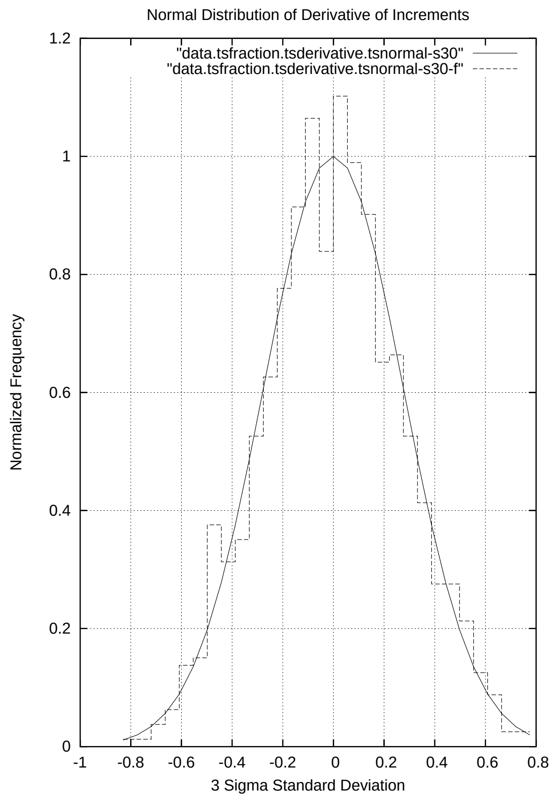

Figure C.512: Non-optimal Logistic Coins Tossing Game, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.508.

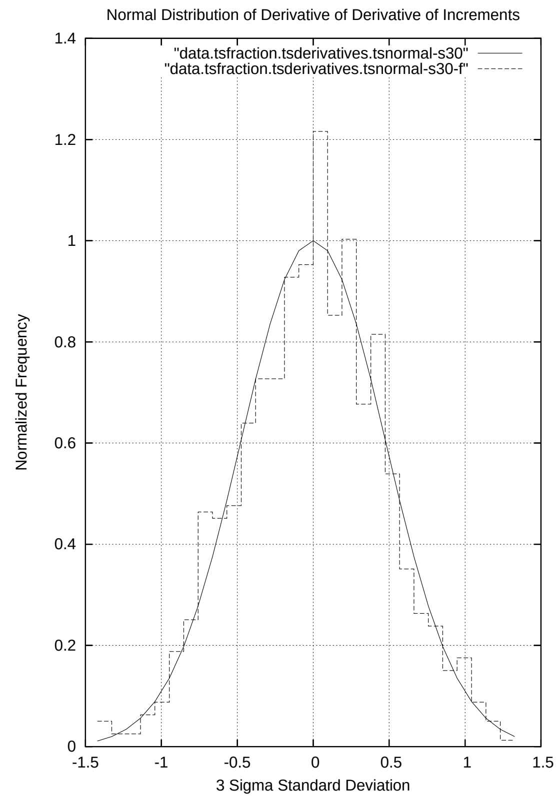

Figure C.513: Non-optimal Logistic Coins Tossing Game, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.508.

mean square of the instantaneous fraction of change[280]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.516 is the instantaneous value of the root mean square of the normalized increments for the Non-optimal Logistic Coins Tossing Game, and Figure C.517 is the instantaneous Shannon probability for the normalized increments.

C.23.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.23.4. Figure C.518 is a graph of the logistic function estimates of the time series data for the Non-optimal Logistic Coins Tossing Game. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require

[280] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

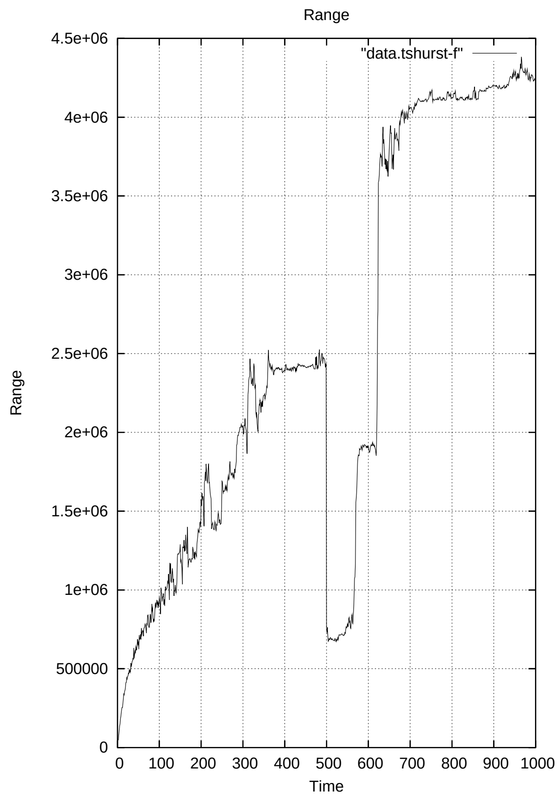

Figure C.514: Non-optimal Logistic Coins Tossing Game, range of the time series data shown in Figure C.507.

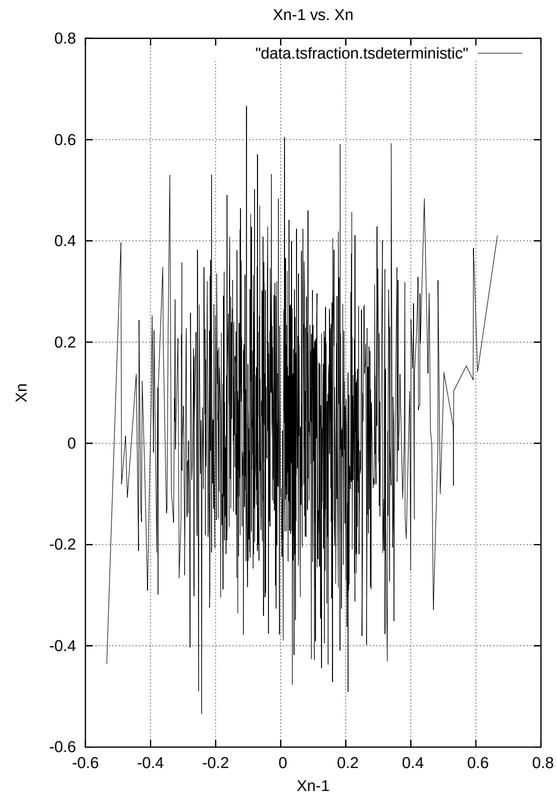

Figure C.515: Non-optimal Logistic Coins Tossing Game, deterministic map of the normalized increments of the time series data shown in Figure C.508.

intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[281]. The methodology should be regarded as “fragile.” It is included for completeness.

Figure C.518 is a graph of the logistic function for the time series data presented in Figure C.507. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.508. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.508. Figure C.519 is the same graph, but with the time scale expanded by a factor of two.

C.23.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.23.5. Figure C.520 is a graph of the Hurst coefficient data time series data shown in Figure C.507. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

[281]For example, in Figures C.518 and C.519, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.23.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

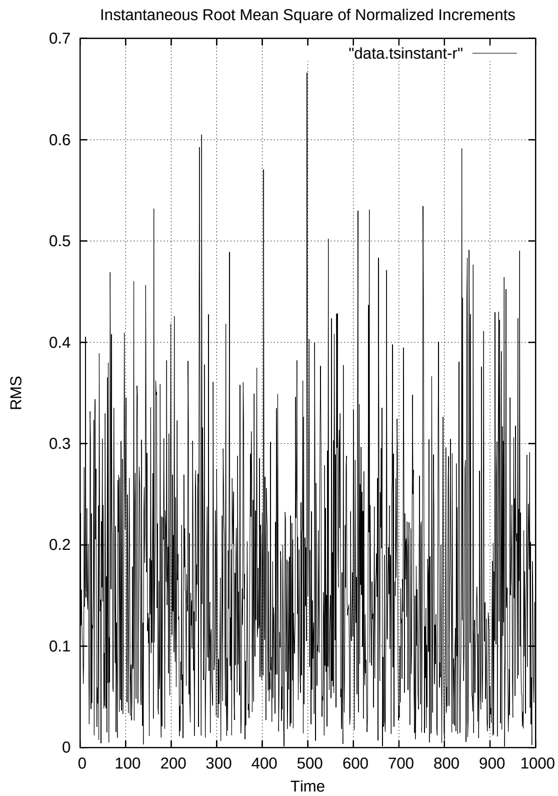

Figure C.516: Non-optimal Logistic Coins Tossing Game, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.507.

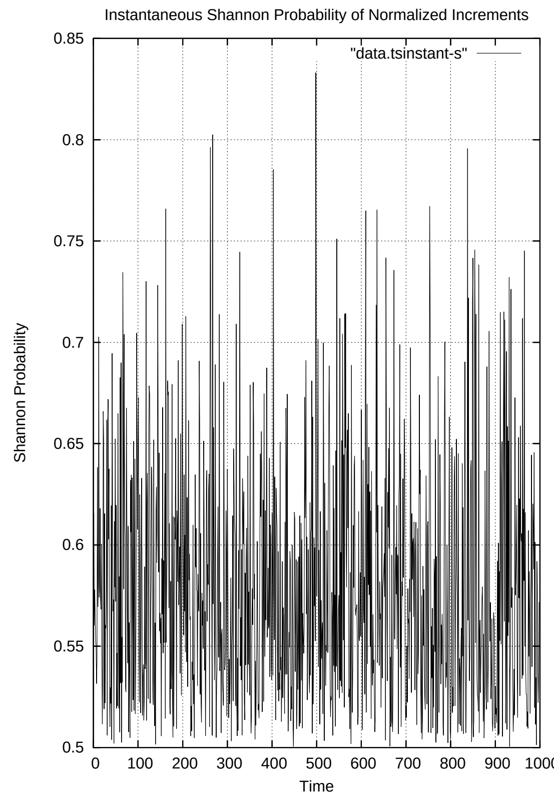

Figure C.517: Non-optimal Logistic Coins Tossing Game, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.507.

Figure C.521 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.508. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.520 implies that the variance of the rate of revenue returns, (per tosses,) in the Non-optimal Logistic Coins Tossing Game, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[282], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.524, and, C.525 compare methods of approximation of

[282]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

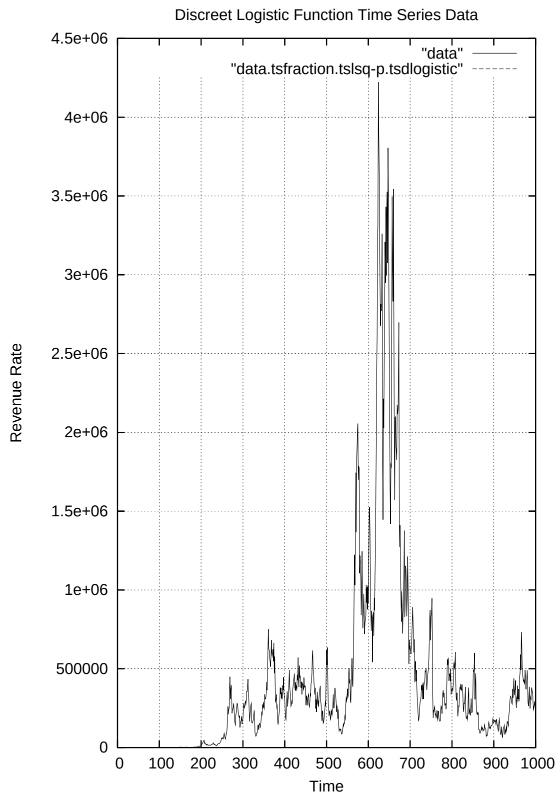

Figure C.518: Non-optimal Logistic Coins Tossing Game, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.508 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

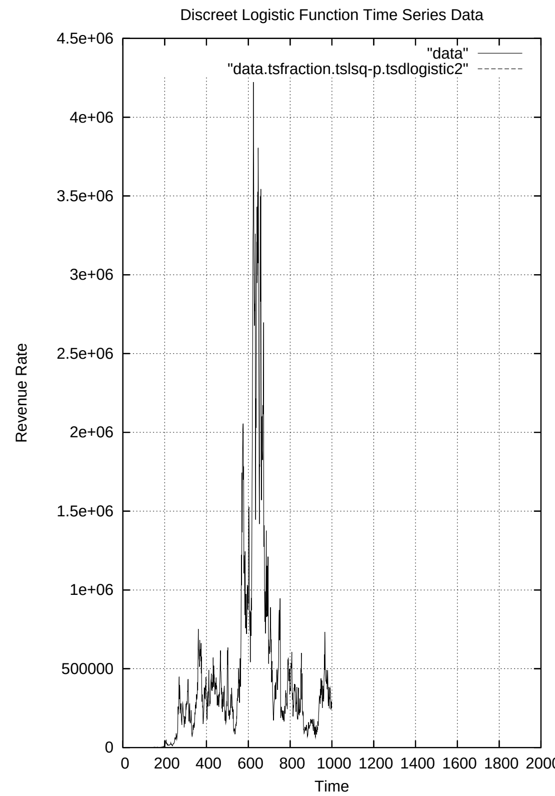

Figure C.519: Non-optimal Logistic Coins Tossing Game, logistic function estimates of Figure C.518 with the time scale expanded by a factor of two.

the "forecastability" of the rate of revenue returns in the Non-optimal Logistic Coins Tossing Game for the near term and far term, respectively [Pet91, pp. 83-84][283]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per tosses.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.520, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.842100, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.865}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.842100} \tag{C.866}$$

[283] The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

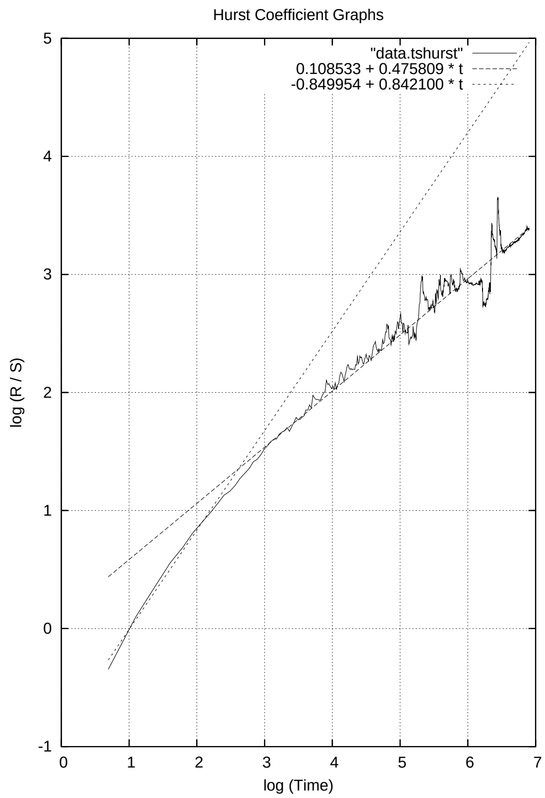

Figure C.520: Non-optimal Logistic Coins Tossing Game, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.508. The slope of the graph is the Hurst coefficient.

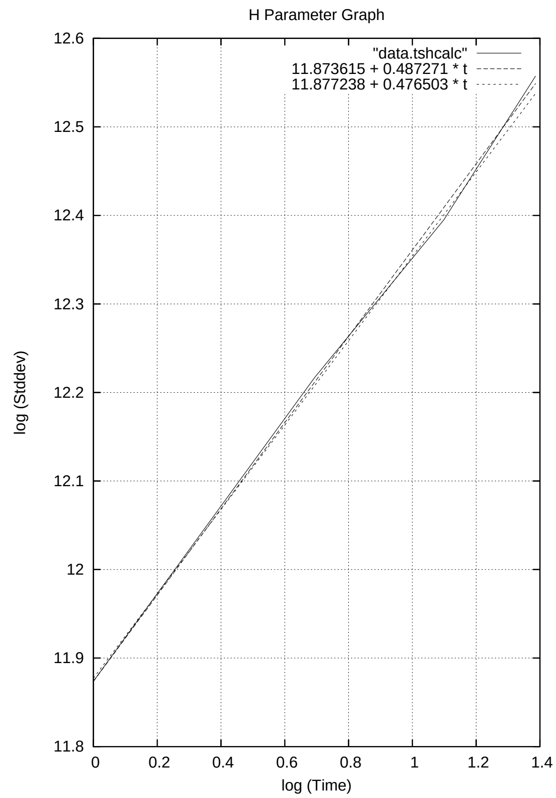

Figure C.521: Non-optimal Logistic Coins Tossing Game, H parameter data for the normalized increments of the time series data shown in Figure C.508 The slope of the graph is the H parameter.

$$\propto \quad (t_2 - t_1)^{1.684200} \tag{C.867}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per tosses,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[284]. A Hurst coefficient of 0.842100, (for the near future, and 0.475809 for the distant future.) implies that the likelihood of the rate of revenue returns, (per tosses,) for any two consecutive tossess being the same is

[284]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.23.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Non-optimal Logistic Coins Tossing Game. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

84.210000% [Pet91, pp. 66] for the near future, and 0.475809 for the distant future. Likewise, there is a 84.210000% chance of the rate of revenue returns, (per tosses,) movements being the same in consecutive time periods—ie., if, in a given tosses, the rate of revenue returns, (per tosses,) is increasing, there is a 84.210000% that the rate of revenue returns, (per tosses,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per tosses,) for the Non-optimal Logistic Coins Tossing Game are over time, since the probability of having n many consecutive tossess of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many tossess of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.868}$$
$$= 0.842100^n \tag{C.869}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.508, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next tosses's rate of revenue returns would be the same as the current tosses's revenue rate. Interestingly, it is $0.032010 \cdot 100$ percent, on the average, with a standard deviation of $0.195609 \cdot 100$ percent, and a root mean square error value of $0.198114 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per tosses,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.870}$$
$$\propto (t_2 - t_1)^{0.842100} \tag{C.871}$$

where R is the range of values in the increments of the rate of revenue returns, (per tosses.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per tosses,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per tosses) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.871 reduces to, [Sch91, pp. 129]:

$$R(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.872}$$
$$\propto \sqrt{(t_2 - t_1)} \tag{C.873}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per tosses,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.874}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per tosses,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[285].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

[285] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.875}$$

$$\propto \frac{X(rt)}{r^{0.842100}} \tag{C.876}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.521, to provide a least squares approximation to the H parameter for the Non-optimal Logistic Coins Tossing Game. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.476503 for the near future, and 0.487271 for the distant future.

Figures C.520 and C.521 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.508. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.508, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.522 and C.523 was made using the -d option.

C.23.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.23.2. This section derives various values based on the “average” of the normalized increments presented in Figure C.509. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.508. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Logistic Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Non-optimal Logistic Coins Tossing Game, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.508, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.032010 + 1)}{\ln(2)} = 0.045457 \tag{C.877}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.508, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.061302 + 1)}{\ln(2)} = 0.085835 \tag{C.878}$$

Note that if the mean is not constant in Figure C.508, this method will not provide accurate results.

And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.507:

$$bits = 0.012612 \tag{C.879}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.507:

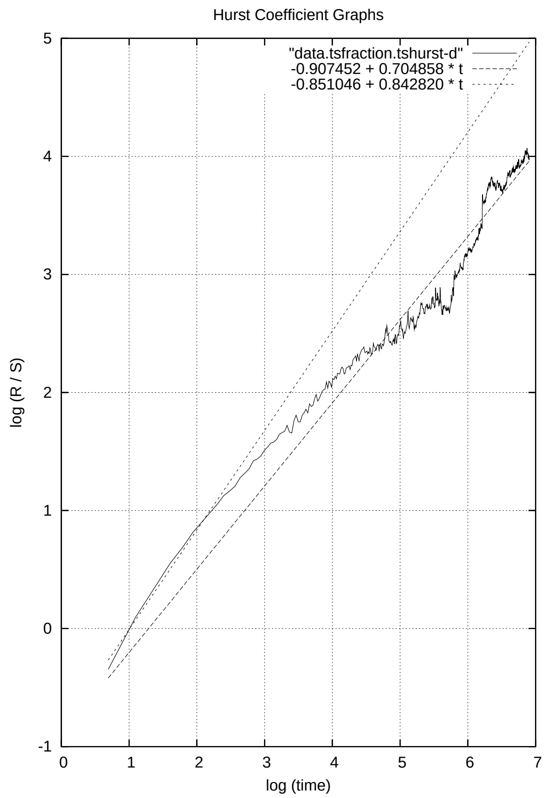

Figure C.522: Non-optimal Logistic Coins Tossing Game, traditional Hurst coefficient data for the time series data shown in Figure C.507. The slope of the graph is the Hurst coefficient, and is 0.842820 for the near term, and 0.704858 for the far term.

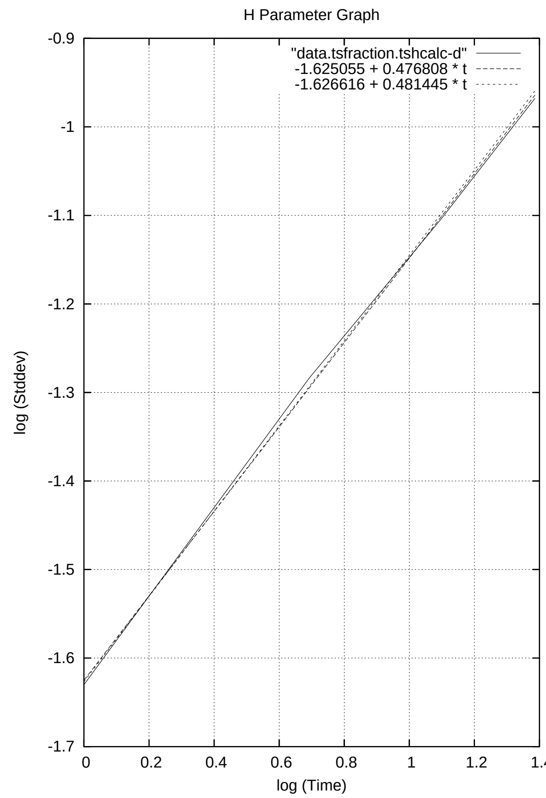

Figure C.523: Non-optimal Logistic Coins Tossing Game, traditional H parameter data for the time series data shown in Figure C.507 The slope of the graph is the H parameter, and is 0.481445 for the near term, and 0.476808 for the far term.

$$bits = 0.018217 \tag{C.880}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.23.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.018217t} \tag{C.881}$$

therefore:

$$C\left(p\right) = 0.018217 \tag{C.882}$$

and, *tsshannon* 0.018217 gives:

$$C\left(0.579290\right) = 0.018217 \tag{C.883}$$

therefore:

$$2^{C(0.579290)} = 2^{0.018217} \quad (C.884)$$
$$= 1.012707 \quad (C.885)$$
$$= 1.270712\% \quad (C.886)$$

and:

$$2p - 1 = (2 \cdot 0.579290) - 1 \quad (C.887)$$
$$= 0.158580 \quad (C.888)$$
$$= 15.858000\% \quad (C.889)$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Non-optimal Logistic Coins Tossing Game executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every tosses, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 15.858000% of its rate of revenue returns, (per tosses.) As a conceptual model, the remaining 84.142000% will be held in "reserve" with a 57.929000% chance of making twice the 15.858000% back, (and a 42.071000% chance of making 0.0,) in one tosses, on the average, for an average growth in its rate of revenue returns, (per tosses,) of 1.270712%, or a doubling of its rate of revenue returns, (per tosses,) in 54.893781 tossess.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 15.858000% per tosses of the rate of revenue returns, (per tosses,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.270712%, per tosses, on average.

Note that the metrics presented in this section are representative of the Non-optimal Logistic Coins Tossing Game as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 15.858000% of its rate of revenue returns, (per tosses,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some tossess, depending on the Non-optimal Logistic Coins Tossing Game's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other tossess, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.270712% per tosses.

As another simple example, a company re-invests 15.858000% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 15.858000% per tosses investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.270712% per tosses.

As an example of "product portfolio" management, suppose a company re-invests 15.858000% of its rate of revenue returns, (per tosses,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 15.858000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 15.858000$ percent for the second product, implying that the company should diversify its product line[286]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then

[286] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near

the re-investment in both products should total the 15.858000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65-1)}{(2 \cdot 0.55-1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Non-optimal Logistic Coins Tossing Game, as a standard bench mark, then the optimal number will be $\frac{1}{0.158580}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.508, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 15.858000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 15.858000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.23.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Non-optimal Logistic Coins Tossing Game, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Non-optimal Logistic Coins Tossing Game time series is 0.032010, and 0.198114respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.815559.

If this value seems consistent number of companies in the Non-optimal Logistic Coins Tossing Game, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Non-optimal Logistic Coins Tossing Game are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.589457, which would be the value which should be used in Section C.23.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.23.5 is greater than the average Shannon probability for the companies participating in the Non-optimal Logistic Coins Tossing Game, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.23.5. The maximum exploitability for the Non-optimal Logistic Coins Tossing Game is derived in Section C.23.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Non-optimal Logistic Coins Tossing Game is 0.589457, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.580787 in the Non-optimal

term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Logistic Coins Tossing Game. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.890}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Non-optimal Logistic Coins Tossing Game would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.23.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.509. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.508. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Non-optimal Logistic Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.23.5, is derived from the Non-optimal Logistic Coins Tossing Game metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.23.9.

An additional exploitable strategy may be time itself. Equations C.867, C.871, and, C.869, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Non-optimal Logistic Coins Tossing Game, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.524, and, C.525 compare methods of approximation of the "forecastability" of rate of revenue returns in the Non-optimal Logistic Coins Tossing Game for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Non-optimal Logistic Coins Tossing Game. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[287]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.524, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over

[287]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

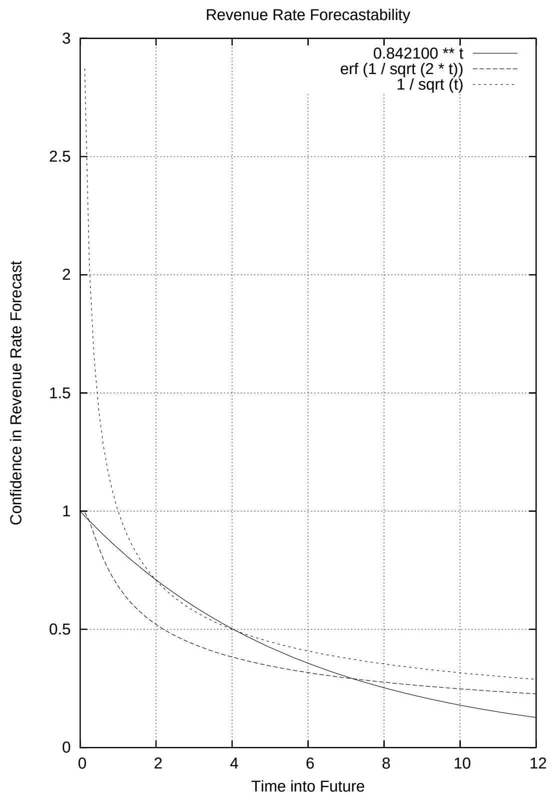

Figure C.524: Non-optimal Logistic Coins Tossing Game, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.842100^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

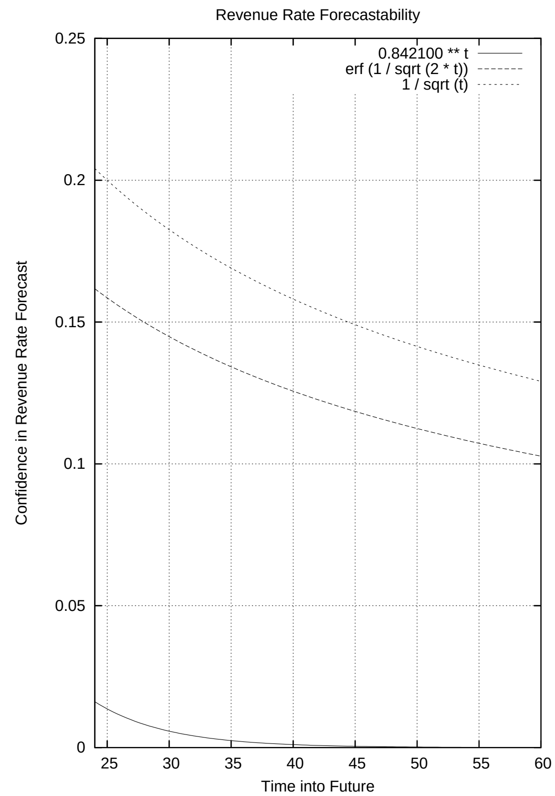

Figure C.525: Non-optimal Logistic Coins Tossing Game, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.869, $0.842100^n = 0.5$ tossess of operations. Since the optimal amount of inventory and, from Equation C.867, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.23.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.23.9. Figure C.526 represents a constructional simulation of the time series data presented in Figure C.507. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—

the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.508. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.508 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.527 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.510.

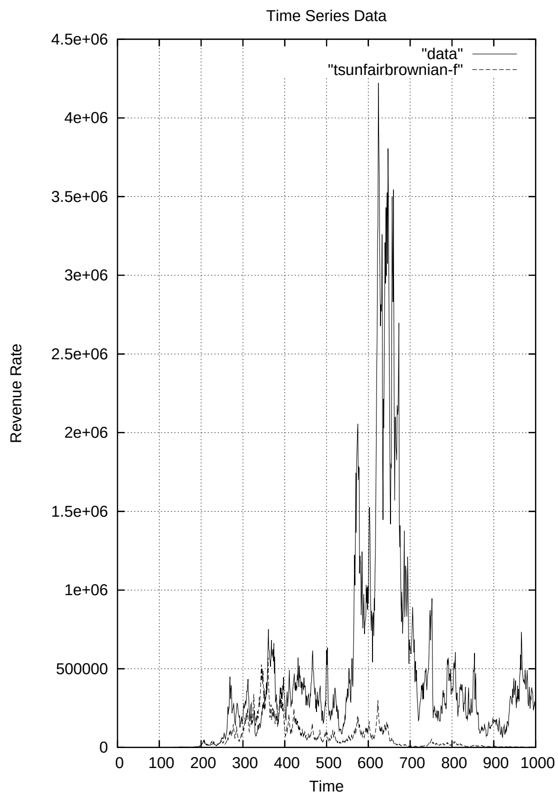

Figure C.526: Non-optimal Logistic Coins Tossing Game, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.198114. This data is superimposed on the data presented in Figure C.507.

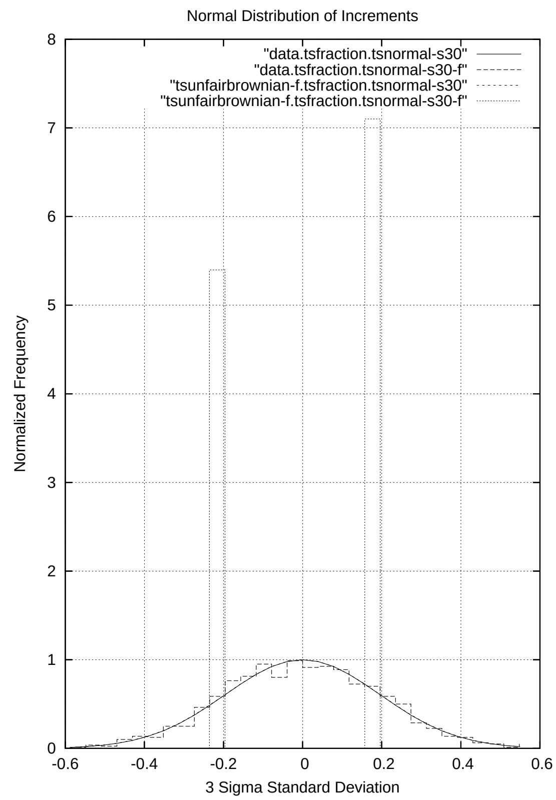

Figure C.527: Non-optimal Logistic Coins Tossing Game, normalized histogram of the normalized increments of the time series data shown in Figure C.526, empirical and simulated. The empirical data has a mean of 0.032010, with a standard deviation of 0.195609. By comparison, the simulated data has a mean of 0.026997 with a standard deviation of 0.196364. This data is superimposed on the data presented in Figure C.510. The area under the four curves is identical.

C.23.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.23.3. One of the issues of analysis, as mentioned in Section C.23.7, is to determine the maximum Shannon probability for the time series presented in Figure C.507. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.528 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.507. Figure C.529 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.507. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.579290, as derived in Section C.23.5 to 0.568000. This process, essentially, extracts the random statistical data from the time series presented in Figure C.507, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.507, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of tossess that the Non-optimal Logistic Coins Tossing Game movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.567568, as compared with the predicted value from the program *tsshannonmax* of 0.568000.

C.23.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.509. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.508. These values will be used in a fixed increment Brownian fractal analysis of the Non-optimal Logistic Coins Tossing Game, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.23.6 and D.23.7. As a subjective evaluation of the "quality" of the analysis of the Non-optimal Logistic Coins Tossing Game, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.507 from Figure C.508, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Logistic Coins Tossing Game movement was positive, as presented in Section C.23.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.891}$$

$$0.567568 \approx \frac{\frac{0.032010}{0.198114} + 1}{2} \tag{C.892}$$

$$0.567568 \approx 0.580787 \tag{C.893}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.567568 \approx 0.580787 \approx 0.568000 \tag{C.894}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.23.5, should be compared. The four methods used were the mean of Figure C.508, the constant in the least squares approximation to Figure C.508, the least squares exponential approximation to Figure C.507, and the logarithmic returns of Figure C.507,

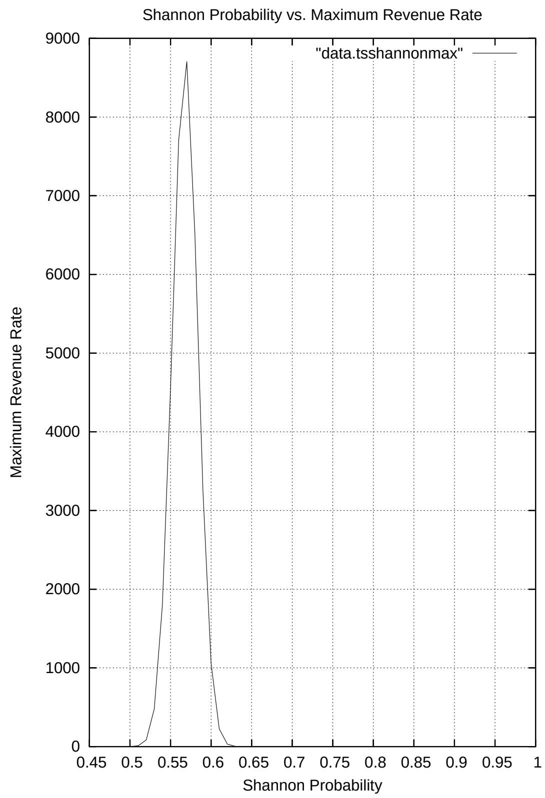

Figure C.528: Non-optimal Logistic Coins Tossing Game, maximum rate of revenue returns, per tosses, vs. Shannon probability. The maximum rate of revenue returns, per tosses, occurs at a Shannon probability of 0.568000.

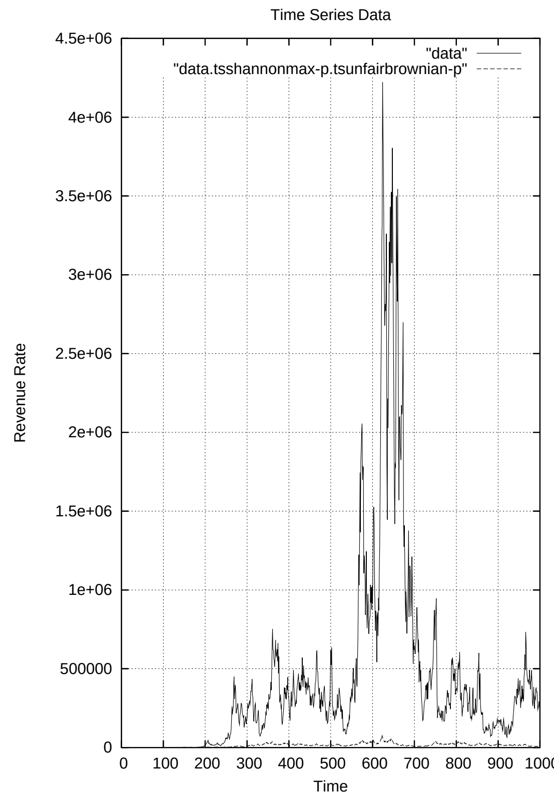

Figure C.529: Non-optimal Logistic Coins Tossing Game, maximum rate of revenue returns, per tosses, at a Shannon probability, of 0.568000, corresponding to a "wager" fraction of 0.136000.

derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.045457 \approx 0.085835 \approx 0.012612 \approx 0.018217 \tag{C.895}$$

It is implied in Section C.23.5, Subsection C.23.5 and in Section C.23.8 that, a Brownian motion with fixed increments fractal may "model" the Non-optimal Logistic Coins Tossing Game. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.896}$$

$$0.198114\,(2 \cdot 0.567568 - 1) \approx \frac{0.195609\,(2 \cdot 0.567568 - 1)}{2\sqrt{0.567568\,(1-0.567568)}} \tag{C.897}$$

$$0.198114 \cdot 0.135135 \approx 0.195609 \cdot 0.136386 \tag{C.898}$$

$$0.026772 \approx 0.026678 \tag{C.899}$$

and, equating to the mean:

$$0.032010 \approx 0.026772 \approx 0.026678 \tag{C.900}$$

where, as in Equation C.893 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.507 from Figure C.508, and the Shannon probability as calculated by counting the total number of tossess that the Non-optimal Logistic Coins Tossing Game movement was positive, as presented in Section C.23.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[288], where the absolute value is presented in Figure C.509, and the root mean square value is presented in Figure C.508:

$$0.159477 \approx 0.198114 \tag{C.901}$$

Note, that if the Non-optimal Logistic Coins Tossing Game could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.507 from Figure C.508 should be zero. It is 0.117600.

C.24 Simulated Industrial Market

For the analysis, the data was in the directory ../markets/tsmarket[289].

The data in this section is presented in tabular form in Section D.24. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Simulated Industrial Market. This is included for "theoretical" comparative purposes.

C.24.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.24.1. Figure C.530 is a graph of the time series data for the Simulated Industrial Market.

Figure C.531 is a graph of the normalized increments of the time series data presented in Figure C.530. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.532 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.531. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the

[288] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[289] As a simulation model, the program *tsmarket* was run to make a time series data file, with the following parameters:

```
tsmarket -p 0.55 -c 11 300 > data
```

to make a time series of 300 elements, with a Shannon probability of 0.55, and 11 companies participating in the market, each with equal market share, and operating optimally. The data is by months.

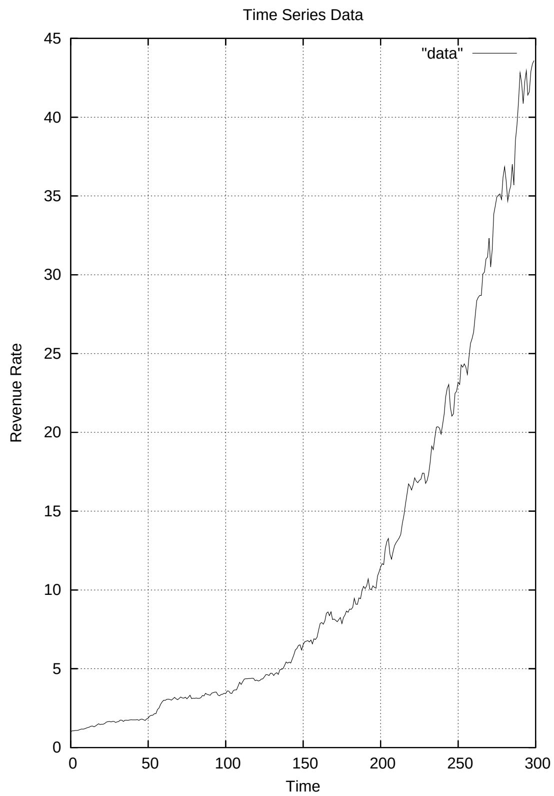

Figure C.530: Simulated Industrial Market, time series data.

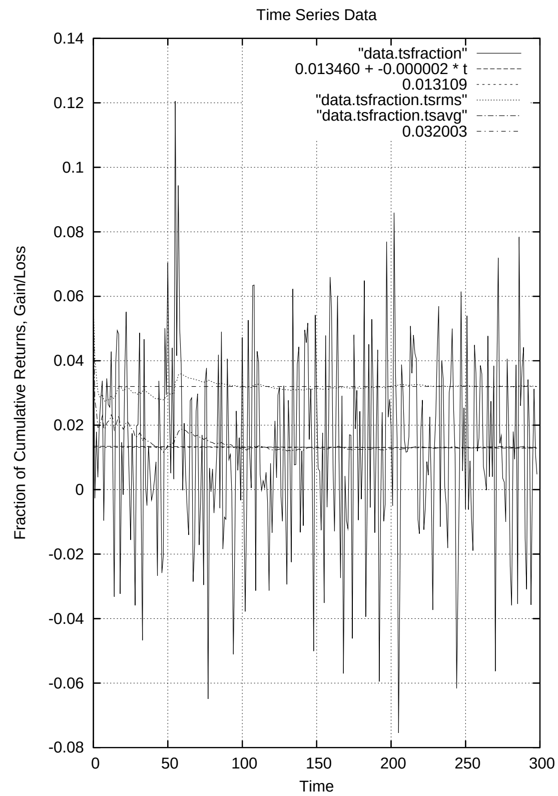

Figure C.531: Simulated Industrial Market, normalized increments of the time series data presented in Figure C.530. The mean is 0.013109 with a standard deviation of 0.029243. The formula for the least squares approximation is $0.013460 + -0.000002t$, and the root mean squared value is 0.032003. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.000002, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[290].

[290]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8

Figure C.533 is the normalized histogram of the normalized increments of the time series data shown in Figure C.531. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

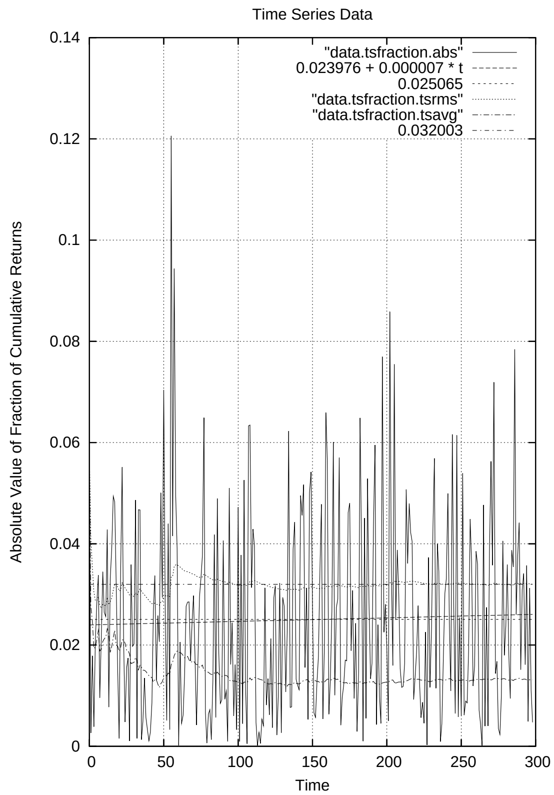

Figure C.532: Simulated Industrial Market, absolute value of the normalized increments of the time series data presented in Figure C.531. The mean is 0.025065 with a standard deviation of 0.019930. The formula for the least squares approximation is $0.023976 + 0.000007t$, and the root mean square value, from Figure C.531, is 0.032003. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.531, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

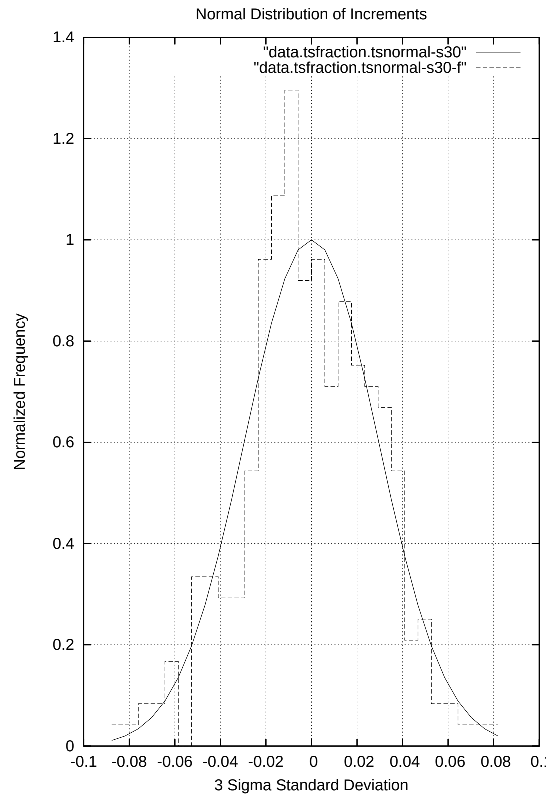

Figure C.533: Simulated Industrial Market, normalized histogram of the normalized increments of the time series data shown in Figure C.531. The data has a mean of 0.013109, with a standard deviation of 0.029243. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 1.229000, with a critical value of 42.557000.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.533.

depending on the accuracy of of "fit" to a Gaussian distribution.

Figure C.534 is the statistical estimate for the data presented in Figure C.531, as derived by the program *tsstatest*, which is briefly described in appendix B.

```
For a mean of 0.013065, with a confidence level of 0.900000
    that the error did not exceed 0.001307, 1624 samples would be required.
    (With 300 samples, the estimated error is 0.003039 = 23.261392 percent.)
For a standard deviation of 0.032003, with a confidence level of 0.900000
    that the error did not exceed 0.003200, 136 samples would be required.
    (With 300 samples, the estimated error is 0.002149 = 6.715087 percent.)
```

Figure C.534: Simulated Industrial Market, statistical estimates of the normalized increments of the time series shown in Figure C.531. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.531.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.579097$, as derived in Section C.24.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.535 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.531. In principle, if the distribution of the normalized increments presented in Figure C.533 is Gaussian in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.536 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.531. In principle, if the distribution of the normalized increments presented in Figure C.533 is an integrated Gaussian distribution in nature, this distribution would be similar to “white noise,” as presented in appendix B, Figure B.43.

Figure C.537 is the range of values of the time series shown in Figure C.530. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.537 would be a square root function[291]. Figure C.538 is the deterministic map of the normalized increments of the time series data shown in Figure C.531. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.533 would seem to indicate that the time series data for the Simulated Industrial Market represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

C.24.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

[291] Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.537 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

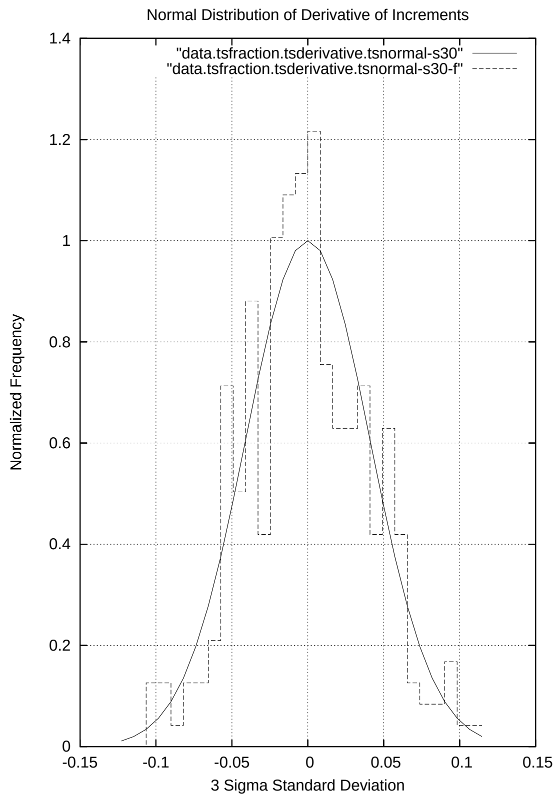

Figure C.535: Simulated Industrial Market, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.531.

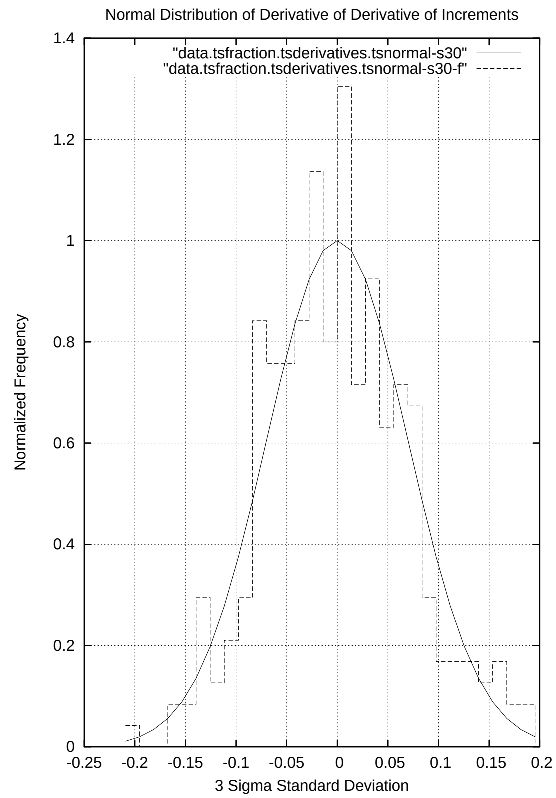

Figure C.536: Simulated Industrial Market, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.531.

mean square of the instantaneous fraction of change[292]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.539 is the instantaneous value of the root mean square of the normalized increments for the Simulated Industrial Market, and Figure C.540 is the instantaneous Shannon probability for the normalized increments.

C.24.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.24.4. Figure C.541 is a graph of the logistic function estimates of the time series data for the Simulated Industrial Market. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require

[292]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

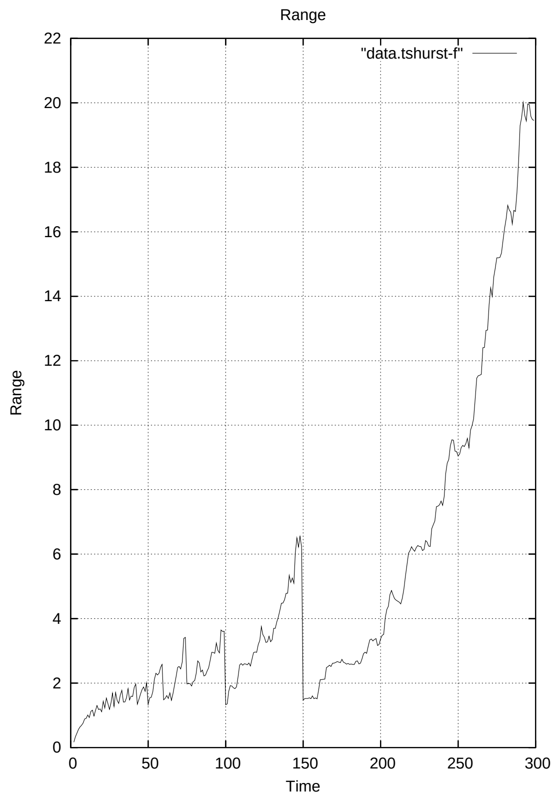

Figure C.537: Simulated Industrial Market, range of the time series data shown in Figure C.530.

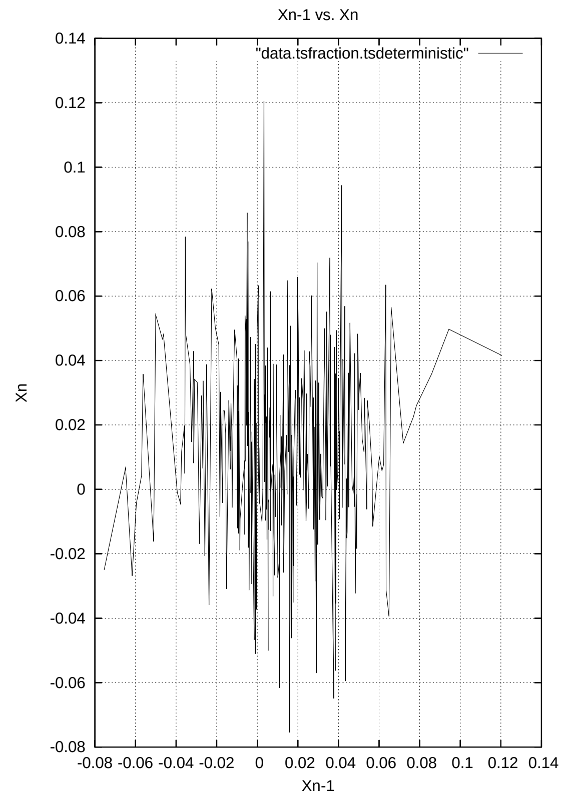

Figure C.538: Simulated Industrial Market, deterministic map of the normalized increments of the time series data shown in Figure C.531.

intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[293]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.541 is a graph of the logistic function for the time series data presented in Figure C.530. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.531. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.531. Figure C.542 is the same graph, but with the time scale expanded by a factor of two.

C.24.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.24.5. Figure C.543 is a graph of the Hurst coefficient data time series data shown in Figure C.530. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

[293]For example, in Figures C.541 and C.542, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.24.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

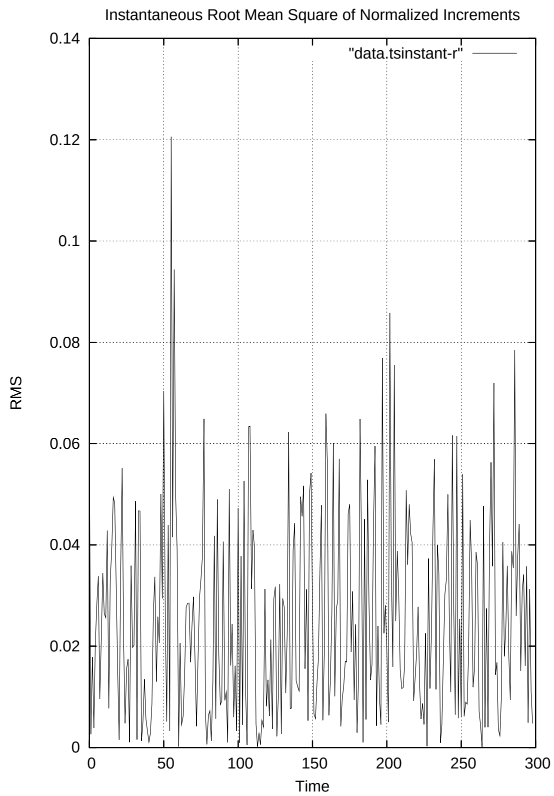

Figure C.539: Simulated Industrial Market, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.530.

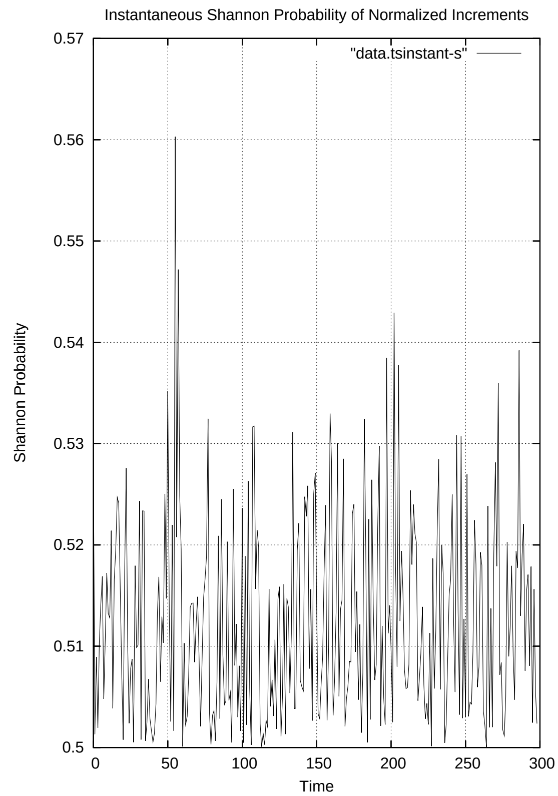

Figure C.540: Simulated Industrial Market, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.530.

Figure C.544 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.531. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.543 implies that the variance of the rate of revenue returns, (per month,) in the Simulated Industrial Market, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[294], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.547, and, C.548 compare methods of approximation of

[294]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

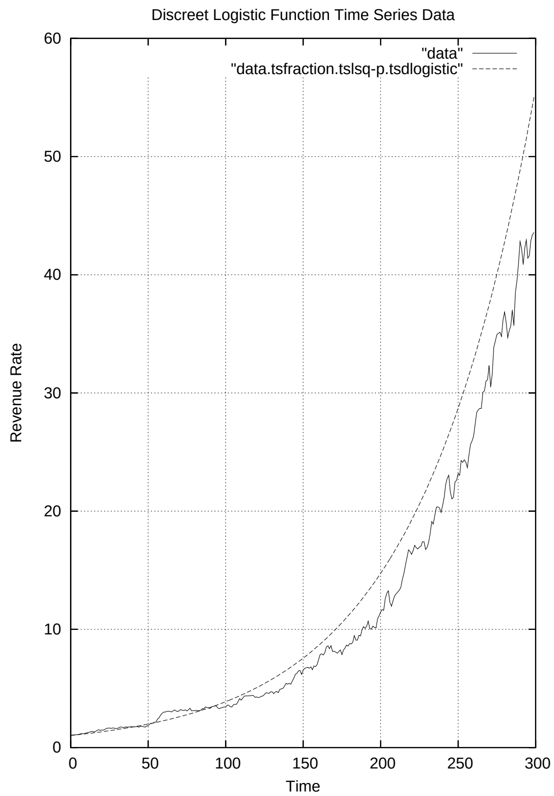

Figure C.541: Simulated Industrial Market, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.531 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

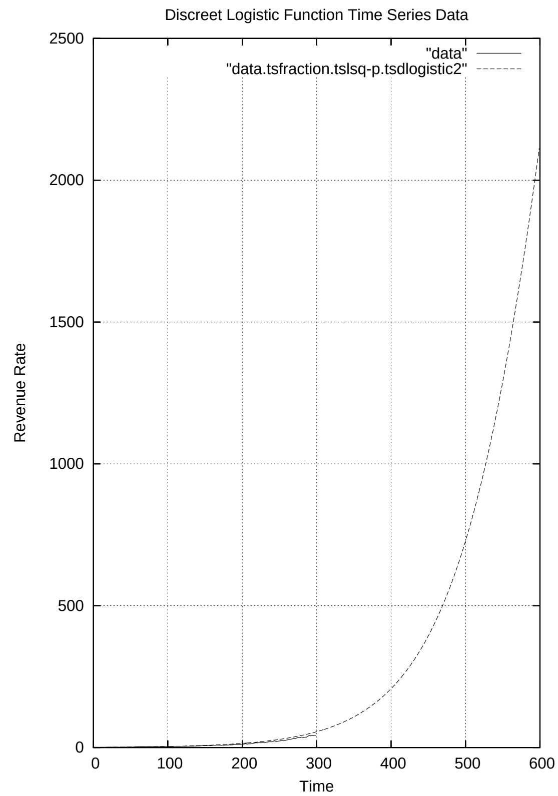

Figure C.542: Simulated Industrial Market, logistic function estimates of Figure C.541 with the time scale expanded by a factor of two.

the "forecastability" of the rate of revenue returns in the Simulated Industrial Market for the near term and far term, respectively [Pet91, pp. 83-84][295]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.543, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.848216, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.902}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.848216} \tag{C.903}$$

$$\propto (t_2 - t_1)^{1.696432} \tag{C.904}$$

[295]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

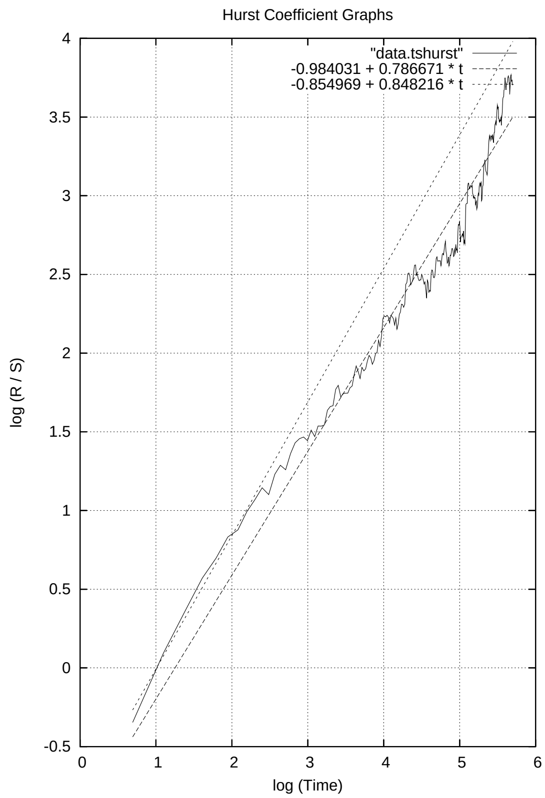

Figure C.543: Simulated Industrial Market, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.531. The slope of the graph is the Hurst coefficient.

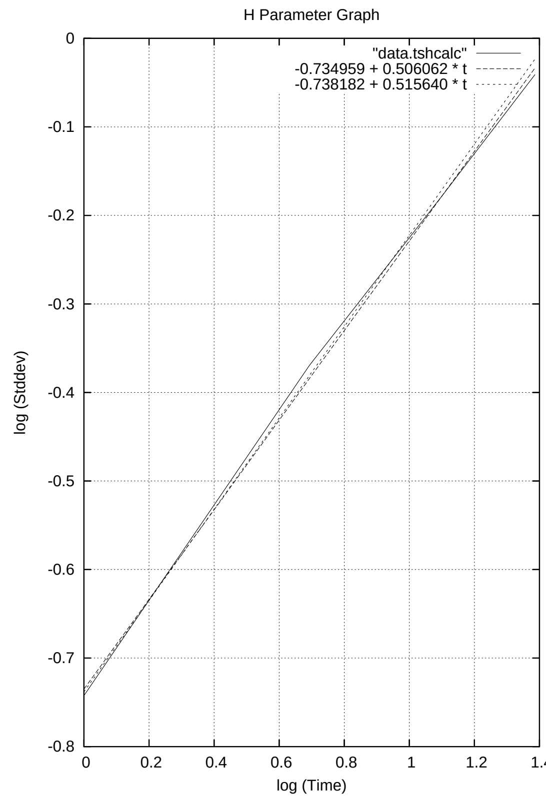

Figure C.544: Simulated Industrial Market, H parameter data for the normalized increments of the time series data shown in Figure C.531 The slope of the graph is the H parameter.

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[296]. A Hurst coefficient of 0.848216, (for the near future, and 0.786671 for the distant future.) implies that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 84.821600% [Pet91, pp. 66] for the near future, and 0.786671 for the distant future. Likewise, there is a 84.821600% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in

[296]Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.24.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Simulated Industrial Market. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

a given month, the rate of revenue returns, (per month,) is increasing, there is a 84.821600% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the Simulated Industrial Market are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \tag{C.905}$$

$$= 0.848216^n \tag{C.906}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.531, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.013109 \cdot 100$ percent, on the average, with a standard deviation of $0.029243 \cdot 100$ percent, and a root mean square error value of $0.032003 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.907}$$

$$\propto (t_2 - t_1)^{0.848216} \tag{C.908}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.908 reduces to, [Sch91, pp. 129]:

$$R(t_2 - t_1) \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.909}$$

$$\propto \sqrt{(t_2 - t_1)} \tag{C.910}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.911}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[297].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

[297]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.912}$$

$$\propto \frac{X(rt)}{r^{0.848216}} \tag{C.913}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.544, to provide a least squares approximation to the H parameter for the Simulated Industrial Market. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.515640 for the near future, and 0.506062 for the distant future.

Figures C.543 and C.544 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.531. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.531, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.545 and C.546 was made using the -d option.

C.24.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.24.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.532. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.531. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Industrial Market, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Simulated Industrial Market, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.531, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.013109 + 1)}{\ln(2)} = 0.018789 \tag{C.914}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.531, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.013460 + 1)}{\ln(2)} = 0.019289 \tag{C.915}$$

Note that if the mean is not constant in Figure C.531, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.530:

$$bits = 0.017469 \tag{C.916}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.530:

$$bits = 0.018128 \tag{C.917}$$

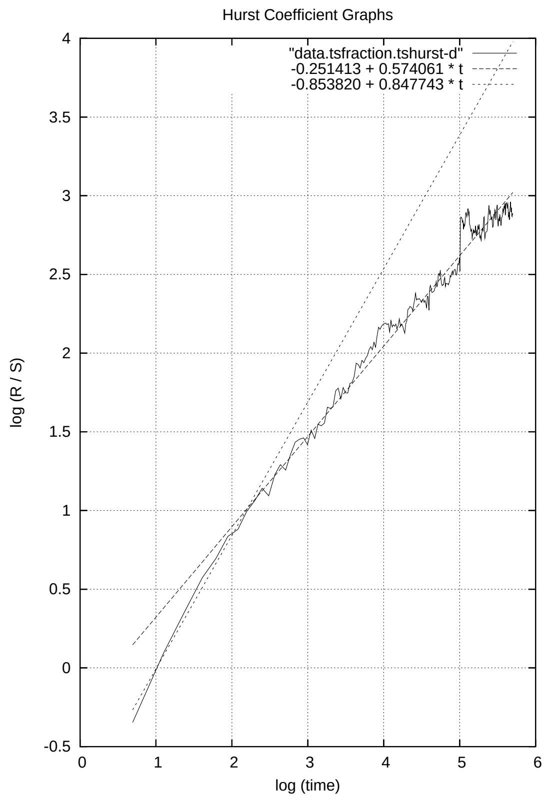

Figure C.545: Simulated Industrial Market, traditional Hurst coefficient data for the time series data shown in Figure C.530. The slope of the graph is the Hurst coefficient, and is 0.847743 for the near term, and 0.574061 for the far term.

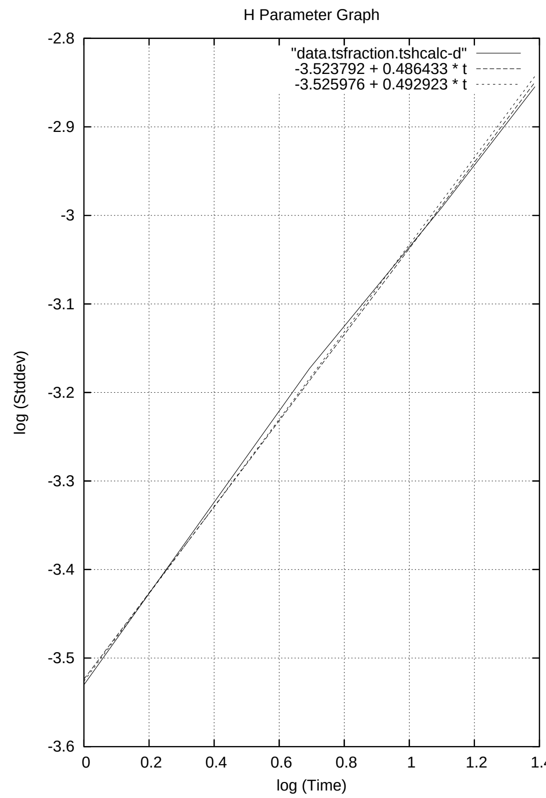

Figure C.546: Simulated Industrial Market, traditional H parameter data for the time series data shown in Figure C.530 The slope of the graph is the H parameter, and is 0.492923 for the near term, and 0.486433 for the far term.

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.24.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.018128t} \tag{C.918}$$

therefore:

$$C\left(p\right) = 0.018128 \tag{C.919}$$

and, *tsshannon* 0.018128 gives:

$$C\left(0.579097\right) = 0.018128 \tag{C.920}$$

therefore:

$$2^{C(0.579097)} = 2^{0.018128} \tag{C.921}$$

$$= 1.012645 \tag{C.922}$$

$$= \quad 1.264465\% \tag{C.923}$$

and:

$$2p - 1 = (2 \cdot 0.579097) - 1 \tag{C.924}$$
$$= 0.158194 \tag{C.925}$$
$$= 15.819400\% \tag{C.926}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Simulated Industrial Market executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 15.819400% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 84.180600% will be held in "reserve" with a 57.909700% chance of making twice the 15.819400% back, (and a 42.090300% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 1.264465%, or a doubling of its rate of revenue returns, (per month,) in 55.163283 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 15.819400% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 1.264465%, per month, on average.

Note that the metrics presented in this section are representative of the Simulated Industrial Market as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 15.819400% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the Simulated Industrial Market's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 1.264465% per month.

As another simple example, a company re-invests 15.819400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 15.819400% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 1.264465% per month.

As an example of "product portfolio" management, suppose a company re-invests 15.819400% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 15.819400$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 15.819400$ percent for the second product, implying that the company should diversify its product line[298].

[298]The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 15.819400%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65-1)}{(2 \cdot 0.55-1)} = 3:1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Simulated Industrial Market, as a standard bench mark, then the optimal number will be $\frac{1}{0.158194}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.531, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 15.819400% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 15.819400% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.24.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Simulated Industrial Market, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Simulated Industrial Market time series is 0.013109, and 0.032003respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 12.799358.

If this value seems consistent number of companies in the Simulated Industrial Market, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Simulated Industrial Market are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.557247, which would be the value which should be used in Section C.24.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.24.5 is greater than the average Shannon probability for the companies participating in the Simulated Industrial Market, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.24.5. The maximum exploitability for the Simulated Industrial Market is derived in Section C.24.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Simulated Industrial Market is 0.557247, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.704809 in the Simulated Industrial Market. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.927}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Simulated Industrial Market would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.24.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.532. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.531. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Industrial Market, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.24.5, is derived from the Simulated Industrial Market metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.24.9.

An additional exploitable strategy may be time itself. Equations C.904, C.908, and, C.906, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Simulated Industrial Market, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.547, and, C.548 compare methods of approximation of the "forecastability" of rate of revenue returns in the Simulated Industrial Market for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Simulated Industrial Market. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[299]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.547, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.906, $0.848216^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.904, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.24.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.24.9. Figure C.549 represents a constructional simulation of the time series data presented in Figure C.530. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data.

[299]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

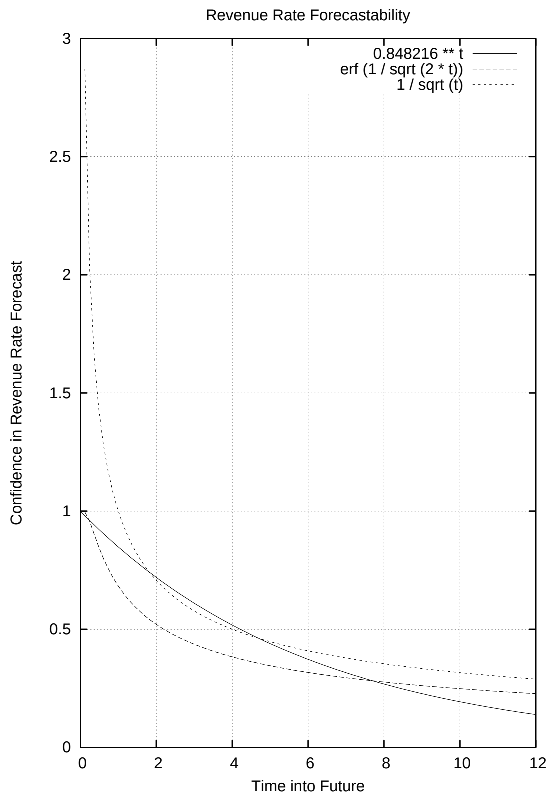

Figure C.547: Simulated Industrial Market, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.848216^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

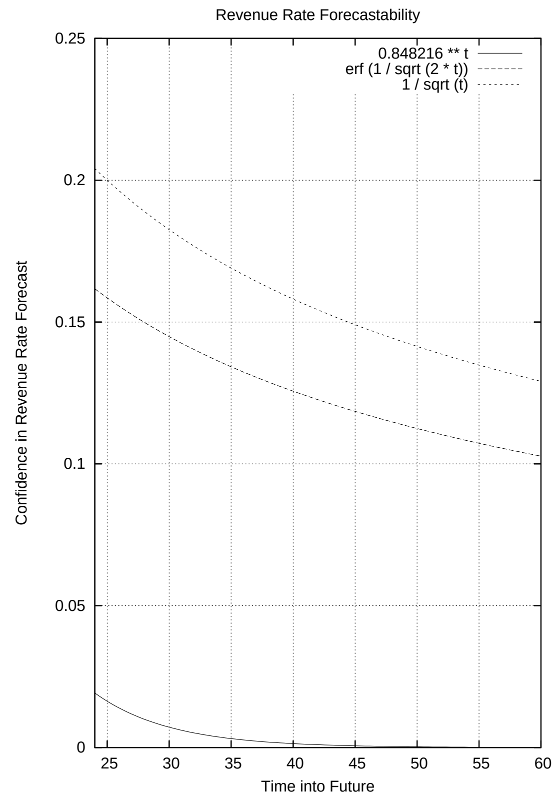

Figure C.548: Simulated Industrial Market, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.531. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.531 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.550 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.533.

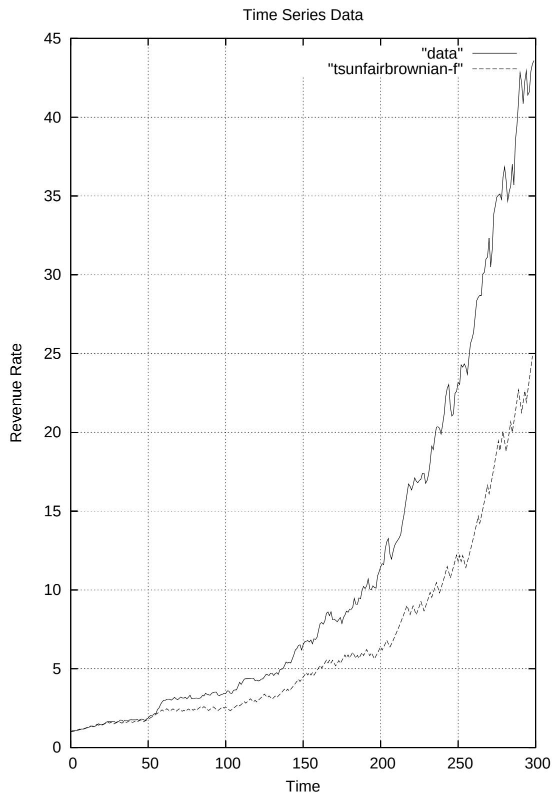

Figure C.549: Simulated Industrial Market, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.032003. This data is superimposed on the data presented in Figure C.530.

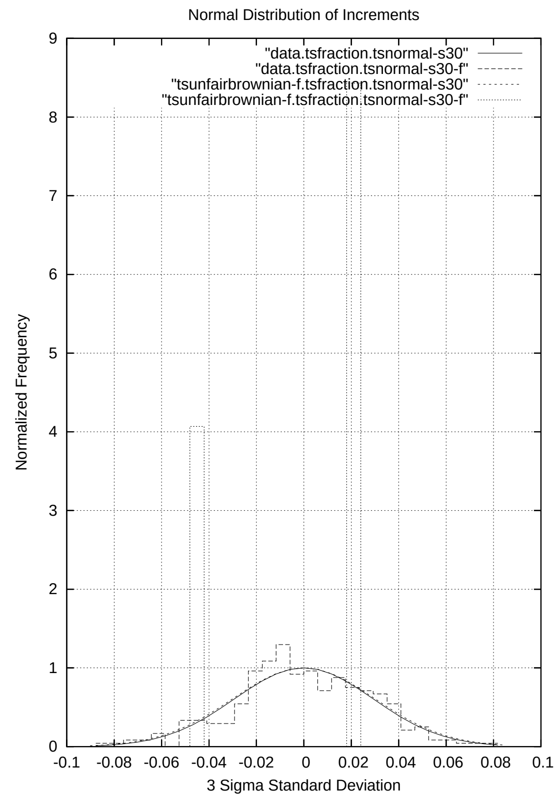

Figure C.550: Simulated Industrial Market, normalized histogram of the normalized increments of the time series data shown in Figure C.549, empirical and simulated. The empirical data has a mean of 0.013109, with a standard deviation of 0.029243. By comparison, the simulated data has a mean of 0.011169 with a standard deviation of 0.030041. This data is superimposed on the data presented in Figure C.533. The area under the four curves is identical.

C.24.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.24.3. One of the issues of analysis, as mentioned in Section C.24.7, is to determine the maximum Shannon probability for the time series presented in Figure C.530. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.551 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.530. Figure C.552 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.530. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.579097, as derived in Section C.24.5 to 0.676667. This process, essentially, extracts the random statistical data from the time series presented in Figure C.530, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

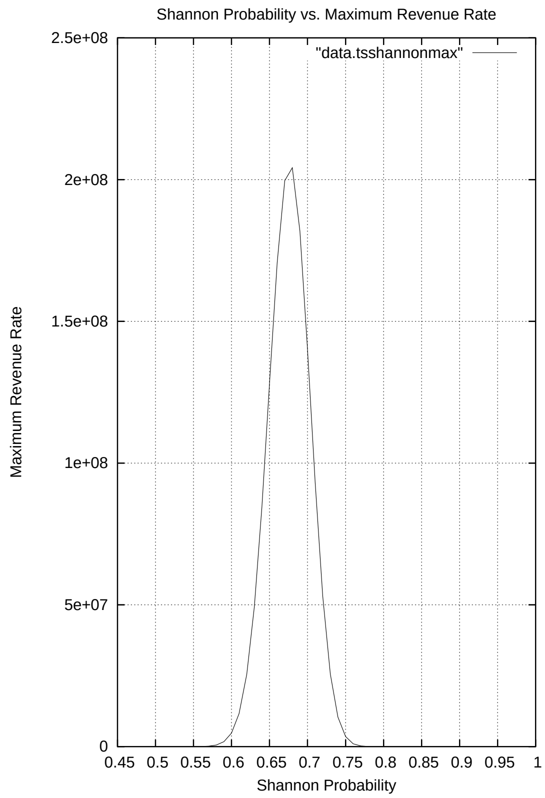

Figure C.551: Simulated Industrial Market, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.676667.

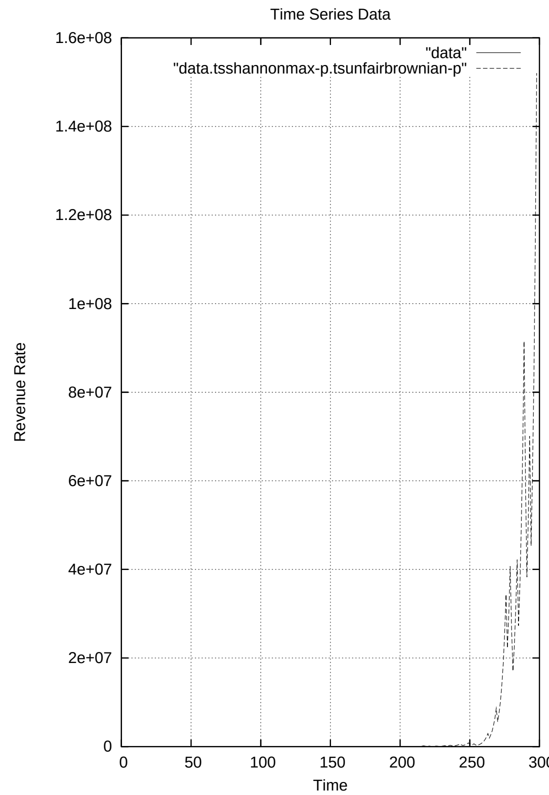

Figure C.552: Simulated Industrial Market, maximum rate of revenue returns, per month, at a Shannon probability, of 0.676667, corresponding to a "wager" fraction of 0.353334.

If it is assumed that the time series data set, presented in Figure C.530, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the Simulated Industrial Market movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.675585, as compared with the predicted value from the program *tsshannonmax* of 0.676667.

C.24.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.532. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.531. These values will be used in a fixed increment Brownian fractal analysis of the Simulated Industrial Market, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.24.6 and D.24.7. As a subjective evaluation of the "quality" of the analysis of the Simulated Industrial Market, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.530 from Figure C.531, and the Shannon probability as calculated by counting the total number of months that the Simulated Industrial Market movement was positive, as presented in Section C.24.9:

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.928}$$

$$0.675585 \approx \frac{\frac{0.013109}{0.032003} + 1}{2} \tag{C.929}$$

$$0.675585 \approx 0.704809 \tag{C.930}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.675585 \approx 0.704809 \approx 0.676667 \tag{C.931}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.24.5, should be compared. The four methods used were the mean of Figure C.531, the constant in the least squares approximation to Figure C.531, the least squares exponential approximation to Figure C.530, and the logarithmic returns of Figure C.530, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.018789 \approx 0.019289 \approx 0.017469 \approx 0.018128 \tag{C.932}$$

It is implied in Section C.24.5, Subsection C.24.5 and in Section C.24.8 that, a Brownian motion with fixed increments fractal may "model" the Simulated Industrial Market. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.933}$$

$$0.032003\,(2 \cdot 0.675585 - 1) \approx \frac{0.029243\,(2 \cdot 0.675585 - 1)}{2\sqrt{0.675585\,(1 - 0.675585)}} \tag{C.934}$$

$$0.032003 \cdot 0.351171 \approx 0.029243 \cdot 0.375057 \tag{C.935}$$

$$0.011239 \approx 0.010968 \tag{C.936}$$

and, equating to the mean:

$$0.013109 \approx 0.011239 \approx 0.010968 \tag{C.937}$$

where, as in Equation C.930 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.530 from Figure C.531, and the Shannon probability as calculated by counting the total number of months that the Simulated Industrial Market movement was positive, as presented in Section C.24.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[300], where the absolute value is presented in Figure C.532, and the root mean square value is presented in Figure C.531:

$$0.025065 \approx 0.032003 \tag{C.938}$$

Note, that if the Simulated Industrial Market could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.530 from Figure C.531 should be zero. It is 0.019930.

C.25 Discreet Logistic Function

For the analysis, the data was in the directory ../markets/tsdlogistic[301].

The data in this section is presented in tabular form in Section D.25. This is included for "theoretical" comparative purposes—of particular interest is the deterministic map in Figure C.561.

C.25.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.25.1. Figure C.553 is a graph of the time series data for the Discreet Logistic Function.

Figure C.554 is a graph of the normalized increments of the time series data presented in Figure C.553. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.555 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.554. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[302].

Figure C.556 is the normalized histogram of the normalized increments of the time series data shown in Figure C.554. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.556.

Figure C.557 is the statistical estimate for the data presented in Figure C.554, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.656864$, as derived in Section C.25.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set

[300]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[301]As a simulation model, the program *tsdlogistic* was run to make a time series data file, with the following parameters:

tsdlogistic 4 1 315 | awk 'if ($1 > 0.0) print $1' > data

to make a time series of 300 elements, with no element equal to zero. The data is by months.

[302]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

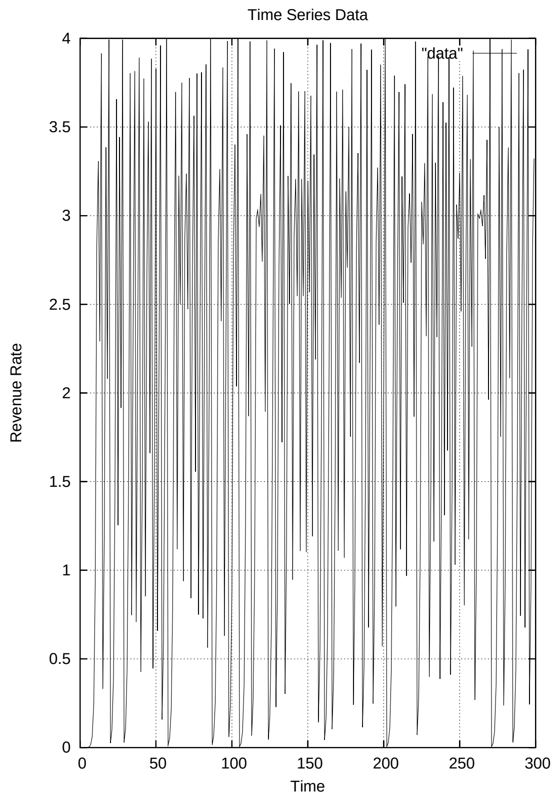

Figure C.553: Discreet Logistic Function, time series data.

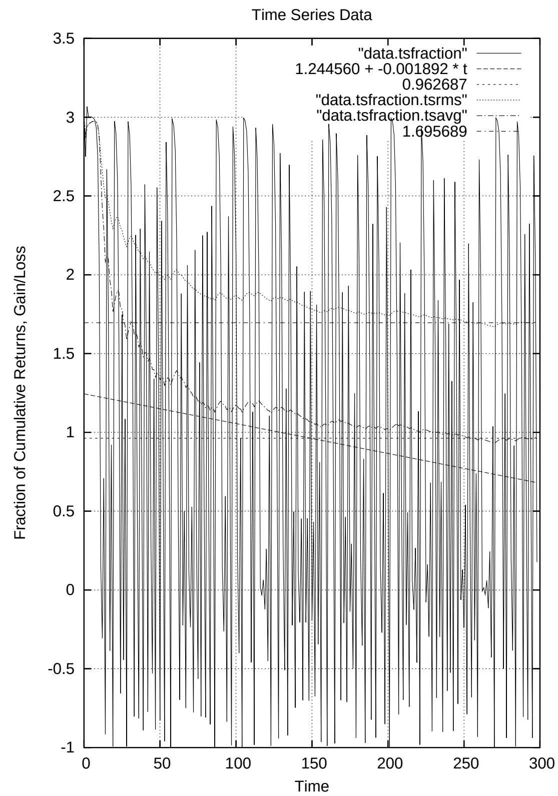

Figure C.554: Discreet Logistic Function, normalized increments of the time series data presented in Figure C.553. The mean is 0.962687 with a standard deviation of 1.398261. The formula for the least squares approximation is $1.244560 + -0.001892t$, and the root mean squared value is 1.695689. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, -0.001892, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.558 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.554. In principle, if the distribution of the normalized increments presented in Figure C.556

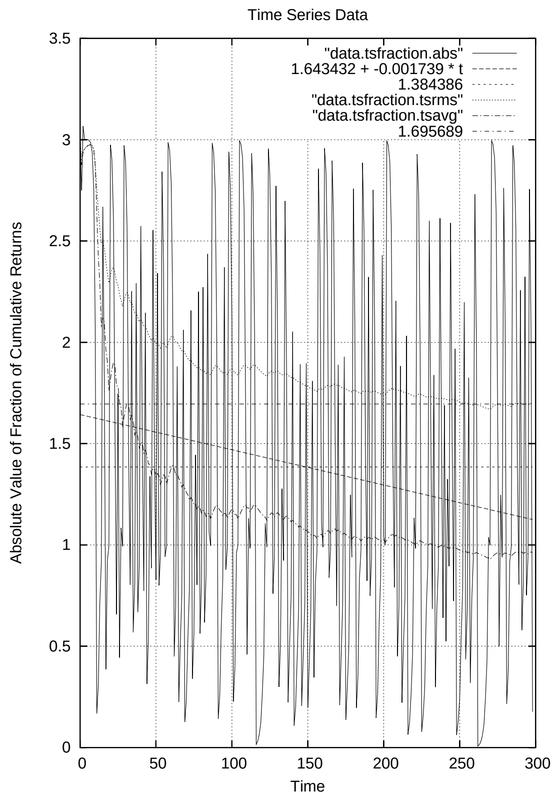

Figure C.555: Discreet Logistic Function, absolute value of the normalized increments of the time series data presented in Figure C.554. The mean is 1.384386 with a standard deviation of 0.980842. The formula for the least squares approximation is $1.643432 + -0.001739t$, and the root mean square value, from Figure C.554, is 1.695689. The graph, labeled "data-.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.554, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

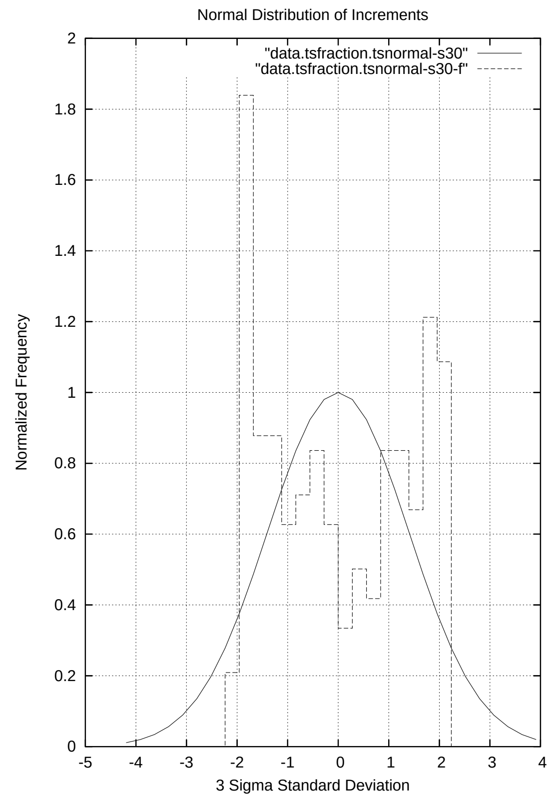

Figure C.556: Discreet Logistic Function, normalized histogram of the normalized increments of the time series data shown in Figure C.554. The data has a mean of 0.962687, with a standard deviation of 1.398261. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 11.091000, with a critical value of 42.557000.

is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.559 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.554. In principle, if the distribution of the normalized increments presented in Figure C.556 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

```
For  a mean  of 0.959478,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.095948,   846  samples  would  be  required.
     (With  300  samples,   the  estimated   error  is 0.161032   = 16.783318   percent.)
For  a standard   deviation   of 1.695689,   with  a confidence   level  of 0.900000
     that  the  error  did  not  exceed  0.169569,   136  samples  would  be  required.
     (With  300  samples,   the  estimated   error  is 0.113867   = 6.715087   percent.)
```

Figure C.557: Discreet Logistic Function, statistical estimates of the normalized increments of the time series shown in Figure C.554. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of ± 10%, or alternately, the error estimate on the time series shown in Figure C.554.

Figure C.560 is the range of values of the time series shown in Figure C.553. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.560 would be a square root function[303]. Figure C.561 is the deterministic map of the normalized increments of the time series data shown in Figure C.554. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

Figure C.556 would seem to indicate that the time series data for the Discreet Logistic Function does not represent a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to discount the assumption that the time series data represents fractional Brownian motion.

C.25.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root mean square of the instantaneous fraction of change[304]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.562 is the instantaneous value of the root mean square of the normalized increments for the Discreet Logistic Function, and Figure C.563 is the instantaneous Shannon probability for the normalized increments.

C.25.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.25.4. Figure C.564 is a graph of the logistic function estimates of the time series data for the Discreet Logistic Function. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require

[303]Note that the “roughness,” or “sawtooth” characteristics of the graph in Figure C.560 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

[304]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of “fit” to a Gaussian distribution.

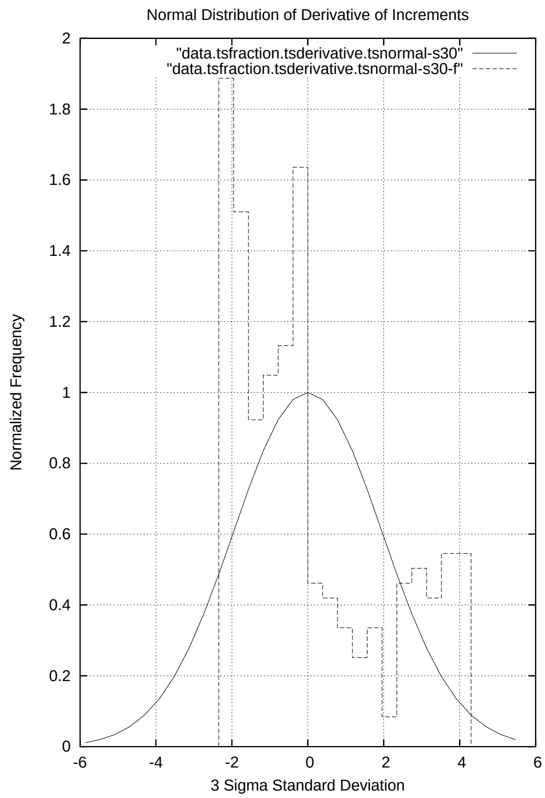

Figure C.558: Discreet Logistic Function, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.554.

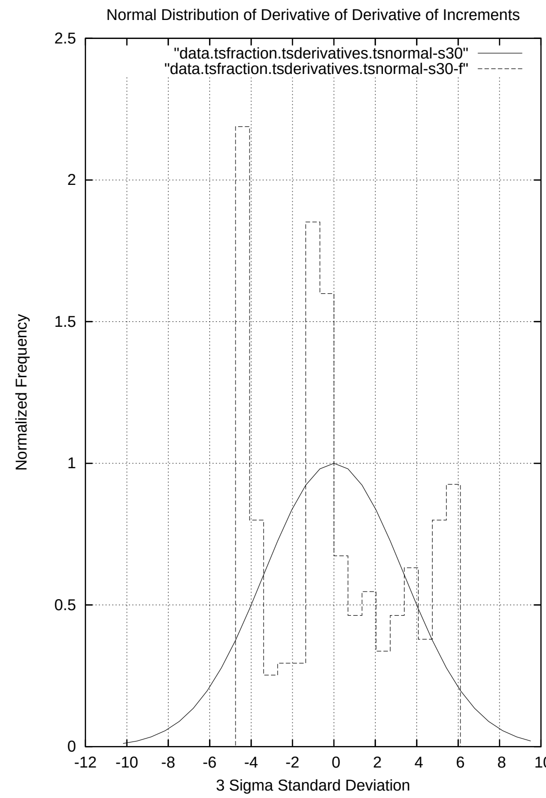

Figure C.559: Discreet Logistic Function, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.554.

intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[305]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.564 is a graph of the logistic function for the time series data presented in Figure C.553. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.554. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.554. Figure C.565 is the same graph, but with the time scale expanded by a factor of two.

C.25.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.25.5. Figure C.566 is a graph of the Hurst coefficient data time series data shown in Figure C.553. The slope of the graph is the Hurst coefficient. The data for this figure

[305]For example, in Figures C.564 and C.565, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.25.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

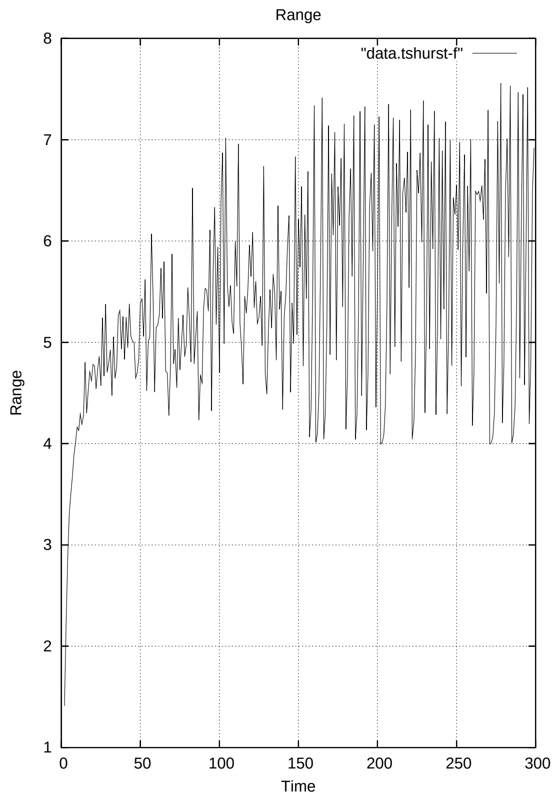

Figure C.560: Discreet Logistic Function, range of the time series data shown in Figure C.553.

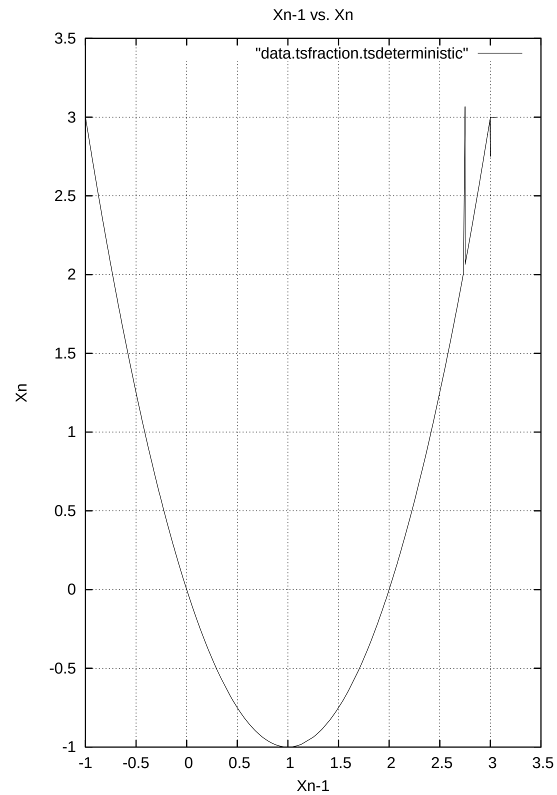

Figure C.561: Discreet Logistic Function, deterministic map of the normalized increments of the time series data shown in Figure C.554.

was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.567 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.554. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.566 implies that the variance of the rate of revenue returns, (per month,) in the Discreet Logistic Function, $V(t_2 - t_1)$, over a period of time is proportional to the period of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[306], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.570, and, C.571 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Discreet Logistic Function for the near term and far term,

[306]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

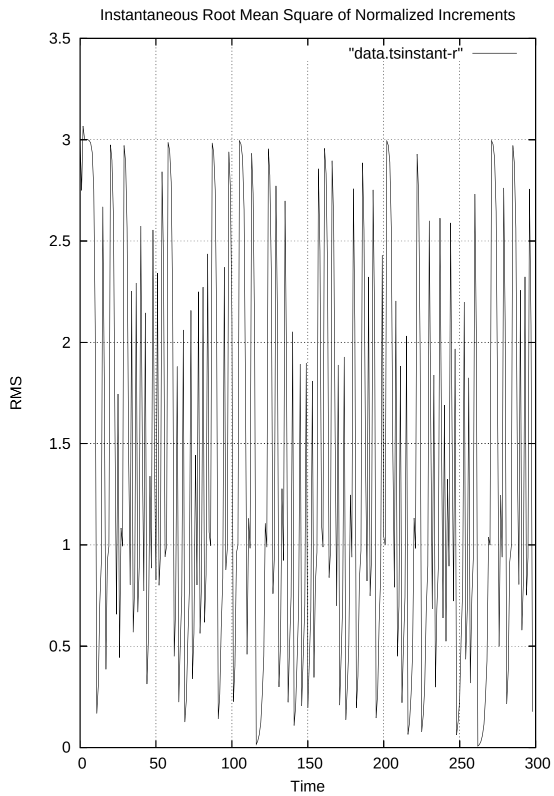

Figure C.562: Discreet Logistic Function, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.553.

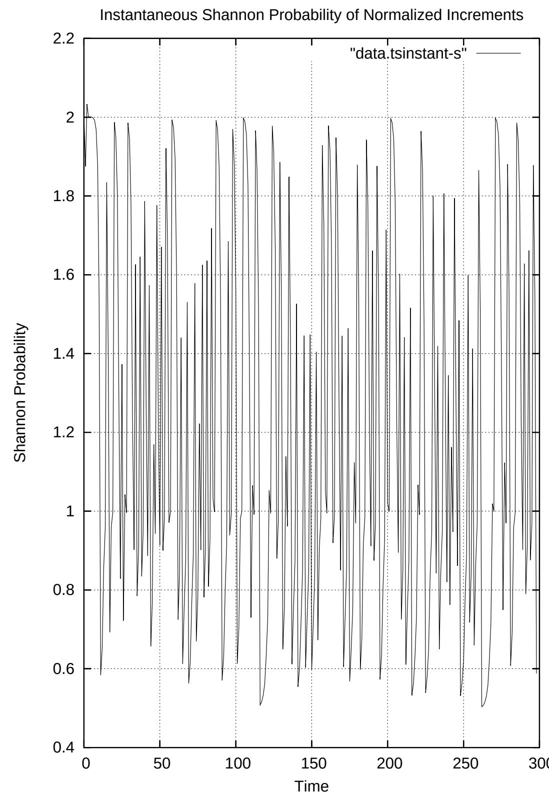

Figure C.563: Discreet Logistic Function, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.553.

respectively [Pet91, pp. 83-84][307]. This seems to be a quantitative statement concerning "windows of opportunity" in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.566, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.614199, so that:

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot H} \tag{C.939}$$

$$V(t_2 - t_1) \propto (t_2 - t_1)^{2 \cdot 0.614199} \tag{C.940}$$

$$\propto (t_2 - t_1)^{1.228398} \tag{C.941}$$

[307]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

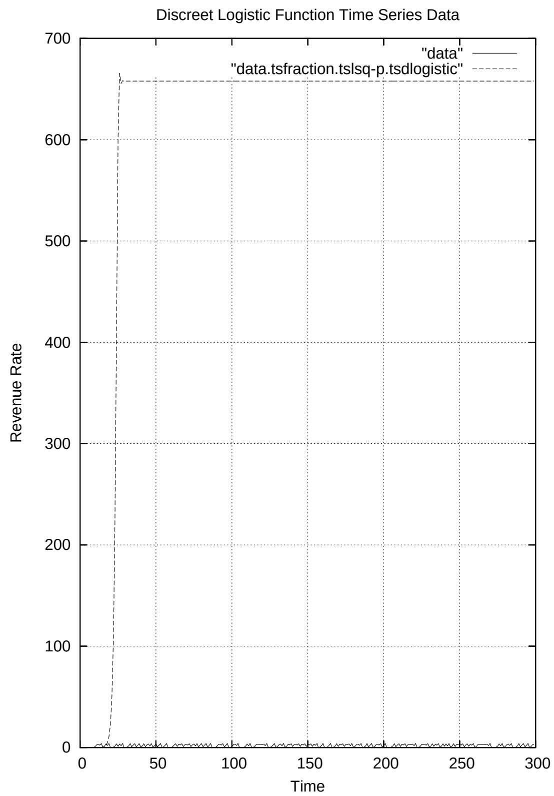

Figure C.564: Discreet Logistic Function, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.554 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

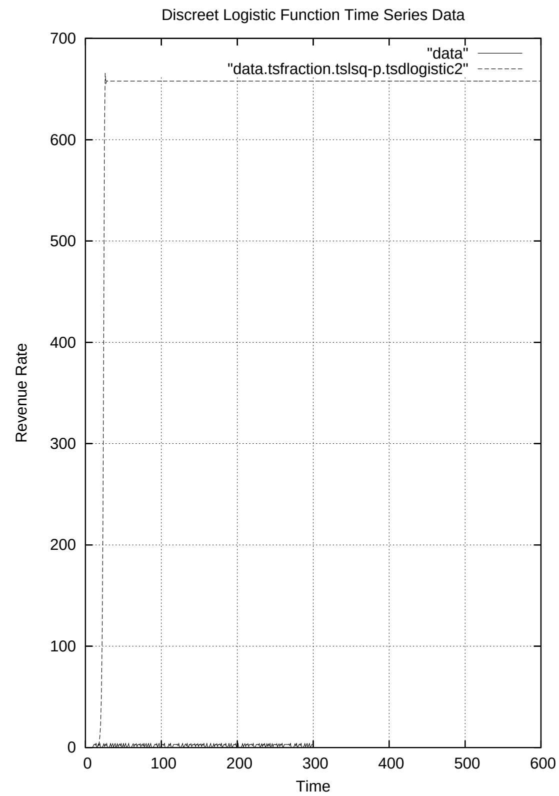

Figure C.565: Discreet Logistic Function, logistic function estimates of Figure C.564 with the time scale expanded by a factor of two.

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[308]. A Hurst coefficient of 0.614199, (for the near future, and 0.122234 for the distant future.) implies that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 61.419900% [Pet91, pp. 66] for the near future, and 0.122234 for the distant future. Likewise, there is a 61.419900%

[308] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.25.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Discreet Logistic Function. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

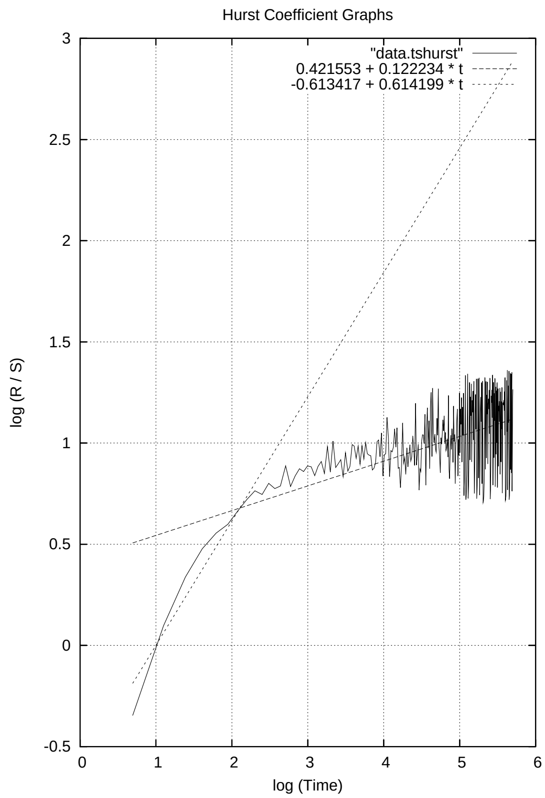

Figure C.566: Discreet Logistic Function, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.554. The slope of the graph is the Hurst coefficient.

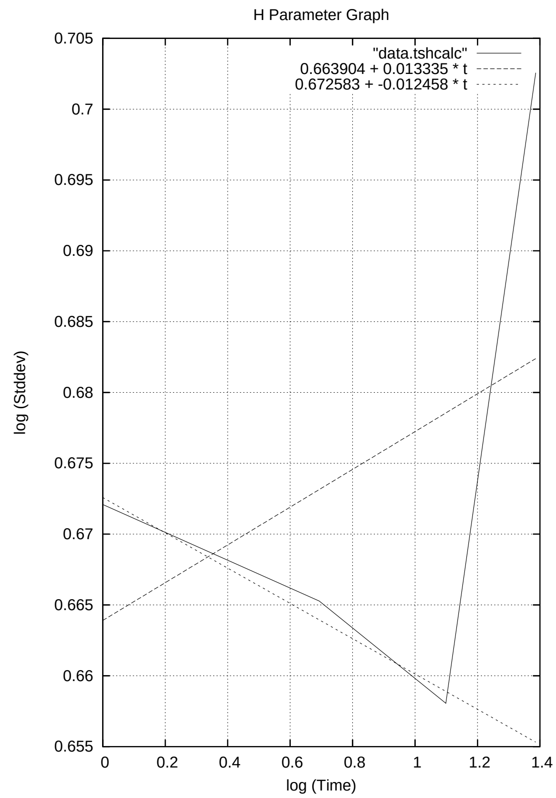

Figure C.567: Discreet Logistic Function, H parameter data for the normalized increments of the time series data shown in Figure C.554 The slope of the graph is the H parameter.

chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 61.419900% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the Discreet Logistic Function are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

$$p_a(n) = H^n \quad \text{(C.942)}$$
$$= 0.614199^n \quad \text{(C.943)}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.554, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.962687 \cdot 100$ percent, on the average, with a standard deviation of $1.398261 \cdot 100$ percent, and a root mean square error value of $1.695689 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^H \tag{C.944}$$
$$\propto \quad \left(t_2 - t_1\right)^{0.614199} \tag{C.945}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.945 reduces to, [Sch91, pp. 129]:

$$R\left(t_2 - t_1\right) \quad \propto \quad \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.946}$$
$$\propto \quad \sqrt{\left(t_2 - t_1\right)} \tag{C.947}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R\left(t_2 - t_1\right)}{S} \propto \left(t_2 - t_1\right)^{\frac{1}{2}} \tag{C.948}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[309].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X\left(t\right) \quad \propto \quad \frac{X\left(rt\right)}{r^H} \tag{C.949}$$
$$\propto \quad \frac{X\left(rt\right)}{r^{0.614199}} \tag{C.950}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.567, to provide a least squares approximation to the H parameter for the Discreet Logistic Function. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is -0.012458 for the near future, and 0.013335 for the distant future.

Figures C.566 and C.567 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.554. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.554, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.568 and C.569 was made using the -d option.

[309] To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

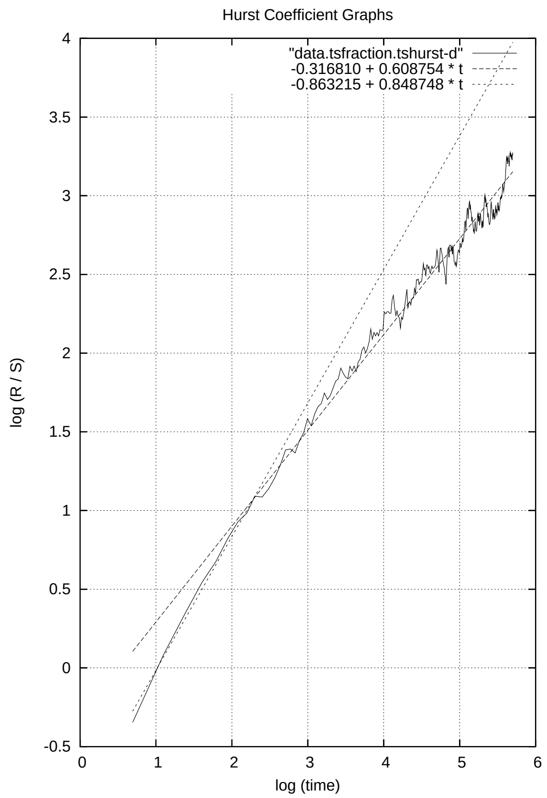

Figure C.568: Discreet Logistic Function, traditional Hurst coefficient data for the time series data shown in Figure C.553. The slope of the graph is the Hurst coefficient, and is 0.848748 for the near term, and 0.608754 for the far term.

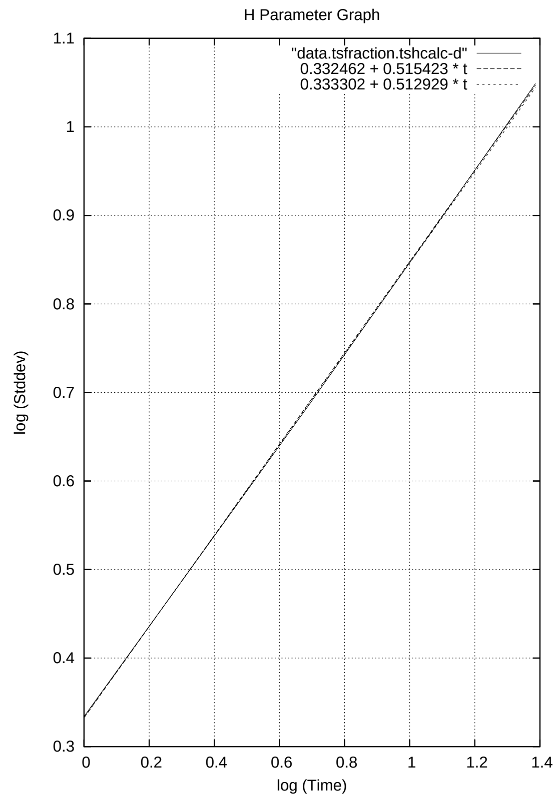

Figure C.569: Discreet Logistic Function, traditional H parameter data for the time series data shown in Figure C.553 The slope of the graph is the H parameter, and is 0.512929 for the near term, and 0.515423 for the far term.

C.25.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.25.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.555. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.554. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Discreet Logistic Function, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Discreet Logistic Function, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in

Chapter B, and is presented in Figure C.554, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.962687 + 1)}{\ln(2)} = 0.972830 \tag{C.951}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.554, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(1.244560 + 1)}{\ln(2)} = 1.166433 \tag{C.952}$$

Note that if the mean is not constant in Figure C.554, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.553:

$$bits = 0.008341 \tag{C.953}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.553:

$$bits = 0.072212 \tag{C.954}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.25.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.072212t} \tag{C.955}$$

therefore:

$$C(p) = 0.072212 \tag{C.956}$$

and, *tsshannon* 0.072212 gives:

$$C(0.656864) = 0.072212 \tag{C.957}$$

therefore:

$$\begin{aligned} 2^{C(0.656864)} &= 2^{0.072212} && \text{(C.958)} \\ &= 1.051327 && \text{(C.959)} \\ &= 5.132739\% && \text{(C.960)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.656864) - 1 && \text{(C.961)} \\ &= 0.313728 && \text{(C.962)} \\ &= 31.372800\% && \text{(C.963)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the “typical” organization operating in the Discreet Logistic Function executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 31.372800% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 68.627200% will be held in “reserve” with a 65.686400% chance of making twice the 31.372800% back, (and a 34.313600% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 5.132739%, or a doubling of its rate of revenue returns, (per month,) in 13.848114 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 31.372800% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 5.132739%, per month, on average.

Note that the metrics presented in this section are representative of the Discreet Logistic Function as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 31.372800% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the Discreet Logistic Function's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 5.132739% per month.

As another simple example, a company re-invests 31.372800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 31.372800% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 5.132739% per month.

As an example of "product portfolio" management, suppose a company re-invests 31.372800% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 31.372800$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 31.372800$ percent for the second product, implying that the company should diversify its product line[310]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 31.372800%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Discreet Logistic Function, as a standard bench mark, then the optimal number will be $\frac{1}{0.313728}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.554, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider

[310] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

the issue of product mix. In this interpretation, 31.372800% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 31.372800% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.25.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Discreet Logistic Function, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Discreet Logistic Function time series is 0.962687, and 1.695689respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 0.334806.

If this value seems consistent number of companies in the Discreet Logistic Function, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Discreet Logistic Function are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.990583, which would be the value which should be used in Section C.25.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.25.5 is greater than the average Shannon probability for the companies participating in the Discreet Logistic Function, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.25.5. The maximum exploitability for the Discreet Logistic Function is derived in Section C.25.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Discreet Logistic Function is 0.990583, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.783863 in the Discreet Logistic Function. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.964}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Discreet Logistic Function would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.25.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.555. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.554. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Discreet Logistic Function, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.25.5, is derived from the Discreet Logistic Function metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.25.9.

An additional exploitable strategy may be time itself. Equations C.941, C.945, and, C.943, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Discreet Logistic Function, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.570, and, C.571 compare methods of approximation of the "forecastability" of rate of revenue returns in the Discreet Logistic Function for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Discreet Logistic Function. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[311]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

As an interesting interpretation of the data presented in Figure C.570, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.943, $0.614199^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.941, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.25.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.25.9. Figure C.572 represents a constructional simulation of the time series data presented in Figure C.553. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.554. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.554 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.573 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.556.

C.25.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.25.3. One of the issues of analysis, as mentioned in Section C.25.7, is to determine the maximum Shannon probability for the time series presented in Figure C.553. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.574 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum

[311] For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

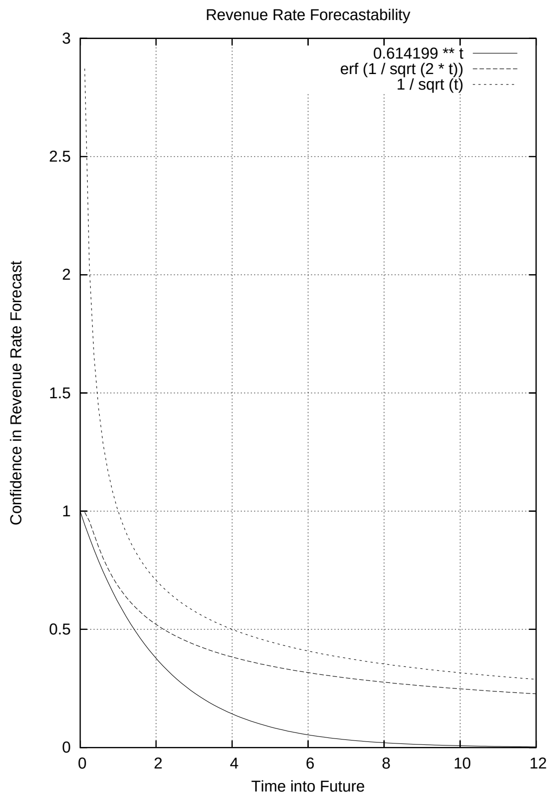

Figure C.570: Discreet Logistic Function, “forecastability” of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.614199^t$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

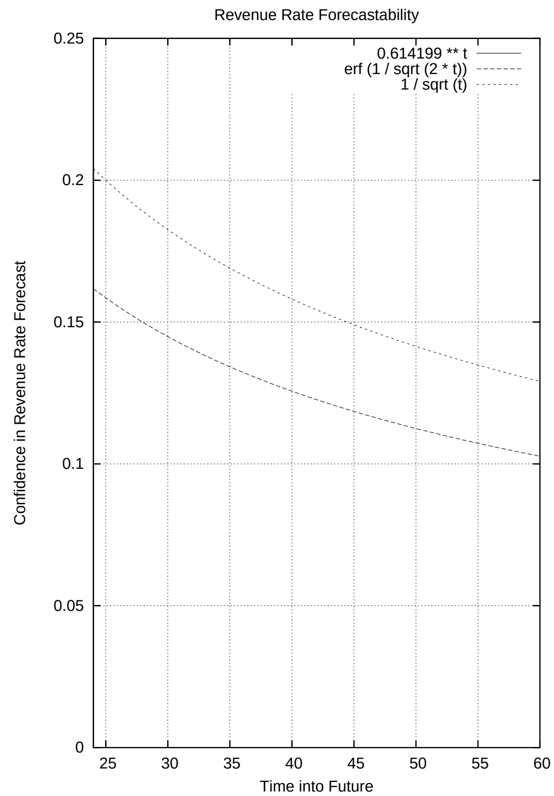

Figure C.571: Discreet Logistic Function, “forecastability” of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of “forecastability” of the rate of revenue returns.

Shannon probability for the time series data presented in Figure C.553. Figure C.575 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.553. This represents a “what if” the investment strategy was changed from a Shannon probability of 0.656864, as derived in Section C.25.5 to 0.666667. This process, essentially, extracts the random statistical data from the time series presented in Figure C.553, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The “quality” of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be “modeled” with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.553, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the Discreet Logistic Function movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.665552, as compared with the predicted value from the program *tsshannonmax* of 0.666667.

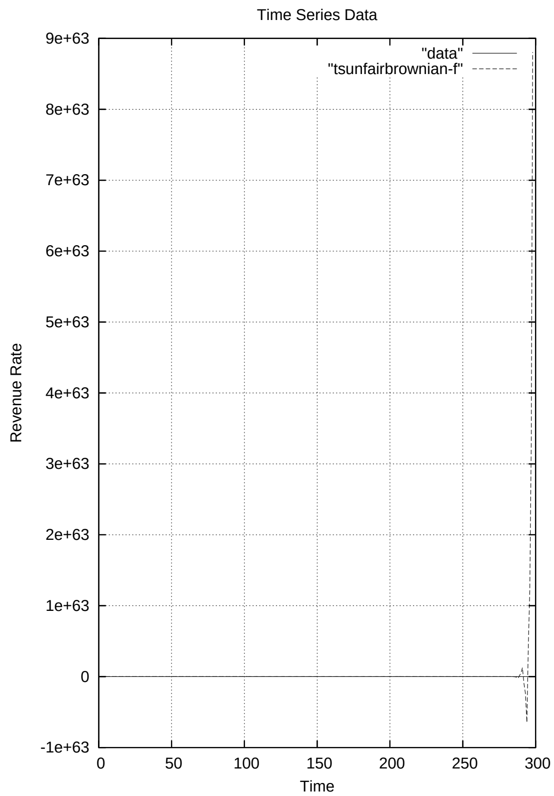

Figure C.572: Discreet Logistic Function, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 1.695689. This data is superimposed on the data presented in Figure C.553.

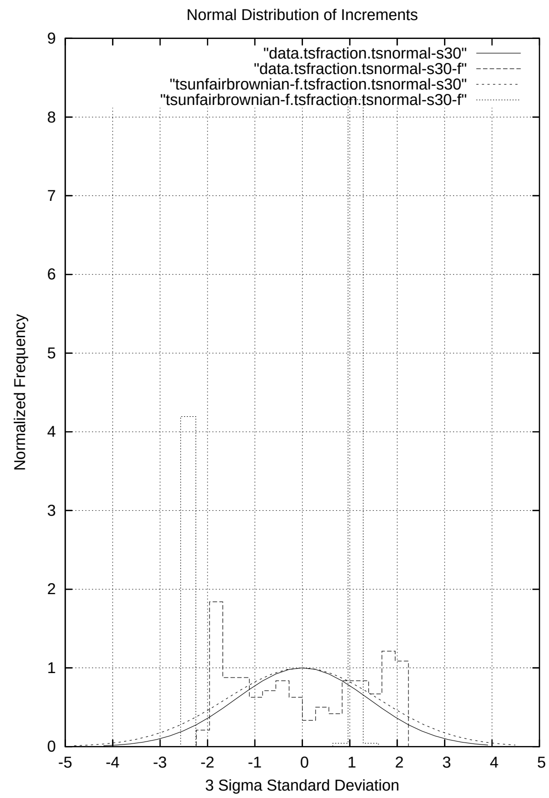

Figure C.573: Discreet Logistic Function, normalized histogram of the normalized increments of the time series data shown in Figure C.572, empirical and simulated. The empirical data has a mean of 0.962687, with a standard deviation of 1.398261. By comparison, the simulated data has a mean of 0.556782 with a standard deviation of 1.603622. This data is superimposed on the data presented in Figure C.556. The area under the four curves is identical.

C.25.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the “average” of the normalized increments presented in Figure C.555. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.554. These values will be used in a fixed increment Brownian fractal analysis of the Discreet Logistic Function, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.25.6 and D.25.7. As a subjective evaluation of the “quality” of the analysis of the Discreet Logistic Function, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.553 from Figure C.554, and the Shannon probability as calculated by counting the total number of months that the Discreet Logistic Function movement was positive, as presented in Section C.25.9:

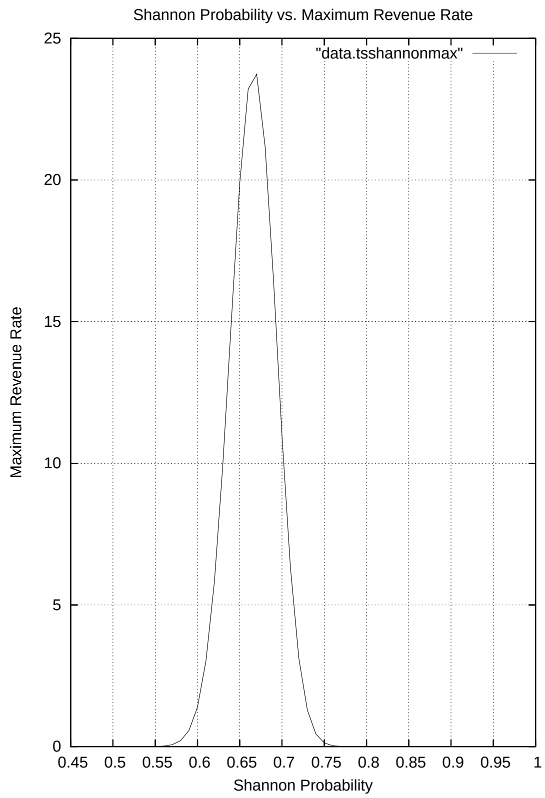

Figure C.574: Discreet Logistic Function, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.666667.

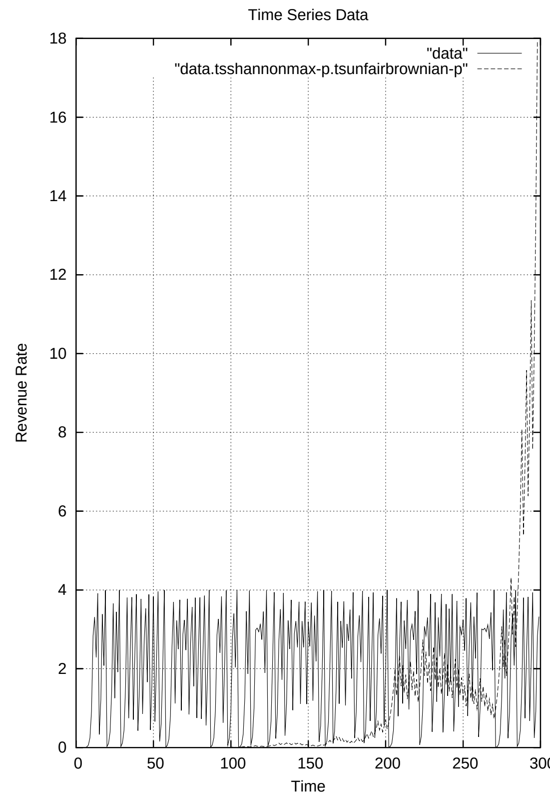

Figure C.575: Discreet Logistic Function, maximum rate of revenue returns, per month, at a Shannon probability, of 0.666667, corresponding to a "wager" fraction of 0.333334.

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.965}$$

$$0.665552 \approx \frac{\frac{0.962687}{1.695689} + 1}{2} \tag{C.966}$$

$$0.665552 \approx 0.783863 \tag{C.967}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

$$0.665552 \approx 0.783863 \approx 0.666667 \tag{C.968}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.25.5, should be compared. The four methods used were the mean of Figure C.554, the constant in the least squares approximation to

Figure C.554, the least squares exponential approximation to Figure C.553, and the logarithmic returns of Figure C.553, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.972830 \approx 1.166433 \approx 0.008341 \approx 0.072212 \tag{C.969}$$

It is implied in Section C.25.5, Subsection C.25.5 and in Section C.25.8 that, a Brownian motion with fixed increments fractal may "model" the Discreet Logistic Function. Using Equation 2.104 from Chapter 2, Section 2.5:

$$rms\,(2P-1) \approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} \tag{C.970}$$

$$1.695689\,(2 \cdot 0.665552 - 1) \approx \frac{1.398261\,(2 \cdot 0.665552 - 1)}{2\sqrt{0.665552\,(1 - 0.665552)}} \tag{C.971}$$

$$1.695689 \cdot 0.331104 \approx 1.398261 \cdot 0.350896 \tag{C.972}$$

$$0.561449 \approx 0.490644 \tag{C.973}$$

and, equating to the mean:

$$0.962687 \approx 0.561449 \approx 0.490644 \tag{C.974}$$

where, as in Equation C.967 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.553 from Figure C.554, and the Shannon probability as calculated by counting the total number of months that the Discreet Logistic Function movement was positive, as presented in Section C.25.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[312], where the absolute value is presented in Figure C.555, and the root mean square value is presented in Figure C.554:

$$1.384386 \approx 1.695689 \tag{C.975}$$

Note, that if the Discreet Logistic Function could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.553 from Figure C.554 should be zero. It is 0.980842.

C.26 Simulated Equity Market Index

For the analysis, the data was in the directory ../markets/tsgaussian.tsmath.tsmath.tsunfraction[313].

The program *tsunfraction*, which is described briefly in appendix B, provides the inverse function of the program *tsunfraction*. This allows a time series that contains normalized increments to be constructed, and then, cumulative summed into a fractal time series by the program *tsunfraction*.

[312]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[313]As a simulation model, the programs *tsgaussian*, *tsmath*, and *tsunfraction* were run to make a time series data file, with the following parameters:

```
tsgaussian 5000 | tsmath -t -m 0.01 | tsmath -t -a 0.0003 | tsunfraction > data
```

to make a time series of 5000 elements, with a Shannon probability of 0.515, to demonstrate an alternative method of constructing fractal time series. The average of the normalized increments is 0.0003, and the root mean square value of the normalize increments is 0.01, which is "typical" for an equity market time series. The data is by months.

The data in this section is presented in tabular form in Section D.26. Note that in this analysis, the rate of revenue returns means the increase or decrease in the cumulative sum of the Simulated Equity Market Index. This is included for "theoretical" comparative purposes.

C.26.1 Time Series Increments Analysis

The data in this section is presented in tabular form in Section D.26.1. Figure C.576 is a graph of the time series data for the Simulated Equity Market Index.

Figure C.577 is a graph of the normalized increments of the time series data presented in Figure C.576. The data presented was made by running the program *tsfraction* on the time series data. The program *tsfraction* is described briefly in Appendix B, and subtracts the previous value from the next value, dividing this difference by the previous value, for each element in the time series data. The new time series contains the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns.

Figure C.578 is a graph of the absolute value of the normalized increments of the time series data presented in Figure C.577. The data presented was made by running the Unix utility sed(1) on the normalized increments time series data to remove the negative signs. This is an absolute value procedure. The resulting time series contains the absolute value of the instantaneous change in the rate of revenue returns, divided by the magnitude of the instantaneous rate of revenue returns[314].

Figure C.579 is the normalized histogram of the normalized increments of the time series data shown in Figure C.577. The abscissa is 3 σ limits, and the area under the two curves is identical. The data for this figure was produced by the program *tsnormal*, which is described briefly in Appendix B.

The program *tsXsquared*, which is briefly described in appendix B, was used to derive the χ^2 statistics for the data presented in Figure C.579.

Figure C.580 is the statistical estimate for the data presented in Figure C.577, as derived by the program *tsstatest*, which is briefly described in appendix B.

Note that the data set size estimations, as produced by the *tsstatest* program, are probably very conservative, depending on the magnitude of the Shannon probability, $P = 0.511475$, as derived in Section C.26.5. See Chapter 2, Section 2.7 for possible alternative methodologies for addressing the analysis of fractal time series with limited data set sizes. Depending on the magnitude of the Shannon probability, P, these estimates can be several orders of magnitude too high.

Figure C.581 is the normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.577. In principle, if the distribution of the normalized increments presented in Figure C.579 is Gaussian in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43. The data was generated by the *tsderivative* program, which is briefly described in appendix B. Figure C.582 is the normalized histogram of the second derivative of the normalized increments of the time series data shown in Figure C.577. In principle, if the distribution of the normalized increments presented in Figure C.579 is an integrated Gaussian distribution in nature, this distribution would be similar to "white noise," as presented in appendix B, Figure B.43.

Figure C.583 is the range of values of the time series shown in Figure C.576. The horizontal axis is time into the future. In principle, if the time series was characterized as fractional Brownian motion the graph in Figure C.583 would be a square root function[315]. Figure C.584 is the deterministic map of the normalized increments of the time series data shown in Figure C.577. The deterministic map is useful for determining if a time series was created by a deterministic mechanism. This, essentially, maps each element in the time series with the previous element in the time series. See, [PJS92, pp. 745].

[314] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[315] Note that the "roughness," or "sawtooth" characteristics of the graph in Figure C.583 are a computational artifact—caused by not using the -m option to the program *tshurst*, which is computationally inefficient.

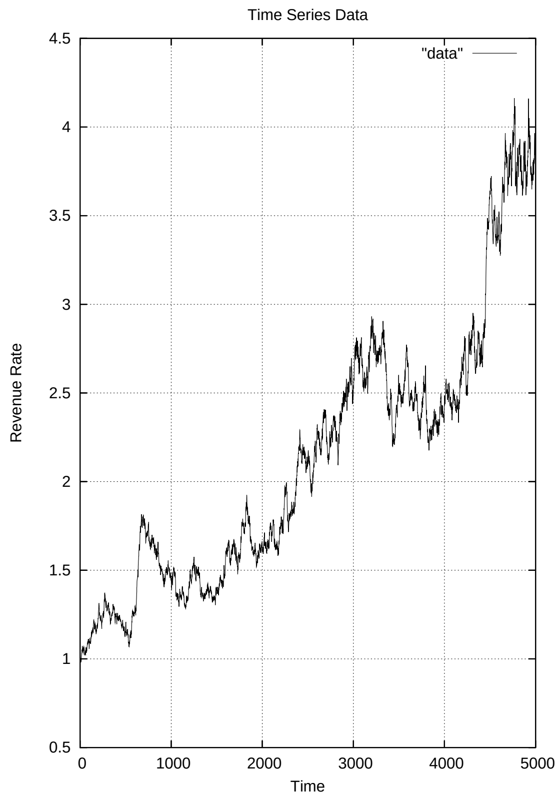

Figure C.576: Simulated Equity Market Index, time series data.

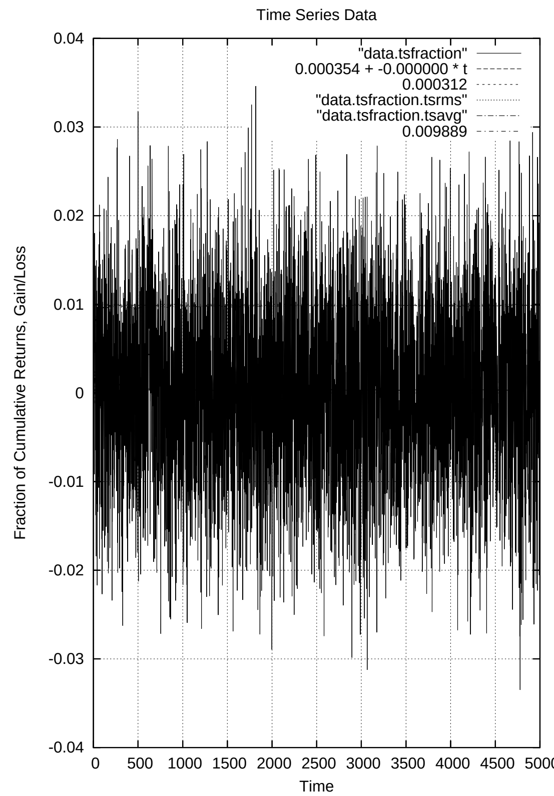

Figure C.577: Simulated Equity Market Index, normalized increments of the time series data presented in Figure C.576. The mean is 0.000312 with a standard deviation of 0.009885. The formula for the least squares approximation is $0.000354 + 0.000000t$, and the root mean squared value is 0.009889. The graph, labeled “data.tsfraction.tsrms,” is the running root mean square, and “data.tsfraction.tsavg” is the running average of the normalized increments. This graph is the fraction of change in the time series, as a function of time. Note that the slope of the mean, 0.000000, is the coefficient of the nonlinearity term in the normalized increments. See Chapter 2, Section 2.8 for a possible application of the logistic function to this data set.

Figure C.579 would seem to indicate that the time series data for the Simulated Equity Market Index represents a cumulative sum/integration of a random process that has a Gaussian distribution, (ie., satisfies the Gaussian increments property of fractional Brownian motion [Cro95, pp. 250],) tending to justify the assumption that the time series data represents fractional Brownian motion.

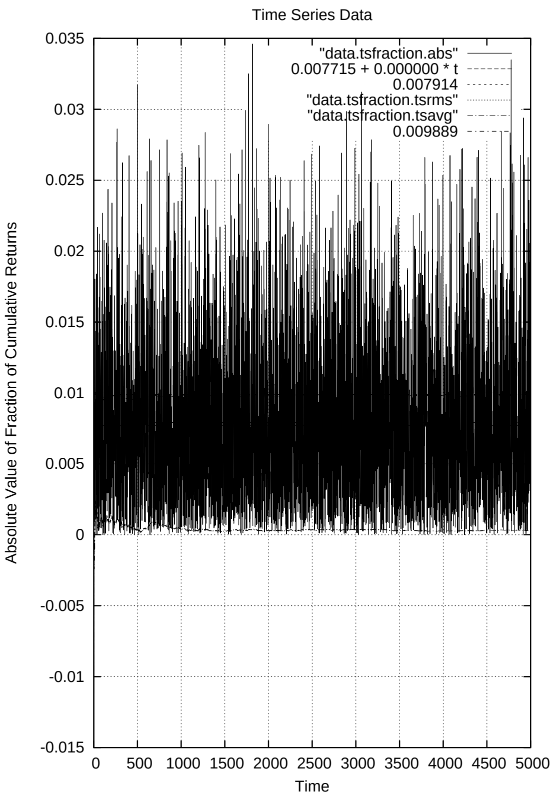

Figure C.578: Simulated Equity Market Index, absolute value of the normalized increments of the time series data presented in Figure C.577. The mean is 0.007914 with a standard deviation of 0.005930. The formula for the least squares approximation is $0.007715 + 0.000000t$, and the root mean square value, from Figure C.577, is 0.009889. The graph, labeled "data.tsfraction.tsrms," is the running root mean square, and "data.tsfraction.tsavg" is the running average of the normalized increments presented in Figure C.577, superimposed here for convenience. This graph is the absolute value of the fraction of change in the time series, as a function of time.

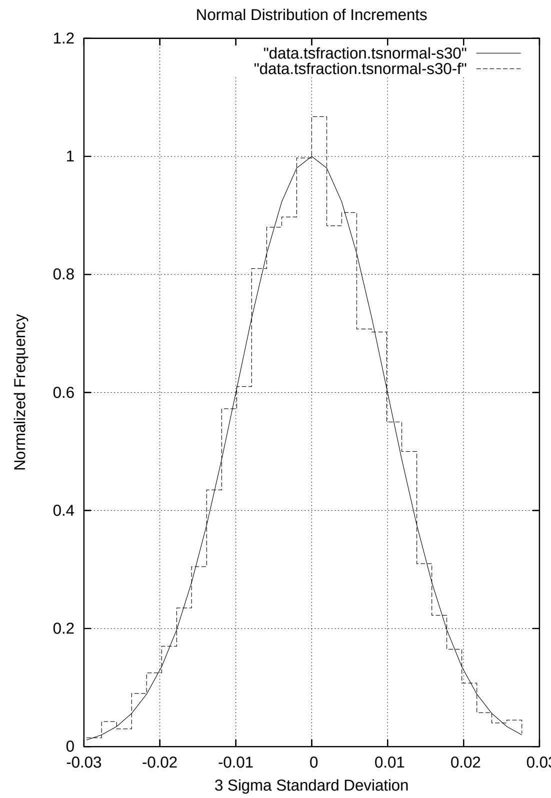

Figure C.579: Simulated Equity Market Index, normalized histogram of the normalized increments of the time series data shown in Figure C.577. The data has a mean of 0.000312, with a standard deviation of 0.009885. The area under the two curves is identical. The χ^2 value of the observed and expected values of the two curves is 0.213000, with a critical value of 42.557000.

C.26.2 Instantaneous Analysis of Normalized Increments

The program *tsinstant*, which is briefly described in Appendix B, is for finding the instantaneous fraction of change in a time series. The value of a sample in the time series is subtracted from the previous sample in the time series, and divided by the value of the previous sample. As explained in Chapter 2, Sections 2.1, 2.3.3, 2.5, 2.6 and, 2.6.1 for Brownian motion, random walk fractals, the absolute value of the instantaneous fraction of change is also the root

```
For a mean of 0.000312, with a confidence level of 0.900000
    that the error did not exceed 0.000031, 271515 samples would be required.
    (With 5001 samples, the estimated error is 0.000230 = 73.683071 percent.)
For a standard deviation of 0.009889, with a confidence level of 0.900000
    that the error did not exceed 0.000989, 136 samples would be required.
    (With 5001 samples, the estimated error is 0.000163 = 1.644689 percent.)
```

Figure C.580: Simulated Equity Market Index, statistical estimates of the normalized increments of the time series shown in Figure C.577. The table was produced with the *tsstatest* program, and illustrates the size of the data set required for a confidence level of 90%, with an error estimate of $\pm$ 10%, or alternately, the error estimate on the time series shown in Figure C.577.

mean square of the instantaneous fraction of change[316]. Squaring this value is the average of the instantaneous fraction of change, and adding unity to the absolute value of the instantaneous fraction of change, and dividing by two, is the Shannon probability of the instantaneous fraction of change.

Figure C.585 is the instantaneous value of the root mean square of the normalized increments for the Simulated Equity Market Index, and Figure C.586 is the instantaneous Shannon probability for the normalized increments.

C.26.3 Logistic Analysis

The data in this section is presented in tabular form in Section D.26.4. Figure C.587 is a graph of the logistic function estimates of the time series data for the Simulated Equity Market Index. The reader is cautioned that these graphs are constructed using the method suggested in Chapter 2, Section 2.8 and enormous precision is required for adequate prediction of the logistic function, [Mod92]. Particularly, the non-linear term will usually require intervention to produce a practical fit to the data. In addition, there are numerical stability issues with logistic function methodologies[317]. The methodology should be regarded as "fragile." It is included for completeness.

Figure C.587 is a graph of the logistic function for the time series data presented in Figure C.576. The data presented was made by running the program *tsdlogistic*, which is described briefly in Appendix B, on the parameters extracted from the time series data as suggested in Figure C.577. The program *tslsq* was used to derive the constant and the slope of the normalized increments of the data presented in Figure C.577. Figure C.588 is the same graph, but with the time scale expanded by a factor of two.

C.26.4 Hurst Coefficient Analysis

The data in this section is presented in tabular form in Section D.26.5. Figure C.589 is a graph of the Hurst coefficient data time series data shown in Figure C.576. The slope of the graph is the Hurst coefficient. The data for this figure was produced by the program *tshurst*, which is described briefly in Appendix B.

Figure C.590 is a graph of the H parameter data for the normalized increments of the time series data shown in Figure C.577. The data for this figure was produced by the program *tshcalc*, which is described briefly in Appendix B.

The approximately linear slope of the graph in Figure C.589 implies that the variance of the rate of revenue returns, (per month,) in the Simulated Equity Market Index, $V(t_2 - t_1)$, over a period of time is proportional to the period

[316]The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is $\approx$ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

[317]For example, in Figures C.587 and C.588, if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs. See Section D.26.4 for the actual derived values. In other cases, the magnitude of b was too large, resulting in a graph that was decreasing as a function of time

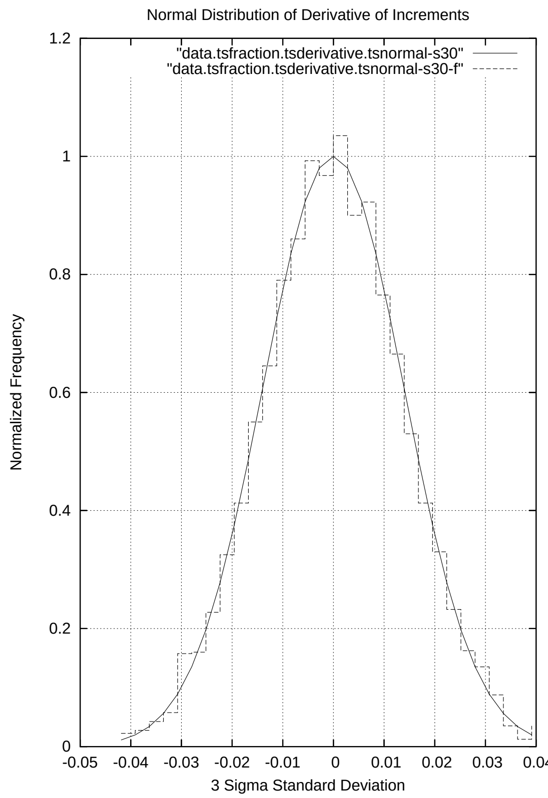

Figure C.581: Simulated Equity Market Index, normalized histogram of the first derivative of the normalized increments of the time series data shown in Figure C.577.

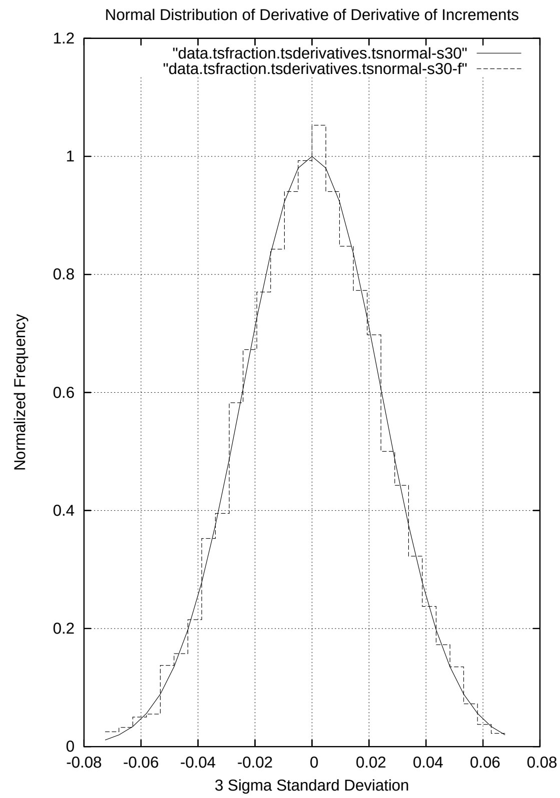

Figure C.582: Simulated Equity Market Index, normalized histogram of second derivative of the the normalized increments of the time series data shown in Figure C.577.

of time raised to twice the Hurst coefficient [Fed88, pp. 180], [Cro95, pp. 246]. This seems to be a quantitative statement concerning how fast, and to what degree, the rate of revenue returns' state of affairs can change over a period of time. An additional implication, for Hurst coefficients sufficiently close to 0.5, is that the probability of the state of affairs repeating sometime in the future goes down with increasing time[318], t, $p(t) = erf(1/\sqrt{2t})$ which is approximately $1/\sqrt{t}$ for $t \gg 1$ [Sch91, pp. 160]. Figures C.593, and, C.594 compare methods of approximation of the "forecastability" of the rate of revenue returns in the Simulated Equity Market Index for the near term and far term, respectively [Pet91, pp. 83-84][319]. This seems to be a quantitative statement concerning "windows of opportunity"

[318]It can be shown that the number of expected market "high" and "low" transitions, N, scales with the square root of time, or $N \propto \sqrt{t}$, meaning that the cumulative distribution of the probability, P, of the duration of a market's "high" or "low" exceeding a given time interval, t, is proportional to the reciprocal of the square root of the time interval, $P \propto 1/\sqrt{t}$, (or, conversely, that the probability of the duration of a market's "high" or "low" exceeding a given time interval is proportional to the reciprocal of the time interval raised to the power $3/2$, ie., $P \propto 1/t^{3/2}$, [Sch91, pp. 153]. What this means is that a histogram of the "zero free" run-lengths of a market being "high" or "low," over a long time, would have a $1/t^{3/2}$ characteristic.)

[319]The author is not comfortable with Peters' interpretation. For example, if the algorithm explained in [Pet91, pp. 82] is used on "white noise" which, by definition, never has any correlations, the short term Hurst coefficient, and thus the "forecastability," is still near unity—a bit of an enigma. This can be verified with the *tswhite* and *tshurst* programs, which are briefly described in Appendix B.

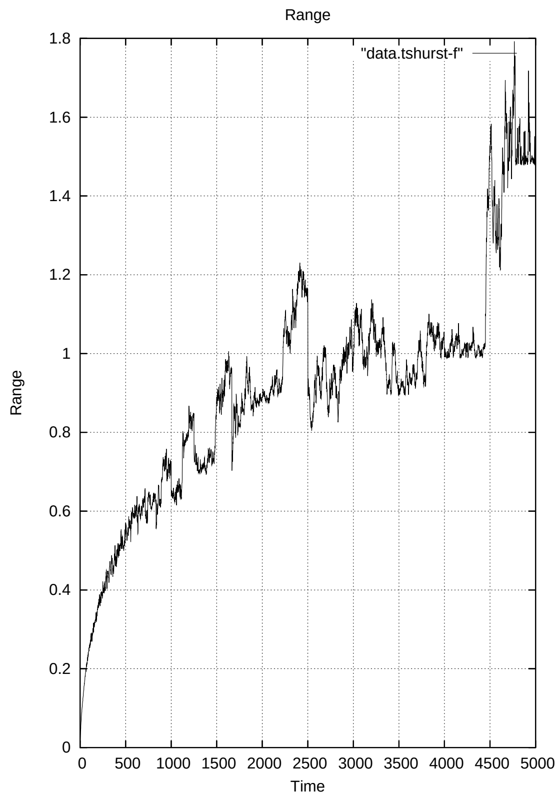

Figure C.583: Simulated Equity Market Index, range of the time series data shown in Figure C.576.

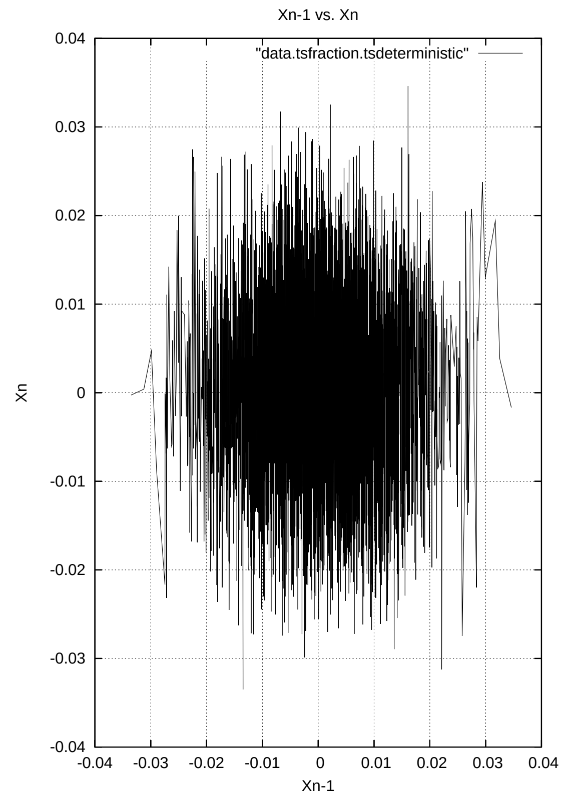

Figure C.584: Simulated Equity Market Index, deterministic map of the normalized increments of the time series data shown in Figure C.577.

in the rate of revenue returns, (per month.) The program *tslsq* was used on the Hurst coefficient data, presented in Figure C.589, to provide a least squares approximation to the Hurst coefficient. The superimposed least squares approximation with on original Hurst coefficient data is presented. The time series data has a Hurst coefficient of 0.841512, so that:

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot H} \tag{C.976}$$

$$V(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{2 \cdot 0.841512} \tag{C.977}$$

$$\propto \quad (t_2 - t_1)^{1.683024} \tag{C.978}$$

where $V(t_2 - t_1)$ is the variance of the increments of the rate of revenue returns, (per month,) over the time interval $t_2 - t_1$, [Fed88, pp. 177], [PJS92, pp. 494]. If $H > \frac{1}{2}$, then the time series is termed as being characterized by "fractional Brownian motion [Fed88, pp. 170]."

In some sense, the Hurst coefficient is a quantitative expression of the "forecastability" of the future based on the past[320]. A Hurst coefficient of 0.841512, (for the near future, and 0.426071 for the distant future.) implies

[320] Actually, in general, when summing fractal entities, the method used should be a root mean square process, dependent on the Hurst Coefficient,

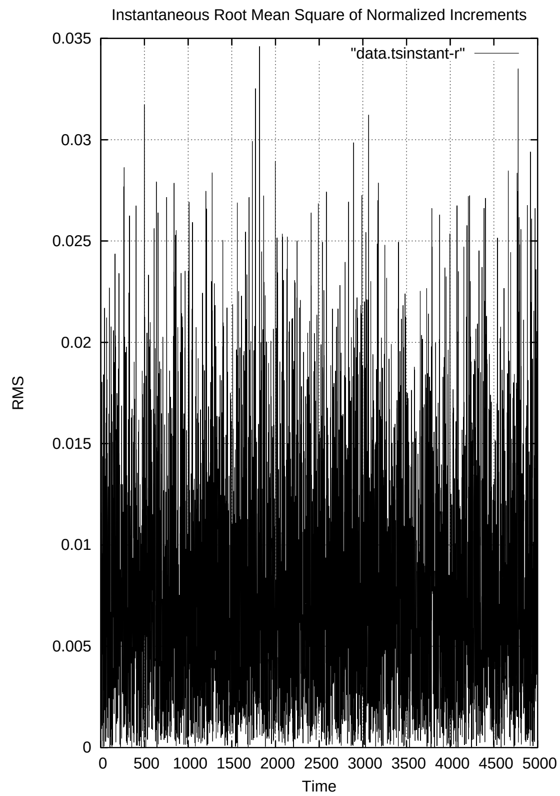

Figure C.585: Simulated Equity Market Index, instantaneous value of the root mean square of the normalized increments, provided by running the program *tsinstant* with the -r option on the data presented in Figure C.576.

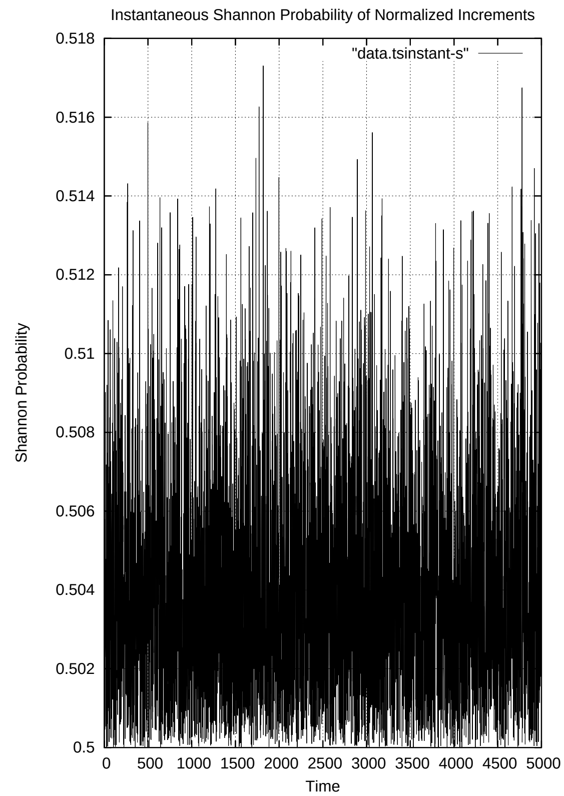

Figure C.586: Simulated Equity Market Index, instantaneous value of the Shannon probability of the normalized increments, provided by running the program *tsinstant* with the -s option on the data presented in Figure C.576.

that the likelihood of the rate of revenue returns, (per month,) for any two consecutive months being the same is 84.151200% [Pet91, pp. 66] for the near future, and 0.426071 for the distant future. Likewise, there is a 84.151200% chance of the rate of revenue returns, (per month,) movements being the same in consecutive time periods—ie., if, in a given month, the rate of revenue returns, (per month,) is increasing, there is a 84.151200% that the rate of revenue returns, (per month,) will increase in the following period, also. In some sense, this is a quantitative statement on how "predictable," or "forecastable" the rate of revenue returns, (per month,) for the Simulated Equity Market Index are over time, since the probability of having n many consecutive months of the same agenda is H^n where H is the Hurst coefficient, or, letting the short term probability of having n many months of the same market agenda, p_a, is:

H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the fractal entities. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See Section C.26.5 for the particulars on using Hurst Coefficient to sum fractal process' for the Simulated Equity Market Index. See also [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

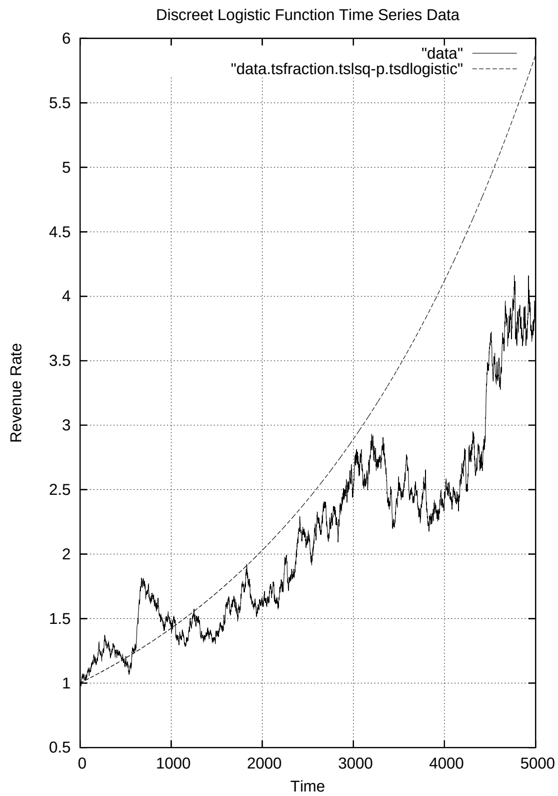

Figure C.587: Simulated Equity Market Index, logistic function estimates, provided by running the *tslsq* program on the normalized increments presented in Figure C.577 with the -p option. These parameters were used as arguments to the *tsdlogistic* program.

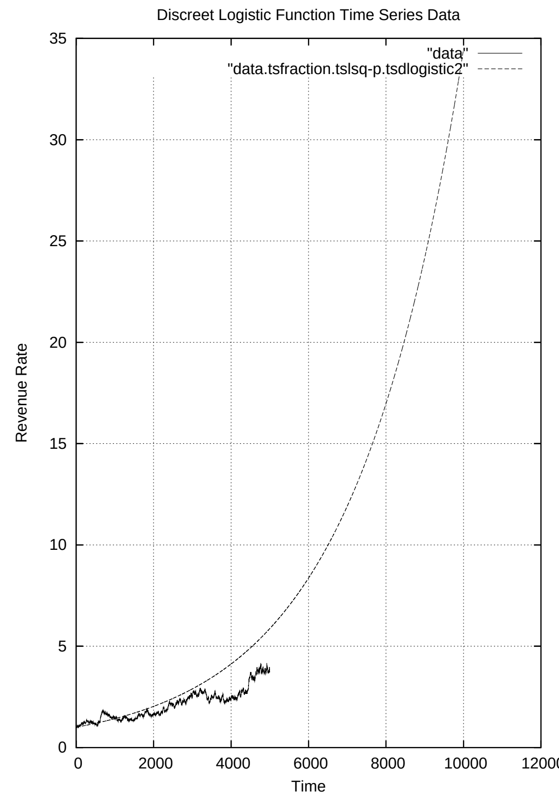

Figure C.588: Simulated Equity Market Index, logistic function estimates of Figure C.587 with the time scale expanded by a factor of two.

$$p_a(n) = H^n \tag{C.979}$$
$$= 0.841512^n \tag{C.980}$$

As an interesting interpretation of the normalized increments of the time series data presented in Figure C.577, if the vertical axis is multiplied by 100, to convert to percent, then the graph represents the error, in percent, that would be made by forecasting, month by month, that the next month's rate of revenue returns would be the same as the current month's revenue rate. Interestingly, it is $0.000312 \cdot 100$ percent, on the average, with a standard deviation of $0.009885 \cdot 100$ percent, and a root mean square error value of $0.009889 \cdot 100$ percent—small values for such a simple forecasting mechanism.

This is, essentially, a statement of the range of values, in the increments of the rate of revenue returns, (per month,) that is to be expected over the time interval, $t_2 - t_1$, R_v, [Fed88, pp. 178], [Ç93, pp. 172]:

$$R_v(t_2 - t_1) \propto (t_2 - t_1)^H \tag{C.981}$$

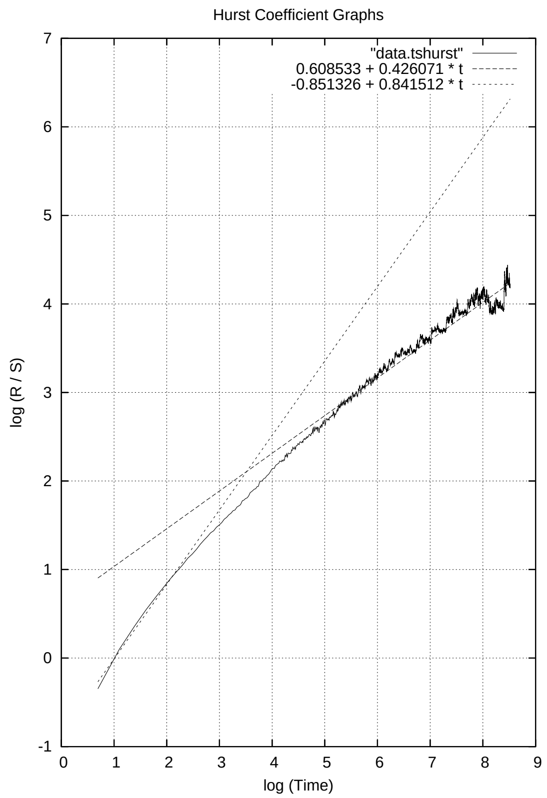

Figure C.589: Simulated Equity Market Index, Hurst coefficient data for the normalized increments of the time series data shown in Figure C.577. The slope of the graph is the Hurst coefficient.

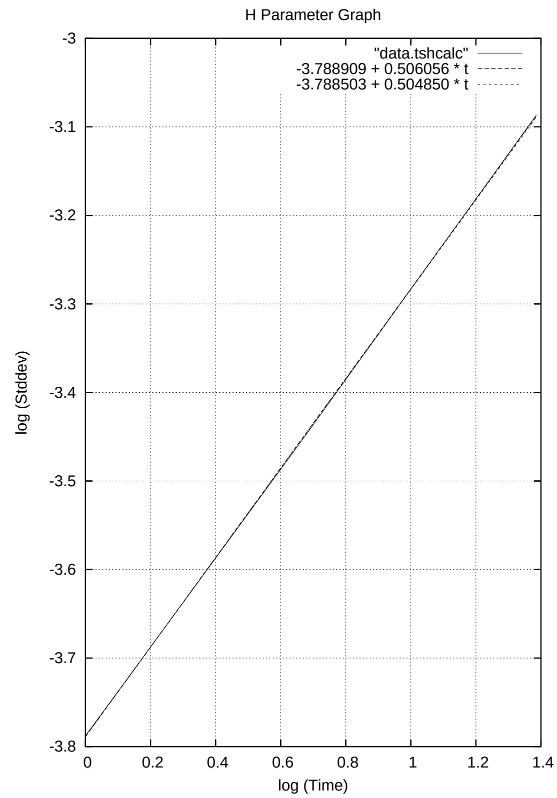

Figure C.590: Simulated Equity Market Index, H parameter data for the normalized increments of the time series data shown in Figure C.577 The slope of the graph is the H parameter.

$$\propto \quad (t_2 - t_1)^{0.841512} \tag{C.982}$$

where R is the range of values in the increments of the rate of revenue returns, (per month.) A Hurst coefficient, H, that is much larger than $\frac{1}{2}$, (but less than 1,) implies a strongly non-Gaussian distribution in the increments of the rate of revenue returns, (per month,) [Fed88, pp. 152, 194], and a Hurst coefficient near $\frac{1}{2}$ implies that the increments of the rate of revenue returns, (per month) is characteristic of an independent process [Fed88, pp. 195]. Extreme caution should be exercised in using Markov statistics in any analysis where the Hurst coefficient is not $\frac{1}{2}$, [Cro95, pp. 124], [Pet91, pp. 106].

As a useful approximation, if H, is approximately $\frac{1}{2}$, Equation C.982 reduces to, [Sch91, pp. 129]:

$$R\,(t_2 - t_1) \quad \propto \quad (t_2 - t_1)^{\frac{1}{2}} \tag{C.983}$$

$$\propto \quad \sqrt{(t_2 - t_1)} \tag{C.984}$$

In the case where the Hurst coefficient, H, is $\frac{1}{2}$, the range of values in the increments of the rate of revenue returns, (per month,) divided by the standard deviation of these values, S, can be anticipated to increase over time according to the following relation, [Fed88, pp. 154], [Sch91, pp. 129]:

$$\frac{R(t_2 - t_1)}{S} \propto (t_2 - t_1)^{\frac{1}{2}} \tag{C.985}$$

which is a useful conceptual approximation, since it involves only the square root function—if the range and the standard deviation of the increments of the rate of revenue returns, (per month,) are known, (and $H \approx \frac{1}{2}$,) then the expected change in $\frac{R}{S}$, will increase with the square root of time[321].

Another useful approximation when rescaling processes that are characterize by Brownian motion, (ie., when $H \approx \frac{1}{2}$,) is that:

$$X(t) \propto \frac{X(rt)}{r^H} \tag{C.986}$$

$$\propto \frac{X(rt)}{r^{0.841512}} \tag{C.987}$$

Where $X(t)$ is the process characterized by Brownian motion, and r is a scaling factor, [PJS92, pp. 494].

The program *tslsq* was used on the H parameter data, presented in Figure C.590, to provide a least squares approximation to the H parameter for the Simulated Equity Market Index. The superimposed least squares approximation on the original H parameter data is presented. By contrast, the H parameter, as derived by the methodology outlined in [Cro95, pp. 249], is 0.504850 for the near future, and 0.506056 for the distant future.

Figures C.589 and C.590 represent Hurst coefficient and H parameter data that are derived from the normalized increments, shown in Figure C.577. In this case, the data is considered a normalized derivative of the time series data presented in Figure C.577, instead of a cumulative sum. The program, *tshurst*, is described briefly in appendix B, and the data for figures C.591 and C.592 was made using the -d option.

C.26.5 Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.26.2. This section derives various values based on the "average" of the normalized increments presented in Figure C.578. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.577. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Equity Market Index, and may, or may not, provide adequate accuracy for projections.

For an organization operating in the Simulated Equity Market Index, the fiscal strategy, commensurate with the aggregate environment, can be derived as follows [Sch91, pp. 128, pp 151], [Rez94, pp. 450], [Pie80, pp. 270]:

Logarithmic Returns

The logarithmic returns can be calculated by various means. Four will be presented here, for comparison.

The logarithmic returns, in bits, $bits$, as computed from the mean, by the program *tsnormal*, which is described in Chapter B, and is presented in Figure C.577, and Equation 2.17 from Section 2.3.2 in Chapter 2:

$$bits = \frac{\ln(0.000312 + 1)}{\ln(2)} = 0.000450 \tag{C.988}$$

By comparison, the logarithmic returns, in bits, $bits$, as computed from the constant in the least squares approximation, using the program *tslsq*, which is briefly described in Chapter B, as presented in Figure C.577, and Equation 2.17 from Section 2.3.2 in Chapter 2:

[321]To be precise, it is actually asymptotically proportional to $\tau^{\frac{1}{2}}$

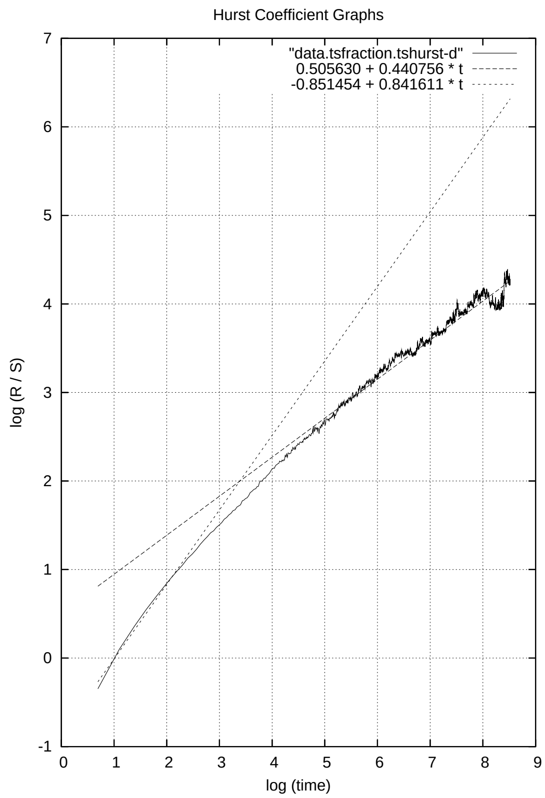

Figure C.591: Simulated Equity Market Index, traditional Hurst coefficient data for the time series data shown in Figure C.576. The slope of the graph is the Hurst coefficient, and is 0.841611 for the near term, and 0.440756 for the far term.

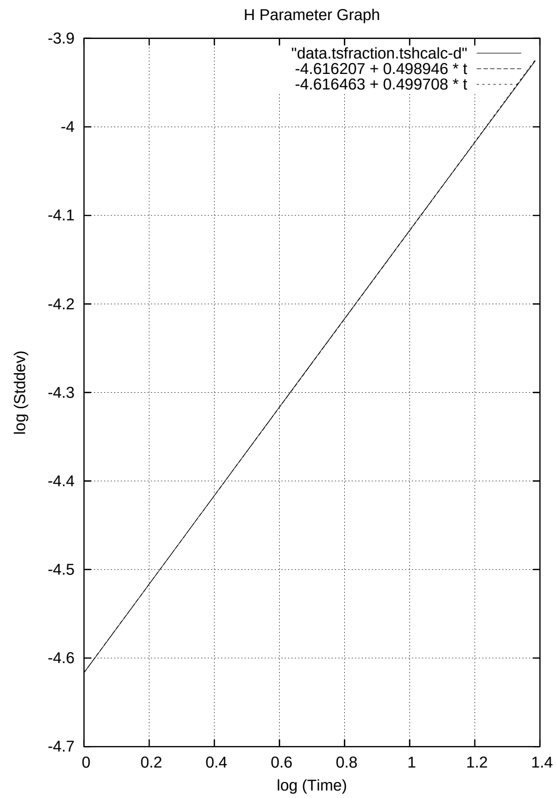

Figure C.592: Simulated Equity Market Index, traditional H parameter data for the time series data shown in Figure C.576 The slope of the graph is the H parameter, and is 0.499708 for the near term, and 0.498946 for the far term.

$$bits = \frac{\ln(0.000354 + 1)}{\ln(2)} = 0.000511 \tag{C.989}$$

Note that if the mean is not constant in Figure C.577, this method will not provide accurate results.
And by yet another comparison, using the program *tslsq*, which is briefly described in Chapter B, with the -e -p options, to provide a formula for the least squares exponential fit to the time series data set presented in Figure C.576:

$$bits = 0.000330 \tag{C.990}$$

And finally, by comparison, from the *tslogreturns* program, which is briefly described in Chapter B, with the -p option, to provide a formula for the logarithmic returns of the time series data set presented in Figure C.576:

$$bits = 0.000380 \tag{C.991}$$

Calculation of Shannon Probability

Ideally, all of the values presented in Section C.26.5 would be equal. Using the logarithmic returns provided by the *tslogreturns* program, to be consistent with [Pet91, pp. 81]

$$2^{0.000380t} \tag{C.992}$$

therefore:

$$C\left(p\right) = 0.000380 \tag{C.993}$$

and, *tsshannon* 0.000380 gives:

$$C\left(0.511475\right) = 0.000380 \tag{C.994}$$

therefore:

$$\begin{aligned} 2^{C(0.511475)} &= 2^{0.000380} && \text{(C.995)} \\ &= 1.000263 && \text{(C.996)} \\ &= 0.026343\% && \text{(C.997)} \end{aligned}$$

and:

$$\begin{aligned} 2p - 1 &= (2 \cdot 0.511475) - 1 && \text{(C.998)} \\ &= 0.022950 && \text{(C.999)} \\ &= 2.295000\% && \text{(C.1000)} \end{aligned}$$

Presuming the simplified assumptions outlined in Section 1.1, the "typical" organization operating in the Simulated Equity Market Index executes a long term fiscal strategy, commensurate with the aggregate environment, that is to invest, every month, in sufficient additional resources and infrastructure, to increase the manufacturing of goods and services by 2.295000% of its rate of revenue returns, (per month.) As a conceptual model, the remaining 97.705000% will be held in "reserve" with a 51.147500% chance of making twice the 2.295000% back, (and a 48.852500% chance of making 0.0,) in one month, on the average, for an average growth in its rate of revenue returns, (per month,) of 0.026343%, or a doubling of its rate of revenue returns, (per month,) in 2631.578947 months.

Example Fixed Increment Approximation Fiscal Strategies

A possible metric on the effectiveness of long term fiscal management could possibly be that if an investment of 2.295000% per month of the rate of revenue returns, (per month,) is made in resources and infrastructure, then the rate of revenue returns would be expected to increase by 0.026343%, per month, on average.

Note that the metrics presented in this section are representative of the Simulated Equity Market Index as an aggregate whole, and may or may not be accurate representations for any particular participant in the environment. Of interest to the participants in the environment would be a similar analysis of each product or service rendered in the marketplace.

As a simple illustrative example, a company operating in this environment might obtain a credit line from a bank that is equal to 2.295000% of its rate of revenue returns, (per month,) to finance additional operations. In this simple scenario, the company would use its revenue base as collateral for the loan. Some months, depending on the Simulated Equity Market Index's environment, the company's rate of revenue returns exceeds what was borrowed from the bank, and the loan is repaid in full. Other months, the company must default, and the bank seizes a portion of the company's revenue base to pay the delinquent loan. However, on the average, the company will expand its rate of revenue returns at 0.026343% per month.

As another simple example, a company re-invests 2.295000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Although some products will be successful and the return on the investment will exceed the 2.295000% per month investment, others will not. However, on the average, the company will expand it gross rate of revenue returns at 0.026343% per month.

As an example of "product portfolio" management, suppose a company re-invests 2.295000% of its rate of revenue returns, (per month,) in development, marketing, sales, and distribution of new products. Further suppose that the company has two products, and a fractal analysis of the individual product rate of revenue return time series indicates that one product has a Shannon probability of 0.65, and the other has a Shannon probability of 0.55. Then the percentage of re-investment in the first product would be $(2 \cdot 0.65 - 1) \cdot 2.295000$, percent of the rate of revenue returns, and $(2 \cdot 0.55 - 1) \cdot 2.295000$ percent for the second product, implying that the company should diversify its product line[322]. Note that this is a "bet hedging" metric methodology, and assumes that the products have uncorrelated revenue return rates. If this re-investment methodology is not feasible, perhaps for strategic financial reasons, then the re-investment in both products should total the 2.295000%, and the investment in each product should be made at a ratio of $\frac{(2 \cdot 0.65 - 1)}{(2 \cdot 0.55 - 1)} = 3 : 1$, respectively. Note that this "bet hedging" can be used to define the optimal number of products that can be supported on the rate of revenue returns. If it assumed that all products are "typical" for the Simulated Equity Market Index, as a standard bench mark, then the optimal number will be $\frac{1}{0.022950}$. Note that this is a "theoretical" value, since not all products are "typical," and there may be strategic reasons, for example product leveraging, that may increase the number of products above the optimum. However, most of the revenue should come from the optimal number of products, since having more products will decrease the amount of the potential investment in each product, and having less than the optimum number of products will increase the risk that many of the products could suffer a "down market" concurrently, impacting the rate of revenue returns. As another interesting interpretation of the optimal "hedging of bets," in product portfolio strategy, and considering the graph of the normalized increments presented in Figure C.577, if the organization is running optimally, then these products will generate, at least in principle, one standard deviation, approximately $0.8413 = 84.13\%$ of the future growth in rate of revenue returns. Naturally, these are approximations, and the values are an approximation to a, probably, complex process, and appropriate scrutiny should be exercised before making specific projections. As yet another example of "product portfolio" management, consider the issue of product mix. In this interpretation, 2.295000% of the product manufactured should be "proprietary," while the rest is "industry standard." As yet another possibility, 2.295000% of the product manufactured should be predatory into new markets, and the remainder in markets that are "traditional" for the company.

C.26.6 Number of Companies

This section evaluates the approximate, or "average," number of companies in the Simulated Equity Market Index, and uses the method outlined in Chapter 2, Section 2.6. Since the average, avg_{ind}, and the root mean square, rms_{ind}, of the normalized increments of the Simulated Equity Market Index time series is 0.000312, and 0.009889respectively, the number of companies participating in the market can be calculated by Equation 2.109 to be 3.190435.

If this value seems consistent number of companies in the Simulated Equity Market Index, within the assumptions outlined in Chapter 2, Section 2.6, then it would seem that there is some circumstantial or indirect evidence that the companies participating in the Simulated Equity Market Index are operating optimally, and the "average" Shannon probability, P for each participating company would be, using Equation 2.110, 0.508832, which would be the value which should be used in Section C.26.5 for each participating company if market expansion was to be consistent with the rest of the industry. However, if the Shannon probability derived in Section C.26.5 is greater than the average

[322] The astute reader would note that the linear addition was used to add the contribution to development of each product. This is a "near term" interpretation. Actually, in general, the method used should be a root mean square process, dependent on the Hurst Coefficient, H, where $P_{total}^H = P_1^H + P_2^H + \cdots$, where P_n is the contribution to each individual product. For a Brownian motion, or random walk type of fractal the Hurst Coefficient is a function of time into the future. For the "near term," the Hurst coefficient is very near unity, meaning the summation process is linear. For the "long term," $H \approx 0.5$, or a standard root mean square summation process should be used. If H is 0.5 then the market is termed a Brownian motion, or random walk process. If it is larger than 0.5, it is termed fractional Brownian motion process. For a random walk process, "near term" and "far term" are quantitatively differentiated on the Hurst Coefficient graph where $1 - \ln(t) = 0.5 \cdot \ln(t)$, or when $\ln(t) = 2$, or $t = 7.389\ldots$ See [Pet91, pp. 67, 83-84] and [Sch91, pp. 129, 159] for particulars on the implications of the Hurst Coefficient and root mean square summation issues.

Shannon probability for the companies participating in the Simulated Equity Market Index, as derived in this section, then the market would, possibly, be exploitable with the fiscal strategy outlined in Section C.26.5. The maximum exploitability for the Simulated Equity Market Index is derived in Section C.26.9, but it is probably of doubtful practicality.

Note that these optimizations would maximize a company's market growth. Since there are probably many companies competing in the market place, this would not necessarily maximize a company's P&L, as described in Chapter 2, Section 2.6.1. The Shannon probability that maximizes market share in the Simulated Equity Market Index is 0.508832, with several alternative solutions listed in the previous paragraph. However, these should be contrasted to the Shannon probability that maximizes a company's P&L which is 0.515775 in the Simulated Equity Market Index. In all cases, the fraction of the P&L that should be "wagered" on the future, f, should be:

$$f = 2P - 1 \tag{C.1001}$$

where P is the particular Shannon probability chosen optimize a particular fiscal strategy. Interestingly, the measured Shannon probability of the Simulated Equity Market Index would tend to indicate that the companies participating in the market have chosen a fiscal strategy that optimizes market growth, as opposed to capital growth.

As interesting interpretation of these exploitive issues, since all three fiscal strategies will result in exponential market growth for every company participating in the market, is that they may represent, perhaps, an example of "increasing returns."

C.26.7 Fixed Increment Approximation for Operational Strategy

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This section derives various values based on the "average" of the normalized increments presented in Figure C.578. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.577. These values will be used in a fixed increment Brownian fractal analysis and simulation of the Simulated Equity Market Index, and may, or may not, provide adequate accuracy for projections.

It should be noted that the analysis of fiscal strategy, presented in Section C.26.5, is derived from the Simulated Equity Market Index metrics and may, or may not, be maximally optimal. For the optimal fiscal strategy, which may be exploitable, see Section C.26.9.

An additional exploitable strategy may be time itself. Equations C.978, C.982, and, C.980, are, essentially, metrics on how fast a decision, which is based on information concerning the current status of the Simulated Equity Market Index, becomes obsolete. Obviously, how long a decision is expected to remain relevant should be addressed as an operational necessity in strategic planning and project management. Figures C.593, and, C.594 compare methods of approximation of the "forecastability" of rate of revenue returns in the Simulated Equity Market Index for the near term and far term [Pet91, pp. 83-84], respectively. As a general rule, caution must be exercised when making decisions that will span a time interval larger than the time interval where the "forecastability" of rate of revenue returns drops below 50%. Beyond this time interval, the chances increase that the competitive and market forces will alter the market environment in a possibly detrimental unanticipated fashion. Obviously, there is significant advantage in "timeliness" of development, manufacturing, and distribution of products and services that are consistent with this temporal agenda. Automation of these processes, if executed consistently with this agenda, should be considered a competitive advantage.

In some sense, this temporal agenda defines the "average" product or service life cycle in the Simulated Equity Market Index. When the "forecastability" of rate of revenue returns drops below 50%, there is an even chance that the rate of revenue returns for the product or service will change in a detrimental fashion. If it is assumed that a product or service life cycle consists of a ramp up, a maintenence interval, and a ramp down, then, if all three life cycle intervals are equal, the product life cycle will be, approximately, three times the time interval where the "forecastability" of rate of revenue returns drops below 50%. Although probably not an accurate prediction of product or service life cycle, the technique may be used as a conceptual approximation to the dynamics of "market windows.[323]" The conceptual approximation will probably predict a "conservative" or "pessimistic" value in relation to actual markets.

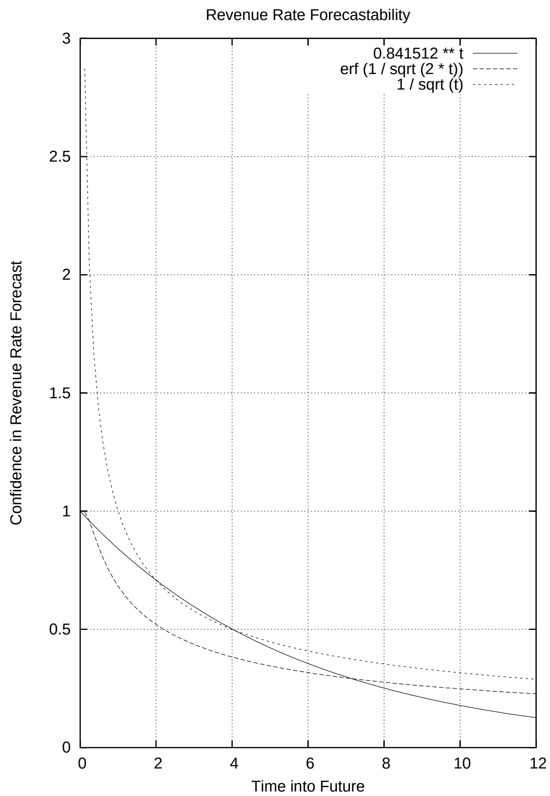

Figure C.593: Simulated Equity Market Index, "forecastability" of near term rate of revenue returns. Although the error function is the most accurate, for the near term, $H^t = 0.841512^t$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

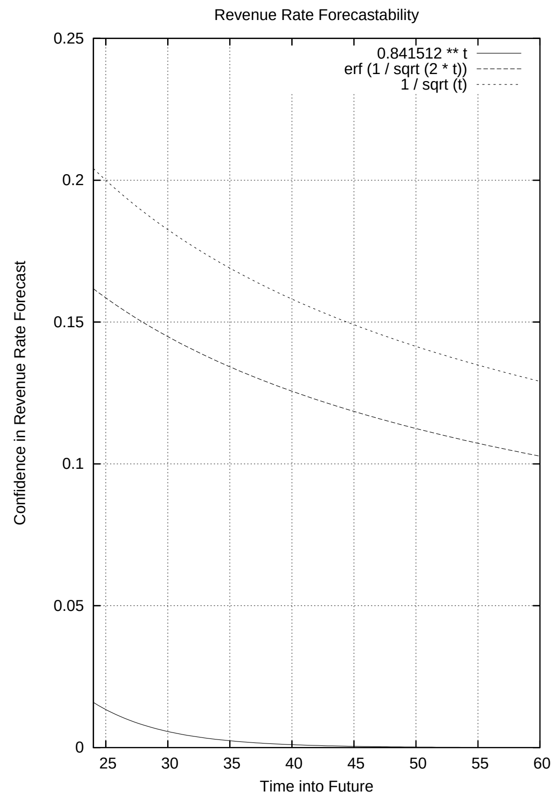

Figure C.594: Simulated Equity Market Index, "forecastability" of far term rate of revenue returns. Although the error function is the most accurate, for the far term, $\frac{1}{\sqrt{t}}$ may be used as a reliable metric of "forecastability" of the rate of revenue returns.

As an interesting interpretation of the data presented in Figure C.593, there may be, perhaps, some applicability to such operational agendas as inventory control. Maintaining too little inventory, obviously, will create a situation where the organization can not exploit market expansion, and maintaining too much inventory, likewise, would over extend the company, creating unnecessary losses when the market contracts. The company should maintain inventory levels that do not exceed, from Equation C.980, $0.841512^n = 0.5$ months of operations. Since the optimal amount of inventory and, from Equation C.978, the variance of change in the rate of revenue returns in the future can be calculated, there may, perhaps, be some applicability to a forecasting methodology that can be incorporated into other areas of operations research, for example the linear algebras using simplex methodologies for optimization of manufacturing

[323]For example, consider the market for table salt. Since it has inelastic supply and demand curves, and is a necessary requirement for life, it would be expected that the Hurst coefficient would be very near unity—ignoring competitive pressures in the market. The predictability of the table salt market would, therefore, be expected to be relatively good, over time.

processes. Traditionally, these forecasts are made by the sales department, and are subject to various subjective biases.

C.26.8 Simulation of Fixed Increment Approximation for Fiscal Strategy

The data in this section is presented in tabular form in Section D.26.9. Figure C.595 represents a constructional simulation of the time series data presented in Figure C.576. The program *tsunfairbrownian*, which is briefly described in appendix B, was used in the reconstruction. The reconstructed data is superimposed on the original time series data. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments—the value of the fixed increment is derived from the root mean square average of the normalized increments presented in Figure C.577. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the normalized increments presented in Figure C.577 represent a relatively complex process, that may not be "modeled" with such a simple methodology.

As a further comparison of the the constructional simulation with the original time series data, Figure C.596 presents a normalized histogram of the normalized increments of the reconstructed time series, superimposed on the normalized histogram presented in Figure C.579.

C.26.9 Simulation of Fixed Increment Approximation for Optimally Maximal Fiscal Strategy

The data in this section is presented in tabular form in Section D.26.3. One of the issues of analysis, as mentioned in Section C.26.7, is to determine the maximum Shannon probability for the time series presented in Figure C.576. Potentially, this could be exploited with an aggressive fiscal strategy. Figure C.597 is a graph of the output of the *tsshannonmax* program, which is described briefly in appendix B. The maximum of this function is the maximum Shannon probability for the time series data presented in Figure C.576. Figure C.598 was constructed using *tsunfairbrownian* program, which is also described in appendix B, with the maximum Shannon probability, and the time series data presented in Figure C.576. This represents a "what if" the investment strategy was changed from a Shannon probability of 0.511475, as derived in Section C.26.5 to 0.516297. This process, essentially, extracts the random statistical data from the time series presented in Figure C.576, and constructs a new time series, using the random statistical data, with a different investment strategy. The program, *tsunfairbrownian*, essentially, constructs the new time series as a Brownian fractal with fixed increments. The "quality" of such a reconstruction should be subject to adequate scepticism and scrutiny since, in all probability, the increments in the original data represent a relatively complex process, that may not be "modeled" with such a simple methodology.

If it is assumed that the time series data set, presented in Figure C.576, constitutes classical Brownian motion, then the Shannon probability can be calculated by counting the total number of months that the Simulated Equity Market Index movement was positive, and dividing by the total number of timescales represented in the time series. This quotient is 0.516200, as compared with the predicted value from the program *tsshannonmax* of 0.516297.

C.26.10 Qualitative Verification of Fixed Increment Approximation Analysis

This section evaluates various values based on the "average" of the normalized increments presented in Figure C.578. These values are an approximation to a, probably, complex process with a distribution shown in Figure C.577. These values will be used in a fixed increment Brownian fractal analysis of the Simulated Equity Market Index, and may, or may not, provide adequate accuracy for projections.

The data in this section is presented in tabular form in sections D.26.6 and D.26.7. As a subjective evaluation of the "quality" of the analysis of the Simulated Equity Market Index, from Chapter 3, Equation 3.8, and using the mean and root mean square values of the normalized increments of the time series data presented in Figure C.576 from Figure C.577, and the Shannon probability as calculated by counting the total number of months that the Simulated Equity Market Index movement was positive, as presented in Section C.26.9:

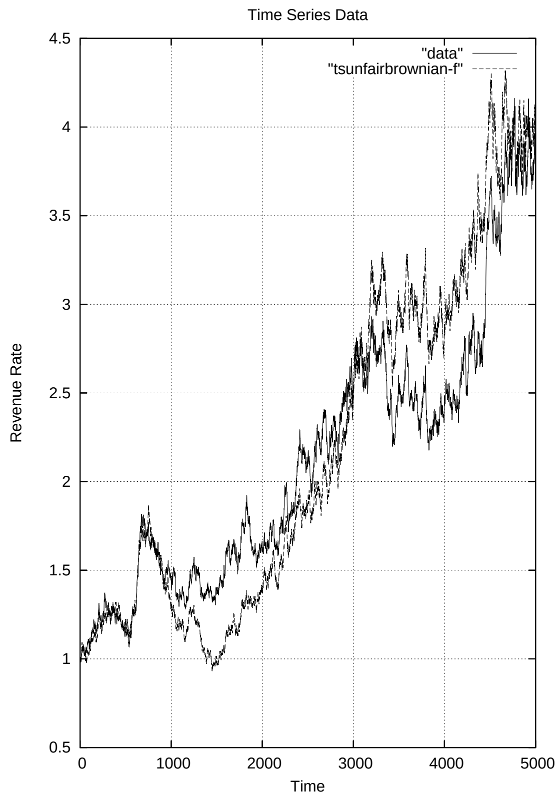

Figure C.595: Simulated Equity Market Index, Time series data, empirical and simulated, using the program *tsunfairbrownian* with f = 0.009889. This data is superimposed on the data presented in Figure C.576.

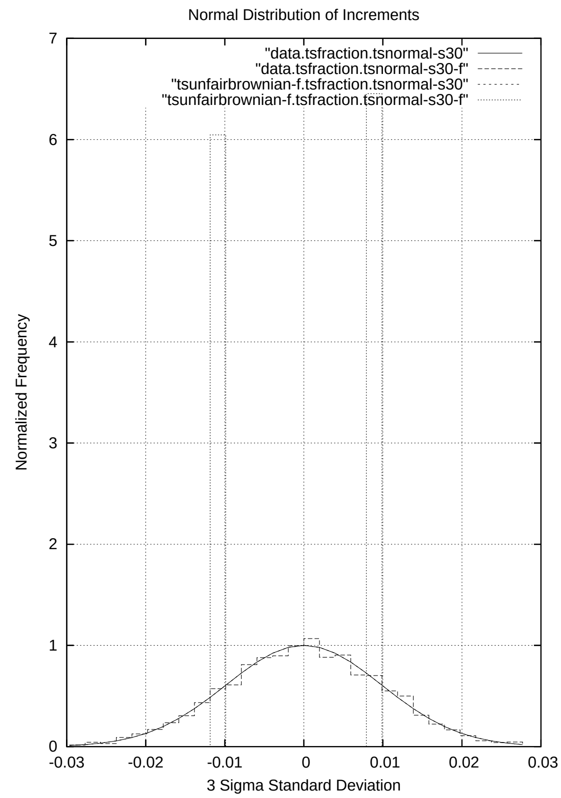

Figure C.596: Simulated Equity Market Index, normalized histogram of the normalized increments of the time series data shown in Figure C.595, empirical and simulated. The empirical data has a mean of 0.000312, with a standard deviation of 0.009885. By comparison, the simulated data has a mean of 0.000322 with a standard deviation of 0.009885. This data is superimposed on the data presented in Figure C.579. The area under the four curves is identical.

$$P \approx \frac{\frac{avg}{rms} + 1}{2} \tag{C.1002}$$

$$0.516200 \approx \frac{\frac{0.000312}{0.009889} + 1}{2} \tag{C.1003}$$

$$0.516200 \approx 0.515775 \tag{C.1004}$$

and comparing these values to the Shannon probability, as found by the *tsshannonmax* program, which iterates for a maximum:

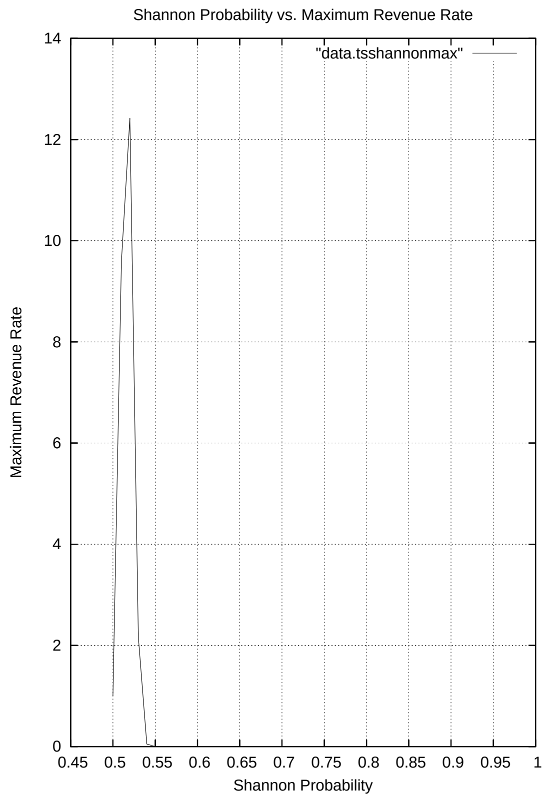

Figure C.597: Simulated Equity Market Index, maximum rate of revenue returns, per month, vs. Shannon probability. The maximum rate of revenue returns, per month, occurs at a Shannon probability of 0.516297.

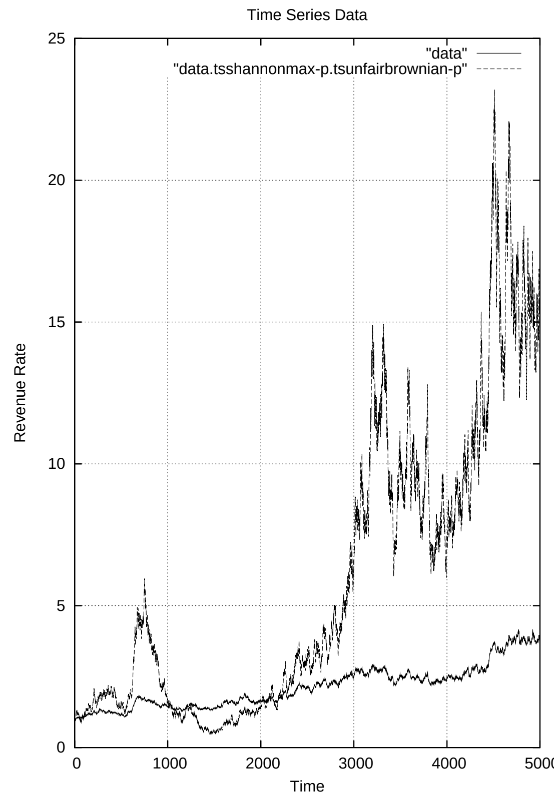

Figure C.598: Simulated Equity Market Index, maximum rate of revenue returns, per month, at a Shannon probability, of 0.516297, corresponding to a "wager" fraction of 0.032594.

$$0.516200 \approx 0.515775 \approx 0.516297 \tag{C.1005}$$

In addition, the different methods of calculating the logarithmic returns, presented in Section C.26.5, should be compared. The four methods used were the mean of Figure C.577, the constant in the least squares approximation to Figure C.577, the least squares exponential approximation to Figure C.576, and the logarithmic returns of Figure C.576, derived as the mean of the logarithms of the quotients of the increments. The values for each of the methods are, respectively:

$$0.000450 \approx 0.000511 \approx 0.000330 \approx 0.000380 \tag{C.1006}$$

It is implied in Section C.26.5, Subsection C.26.5 and in Section C.26.8 that, a Brownian motion with fixed increments fractal may "model" the Simulated Equity Market Index. Using Equation 2.104 from Chapter 2, Section 2.5:

$$
\begin{aligned}
rms\,(2P-1) &\approx \frac{\sigma\,(2P-1)}{2\sqrt{P\,(1-P)}} && \text{(C.1007)}\\
0.009889\,(2\cdot 0.516200-1) &\approx \frac{0.009885\,(2\cdot 0.516200-1)}{2\sqrt{0.516200\,(1-0.516200)}} && \text{(C.1008)}\\
0.009889\cdot 0.032400 &\approx 0.009885\cdot 0.032417 && \text{(C.1009)}\\
0.000320 &\approx 0.000320 && \text{(C.1010)}
\end{aligned}
$$

and, equating to the mean:

$$0.000312 \approx 0.000320 \approx 0.000320 \qquad \text{(C.1011)}$$

where, as in Equation C.1004 using the mean, root mean square, and standard deviation values of the normalized increments of the time series data presented in Figure C.576 from Figure C.577, and the Shannon probability as calculated by counting the total number of months that the Simulated Equity Market Index movement was positive, as presented in Section C.26.9.

As a final qualitative comparison, the absolute value of the normalized increments should be the same as the root mean square value[324], where the absolute value is presented in Figure C.578, and the root mean square value is presented in Figure C.577:

$$0.007914 \approx 0.009889 \qquad \text{(C.1012)}$$

Note, that if the Simulated Equity Market Index could be "modeled" as a Brownian motion with fixed increments fractal, then the standard deviation of the absolute value of the normalized increments of the time series data presented in Figure C.576 from Figure C.577 should be zero. It is 0.005930.

[324] The absolute value of the normalized increments, when averaged, is related to the root mean square of the increments by a constant. If the normalized increments are a fixed increment, the constant is unity. If the normalized increments have a Gaussian distribution, the constant is ≈ 0.8 depending on the accuracy of of "fit" to a Gaussian distribution.

Appendix D

Condensed Fractal Analysis of Various Market Segments in the North American Electronics Industry

This appendix presents, in condensed tabular form, the numerical metrics that were derived in appendix C.

D.1 North American Integrated Circuit Market

For the analysis, the data was in the directory ../markets/ic.namerica[1].

The data in this section is presented in Section C.1.

D.1.1 North American Integrated Circuit Market, normalized increments

The data in table D.1 is condensed from Section C.1.1.

Table D.1: North American Integrated Circuit Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.045132	0.075442	0.087396	0.042616	0.000081	0.070147	0.052548	0.087396	0.081978	-0.000382

D.1.2 North American Integrated Circuit Market, Logarithmic Returns, in Bits

The data in table D.2 is condensed from Section C.1.5.

D.1.3 North American Integrated Circuit Market, Shannon probabilities

The data in table D.3 is condensed from sections C.1.5 and C.1.10.

[1] Data from the Semiconductor Industry Association, 1979—1994, by quarters, in millions of dollars, US.

Table D.2: North American Integrated Circuit Market, Logarithmic Returns, in Bits.

Calculated from Table D.1		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.063685	0.060208	0.046835	0.058857

Table D.3: North American Integrated Circuit Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.746032	0.758204	0.750000	0.641843

D.1.4 North American Integrated Circuit Market, Logistic Analysis

The data in table D.4 is condensed from Section C.1.3[2].

Table D.4: North American Integrated Circuit Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.042616	0.000081

D.1.5 North American Integrated Circuit Market, Hurst Coefficients and H Parameters

The data in table D.5 is condensed from Section C.1.4.

Table D.5: North American Integrated Circuit Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.997635	0.720515	0.919129	0.929728

D.1.6 North American Integrated Circuit Market, verification of the increments

The data in table D.7 is condensed from Section C.1.11.

D.1.7 North American Integrated Circuit Market, verification of the increments

The data in table D.8 is condensed from Section C.1.11.

[2]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.6: North American Integrated Circuit Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.993773	0.705810	0.771912	0.742452

Table D.7: North American Integrated Circuit Market, verification the of the increments, the mean, σ is the standard deviation from table D.1, 0.075442, and P is the maximum Shannon probability from table D.3, 0.746032. In principle, the values should equate.

Mean	$\text{rms}(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.045132	0.043004	0.042642

Table D.8: North American Integrated Circuit Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[4].

Mean of the absolute value	rms
0.070147	0.087396

D.1.8 North American Integrated Circuit Market, χ^2 values of the increments

The data in table D.9 is condensed from Section C.4.

Table D.9: North American Integrated Circuit Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
6.219000	42.557000

D.1.9 North American Integrated Circuit Market, time series data, empirical and simulated

The data in table D.10 is condensed from Section C.1.9.

D.1.10 North American Integrated Circuit Market, number of participating companies

The data in table D.11 is condensed from Section C.1.6.

D.1.11 North American Integrated Circuit Market, Shannon probability optimizations

The data in table D.12 is condensed from Section C.1.6.

Table D.10: North American Integrated Circuit Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.045132	0.075442	0.042288	0.077108

Table D.11: North American Integrated Circuit Market, number of participating companies.

Number	Shannon probability
5.908830	0.606221

Table D.12: North American Integrated Circuit Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.758204	0.606221

D.2 World Semiconductor Market

For the analysis, the data was in the directory ../markets/semiconductors.world[5].

The data in this section is presented in Section C.1.

D.2.1 World Semiconductor Market, normalized increments

The data in table D.13 is condensed from Section C.1.1.

Table D.13: World Semiconductor Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.044437	0.064421	0.077739	0.039513	0.000197	0.061981	0.047389	0.077739	0.078868	-0.000675

D.2.2 World Semiconductor Market, Logarithmic Returns, in Bits

The data in table D.14 is condensed from Section C.1.5.

D.2.3 World Semiconductor Market, Shannon probabilities

The data in table D.15 is condensed from sections C.1.5 and C.1.10.

[5]Data from the Semiconductor Industry Association, 1982—1994, by quarters, in millions of dollars, US.

Table D.14: World Semiconductor Market, Logarithmic Returns, in Bits.

Calculated from Table D.13		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.062725	0.055908	0.053777	0.058816

Table D.15: World Semiconductor Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.823529	0.785809	0.826923	0.641794

D.2.4 World Semiconductor Market, Logistic Analysis

The data in table D.16 is condensed from Section C.1.3[6].

Table D.16: World Semiconductor Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.039513	0.000197

D.2.5 World Semiconductor Market, Hurst Coefficients and H Parameters

The data in table D.17 is condensed from Section C.1.4.

Table D.17: World Semiconductor Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
1.025249	0.725956	0.871338	0.885411

D.2.6 World Semiconductor Market, verification of the increments

The data in table D.19 is condensed from Section C.1.11.

D.2.7 World Semiconductor Market, verification of the increments

The data in table D.20 is condensed from Section C.1.11.

[6]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.18: World Semiconductor Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
1.028920	0.644727	0.745601	0.712999

Table D.19: World Semiconductor Market, verification the of the increments, the mean, σ is the standard deviation from table D.13, 0.064421, and P is the maximum Shannon probability from table D.15, 0.823529. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.044437	0.050302	0.054672

Table D.20: World Semiconductor Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[8].

Mean of the absolute value	rms
0.061981	0.077739

D.2.8 World Semiconductor Market, χ^2 values of the increments

The data in table D.21 is condensed from Section C.4.

Table D.21: World Semiconductor Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
9.194000	42.557000

D.2.9 World Semiconductor Market, time series data, empirical and simulated

The data in table D.22 is condensed from Section C.1.9.

D.2.10 World Semiconductor Market, number of participating companies

The data in table D.23 is condensed from Section C.1.6.

D.2.11 World Semiconductor Market, Shannon probability optimizations

The data in table D.24 is condensed from Section C.1.6.

Table D.22: World Semiconductor Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.044437	0.064421	0.049753	0.060339

Table D.23: World Semiconductor Market, number of participating companies.

Number	Shannon probability
7.353038	0.605400

Table D.24: World Semiconductor Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.785809	0.605400

D.3 North American Semiconductor Market

For the analysis, the data was in the directory ../markets/semiconductors.namerica[9].

The data in this section is presented in Section C.1.

D.3.1 North American Semiconductor Market, normalized increments

The data in table D.25 is condensed from Section C.1.1.

Table D.25: North American Semiconductor Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.040216	0.069677	0.079970	0.032989	0.000233	0.064520	0.047627	0.079970	0.072161	-0.000246

D.3.2 North American Semiconductor Market, Logarithmic Returns, in Bits

The data in table D.26 is condensed from Section C.1.5.

D.3.3 North American Semiconductor Market, Shannon probabilities

The data in table D.27 is condensed from sections C.1.5 and C.1.10.

[9]Data from the Semiconductor Industry Association, 1979—1994, by quarters, in millions of dollars, US.

Table D.26: North American Semiconductor Market, Logarithmic Returns, in Bits.

Calculated from Table D.25		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.056883	0.046825	0.042107	0.052703

Table D.27: North American Semiconductor Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.746032	0.751444	0.750000	0.634320

D.3.4 North American Semiconductor Market, Logistic Analysis

The data in table D.28 is condensed from Section C.1.3[10].

Table D.28: North American Semiconductor Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.032989	0.000233

D.3.5 North American Semiconductor Market, Hurst Coefficients and H Parameters

The data in table D.29 is condensed from Section C.1.4.

Table D.29: North American Semiconductor Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.998014	0.714241	0.914381	0.920999

D.3.6 North American Semiconductor Market, verification of the increments

The data in table D.31 is condensed from Section C.1.11.

D.3.7 North American Semiconductor Market, verification of the increments

The data in table D.32 is condensed from Section C.1.11.

[10]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.30: North American Semiconductor Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.991765	0.694731	0.777271	0.748847

Table D.31: North American Semiconductor Market, verification the of the increments, the mean, σ is the standard deviation from table D.25, 0.069677, and P is the maximum Shannon probability from table D.27, 0.746032. In principle, the values should equate.

Mean	$\text{rms}(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.040216	0.039350	0.039383

Table D.32: North American Semiconductor Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[12].

Mean of the absolute value	rms
0.064520	0.079970

D.3.8 North American Semiconductor Market, χ^2 values of the increments

The data in table D.33 is condensed from Section C.4.

Table D.33: North American Semiconductor Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
7.163000	42.557000

D.3.9 North American Semiconductor Market, time series data, empirical and simulated

The data in table D.34 is condensed from Section C.1.9.

D.3.10 North American Semiconductor Market, number of participating companies

The data in table D.35 is condensed from Section C.1.6.

D.3.11 North American Semiconductor Market, Shannon probability optimizations

The data in table D.36 is condensed from Section C.1.6.

Table D.34: North American Semiconductor Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.040216	0.069677	0.038695	0.070556

Table D.35: North American Semiconductor Market, number of participating companies.

Number	Shannon probability
6.288465	0.600270

Table D.36: North American Semiconductor Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.751444	0.600270

D.4 United States Electronic Component Shipments

For the analysis, the data was in the directory ../markets/electronic.components.shipments[13].

The data in this section is presented in Section C.1.

D.4.1 United States Electronic Component Shipments, normalized increments

The data in table D.37 is condensed from Section C.1.1.

Table D.37: United States Electronic Component Shipments, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.009362	0.028297	0.029736	0.010583	-0.000013	0.022871	0.019053	0.029736	0.022658	0.000002

D.4.2 United States Electronic Component Shipments, Logarithmic Returns, in Bits

The data in table D.38 is condensed from Section C.1.5.

D.4.3 United States Electronic Component Shipments, Shannon probabilities

The data in table D.39 is condensed from sections C.1.5 and C.1.10.

[13] Data from the United States Department of Commerce, 1979—1994, by months, in millions of dollars, US.

Table D.38: United States Electronic Component Shipments, Logarithmic Returns, in Bits.

Calculated from Table D.37		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.013444	0.015188	0.010340	0.012810

Table D.39: United States Electronic Component Shipments, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.656250	0.657419	0.658031	0.566532

D.4.4 United States Electronic Component Shipments, Logistic Analysis

The data in table D.40 is condensed from Section C.1.3[14].

Table D.40: United States Electronic Component Shipments, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.010583	-0.000013

D.4.5 United States Electronic Component Shipments, Hurst Coefficients and H Parameters

The data in table D.41 is condensed from Section C.1.4.

Table D.41: United States Electronic Component Shipments, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.755693	0.621033	0.226422	0.331203

D.4.6 United States Electronic Component Shipments, verification of the increments

The data in table D.43 is condensed from Section C.1.11.

D.4.7 United States Electronic Component Shipments, verification of the increments

The data in table D.44 is condensed from Section C.1.11.

[14]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.42: United States Electronic Component Shipments, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.759013	0.677966	0.277732	0.357729

Table D.43: United States Electronic Component Shipments, verification the of the increments, the mean, σ is the standard deviation from table D.37, 0.028297, and P is the maximum Shannon probability from table D.39, 0.656250. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.009362	0.009292	0.009309

Table D.44: United States Electronic Component Shipments, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[16].

Mean of the absolute value	rms
0.022871	0.029736

D.4.8 United States Electronic Component Shipments, χ^2 values of the increments

The data in table D.45 is condensed from Section C.4.

Table D.45: United States Electronic Component Shipments, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
4.711000	42.557000

D.4.9 United States Electronic Component Shipments, time series data, empirical and simulated

The data in table D.46 is condensed from Section C.1.9.

D.4.10 United States Electronic Component Shipments, number of participating companies

The data in table D.47 is condensed from Section C.1.6.

Table D.46: United States Electronic Component Shipments, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.009362	0.028297	0.009185	0.028356

Table D.47: United States Electronic Component Shipments, number of participating companies.

Number	Shannon probability
10.587747	0.548379

D.4.11 United States Electronic Component Shipments, Shannon probability optimizations

The data in table D.48 is condensed from Section C.1.6.

Table D.48: United States Electronic Component Shipments, Shannon probability optimization.

optimize capital growth	optimize market growth
0.657419	0.548379

D.5 United States Electronic Component Production

For the analysis, the data was in the directory ../markets/electronic.components.production[17].

The data in this section is presented in Section C.1.

D.5.1 United States Electronic Component Production, normalized increments

The data in table D.49 is condensed from Section C.1.1.

Table D.49: United States Electronic Component Production, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.008198	0.015216	0.017247	0.003048	0.000058	0.013756	0.010433	0.017247	0.013871	-0.000001

[17] Data from the United States Department of Commerce, 1980—1994, by months, as an index, 1987 = 100.

D.5.2 United States Electronic Component Production, Logarithmic Returns, in Bits

The data in table D.50 is condensed from Section C.1.5.

Table D.50: United States Electronic Component Production, Logarithmic Returns, in Bits.

Calculated from Table D.49		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.011779	0.004391	0.009588	0.011551

D.5.3 United States Electronic Component Production, Shannon probabilities

The data in table D.51 is condensed from sections C.1.5 and C.1.10.

Table D.51: United States Electronic Component Production, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.754190	0.737665	0.755556	0.563187

D.5.4 United States Electronic Component Production, Logistic Analysis

The data in table D.52 is condensed from Section C.1.3[18].

Table D.52: United States Electronic Component Production, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.003048	0.000058

D.5.5 United States Electronic Component Production, Hurst Coefficients and H Parameters

The data in table D.53 is condensed from Section C.1.4.

D.5.6 United States Electronic Component Production, verification of the increments

The data in table D.55 is condensed from Section C.1.11.

[18]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, *b*, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.53: United States Electronic Component Production, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.866652	0.776445	0.864735	0.882559

Table D.54: United States Electronic Component Production, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.869086	0.736363	0.762580	0.783817

Table D.55: United States Electronic Component Production, verification the of the increments, the mean, σ is the standard deviation from table D.49, 0.015216, and P is the maximum Shannon probability from table D.51, 0.754190. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.008198	0.008768	0.008983

D.5.7 United States Electronic Component Production, verification of the increments

The data in table D.56 is condensed from Section C.1.11.

Table D.56: United States Electronic Component Production, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[20].

Mean of the absolute value	rms
0.013756	0.017247

D.5.8 United States Electronic Component Production, χ^2 values of the increments

The data in table D.57 is condensed from Section C.4.

D.5.9 United States Electronic Component Production, time series data, empirical and simulated

The data in table D.58 is condensed from Section C.1.9.

D.5.10 United States Electronic Component Production, number of participating companies

The data in table D.59 is condensed from Section C.1.6.

Table D.57: United States Electronic Component Production, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
6.808000	42.557000

Table D.58: United States Electronic Component Production, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.008198	0.015216	0.008720	0.014922

Table D.59: United States Electronic Component Production, number of participating companies.

Number	Shannon probability
27.560100	0.545271

D.5.11 United States Electronic Component Production, Shannon probability optimizations

The data in table D.60 is condensed from Section C.1.6.

Table D.60: United States Electronic Component Production, Shannon probability optimization.

optimize capital growth	optimize market growth
0.737665	0.545271

D.6 United States Electronics Market

For the analysis, the data was in the directory ../markets/electronics[21].

The data in this section is presented in Section C.1.

D.6.1 United States Electronics Market, normalized increments

The data in table D.61 is condensed from Section C.1.1.

D.6.2 United States Electronics Market, Logarithmic Returns, in Bits

The data in table D.62 is condensed from Section C.1.5.

[21]Data from the United States Department of Commerce, 1980—1994, by months, in millions of dollars, US.

Table D.61: United States Electronics Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.007862	0.062079	0.062404	0.008318	-0.000005	0.048513	0.039361	0.062404	0.061949	-0.000150

Table D.62: United States Electronics Market, Logarithmic Returns, in Bits.

Calculated from Table D.61		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.011298	0.011951	0.007056	0.008559

D.6.3 United States Electronics Market, Shannon probabilities

The data in table D.63 is condensed from sections C.1.5 and C.1.10.

Table D.63: United States Electronics Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.522222	0.562993	0.524862	0.554410

D.6.4 United States Electronics Market, Logistic Analysis

The data in table D.64 is condensed from Section C.1.3[22].

Table D.64: United States Electronics Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.008318	-0.000005

D.6.5 United States Electronics Market, Hurst Coefficients and H Parameters

The data in table D.65 is condensed from Section C.1.4.

[22]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.65: United States Electronics Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.684410	0.399911	0.000643	0.045148

Table D.66: United States Electronics Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.688302	0.479826	0.028858	0.046059

D.6.6 United States Electronics Market, verification of the increments

The data in table D.67 is condensed from Section C.1.11.

Table D.67: United States Electronics Market, verification the of the increments, the mean, σ is the standard deviation from table D.61, 0.062079, and P is the maximum Shannon probability from table D.63, 0.522222. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.007862	0.002774	0.002762

D.6.7 United States Electronics Market, verification of the increments

The data in table D.68 is condensed from Section C.1.11.

Table D.68: United States Electronics Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[24].

Mean of the absolute value	rms
0.048513	0.062404

D.6.8 United States Electronics Market, χ^2 values of the increments

The data in table D.69 is condensed from Section C.4.

D.6.9 United States Electronics Market, time series data, empirical and simulated

The data in table D.70 is condensed from Section C.1.9.

Table D.69: United States Electronics Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
2.817000	42.557000

Table D.70: United States Electronics Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.007862	0.062079	0.003138	0.062500

D.6.10 United States Electronics Market, number of participating companies

The data in table D.71 is condensed from Section C.1.6.

Table D.71: United States Electronics Market, number of participating companies.

Number	Shannon probability
2.018869	0.544334

D.6.11 United States Electronics Market, Shannon probability optimizations

The data in table D.72 is condensed from Section C.1.6.

Table D.72: United States Electronics Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.562993	0.544334

D.7 United States Office Computer Market

For the analysis, the data was in the directory ../markets/computer.office[25].

The data in this section is presented in Section C.1.

D.7.1 United States Office Computer Market, normalized increments

The data in table D.73 is condensed from Section C.1.1.

[25] Data from the United States Department of Commerce, 1982—1994, by months, as an index, 1987 = 100.

Table D.73: United States Office Computer Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope	Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope
0.016771	0.028983	0.033404	0.023041	-0.000081	0.024562	0.022713	0.033404	0.038289	-0.000178

D.7.2 United States Office Computer Market, Logarithmic Returns, in Bits

The data in table D.74 is condensed from Section C.1.5.

Table D.74: United States Office Computer Market, Logarithmic Returns, in Bits.

Calculated from Table D.73		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.023995	0.032864	0.019653	0.023266

D.7.3 United States Office Computer Market, Shannon probabilities

The data in table D.75 is condensed from sections C.1.5 and C.1.10.

Table D.75: United States Office Computer Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	From program: *tsshannon*
0.780645	0.751033	0.782051	0.589554

D.7.4 United States Office Computer Market, Logistic Analysis

The data in table D.76 is condensed from Section C.1.3[26].

Table D.76: United States Office Computer Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.023041	-0.000081

[26]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

D.7.5 United States Office Computer Market, Hurst Coefficients and H Parameters

The data in table D.77 is condensed from Section C.1.4.

Table D.77: United States Office Computer Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.888451	0.723276	0.747093	0.750488

Table D.78: United States Office Computer Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.881397	0.760996	0.458933	0.484129

D.7.6 United States Office Computer Market, verification of the increments

The data in table D.79 is condensed from Section C.1.11.

Table D.79: United States Office Computer Market, verification the of the increments, the mean, σ is the standard deviation from table D.73, 0.028983, and P is the maximum Shannon probability from table D.75, 0.780645. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.016771	0.018749	0.019656

D.7.7 United States Office Computer Market, verification of the increments

The data in table D.80 is condensed from Section C.1.11.

Table D.80: United States Office Computer Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[28].

Mean of the absolute value	rms
0.024562	0.033404

D.7.8 United States Office Computer Market, χ^2 values of the increments

The data in table D.81 is condensed from Section C.4.

Table D.81: United States Office Computer Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
8.722000	42.557000

D.7.9 United States Office Computer Market, time series data, empirical and simulated

The data in table D.82 is condensed from Section C.1.9.

Table D.82: United States Office Computer Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.016771	0.028983	0.018654	0.027800

D.7.10 United States Office Computer Market, number of participating companies

The data in table D.83 is condensed from Section C.1.6.

Table D.83: United States Office Computer Market, number of participating companies.

Number	Shannon probability
15.030105	0.564751

D.7.11 United States Office Computer Market, Shannon probability optimizations

The data in table D.84 is condensed from Section C.1.6.

Table D.84: United States Office Computer Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.751033	0.564751

D.8 United States Information Systems Market

For the analysis, the data was in the directory ../markets/information.systems[29].

[29]Data from the United States Department of Commerce, 1979—1994, by months, in millions of dollars, US.

The data in this section is presented in Section C.1.

D.8.1 United States Information Systems Market, normalized increments

The data in table D.85 is condensed from Section C.1.1.

Table D.85: United States Information Systems Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.008052	0.038579	0.039311	0.010041	-0.000021	0.029745	0.025769	0.039311	0.035145	-0.000057

D.8.2 United States Information Systems Market, Logarithmic Returns, in Bits

The data in table D.86 is condensed from Section C.1.5.

Table D.86: United States Information Systems Market, Logarithmic Returns, in Bits.

Calculated from Table D.85		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.011570	0.014414	0.007623	0.010456

D.8.3 United States Information Systems Market, Shannon probabilities

The data in table D.87 is condensed from sections C.1.5 and C.1.10.

Table D.87: United States Information Systems Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.602094	0.602414	0.604167	0.560125

D.8.4 United States Information Systems Market, Logistic Analysis

The data in table D.88 is condensed from Section C.1.3[30].

D.8.5 United States Information Systems Market, Hurst Coefficients and H Parameters

The data in table D.89 is condensed from Section C.1.4.

[30]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, *b*, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.88: United States Information Systems Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.010041	-0.000021

Table D.89: United States Information Systems Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.710108	0.633980	0.171737	0.247886

Table D.90: United States Information Systems Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.707509	0.659558	0.102042	0.184942

D.8.6 United States Information Systems Market, verification of the increments

The data in table D.91 is condensed from Section C.1.11.

Table D.91: United States Information Systems Market, verification the of the increments, the mean, σ is the standard deviation from table D.85, 0.038579, and P is the maximum Shannon probability from table D.87, 0.602094. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.008052	0.008027	0.008047

D.8.7 United States Information Systems Market, verification of the increments

The data in table D.92 is condensed from Section C.1.11.

Table D.92: United States Information Systems Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[32].

Mean of the absolute value	rms
0.029745	0.039311

D.8.8 United States Information Systems Market, χ^2 values of the increments

The data in table D.93 is condensed from Section C.4.

Table D.93: United States Information Systems Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
2.862000	42.557000

D.8.9 United States Information Systems Market, time series data, empirical and simulated

The data in table D.94 is condensed from Section C.1.9.

Table D.94: United States Information Systems Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.008052	0.038579	0.007862	0.038619

D.8.10 United States Information Systems Market, number of participating companies

The data in table D.95 is condensed from Section C.1.6.

Table D.95: United States Information Systems Market, number of participating companies.

Number	Shannon probability
5.210454	0.544866

D.8.11 United States Information Systems Market, Shannon probability optimizations

The data in table D.96 is condensed from Section C.1.6.

Table D.96: United States Information Systems Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.602414	0.544866

D.9 Dow Jones Average

For the analysis, the data was in the directory ../markets/dj[33].

The data in this section is presented in Section C.1.

D.9.1 Dow Jones Average, normalized increments

The data in table D.97 is condensed from Section C.1.1.

Table D.97: Dow Jones Average, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope	Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope
0.008836	0.034803	0.035806	0.009746	-0.000011	0.025721	0.024985	0.035806	0.032502	-0.000082

D.9.2 Dow Jones Average, Logarithmic Returns, in Bits

The data in table D.98 is condensed from Section C.1.5.

Table D.98: Dow Jones Average, Logarithmic Returns, in Bits.

Calculated from Table D.97		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.012692	0.013992	0.014084	0.011753

D.9.3 Dow Jones Average, Shannon probabilities

The data in table D.99 is condensed from sections C.1.5 and C.1.10.

Table D.99: Dow Jones Average, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.634731	0.623387	0.636905	0.563735

D.9.4 Dow Jones Average, Logistic Analysis

The data in table D.100 is condensed from Section C.1.3[34].

[33]Data from Dow Jones News Information Retrieval Service, 1981—1994, by months, as an index.

[34]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.100: Dow Jones Average, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.009746	-0.000011

D.9.5 Dow Jones Average, Hurst Coefficients and H Parameters

The data in table D.101 is condensed from Section C.1.4.

Table D.101: Dow Jones Average, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.891560	0.566146	0.650691	0.618519

Table D.102: Dow Jones Average, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.894610	0.680456	0.668906	0.644657

D.9.6 Dow Jones Average, verification of the increments

The data in table D.103 is condensed from Section C.1.11.

Table D.103: Dow Jones Average, verification the of the increments, the mean, σ is the standard deviation from table D.97, 0.034803, and P is the maximum Shannon probability from table D.99, 0.634731. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.008836	0.009648	0.009738

D.9.7 Dow Jones Average, verification of the increments

The data in table D.104 is condensed from Section C.1.11.

D.9.8 Dow Jones Average, χ^2 values of the increments

The data in table D.105 is condensed from Section C.4.

Table D.104: Dow Jones Average, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[36].

Mean of the absolute value	rms
0.025721	0.035806

Table D.105: Dow Jones Average, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
8.043000	42.557000

D.9.9 Dow Jones Average, time series data, empirical and simulated

The data in table D.106 is condensed from Section C.1.9.

Table D.106: Dow Jones Average, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.008836	0.034803	0.009922	0.034508

D.9.10 Dow Jones Average, number of participating companies

The data in table D.107 is condensed from Section C.1.6.

Table D.107: Dow Jones Average, number of participating companies.

Number	Shannon probability
6.891981	0.547000

D.9.11 Dow Jones Average, Shannon probability optimizations

The data in table D.108 is condensed from Section C.1.6.

D.10 Cirrus Logic Stock

For the analysis, the data was in the directory ../markets/crus[37].

[37] Data from ftp://ftp.ai.mit.edu/pub/stocks/results/, by days, in dollars, US.

Table D.108: Dow Jones Average, Shannon probability optimization.

optimize capital growth	optimize market growth
0.623387	0.547000

The data in this section is presented in Section C.1.

D.10.1 Cirrus Logic Stock, normalized increments

The data in table D.109 is condensed from Section C.1.1.

Table D.109: Cirrus Logic Stock, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.002031	0.046039	0.046012	0.010587	-0.000054	0.030075	0.034876	0.046012	0.023205	0.000043

D.10.2 Cirrus Logic Stock, Logarithmic Returns, in Bits

The data in table D.110 is condensed from Section C.1.5.

Table D.110: Cirrus Logic Stock, Logarithmic Returns, in Bits.

Calculated from Table D.109		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.002927	0.015194	0.001768	0.001313

D.10.3 Cirrus Logic Stock, Shannon probabilities

The data in table D.111 is condensed from sections C.1.5 and C.1.10.

Table D.111: Cirrus Logic Stock, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.575000	0.522070	0.576324	0.521329

D.10.4 Cirrus Logic Stock, Logistic Analysis

The data in table D.112 is condensed from Section C.1.3[38].

Table D.112: Cirrus Logic Stock, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.010587	-0.000054

D.10.5 Cirrus Logic Stock, Hurst Coefficients and H Parameters

The data in table D.113 is condensed from Section C.1.4.

Table D.113: Cirrus Logic Stock, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.887589	0.706220	0.489541	0.478001

Table D.114: Cirrus Logic Stock, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.887433	0.678733	0.496581	0.487977

D.10.6 Cirrus Logic Stock, verification of the increments

The data in table D.115 is condensed from Section C.1.11.

Table D.115: Cirrus Logic Stock, verification the of the increments, the mean, σ is the standard deviation from table D.109, 0.046039, and P is the maximum Shannon probability from table D.111, 0.575000. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.002031	0.006902	0.006985

[38]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

D.10.7 Cirrus Logic Stock, verification of the increments

The data in table D.116 is condensed from Section C.1.11.

Table D.116: Cirrus Logic Stock, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[40].

Mean of the absolute value	rms
0.030075	0.046012

D.10.8 Cirrus Logic Stock, χ^2 values of the increments

The data in table D.117 is condensed from Section C.4.

Table D.117: Cirrus Logic Stock, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
4.817000	42.557000

D.10.9 Cirrus Logic Stock, time series data, empirical and simulated

The data in table D.118 is condensed from Section C.1.9.

Table D.118: Cirrus Logic Stock, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.002031	0.046039	0.007068	0.045537

D.10.10 Cirrus Logic Stock, number of participating companies

The data in table D.119 is condensed from Section C.1.6.

Table D.119: Cirrus Logic Stock, number of participating companies.

Number	Shannon probability
0.959329	0.522533

D.10.11 Cirrus Logic Stock, Shannon probability optimizations

The data in table D.120 is condensed from Section C.1.6.

Table D.120: Cirrus Logic Stock, Shannon probability optimization.

optimize capital growth	optimize market growth
0.522070	0.522533

D.11 United States Gross Domestic Product

For the analysis, the data was in the directory ../markets/us.gdp[41].

The data in this section is presented in Section C.1.

D.11.1 United States Gross Domestic Product, normalized increments

The data in table D.121 is condensed from Section C.1.1.

Table D.121: United States Gross Domestic Product, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope	Mean	Standard deviation	rms	Least Squares Constant	Least Squares Slope
0.005789	0.008347	0.010103	0.003515	0.000073	0.008280	0.005836	0.010103	0.009628	-0.000044

D.11.2 United States Gross Domestic Product, Logarithmic Returns, in Bits

The data in table D.122 is condensed from Section C.1.5.

Table D.122: United States Gross Domestic Product, Logarithmic Returns, in Bits.

Calculated from Table D.121		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.008328	0.005062	0.008994	0.008149

D.11.3 United States Gross Domestic Product, Shannon probabilities

The data in table D.123 is condensed from sections C.1.5 and C.1.10.

[41] Data from the United States Department of Commerce, 1979—1994, by months, in billions of 1987 dollars, US.

Table D.123: United States Gross Domestic Product, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.857143	0.786499	0.859375	0.553093

D.11.4 United States Gross Domestic Product, Logistic Analysis

The data in table D.124 is condensed from Section C.1.3[42].

Table D.124: United States Gross Domestic Product, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.003515	0.000073

D.11.5 United States Gross Domestic Product, Hurst Coefficients and H Parameters

The data in table D.125 is condensed from Section C.1.4.

Table D.125: United States Gross Domestic Product, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.935237	0.858488	0.741427	0.734478

Table D.126: United States Gross Domestic Product, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.935962	0.846454	0.723502	0.712101

D.11.6 United States Gross Domestic Product, verification of the increments

The data in table D.127 is condensed from Section C.1.11.

D.11.7 United States Gross Domestic Product, verification of the increments

The data in table D.128 is condensed from Section C.1.11.

[42]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.127: United States Gross Domestic Product, verification the of the increments, the mean, σ is the standard deviation from table D.121, 0.008347, and P is the maximum Shannon probability from table D.123, 0.857143. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.005789	0.007216	0.008519

Table D.128: United States Gross Domestic Product, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[44].

Mean of the absolute value	rms
0.008280	0.010103

D.11.8 United States Gross Domestic Product, χ^2 values of the increments

The data in table D.129 is condensed from Section C.4.

Table D.129: United States Gross Domestic Product, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
8.124000	42.557000

D.11.9 United States Gross Domestic Product, time series data, empirical and simulated

The data in table D.130 is condensed from Section C.1.9.

Table D.130: United States Gross Domestic Product, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.005789	0.008347	0.007170	0.007176

D.11.10 United States Gross Domestic Product, number of participating companies

The data in table D.131 is condensed from Section C.1.6.

Table D.131: United States Gross Domestic Product, number of participating companies.

Number	Shannon probability
56.715641	0.538043

D.11.11 United States Gross Domestic Product, Shannon probability optimizations

The data in table D.132 is condensed from Section C.1.6.

Table D.132: United States Gross Domestic Product, Shannon probability optimization.

optimize capital growth	optimize market growth
0.786499	0.538043

D.12 United States Employment Figures

For the analysis, the data was in the directory ../markets/us.employment[45].

The data in this section is presented in Section C.1.

D.12.1 United States Employment Figures, normalized increments

The data in table D.133 is condensed from Section C.1.1.

Table D.133: United States Employment Figures, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.001327	0.002254	0.002611	0.000929	0.000004	0.002171	0.001454	0.002611	0.002656	-0.000005

D.12.2 United States Employment Figures, Logarithmic Returns, in Bits

The data in table D.134 is condensed from Section C.1.5.

D.12.3 United States Employment Figures, Shannon probabilities

The data in table D.135 is condensed from sections C.1.5 and C.1.10.

[45] Data from the United States Bureau of Labor and Statistics, 1980—1994, by months, in thousands of persons.

Table D.134: United States Employment Figures, Logarithmic Returns, in Bits.

Calculated from Table D.133		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.001913	0.001340	0.002205	0.001900

Table D.135: United States Employment Figures, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.758242	0.754117	0.759563	0.525655

D.12.4 United States Employment Figures, Logistic Analysis

The data in table D.136 is condensed from Section C.1.3[46].

Table D.136: United States Employment Figures, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.000929	0.000004

D.12.5 United States Employment Figures, Hurst Coefficients and H Parameters

The data in table D.137 is condensed from Section C.1.4.

Table D.137: United States Employment Figures, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.879967	0.986346	0.831138	0.845292

D.12.6 United States Employment Figures, verification of the increments

The data in table D.139 is condensed from Section C.1.11.

D.12.7 United States Employment Figures, verification of the increments

The data in table D.140 is condensed from Section C.1.11.

[46]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.138: United States Employment Figures, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.880810	0.978765	0.822934	0.835377

Table D.139: United States Employment Figures, verification the of the increments, the mean, σ is the standard deviation from table D.133, 0.002254, and P is the maximum Shannon probability from table D.135, 0.758242. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.001327	0.001349	0.001360

Table D.140: United States Employment Figures, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[48].

Mean of the absolute value	rms
0.002171	0.002611

D.12.8 United States Employment Figures, χ^2 values of the increments

The data in table D.141 is condensed from Section C.4.

Table D.141: United States Employment Figures, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
4.227000	42.557000

D.12.9 United States Employment Figures, time series data, empirical and simulated

The data in table D.142 is condensed from Section C.1.9.

D.12.10 United States Employment Figures, number of participating companies

The data in table D.143 is condensed from Section C.1.6.

D.12.11 United States Employment Figures, Shannon probability optimizations

The data in table D.144 is condensed from Section C.1.6.

Table D.142: United States Employment Figures, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.001327	0.002254	0.001342	0.002246

Table D.143: United States Employment Figures, number of participating companies.

Number	Shannon probability
194.651242	0.518214

Table D.144: United States Employment Figures, Shannon probability optimization.

optimize capital growth	optimize market growth
0.754117	0.518214

D.13 United States Leading Economic Indicators

For the analysis, the data was in the directory ../markets/us.indicators[49].

The data in this section is presented in Section C.1.

D.13.1 United States Leading Economic Indicators, normalized increments

The data in table D.145 is condensed from Section C.1.1.

Table D.145: United States Leading Economic Indicators, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.000733	0.005089	0.005128	0.000683	0.000001	0.003869	0.003375	0.005128	0.006230	-0.000027

D.13.2 United States Leading Economic Indicators, Logarithmic Returns, in Bits

The data in table D.146 is condensed from Section C.1.5.

D.13.3 United States Leading Economic Indicators, Shannon probabilities

The data in table D.147 is condensed from sections C.1.5 and C.1.10.

[49] Data from the United States Department of Commerce, 1980—1994, by months, as an index of 1987 = 100.

Table D.146: United States Leading Economic Indicators, Logarithmic Returns, in Bits.

Calculated from Table D.145		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.001057	0.000985	0.001105	0.001033

Table D.147: United States Leading Economic Indicators, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.620112	0.571470	0.622222	0.518919

D.13.4 United States Leading Economic Indicators, Logistic Analysis

The data in table D.148 is condensed from Section C.1.3[50].

Table D.148: United States Leading Economic Indicators, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.000683	0.000001

D.13.5 United States Leading Economic Indicators, Hurst Coefficients and H Parameters

The data in table D.149 is condensed from Section C.1.4.

Table D.149: United States Leading Economic Indicators, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.931126	0.714236	0.745674	0.730044

D.13.6 United States Leading Economic Indicators, verification of the increments

The data in table D.151 is condensed from Section C.1.11.

D.13.7 United States Leading Economic Indicators, verification of the increments

The data in table D.152 is condensed from Section C.1.11.

[50]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.150: United States Leading Economic Indicators, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.930925	0.708761	0.726568	0.703224

Table D.151: United States Leading Economic Indicators, verification the of the increments, the mean, σ is the standard deviation from table D.145, 0.005089, and P is the maximum Shannon probability from table D.147, 0.620112. In principle, the values should equate.

Mean	$\text{rms}(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.000733	0.001232	0.001259

Table D.152: United States Leading Economic Indicators, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[52].

Mean of the absolute value	rms
0.003869	0.005128

D.13.8 United States Leading Economic Indicators, χ^2 values of the increments

The data in table D.153 is condensed from Section C.4.

Table D.153: United States Leading Economic Indicators, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
4.790000	42.557000

D.13.9 United States Leading Economic Indicators, time series data, empirical and simulated

The data in table D.154 is condensed from Section C.1.9.

D.13.10 United States Leading Economic Indicators, number of participating companies

The data in table D.155 is condensed from Section C.1.6.

D.13.11 United States Leading Economic Indicators, Shannon probability optimizations

The data in table D.156 is condensed from Section C.1.6.

Table D.154: United States Leading Economic Indicators, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.000733	0.005089	0.001210	0.004997

Table D.155: United States Leading Economic Indicators, number of participating companies.

Number	Shannon probability
27.874555	0.513537

Table D.156: United States Leading Economic Indicators, Shannon probability optimization.

optimize capital growth	optimize market growth
0.571470	0.513537

D.14 United States M2

For the analysis, the data was in the directory ../markets/us.m2[53].

The data in this section is presented in Section C.1.

D.14.1 United States M2, normalized increments

The data in table D.157 is condensed from Section C.1.1.

Table D.157: United States M2, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.001492	0.004594	0.004817	0.004922	-0.000041	0.003529	0.003289	0.004817	0.005573	-0.000025

D.14.2 United States M2, Logarithmic Returns, in Bits

The data in table D.158 is condensed from Section C.1.5.

D.14.3 United States M2, Shannon probabilities

The data in table D.159 is condensed from sections C.1.5 and C.1.10.

[53]Data from the United States Federal Reserve Board, 1980—1994, by months, in billions of 1987 dollars, US.

Table D.158: United States M2, Logarithmic Returns, in Bits.

Calculated from Table D.157		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.002151	0.007084	0.002294	0.002123

Table D.159: United States M2, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.568862	0.654868	0.571429	0.527119

D.14.4 United States M2, Logistic Analysis

The data in table D.160 is condensed from Section C.1.3[54].

Table D.160: United States M2, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.004922	-0.000041

D.14.5 United States M2, Hurst Coefficients and H Parameters

The data in table D.161 is condensed from Section C.1.4.

Table D.161: United States M2, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.956159	0.917851	0.832486	0.823599

D.14.6 United States M2, verification of the increments

The data in table D.163 is condensed from Section C.1.11.

D.14.7 United States M2, verification of the increments

The data in table D.164 is condensed from Section C.1.11.

[54]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.162: United States M2, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.955812	0.915770	0.834956	0.824890

Table D.163: United States M2, verification the of the increments, the mean, σ is the standard deviation from table D.157, 0.004594, and P is the maximum Shannon probability from table D.159, 0.568862. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.001492	0.000663	0.000639

Table D.164: United States M2, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[56].

Mean of the absolute value	rms
0.003529	0.004817

D.14.8 United States M2, χ^2 values of the increments

The data in table D.165 is condensed from Section C.4.

Table D.165: United States M2, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
3.176000	42.557000

D.14.9 United States M2, time series data, empirical and simulated

The data in table D.166 is condensed from Section C.1.9.

D.14.10 United States M2, number of participating companies

The data in table D.167 is condensed from Section C.1.6.

D.14.11 United States M2, Shannon probability optimizations

The data in table D.168 is condensed from Section C.1.6.

Table D.166: United States M2, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.001492	0.004594	0.000696	0.004781

Table D.167: United States M2, number of participating companies.

Number	Shannon probability
64.300675	0.519313

Table D.168: United States M2, Shannon probability optimization.

optimize capital growth	optimize market growth
0.654868	0.519313

D.15 United States Treasury Bill Returns

For the analysis, the data was in the directory ../markets/us.tbill[57].

The data in this section is presented in Section C.1.

D.15.1 United States Treasury Bill Returns, normalized increments

The data in table D.169 is condensed from Section C.1.1.

Table D.169: United States Treasury Bill Returns, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.005895	0.002409	0.006365	0.009310	-0.000041	0.005895	0.002409	0.006365	0.009310	-0.000041

D.15.2 United States Treasury Bill Returns, Logarithmic Returns, in Bits

The data in table D.170 is condensed from Section C.1.5.

D.15.3 United States Treasury Bill Returns, Shannon probabilities

The data in table D.171 is condensed from sections C.1.5 and C.1.10.

[57]Data from the United States Federal Reserve Board, 1980—1994, by months, in percent. The time series, which was Treasury Bill rate of returns, in percent per year, was converted to cumulative growth per month by converting each element in the time series to a fraction, dividing by 12, and adding 1. The previous value of cumulative returns was multiplied by this number for the next value of cumulative returns.

Table D.170: United States Treasury Bill Returns, Logarithmic Returns, in Bits.

Calculated from Table D.169		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.008480	0.013369	0.008215	0.008425

Table D.171: United States Treasury Bill Returns, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.999990	0.963079	1.000000	0.553983

D.15.4 United States Treasury Bill Returns, Logistic Analysis

The data in table D.172 is condensed from Section C.1.3[58].

Table D.172: United States Treasury Bill Returns, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.009310	-0.000041

D.15.5 United States Treasury Bill Returns, Hurst Coefficients and H Parameters

The data in table D.173 is condensed from Section C.1.4.

Table D.173: United States Treasury Bill Returns, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
1.086831	0.918814	0.987020	0.983813

D.15.6 United States Treasury Bill Returns, verification of the increments

The data in table D.175 is condensed from Section C.1.11.

D.15.7 United States Treasury Bill Returns, verification of the increments

The data in table D.176 is condensed from Section C.1.11.

[58]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.174: United States Treasury Bill Returns, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
1.074084	0.953633	0.975182	0.968718

Table D.175: United States Treasury Bill Returns, verification the of the increments, the mean, σ is the standard deviation from table D.169, 0.002409, and P is the maximum Shannon probability from table D.171, 0.999990. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.005895	0.006365	0.380891

Table D.176: United States Treasury Bill Returns, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[60].

Mean of the absolute value	rms
0.005895	0.006365

D.15.8 United States Treasury Bill Returns, χ^2 values of the increments

The data in table D.177 is condensed from Section C.4.

Table D.177: United States Treasury Bill Returns, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
7.453000	42.557000

D.15.9 United States Treasury Bill Returns, time series data, empirical and simulated

The data in table D.178 is condensed from Section C.1.9.

D.15.10 United States Treasury Bill Returns, number of participating companies

The data in table D.179 is condensed from Section C.1.6.

D.15.11 United States Treasury Bill Returns, Shannon probability optimizations

The data in table D.180 is condensed from Section C.1.6.

Table D.178: United States Treasury Bill Returns, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.005895	0.002409	0.006365	0.000000

Table D.179: United States Treasury Bill Returns, number of participating companies.

Number	Shannon probability
145.508041	0.538389

Table D.180: United States Treasury Bill Returns, Shannon probability optimization.

optimize capital growth	optimize market growth
0.963079	0.538389

D.16 Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin[61].

The data in this section is presented in Section C.1.

D.16.1 Coin Tossing Game, normalized increments

The data in table D.181 is condensed from Section C.1.1.

Table D.181: Coin Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.020736	0.199256	0.200000	0.013699	0.000047	0.200000	0.000001	0.200000	0.200000	0.000000

D.16.2 Coin Tossing Game, Logarithmic Returns, in Bits

The data in table D.182 is condensed from Section C.1.5.

D.16.3 Coin Tossing Game, Shannon probabilities

The data in table D.183 is condensed from sections C.1.5 and C.1.10.

[61] As a simulation model, the program *tscoin* was run to make a time series data file. The data is by tosses.

Table D.182: Coin Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.181		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.029610	0.019629	0.001451	0.000874

Table D.183: Coin Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.551839	0.551840	0.553333	0.517402

D.16.4 Coin Tossing Game, Logistic Analysis

The data in table D.184 is condensed from Section C.1.3[62].

Table D.184: Coin Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.013699	0.000047

D.16.5 Coin Tossing Game, Hurst Coefficients and H Parameters

The data in table D.185 is condensed from Section C.1.4.

Table D.185: Coin Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.853212	0.506256	0.448807	0.461555

D.16.6 Coin Tossing Game, verification of the increments

The data in table D.187 is condensed from Section C.1.11.

D.16.7 Coin Tossing Game, verification of the increments

The data in table D.188 is condensed from Section C.1.11.

[62]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.186: Coin Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.858532	0.571735	0.482986	0.488796

Table D.187: Coin Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.181, 0.199256, and P is the maximum Shannon probability from table D.183, 0.551839. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.020736	0.020736	0.020771

Table D.188: Coin Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[64].

Mean of the absolute value	rms
0.200000	0.200000

D.16.8 Coin Tossing Game, χ^2 values of the increments

The data in table D.189 is condensed from Section C.4.

Table D.189: Coin Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
117.483000	42.557000

D.16.9 Coin Tossing Game, time series data, empirical and simulated

The data in table D.190 is condensed from Section C.1.9.

D.16.10 Coin Tossing Game, number of participating companies

The data in table D.191 is condensed from Section C.1.6.

D.16.11 Coin Tossing Game, Shannon probability optimizations

The data in table D.192 is condensed from Section C.1.6.

Table D.190: Coin Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.020736	0.199256	0.020134	0.199319

Table D.191: Coin Tossing Game, number of participating companies.

Number	Shannon probability
0.518400	0.572000

Table D.192: Coin Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.551840	0.572000

D.17 Non Optimal Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tsunfairbrownian[65].

The data in this section is presented in Section C.1.

D.17.1 Non Optimal Coin Tossing Game, normalized increments

The data in table D.193 is condensed from Section C.1.1.

Table D.193: Non Optimal Coin Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.011477	0.027765	0.030000	0.007028	0.000030	0.030000	0.000000	0.030000	0.030000	0.000000

D.17.2 Non Optimal Coin Tossing Game, Logarithmic Returns, in Bits

The data in table D.194 is condensed from Section C.1.5.

D.17.3 Non Optimal Coin Tossing Game, Shannon probabilities

The data in table D.195 is condensed from sections C.1.5 and C.1.10.

[65] As a simulation model, the program *tscoin* was run to make a time series data file. The data is by tossess.

Table D.194: Non Optimal Coin Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.193		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.016464	0.010104	0.014992	0.015859

Table D.195: Non Optimal Coin Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.691275	0.691283	0.692308	0.574001

D.17.4 Non Optimal Coin Tossing Game, Logistic Analysis

The data in table D.196 is condensed from Section C.1.3[66].

Table D.196: Non Optimal Coin Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.007028	0.000030

D.17.5 Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters

The data in table D.197 is condensed from Section C.1.4.

Table D.197: Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.836828	0.737672	0.700050	0.721608

D.17.6 Non Optimal Coin Tossing Game, verification of the increments

The data in table D.199 is condensed from Section C.1.11.

D.17.7 Non Optimal Coin Tossing Game, verification of the increments

The data in table D.200 is condensed from Section C.1.11.

[66]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.198: Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.821860	0.541955	0.447590	0.464812

Table D.199: Non Optimal Coin Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.193, 0.027765, and P is the maximum Shannon probability from table D.195, 0.691275. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.011477	0.011477	0.011496

Table D.200: Non Optimal Coin Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[68].

Mean of the absolute value	rms
0.030000	0.030000

D.17.8 Non Optimal Coin Tossing Game, χ^2 values of the increments

The data in table D.201 is condensed from Section C.4.

Table D.201: Non Optimal Coin Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
130.435000	42.557000

D.17.9 Non Optimal Coin Tossing Game, time series data, empirical and simulated

The data in table D.202 is condensed from Section C.1.9.

D.17.10 Non Optimal Coin Tossing Game, number of participating companies

The data in table D.203 is condensed from Section C.1.6.

D.17.11 Non Optimal Coin Tossing Game, Shannon probability optimizations

The data in table D.204 is condensed from Section C.1.6.

Table D.202: Non Optimal Coin Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.011477	0.027765	0.011414	0.027791

Table D.203: Non Optimal Coin Tossing Game, number of participating companies.

Number	Shannon probability
12.752222	0.553565

Table D.204: Non Optimal Coin Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.691283	0.553565

D.18 Time Sampled Non Optimal Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tsunfairbrownian.tssample[69].

The data in this section is presented in Section C.1.

D.18.1 Time Sampled Non Optimal Coin Tossing Game, normalized increments

The data in table D.205 is condensed from Section C.1.1.

Table D.205: Time Sampled Non Optimal Coin Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.179112	0.221159	0.284303	0.152372	0.000180	0.233375	0.162645	0.284303	0.214675	0.000126

D.18.2 Time Sampled Non Optimal Coin Tossing Game, Logarithmic Returns, in Bits

The data in table D.206 is condensed from Section C.1.5.

D.18.3 Time Sampled Non Optimal Coin Tossing Game, Shannon probabilities

The data in table D.207 is condensed from sections C.1.5 and C.1.10.

[69] As a simulation model, the program *tscoin* was run to make a time series data file. The data is by tosses.

Table D.206: Time Sampled Non Optimal Coin Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.205		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.237701	0.204607	0.210487	0.210768

Table D.207: Time Sampled Non Optimal Coin Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.805369	0.815002	0.806020	0.763464

D.18.4 Time Sampled Non Optimal Coin Tossing Game, Logistic Analysis

The data in table D.208 is condensed from Section C.1.3[70].

Table D.208: Time Sampled Non Optimal Coin Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.152372	0.000180

D.18.5 Time Sampled Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters

The data in table D.209 is condensed from Section C.1.4.

Table D.209: Time Sampled Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.869484	0.734095	0.958160	0.939536

D.18.6 Time Sampled Non Optimal Coin Tossing Game, verification of the increments

The data in table D.211 is condensed from Section C.1.11.

[70]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.210: Time Sampled Non Optimal Coin Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.857884	0.519188	0.498680	0.500712

Table D.211: Time Sampled Non Optimal Coin Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.205, 0.221159, and P is the maximum Shannon probability from table D.207, 0.805369. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.179112	0.173635	0.170579

D.18.7 Time Sampled Non Optimal Coin Tossing Game, verification of the increments

The data in table D.212 is condensed from Section C.1.11.

Table D.212: Time Sampled Non Optimal Coin Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[72].

Mean of the absolute value	rms
0.233375	0.284303

D.18.8 Time Sampled Non Optimal Coin Tossing Game, χ^2 values of the increments

The data in table D.213 is condensed from Section C.4.

Table D.213: Time Sampled Non Optimal Coin Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
51.060000	42.557000

D.18.9 Time Sampled Non Optimal Coin Tossing Game, time series data, empirical and simulated

The data in table D.214 is condensed from Section C.1.9.

Table D.214: Time Sampled Non Optimal Coin Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.179112	0.221159	0.173262	0.225788

D.18.10 Time Sampled Non Optimal Coin Tossing Game, number of participating companies

The data in table D.215 is condensed from Section C.1.6.

Table D.215: Time Sampled Non Optimal Coin Tossing Game, number of participating companies.

Number	Shannon probability
2.215959	0.711608

D.18.11 Time Sampled Non Optimal Coin Tossing Game, Shannon probability optimizations

The data in table D.216 is condensed from Section C.1.6.

Table D.216: Time Sampled Non Optimal Coin Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.815002	0.711608

D.19 Time Sampled Coin Tossing Game

For the analysis, the data was in the directory ../markets/tscoin.tssample[73].

The data in this section is presented in Section C.1.

D.19.1 Time Sampled Coin Tossing Game, normalized increments

The data in table D.217 is condensed from Section C.1.1.

D.19.2 Time Sampled Coin Tossing Game, Logarithmic Returns, in Bits

The data in table D.218 is condensed from Section C.1.5.

[73] As a simulation model, the program *tscoin* was run to make a time series data file. The data is by tosses.

Table D.217: Time Sampled Coin Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.027085	0.205494	0.206930	-0.019736	0.000314	0.171592	0.115850	0.206930	0.158506	0.000088

Table D.218: Time Sampled Coin Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.217		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.038556	-0.028758	0.011235	0.009520

D.19.3 Time Sampled Coin Tossing Game, Shannon probabilities

The data in table D.219 is condensed from sections C.1.5 and C.1.10.

Table D.219: Time Sampled Coin Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.548495	0.565445	0.550000	0.557377

D.19.4 Time Sampled Coin Tossing Game, Logistic Analysis

The data in table D.220 is condensed from Section C.1.3[74].

Table D.220: Time Sampled Coin Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
-0.019736	0.000314

D.19.5 Time Sampled Coin Tossing Game, Hurst Coefficients and H Parameters

The data in table D.221 is condensed from Section C.1.4.

[74] Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

Table D.221: Time Sampled Coin Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.835189	0.604532	0.453453	0.476672

Table D.222: Time Sampled Coin Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.829636	0.611530	0.500719	0.499231

D.19.6 Time Sampled Coin Tossing Game, verification of the increments

The data in table D.223 is condensed from Section C.1.11.

Table D.223: Time Sampled Coin Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.217, 0.205494, and P is the maximum Shannon probability from table D.219, 0.548495. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.027085	0.020070	0.020025

D.19.7 Time Sampled Coin Tossing Game, verification of the increments

The data in table D.224 is condensed from Section C.1.11.

Table D.224: Time Sampled Coin Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[76].

Mean of the absolute value	rms
0.171592	0.206930

D.19.8 Time Sampled Coin Tossing Game, χ^2 values of the increments

The data in table D.225 is condensed from Section C.4.

D.19.9 Time Sampled Coin Tossing Game, time series data, empirical and simulated

The data in table D.226 is condensed from Section C.1.9.

Table D.225: Time Sampled Coin Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
46.304000	42.557000

Table D.226: Time Sampled Coin Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.027085	0.205494	0.020832	0.206224

D.19.10 Time Sampled Coin Tossing Game, number of participating companies

The data in table D.227 is condensed from Section C.1.6.

Table D.227: Time Sampled Coin Tossing Game, number of participating companies.

Number	Shannon probability
0.632531	0.582288

D.19.11 Time Sampled Coin Tossing Game, Shannon probability optimizations

The data in table D.228 is condensed from Section C.1.6.

Table D.228: Time Sampled Coin Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.565445	0.582288

D.20 Simulated Shannon Probability of 0.6

For the analysis, the data was in the directory ../markets/tsunfairbrownian.exponential[77].

The data in this section is presented in Section C.1.

D.20.1 Simulated Shannon Probability of 0.6, normalized increments

The data in table D.229 is condensed from Section C.1.1.

[77] As a simulation model, the program *tsunfairbrownian* was run to make a time series data file. The data is by time unitss.

Table D.229: Simulated Shannon Probability of 0.6, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.039968	0.195985	0.200000	0.039872	0.000000	0.200000	0.000000	0.200000	0.200000	0.000000

D.20.2 Simulated Shannon Probability of 0.6, Logarithmic Returns, in Bits

The data in table D.230 is condensed from Section C.1.5.

Table D.230: Simulated Shannon Probability of 0.6, Logarithmic Returns, in Bits.

Calculated from Table D.229		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.056539	0.056406	0.029049	0.028997

D.20.3 Simulated Shannon Probability of 0.6, Shannon probabilities

The data in table D.231 is condensed from sections C.1.5 and C.1.10.

Table D.231: Simulated Shannon Probability of 0.6, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.599920	0.599920	0.600000	0.599910

D.20.4 Simulated Shannon Probability of 0.6, Logistic Analysis

The data in table D.232 is condensed from Section C.1.3[78].

Table D.232: Simulated Shannon Probability of 0.6, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.039872	0.000000

[78]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.1.3.

D.20.5 Simulated Shannon Probability of 0.6, Hurst Coefficients and H Parameters

The data in table D.233 is condensed from Section C.1.4.

Table D.233: Simulated Shannon Probability of 0.6, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.539904	0.668253	-0.118068	0.027409

Table D.234: Simulated Shannon Probability of 0.6, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.526347	0.029183	-0.196158	-0.037908

D.20.6 Simulated Shannon Probability of 0.6, verification of the increments

The data in table D.235 is condensed from Section C.1.11.

Table D.235: Simulated Shannon Probability of 0.6, verification the of the increments, the mean, σ is the standard deviation from table D.229, 0.195985, and P is the maximum Shannon probability from table D.231, 0.599920. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.039968	0.039968	0.039972

D.20.7 Simulated Shannon Probability of 0.6, verification of the increments

The data in table D.236 is condensed from Section C.1.11.

Table D.236: Simulated Shannon Probability of 0.6, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[80].

Mean of the absolute value	rms
0.200000	0.200000

D.20.8 Simulated Shannon Probability of 0.6, χ^2 values of the increments

The data in table D.237 is condensed from Section C.4.

Table D.237: Simulated Shannon Probability of 0.6, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
131.562000	42.557000

D.20.9 Simulated Shannon Probability of 0.6, time series data, empirical and simulated

The data in table D.238 is condensed from Section C.1.9.

Table D.238: Simulated Shannon Probability of 0.6, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.039968	0.195985	0.040016	0.195976

D.20.10 Simulated Shannon Probability of 0.6, number of participating companies

The data in table D.239 is condensed from Section C.1.6.

Table D.239: Simulated Shannon Probability of 0.6, number of participating companies.

Number	Shannon probability
0.999200	0.599960

D.20.11 Simulated Shannon Probability of 0.6, Shannon probability optimizations

The data in table D.240 is condensed from Section C.1.6.

Table D.240: Simulated Shannon Probability of 0.6, Shannon probability optimization.

optimize capital growth	optimize market growth
0.599920	0.599960

D.21 Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins[81].

[81] As a simulation model, the program *tscoins* was run to make a time series data file. The data is by tosses.

The data in this section is presented in Section C.21.

D.21.1 Coins Tossing Game, normalized increments

The data in table D.241 is condensed from Section C.21.1.

Table D.241: Coins Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.062916	0.196353	0.205874	0.073339	-0.000070	0.169184	0.117504	0.205874	0.167643	0.000010

D.21.2 Coins Tossing Game, Logarithmic Returns, in Bits

The data in table D.242 is condensed from Section C.21.5.

Table D.242: Coins Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.241		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.088028	0.102106	0.063132	0.061793

D.21.3 Coins Tossing Game, Shannon probabilities

The data in table D.243 is condensed from sections C.21.5 and C.21.9.

Table D.243: Coins Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.645485	0.652802	0.646667	0.645287

D.21.4 Coins Tossing Game, Logistic Analysis

The data in table D.244 is condensed from Section C.21.3[82].

D.21.5 Coins Tossing Game, Hurst Coefficients and H Parameters

The data in table D.245 is condensed from Section C.21.4.

[82]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.21.3.

Table D.244: Coins Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.073339	-0.000070

Table D.245: Coins Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.856676	0.642211	0.464712	0.439712

Table D.246: Coins Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.850254	0.452008	0.474175	0.463850

D.21.6 Coins Tossing Game, verification of the increments

The data in table D.247 is condensed from Section C.21.10.

Table D.247: Coins Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.241, 0.196353, and P is the maximum Shannon probability from table D.243, 0.645485. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.062916	0.059903	0.059717

D.21.7 Coins Tossing Game, verification of the increments

The data in table D.248 is condensed from Section C.21.10.

Table D.248: Coins Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[84].

Mean of the absolute value	rms
0.169184	0.205874

D.21.8 Coins Tossing Game, χ^2 values of the increments

The data in table D.249 is condensed from Section C.464.

Table D.249: Coins Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
1.028000	42.557000

D.21.9 Coins Tossing Game, time series data, empirical and simulated

The data in table D.250 is condensed from Section C.21.8.

Table D.250: Coins Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.062916	0.196353	0.060795	0.197024

D.21.10 Coins Tossing Game, number of participating companies

The data in table D.251 is condensed from Section C.21.6.

Table D.251: Coins Tossing Game, number of participating companies.

Number	Shannon probability
1.484424	0.625415

D.21.11 Coins Tossing Game, Shannon probability optimizations

The data in table D.252 is condensed from Section C.21.6.

Table D.252: Coins Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.652802	0.625415

D.22 Non Optimal Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins-f[85].

The data in this section is presented in Section C.22.

D.22.1 Non Optimal Coins Tossing Game, normalized increments

The data in table D.253 is condensed from Section C.22.1.

Table D.253: Non Optimal Coins Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.009437	0.029453	0.030881	0.011001	-0.000010	0.025378	0.017626	0.030881	0.025146	0.000002

D.22.2 Non Optimal Coins Tossing Game, Logarithmic Returns, in Bits

The data in table D.254 is condensed from Section C.22.5.

Table D.254: Non Optimal Coins Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.253		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.013551	0.015784	0.013218	0.012895

D.22.3 Non Optimal Coins Tossing Game, Shannon probabilities

The data in table D.255 is condensed from sections C.22.5 and C.22.9.

Table D.255: Non Optimal Coins Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.645485	0.652796	0.646667	0.566751

D.22.4 Non Optimal Coins Tossing Game, Logistic Analysis

The data in table D.256 is condensed from Section C.22.3[86].

[85]As a simulation model, the program *tscoins* was run to make a time series data file. The data is by tosses.

[86]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.22.3.

Table D.256: Non Optimal Coins Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.011001	-0.000010

D.22.5 Non Optimal Coins Tossing Game, Hurst Coefficients and H Parameters

The data in table D.257 is condensed from Section C.22.4.

Table D.257: Non Optimal Coins Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.849887	0.680509	0.552238	0.545487

Table D.258: Non Optimal Coins Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.850253	0.452008	0.474175	0.463851

D.22.6 Non Optimal Coins Tossing Game, verification of the increments

The data in table D.259 is condensed from Section C.22.10.

Table D.259: Non Optimal Coins Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.253, 0.029453, and P is the maximum Shannon probability from table D.255, 0.645485. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.009437	0.008985	0.008958

D.22.7 Non Optimal Coins Tossing Game, verification of the increments

The data in table D.260 is condensed from Section C.22.10.

D.22.8 Non Optimal Coins Tossing Game, χ^2 values of the increments

The data in table D.261 is condensed from Section C.487.

Table D.260: Non Optimal Coins Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[88].

Mean of the absolute value	rms
0.025378	0.030881

Table D.261: Non Optimal Coins Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
1.028000	42.557000

D.22.9 Non Optimal Coins Tossing Game, time series data, empirical and simulated

The data in table D.262 is condensed from Section C.22.8.

Table D.262: Non Optimal Coins Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.009437	0.029453	0.009119	0.029553

D.22.10 Non Optimal Coins Tossing Game, number of participating companies

The data in table D.263 is condensed from Section C.22.6.

Table D.263: Non Optimal Coins Tossing Game, number of participating companies.

Number	Shannon probability
9.895808	0.548572

D.22.11 Non Optimal Coins Tossing Game, Shannon probability optimizations

The data in table D.264 is condensed from Section C.22.6.

Table D.264: Non Optimal Coins Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.652796	0.548572

D.23 Non Optimal Logistic Coins Tossing Game

For the analysis, the data was in the directory ../markets/tscoins-b[89].

The data in this section is presented in Section C.23.

D.23.1 Non Optimal Logistic Coins Tossing Game, normalized increments

The data in table D.265 is condensed from Section C.23.1.

Table D.265: Non Optimal Logistic Coins Tossing Game, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.032010	0.195609	0.198114	0.061302	-0.000059	0.159477	0.117600	0.198114	0.167886	-0.000017

D.23.2 Non Optimal Logistic Coins Tossing Game, Logarithmic Returns, in Bits

The data in table D.266 is condensed from Section C.23.5.

Table D.266: Non Optimal Logistic Coins Tossing Game, Logarithmic Returns, in Bits.

Calculated from Table D.265		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.045457	0.085835	0.012612	0.018217

D.23.3 Non Optimal Logistic Coins Tossing Game, Shannon probabilities

The data in table D.267 is condensed from sections C.23.5 and C.23.9.

D.23.4 Non Optimal Logistic Coins Tossing Game, Logistic Analysis

The data in table D.268 is condensed from Section C.23.3[90].

[89] As a simulation model, the program *tscoins* was run to make a time series data file. The data is by tosses.

[90] Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.23.3.

Table D.267: Non Optimal Logistic Coins Tossing Game, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.567568	0.580787	0.568000	0.579290

Table D.268: Non Optimal Logistic Coins Tossing Game, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.061302	-0.000059

D.23.5 Non Optimal Logistic Coins Tossing Game, Hurst Coefficients and H Parameters

The data in table D.269 is condensed from Section C.23.4.

Table D.269: Non Optimal Logistic Coins Tossing Game, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.842100	0.475809	0.476503	0.487271

Table D.270: Non Optimal Logistic Coins Tossing Game, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.842820	0.704858	0.481445	0.476808

D.23.6 Non Optimal Logistic Coins Tossing Game, verification of the increments

The data in table D.271 is condensed from Section C.23.10.

Table D.271: Non Optimal Logistic Coins Tossing Game, verification the of the increments, the mean, σ is the standard deviation from table D.265, 0.195609, and P is the maximum Shannon probability from table D.267, 0.567568. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.032010	0.026772	0.026678

D.23.7 Non Optimal Logistic Coins Tossing Game, verification of the increments

The data in table D.272 is condensed from Section C.23.10.

Table D.272: Non Optimal Logistic Coins Tossing Game, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[92].

Mean of the absolute value	rms
0.159477	0.198114

D.23.8 Non Optimal Logistic Coins Tossing Game, χ^2 values of the increments

The data in table D.273 is condensed from Section C.510.

Table D.273: Non Optimal Logistic Coins Tossing Game, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
0.367000	42.557000

D.23.9 Non Optimal Logistic Coins Tossing Game, time series data, empirical and simulated

The data in table D.274 is condensed from Section C.23.8.

Table D.274: Non Optimal Logistic Coins Tossing Game, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.032010	0.195609	0.026997	0.196364

D.23.10 Non Optimal Logistic Coins Tossing Game, number of participating companies

The data in table D.275 is condensed from Section C.23.6.

D.23.11 Non Optimal Logistic Coins Tossing Game, Shannon probability optimizations

The data in table D.276 is condensed from Section C.23.6.

Table D.275: Non Optimal Logistic Coins Tossing Game, number of participating companies.

Number	Shannon probability
0.815559	0.589457

Table D.276: Non Optimal Logistic Coins Tossing Game, Shannon probability optimization.

optimize capital growth	optimize market growth
0.580787	0.589457

D.24 Simulated Industrial Market

For the analysis, the data was in the directory ../markets/tsmarket[93].

The data in this section is presented in Section C.24.

D.24.1 Simulated Industrial Market, normalized increments

The data in table D.277 is condensed from Section C.24.1.

Table D.277: Simulated Industrial Market, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.013109	0.029243	0.032003	0.013460	-0.000002	0.025065	0.019930	0.032003	0.023976	0.000007

D.24.2 Simulated Industrial Market, Logarithmic Returns, in Bits

The data in table D.278 is condensed from Section C.24.5.

Table D.278: Simulated Industrial Market, Logarithmic Returns, in Bits.

Calculated from Table D.277		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.018789	0.019289	0.017469	0.018128

D.24.3 Simulated Industrial Market, Shannon probabilities

The data in table D.279 is condensed from sections C.24.5 and C.24.9.

[93]As a simulation model, the program *tsmarket* was run to make a time series data file. The data is by months.

Table D.279: Simulated Industrial Market, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program: *tsshannonmax*	*tsshannon*
0.675585	0.704809	0.676667	0.579097

D.24.4 Simulated Industrial Market, Logistic Analysis

The data in table D.280 is condensed from Section C.24.3[94].

Table D.280: Simulated Industrial Market, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.013460	-0.000002

D.24.5 Simulated Industrial Market, Hurst Coefficients and H Parameters

The data in table D.281 is condensed from Section C.24.4.

Table D.281: Simulated Industrial Market, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.848216	0.786671	0.515640	0.506062

Table D.282: Simulated Industrial Market, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.847743	0.574061	0.492923	0.486433

D.24.6 Simulated Industrial Market, verification of the increments

The data in table D.283 is condensed from Section C.24.10.

D.24.7 Simulated Industrial Market, verification of the increments

The data in table D.284 is condensed from Section C.24.10.

[94] Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.24.3.

Table D.283: Simulated Industrial Market, verification the of the increments, the mean, σ is the standard deviation from table D.277, 0.029243, and P is the maximum Shannon probability from table D.279, 0.675585. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.013109	0.011239	0.010968

Table D.284: Simulated Industrial Market, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[96].

Mean of the absolute value	rms
0.025065	0.032003

D.24.8 Simulated Industrial Market, χ^2 values of the increments

The data in table D.285 is condensed from Section C.533.

Table D.285: Simulated Industrial Market, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
1.229000	42.557000

D.24.9 Simulated Industrial Market, time series data, empirical and simulated

The data in table D.286 is condensed from Section C.24.8.

Table D.286: Simulated Industrial Market, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.013109	0.029243	0.011169	0.030041

D.24.10 Simulated Industrial Market, number of participating companies

The data in table D.287 is condensed from Section C.24.6.

Table D.287: Simulated Industrial Market, number of participating companies.

Number	Shannon probability
12.799358	0.557247

D.24.11 Simulated Industrial Market, Shannon probability optimizations

The data in table D.288 is condensed from Section C.24.6.

Table D.288: Simulated Industrial Market, Shannon probability optimization.

optimize capital growth	optimize market growth
0.704809	0.557247

D.25 Discreet Logistic Function

For the analysis, the data was in the directory ../markets/tsdlogistic[97].

The data in this section is presented in Section C.25.

D.25.1 Discreet Logistic Function, normalized increments

The data in table D.289 is condensed from Section C.25.1.

Table D.289: Discreet Logistic Function, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.962687	1.398261	1.695689	1.244560	-0.001892	1.384386	0.980842	1.695689	1.643432	-0.001739

D.25.2 Discreet Logistic Function, Logarithmic Returns, in Bits

The data in table D.290 is condensed from Section C.25.5.

D.25.3 Discreet Logistic Function, Shannon probabilities

The data in table D.291 is condensed from sections C.25.5 and C.25.9.

[97] As a simulation model, the program *tsdlogistic* was run to make a time series data file. The data is by months.

Table D.290: Discreet Logistic Function, Logarithmic Returns, in Bits.

Calculated from Table D.289		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.972830	1.166433	0.008341	0.072212

Table D.291: Discreet Logistic Function, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.665552	0.783863	0.666667	0.656864

D.25.4 Discreet Logistic Function, Logistic Analysis

The data in table D.292 is condensed from Section C.25.3[98].

Table D.292: Discreet Logistic Function, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
1.244560	-0.001892

D.25.5 Discreet Logistic Function, Hurst Coefficients and H Parameters

The data in table D.293 is condensed from Section C.25.4.

Table D.293: Discreet Logistic Function, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.614199	0.122234	-0.012458	0.013335

D.25.6 Discreet Logistic Function, verification of the increments

The data in table D.295 is condensed from Section C.25.10.

D.25.7 Discreet Logistic Function, verification of the increments

The data in table D.296 is condensed from Section C.25.10.

[98]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.25.3.

Table D.294: Discreet Logistic Function, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.848748	0.608754	0.512929	0.515423

Table D.295: Discreet Logistic Function, verification the of the increments, the mean, σ is the standard deviation from table D.289, 1.398261, and P is the maximum Shannon probability from table D.291, 0.665552. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.962687	0.561449	0.490644

Table D.296: Discreet Logistic Function, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[100].

Mean of the absolute value	rms
1.384386	1.695689

D.25.8 Discreet Logistic Function, χ^2 values of the increments

The data in table D.297 is condensed from Section C.556.

Table D.297: Discreet Logistic Function, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
11.091000	42.557000

D.25.9 Discreet Logistic Function, time series data, empirical and simulated

The data in table D.298 is condensed from Section C.25.8.

D.25.10 Discreet Logistic Function, number of participating companies

The data in table D.299 is condensed from Section C.25.6.

D.25.11 Discreet Logistic Function, Shannon probability optimizations

The data in table D.300 is condensed from Section C.25.6.

Table D.298: Discreet Logistic Function, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.962687	1.398261	0.556782	1.603622

Table D.299: Discreet Logistic Function, number of participating companies.

Number	Shannon probability
0.334806	0.990583

Table D.300: Discreet Logistic Function, Shannon probability optimization.

optimize capital growth	optimize market growth
0.783863	0.990583

D.26 Simulated Equity Market Index

For the analysis, the data was in the directory ../markets/tsgaussian.tsmath.tsmath.tsunfraction[101].

The data in this section is presented in Section C.26.

D.26.1 Simulated Equity Market Index, normalized increments

The data in table D.301 is condensed from Section C.26.1.

Table D.301: Simulated Equity Market Index, normalized increments.

Normalized					Normalized Absolute Value				
Mean	Standard deviation	rms	Least Squares		Mean	Standard deviation	rms	Least Squares	
			Constant	Slope				Constant	Slope
0.000312	0.009885	0.009889	0.000354	0.000000	0.007914	0.005930	0.009889	0.007715	0.000000

D.26.2 Simulated Equity Market Index, Logarithmic Returns, in Bits

The data in table D.302 is condensed from Section C.26.5.

D.26.3 Simulated Equity Market Index, Shannon probabilities

The data in table D.303 is condensed from sections C.26.5 and C.26.9.

[101] As a simulation model, the programs *tsgaussian*, *tsmath*, and *tsunfraction* were run to make a time series data file. The data is by months.

Table D.302: Simulated Equity Market Index, Logarithmic Returns, in Bits.

Calculated from Table D.301		From program:	
Mean	Least squares	*tslsq*	*tslogreturns*
0.000450	0.000511	0.000330	0.000380

Table D.303: Simulated Equity Market Index, Shannon probabilities.

Maximum			Operational
Fraction of positive increments	$\frac{\frac{\text{mean}}{\text{rms}}+1}{2}$	From program:	
		tsshannonmax	*tsshannon*
0.516200	0.515775	0.516297	0.511475

D.26.4 Simulated Equity Market Index, Logistic Analysis

The data in table D.304 is condensed from Section C.26.3[102].

Table D.304: Simulated Equity Market Index, Logistic Analysis, $x_t = x_{t-1}\left(a + b \cdot x_{t-1}\right)$.

a	b
0.000354	0.000000

D.26.5 Simulated Equity Market Index, Hurst Coefficients and H Parameters

The data in table D.305 is condensed from Section C.26.4.

Table D.305: Simulated Equity Market Index, Hurst Coefficients and H Parameters.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.841512	0.426071	0.504850	0.506056

D.26.6 Simulated Equity Market Index, verification of the increments

The data in table D.307 is condensed from Section C.26.10.

D.26.7 Simulated Equity Market Index, verification of the increments

The data in table D.308 is condensed from Section C.26.10.

[102]Note that there are numerical stability issues with the methodology used to derive the constants—if the non-linear term, b, was greater than zero, it was set to zero to produce the graphs in Section C.26.3.

Table D.306: Simulated Equity Market Index, Hurst Coefficients and H Parameters, as a Derivative.

Hurst Coefficients		H Parameters	
Near term	Far term	Near term	Far term
0.841611	0.440756	0.499708	0.498946

Table D.307: Simulated Equity Market Index, verification the of the increments, the mean, σ is the standard deviation from table D.301, 0.009885, and P is the maximum Shannon probability from table D.303, 0.516200. In principle, the values should equate.

Mean	rms$(2P-1)$	$\frac{\sigma(2P-1)}{2\sqrt{P(P-1)}}$
0.000312	0.000320	0.000320

Table D.308: Simulated Equity Market Index, verification the of increments. In principle, the mean of the absolute value of the increments and the root mean square of the increments should equate[104].

Mean of the absolute value	rms
0.007914	0.009889

D.26.8 Simulated Equity Market Index, χ^2 values of the increments

The data in table D.309 is condensed from Section C.579.

Table D.309: Simulated Equity Market Index, χ^2 values of the increments. In principle, if the distribution of the normalized increments is a Gaussian distribution, the χ^2 value will be significantly less than the critical value.

χ^2	Critical Value
0.213000	42.557000

D.26.9 Simulated Equity Market Index, time series data, empirical and simulated

The data in table D.310 is condensed from Section C.26.8.

D.26.10 Simulated Equity Market Index, number of participating companies

The data in table D.311 is condensed from Section C.26.6.

D.26.11 Simulated Equity Market Index, Shannon probability optimizations

The data in table D.312 is condensed from Section C.26.6.

Table D.310: Simulated Equity Market Index, time series data, empirical and simulated, analysis of the normalized increments.

Empirical		Simulated	
Mean	Standard deviation	Mean	Standard deviation
0.000312	0.009885	0.000322	0.009885

Table D.311: Simulated Equity Market Index, number of participating companies.

Number	Shannon probability
3.190435	0.508832

Table D.312: Simulated Equity Market Index, Shannon probability optimization.

optimize capital growth	optimize market growth
0.515775	0.508832

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Colophon

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